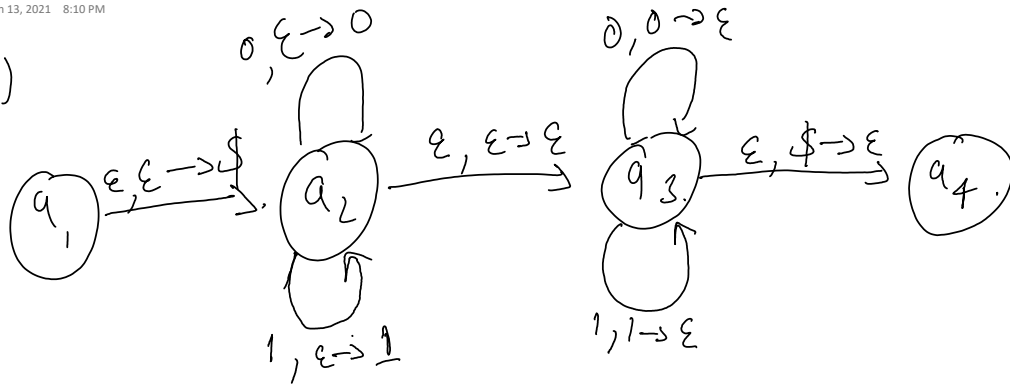
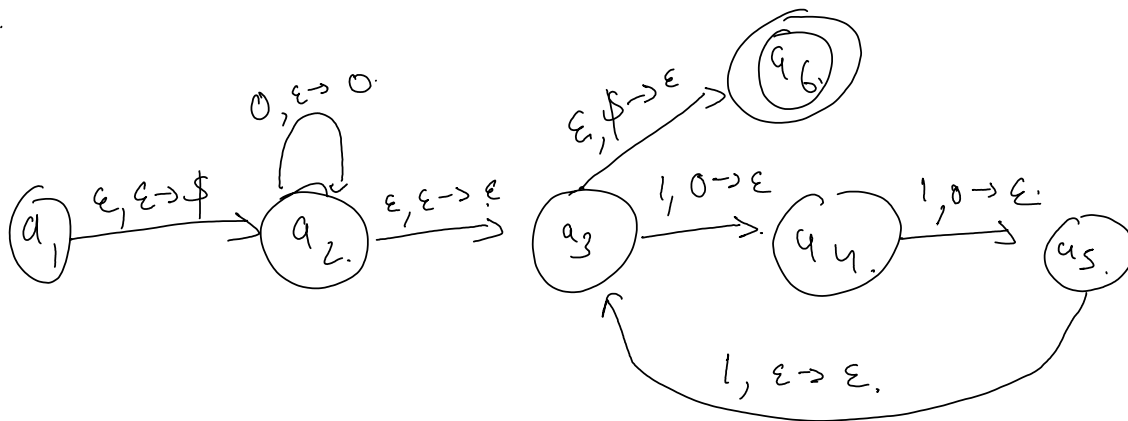
