

## Assignment 7

Ex 1

$$T = \{ (1, 4)^T, 2 \}, (2, 4)^T, 5, (0, 1)^T, 1, (1, 2)^T, 5 \}.$$

(a) Using KNN regression for  $k=3$ , we get,  
Prediction

$$f(0 \leq x < 1, y) = 8/3$$

$$f(1 \leq x < 2, y) = 8/3$$

$$f(2 \leq x < 3, y) = 8/3$$

$$f(3 \leq x < 4, y) = 8/3$$

$$f(4 \leq x < 5, y) = 4$$

$$\begin{aligned} \text{Error} &= \left( (2 - 8/3)^2 + (4 - 5)^2 + (8/3 - 1)^2 + (5 - 4)^2 \right) \times 1/4 \\ &= \frac{47}{36} \end{aligned}$$

(b) 
$$y = \begin{pmatrix} 2 \\ 5 \\ 1 \\ 5 \end{pmatrix}, \quad X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$X^T X \beta = X^T y$$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 5 \\ 4 & 6 & 5 \\ 5 & 5 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 & 5 \\ 4 & 6 & 5 \\ 5 & 5 & 7 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \\ 5 \end{bmatrix}$$

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$$\begin{bmatrix} 4 & 4 & 5 \\ 4 & 6 & 5 \\ 5 & 5 & 7 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 13 \\ 12 \\ 18 \end{bmatrix}$$

$$\beta_1 = \cancel{+6.67} - 5/3$$

$$\beta_2 = 2$$

$$\beta_3 = 7/3$$

$$\text{Prediction} = -5/3 + 2X_1 + 7/3 X_2$$

$$f(1,1) = 8/3, \quad f(2,1) = 14/3, \quad f(0,1) = 2/3, \quad f(1,2) = 5$$

$$\text{Error} = \left( \left( \frac{8}{3} - 2 \right)^2 + \left( \frac{14}{3} - 5 \right)^2 + \left( \frac{2}{3} - 1 \right)^2 + (5 - 5)^2 \right) \times \frac{1}{4} = \frac{29}{144}$$



Ex 2

①  $T = \{(-2, 4), (2, 4), (1, 1), (-1, 1), (0, 0), (3, 9)\}$

$$f(-\infty \text{ to } -0.5) = 2.5 \quad \{ \text{Non-inclusive} \}$$

$$f(-0.5 \text{ to } 1) = 0.5$$

$$f(1 \text{ to } 2) = 2.5$$

$$f(2 \text{ to } \infty) = 6.5$$

$$T_1 = \{(-2, 4), (2, 4), (1, 1)\}$$

$$T_2 = \{(-1, 1), (0, 0), (3, 9)\}$$

$$E_1 = \left( (2.5 - 4)^2 + (6.5 - 4)^2 + (1 - 2.5)^2 \right)^{\frac{1}{3}}$$
$$= \frac{43}{12}$$

$$E_2 = \left( (0.5 - 1)^2 + (0 - 0.5)^2 + (6.5 - 9)^2 \right)^{\frac{1}{3}}$$
$$= \frac{9}{4}$$

$$E = \left( \frac{9}{4} + \frac{43}{12} \right) \times \frac{1}{2} = \frac{35}{12} //$$



①

Crossing out  $(-2, 4)$ , we get  $2x+1$

Crossing out  $(2, 4)$ , we get  $1.08x+2.78$

Crossing out  $(1, 1)$ , we get  $1.09x+3.16$

Crossing out  $(-1, 1)$  we get  $0.91x+2.86$

Crossing out  $(0, 0)$  we get  $0.90x+3.25$

Crossing out  $(3, 9)$  we get  $2$

Using  $(2x+1)$

$$(5-4)^2 + (3-1)^2 + (1-1)^2 + (1-0)^2 + (7-9)^2 = 10$$

Using  $1.08x+2.78$

$$(0.62-4)^2 + (3.86-1)^2 + (1.7-1)^2 + (2.78-0)^2 + (6.02-9)^2$$

$$= 86.70$$

Using  $1.09x+3.16$

$$(0.98-4)^2 + (5.34-4)^2 + (2.07-1)^2 + (3.16-0)^2 + (6.43-9)^2 = 28.6514$$

$$0.91X + 2.86$$

$$(1.04-4)^2 + (4.68-4)^2 + (6.77-1)^2 +$$

$$(2.86-0)^2 + (6.59-9)^2 = 36.70$$

$$0.90X + 3.25$$

$$(1.45-4)^2 + (5.05-4)^2 + (4.15-1)^2 + (2.35-1)^2 +$$

$$(5.95-9)^2 = 28.65$$

$$(2-4)^2 + (2-4)^2 + (2-1)^2 + (2-1)^2 + 2^2 = 16$$

$$E = \frac{16 + 28.65 + 36.70 + 28.65 + 36.70 + 10}{6}$$

$$= 26.116$$