Assignment 6

Exercise 1

ⓐ Prove that $L_a = \{a^{2^n} \mid n \in \mathbb{N}\}$ is not contextfree.

Suppose that La is context free language.
There exists a number p, the pumping length,
where if s is any string in A of length at least
p, then s may be divided into $s = \cancel{uvz} \; uvxyz$,
Given i
1) $uv^ixy^iz \in A$
2) $|vy| > 0$
3) $|vxy| \leq p$.

let a^{2^p} be the worstring over La.
According to the given conditions,
$2^p > p$, $p > 0$
$|u| + i|v| + |x| + i|y| + |z| = 2^p$.

~~For $i > 0$, we see that $|u| + |v| + |x| + |y| + |z| \leq 2^p$~~

Now, pumping i by 1
$|u| + (i+1)|v| + |x| + (i+1)|y| + |z|$
$= |u| + i|v| + |v| + |x| + i|y| + |y| + |z|$
$= |u| + i|v| + |x| + i|y| + |z| + |v| + |y|$
we know that $|vxy| \leq p \Rightarrow |v| + |y| \leq p$.
$= 2^p + a$, $a \leq p$.

$\therefore 2^p < uv^{i+1}xy^{i+2}z < 2^{p+1}$. Contradiction

$\therefore$ Therefore, it is a non-context free language.

(b) $k \geq 1$,

$L_k = \{a^n b^{kn} c^n \mid n \in \mathbb{N}\}$ is a non-context free.

~~If L_k~~ let us suppose that L_k is a content-free language.

~~And let the~~

There exists a pumping length p such that, $|s| = p$,

$s = uv^ixy^iz \in L_k$

$\forall i \geq 0$

$|vy| > 0$

$|vxy| \leq p$

Suppose, $s = a^p b^{kp} c^p$.

Case 1

v, and y contains only one type of symbol ~~, i.e either a or b, nor both~~,

If we pump it by $(k-1)$ times, we can see that

~~$|a| < |b|$, and $|c| < |b|$~~

If v contains b, then obviously $|b| > |a|$ and

if v contains a, then, and supposing that it contained all n a_s, $n(k-1) < nk$. Hence ~~a~~ $|b| > |a|$ will hold true again.

Similarly for y, we get $|b| > |c|$. ~~and~~

Case 2.

at least one of v and y contains ~~both~~ two types of symbol.

In this case we observe that if we pump the string, we get a mix-matched pattern of a, b, and c, which is not the desired pattern.

Therefore L_k must be non-context free.

Exercise 2.

The sequence of configuration for input string 101#101 is given below;

$q_1 101\#101$

$x q_3 01\#101$

$x 0 q_3 1\#101$

$x 0 1 q_3 \#101$

$x 0 1 \# q_5 101$

$x 0 1 q_6 \# x 0 1$

$x 0 q_7 1 \# x 0 1$

$x q_7 0 1 \# x 0 1$

$q_7 x 0 1 \# x 0 1$

$x q_1 0 1 \# x 0 1$

$x x q_2 1 \# x 0 1$

$x x 1 q_2 \# x 0 1$

$x x 1 \# q_4 x 0 1$

$x x 1 \# x q_4 0 1$

$x x 1 \# q_6 x x 1$

$x x 1 q_6 \# x x 1$

$x x q_7 1 \# x x 1$

$x x q_1 1 \# x x 1$

$x x x q_3 \# x x 1$

$x x x \# q_5 x x 1$

$x x x \# x q_5 x 1$

$x x x \# x x q_5 1$

$x x x \# x q_6 x x$

$x x x \# q_6 x x x$

$x x x q_6 \# x x x$

$x x q_6 x \# x x x$

$x q_6 x x \# x x x$

$q_6 x x x \# x x x$ //.

Exercise 3

$w \in \Sigma = \{0, 1\}^*$, where w has same number of zeros and ones.

1) Move the tape head to the right. If 1 is encountered, move to the end of the tape and write "x".
2) If "x" is encountered, then, move to the end to the tape and write "#"
3) Move to the left until 0 is encountered. If 0 is ~~encountere~~ found, then move to the right until "x" is found and replace with "?".

1) If 0 is ~~encountered~~ found, replace it with "x" and search for the left most 1. If found, replace it with "x" and return to tape head. ~~If not found, keep moving right. If blank read, then reject it.~~ If not found, move right
2) If 1 is found, replace it with "x" and search for left most 0. If found, replace it with "x" and return to tape head. If not found, keep moving right. ~~If blank k read while going right, then reject~~.
3) If blank symbol is read, then accept the string.