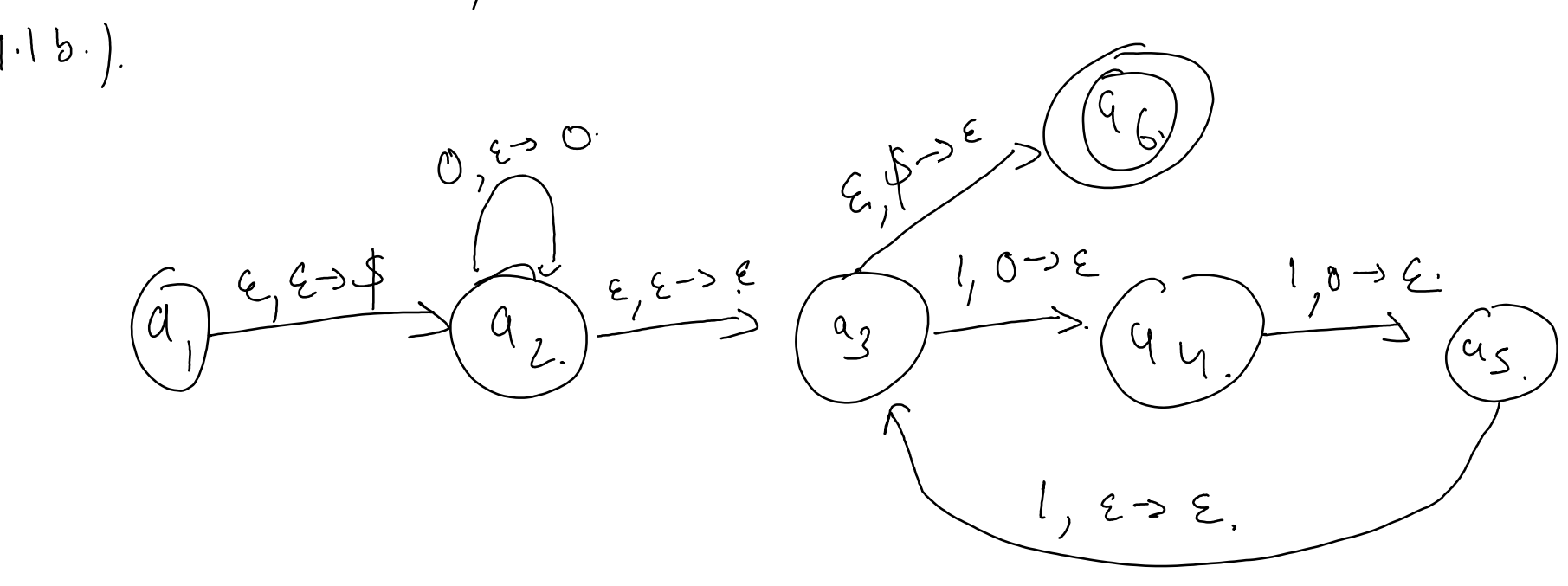
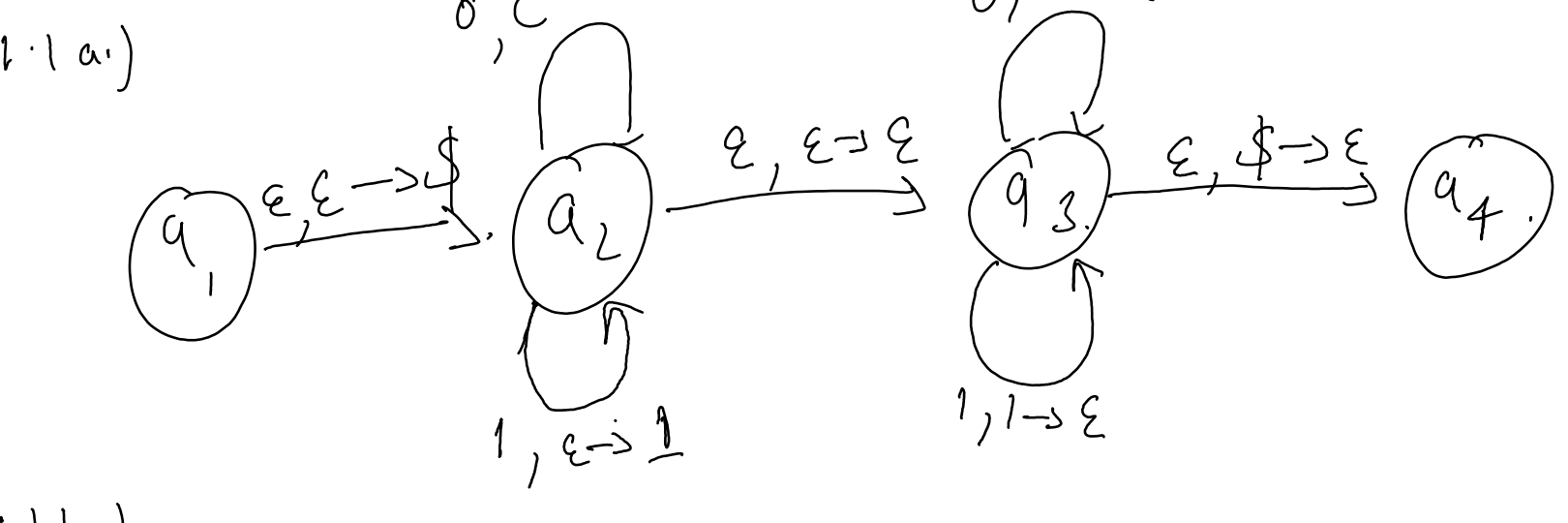


