RIS Lab Report 2

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1 Task 1.4

Estimate k_t in SI units for this motor (Table 1.1) from the data provided using (1.4), the stall current, and the stall torque.

$$\begin{aligned} 1\,oz &= 0.283495 \times 9.8 \\ &= 0.2778\,N \end{aligned} \tag{1}$$

$$\begin{aligned} 1\,oz - in &= 0.2778 \times 0.0254 \\ &= 7.056 \times 10^{-3}\,Nm \end{aligned} \tag{2}$$

Now inserting the values into formula

$$\begin{aligned} k_t &= \frac{\tau_g}{i} \\ &= \frac{7.056 \times 10^{-3} \times 84}{5} \\ &= 0.1185\,NmA^{-1} \end{aligned} \tag{3}$$

2 Task 1.5

Estimate k_e in SI units for this motor from the data provided using (1.7). We will get a better estimate later.

$$\begin{aligned} 500\,RPM &= \frac{500 \times 2\pi}{60} \\ &= 52.35\,rad\,s^{-1} \end{aligned} \tag{1}$$

$$\begin{aligned} k_e &\leq \frac{v_a}{\omega_0} \\ &\leq \frac{12}{52.35} \\ &\leq 0.2291\,V\ s\,rad^{-1} \end{aligned} \tag{2}$$

3 Task 1.6

Estimate b in SI units for the motor in Table 1.1.

$$\begin{aligned} b &= \frac{k_t \, . \, i_0}{\omega_0} \\ &= \frac{0.1185 \times 0.3}{52.35} \\ &= 6.79 \times 10^{-4} \, N \, m \, s \, rad^{-1} \end{aligned} \tag{1}$$

4 Task 1.7

Compute R_a from the motor data given. Using this value, obtain a better estimate for k_e than the one we obtained earlier in (1.7)? Hint: Consider (1.18) in the no-load steady-state condition.

$$\begin{aligned} R_a &= \frac{v_a}{i_s} \\ &= \frac{12}{5} \\ &= 2.4 \, \Omega \end{aligned} \tag{1}$$

Using $\omega_0 = 52.35 \, rad \, s^{-1}$, and a steady state. no load

$$\begin{aligned} v_a - i.R_a - L_a \frac{di}{dt} &= k_e \theta_g \\ v_a - i.R_a &= k_e \theta_g \\ k_e &= \frac{v_a - i.R_a}{\theta_g} \\ &= \frac{12 - 0.3 \times 2.4}{52.35} \\ &= 0.2154 \, V \, s \, rad^{-1} \end{aligned} \tag{2}$$

5 Task 1.8.

On a sheet of paper, derive (1.21b) from (1.17) and (1.20) as explained. These transfer-functions $H_{\theta\, v}(s)$ and $H_{\theta\, l}(s)$ are important for position-control (a.k.a. servo mode) of the motor.

Task 1.8

$$(Js^2+bs)\,\Theta_g(s) = k_t I(s) - T_L(s) \quad \text{---} \ (1.17)$$

$$\frac{(Js^2+bs)\,\Theta_g(s) + T_L(s)}{k_t} = I(s) \quad \text{---} \ (i)$$

$$V_a(s) - (R_a + L_a s) I(s) = k_e s \Theta_g(s)$$

$$V_a(s) - k_e s \Theta_g(s) = (R_a + L_a s) I(s)$$

$$I(s) = \frac{V_a(s) - k_e s \Theta_g(s)}{R_a + L_a s} \quad \text{---} \ (ii)$$

$$\frac{(Js^2+bs)\,\Theta_g(s) + T_L(s)}{k_t} = \frac{V_a(s) - k_e s \Theta_g(s)}{R_a + L_a s} \quad \text{--- from (i) and (ii)}$$

or, $(Js^2+bs)\Theta_g(s)\cdot(R_a+L_a s) + (R_a+L_a s)T_L(s) = k_t V_a(s) - k_t k_e s \Theta_g(s)$

or, $(Js^2+bs)(R_a+L_a s)\Theta_g(s) + k_t k_e s \Theta_g(s) = k_t V_a(s) - (R_a+L_a s)T_L(s)$

or, $\Theta_g(s)\Big(s(Js+b)(R_a+L_a s) + s(k_t k_e)\Big) = k_t V_a(s) - (R_a+L_a s)T_L(s)$

or, $\Theta_g(s)\Big(s\big\{(Js+b)(R_a+L_a s) + k_t k_e\big\}\Big) = k_t V_a(s) - (R_a+L_a s)T_L(s)$

From 1.21a, $D(s) = (R_a+L_a s)(Js+b) + k_e k_t$

or, $\Theta_g(s)\; s\, D(s) = k_t V_a(s) - (R_a+L_a s)T_L(s)$

or, $\Theta_g(s) = \dfrac{k_t V_a(s)}{sD(s)} - \dfrac{R_a+L_a s}{s\,D(s)}\cdot T_L(s)$ Proved

6 Task 1.9

Show that if the inductance L_a is small and can be ignored, $H_{\omega v}(s)$ can be written as a first order system

$$H_{\omega v}(s) \approx \frac{K}{s+\alpha}$$

What are α and K in terms of the other parameters?

$$H_{wv}(s) \triangleq \frac{k_t}{D(s)} \tag{1}$$

$$D(s) \triangleq (R_a + L_a s)(Js + b) + k_e k_t \tag{2}$$

Combining the equations we end up with,

$$\begin{aligned} H_{wv}(s) &= \frac{k_t}{(R_a + L_a s)(Js + b) + k_e k_t} \\ &= \frac{k_t}{R_a(Js + b) + k_e k_t} \\ &= \frac{k_t}{R_a Js + R_a b + k_e k_t} \\ &= \frac{\frac{k_t}{R_a J}}{(s + \frac{b}{J} + \frac{k_e k_t}{R_a J})} \end{aligned} \tag{3}$$

Therefore,

$$K = \frac{k_t}{R_a J} \tag{4}$$

$$\alpha = \frac{b}{J} + \frac{k_e k_t}{R_a J} \tag{5}$$