1.1 a.)

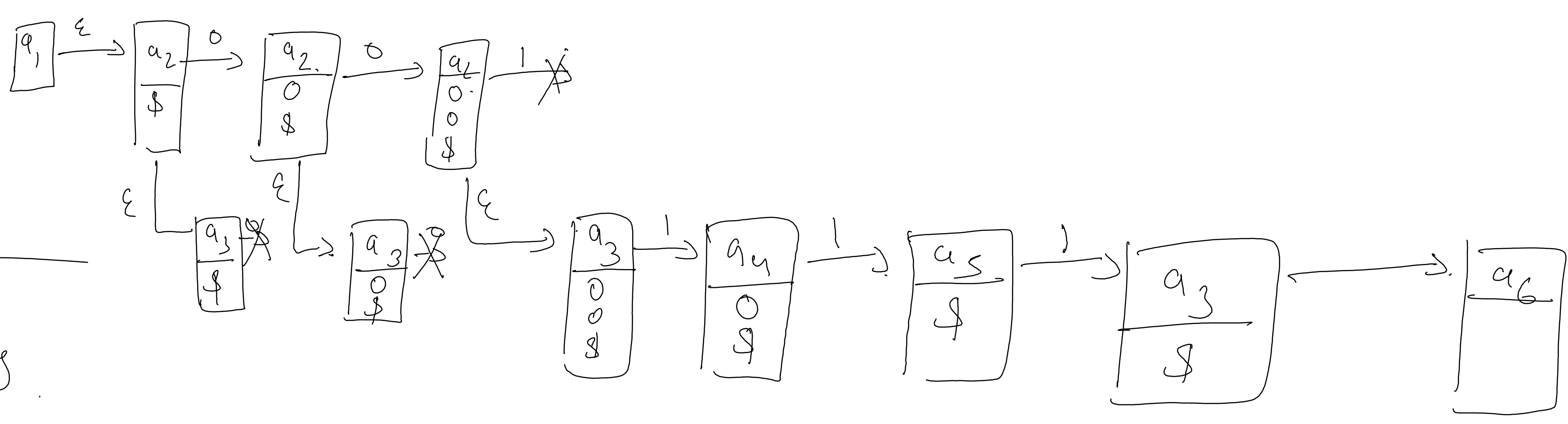


1.1 b.)





ε 00111



Ex 2.)  
 $L(M_1) = \{a^n b^{n_2} c^{n_3} \mid n_1 + n_2 = n_3 \wedge \{n_1, n_2, n_3\} \in P(N)\}$   
 $L(M_2) = \{0^n 1^n \mid n \in N\}$

Ex 3.)  
Ans a)  
We know, the terminals of the CFG, formed by the automation is the same as the input set of the PDA.  
So,  $\Sigma = \{0, 1\}$

And, the start variable of the CFG is.  
 $S = A_{q_0, q_{accept}}$   
 $= A_{11}$   
Now, writing the variables of the CFG,  
 $V = \{A_{11}, A_{22}, A_{33}, A_{12}, A_{21}, A_{23}, A_{32}, A_{13}, A_{31}\}$   
Now, deriving the rules:  
 $\therefore \delta(1, \epsilon, \epsilon) = (2, \$) \text{ and } \delta(3, \epsilon, \$) = (1, \epsilon)$   
 $A_{11} = \epsilon A_{23} \epsilon$   
 $= A_{23}$   
 $\therefore \delta(2, 1, \epsilon) = (2, 1), \delta(2, 0, 1) = (3, \epsilon) \text{ and } \delta(3, 0, 1) = (3, \epsilon)$

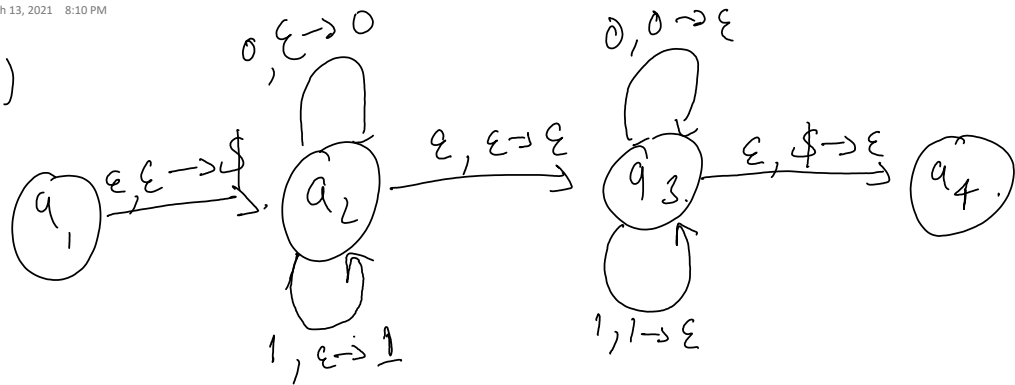
$A_{23} = 1 A_{22} 0 \mid 1 A_{23} 0$   
Now, using the third and the second parts of the lemma to derive further rules.  
Since the number of rules that can be derived from the second condition is large, we leave it just give it is as  
 $A_{pq} = A_{pr} A_{rq} \text{ [for } p, q, r \in \{1, 2, 3\}]$

The rules derived from the second condition are:  
 $A_{11} = \epsilon \quad A_{22} = \epsilon \quad A_{33} = \epsilon$

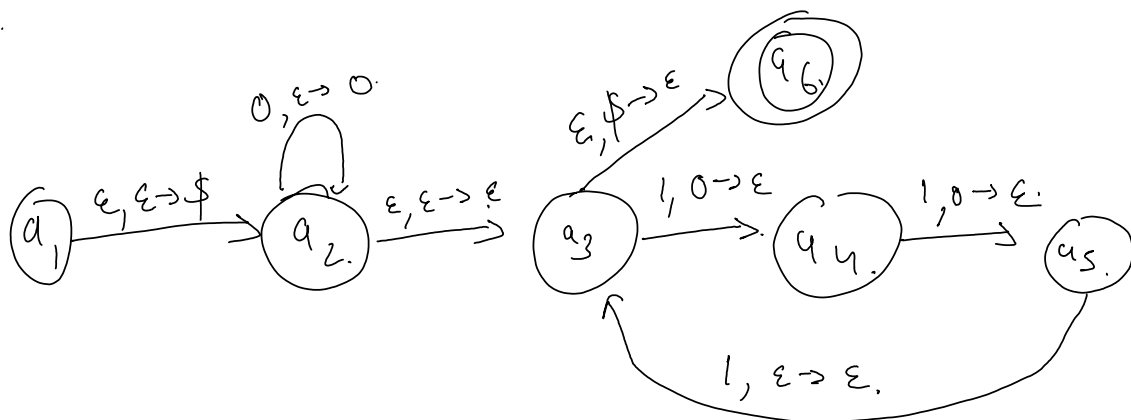
$\therefore$  The CFG generated is given by.  
 $V = \{A_{11}, A_{22}, A_{33}, A_{12}, A_{21}, A_{23}, A_{32}, A_{13}, A_{31}\}$   
 $\Sigma = \{0, 1\}$   
 $S = A_{11}$   
 $R =$   
 $A_{11} = A_{23} \mid \epsilon$   
 $A_{23} = 1 A_{22} 0 \mid 1 A_{23} 0$   
 $A_{pq} = A_{pr} A_{rq} \text{ [for } p, q, r \in \{1, 2, 3\}]$   
 $A_{22} = \epsilon$   
 $A_{33} = \epsilon$

Ans b.)  
So,  
 $S = A_{11}$   
 $= A_{11} A_{11} \text{ [Since } A_{11} = A_{11} A_{11} \text{ is one of the rules in.}$   
 $A_{pq} = A_{pr} A_{rq} \text{ for } p, q, r \in \{1, 2, 3\}, \text{ here } q = p = r = 1]$   
 $= A_{23} A_{23}$   
 $= 1 A_{23} 0 1 A_{23} 0$

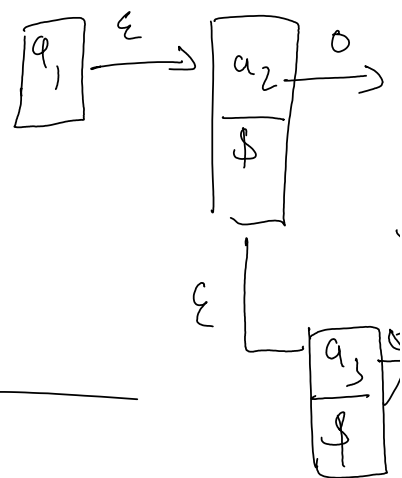
1.1 a.)



1.1 b.)



ε 00111



Ex 2.)

$$L(M_1) = \{ a^{n_1} b^{n_2} c^{n_3} \mid n_1 + n_2 = n_3 \wedge \{n_1, n_2, n_3\} \in P(\mathbb{N}) \}$$

$$L(M_2) = \{ 0^n 1^n \mid n \in \mathbb{N} \}$$

Ex 3.)

Ans a.)

We know, the terminals of the CFG, formed by the automaton, is the same as the input set of the PDA.

So,

$$\Sigma = \{0, 1\}$$

And, the start variable of the CFG is.

$$S = A_{q_0, q_{\text{accept}}} \\ = A_{11}$$

Now, writing the variables of the CFG,

$$V = \{A_{11}, A_{22}, A_{33}, A_{12}, A_{21}, A_{23}, A_{32}, A_{13}, A_{21}\}$$

Now, deriving the rules:

$$\therefore \delta(1, \epsilon, \epsilon) = (2, \$) \quad \text{and} \quad \delta(3, \epsilon, \$) = (1, \epsilon)$$

$$A_{11} = \epsilon A_{23} \epsilon \\ = A_{23}$$

$$\therefore \delta(2, 1, \epsilon) = (2, 1) \quad \delta(2, 0, 1) = (3, \epsilon) \quad \text{and} \quad \delta(3, 0, 1) = (3, \epsilon)$$

$$A_{23} = 1A_{22}0 \mid 1A_{23}0$$

Now, using the third and the second parts of the condition, to derive further rules.

Since the number of rules that can be derived from the large, we leave it just like it is as

$$A_{pq} = A_{pr} A_{rq} \quad [\text{for } p, q, r \in \{1, 2, 3\}]$$

The rules derived from the second condition are:

$$A_{11} = \epsilon \quad A_{22} = \epsilon \quad A_{33} = \epsilon$$

∴ The CFG generated is given by.

$$V = \{A_{11}, A_{22}, A_{33}, A_{12}, A_{21}, A_{23}, A_{32}, A_{13}, A_{31}\}$$

$$\Sigma = \{0, 1\}$$

$$S = A_{11}$$

$$R =$$

$$A_{11} = A_{23} \mid \epsilon$$

$$A_{23} = 1 A_{22} 0 \mid 1 A_{23} 0.$$

$$A_{pq} = A_{pr} A_{rq} \quad [\text{for } p, q, r \in \{1, 2, 3\}].$$

$$A_{22} = \epsilon$$

$$A_{33} = \epsilon$$

Ans b.)

So,

$$S = A_{11}$$

$$= A_{11} A_{11} \quad [\text{Since } A_{11} = A_{11} A_{11} \text{ is one of the rules } A_{pq} = A_{pr} A_{rq} \text{ for } p, q, r \in \{1, 2, 3\}, \text{ here } p = q = r = 1.]$$

$$= A_{23} A_{23}$$

$$= 1 A_{23} 0 1 A_{23} 0$$

$$= 1 1 A_{22} 0 0 1 1 A_{23} 0 0$$

$$= 1 1 \epsilon 0 0 1 1 A_{22} 0 0 0$$

$$= 11001112000$$

$$= 110011000$$

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