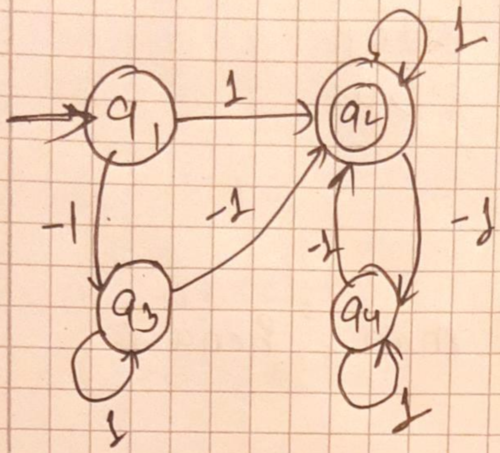
Homework

submitted by: Maulik Chhetri, Mahiem Agrawal, Subigya Paudel

Exercise 1

$L_1 = \{ \omega \in \Sigma^* : \omega = \omega_1, \dots \omega_n \text{ and } \prod_{i=1}^{n} \omega_i = 1 \}$.



The following NFA STD accepts L_1.
Hence it is a regular language

$L_2 = \{ \omega \in \Sigma^*, \omega = \omega_1 \dots \omega_n \text{ and } \sum_{i=1}^{n} \omega_i = 0 \}$

Assume that L_2 is regular.
By pumping lemma, there exists pumping length p of string s such that,

$s = xy^iz, \ \forall i \geq 0$

$|xy| \leq p$

$|y| > 0$

Let $s = 1^p(-1)^p$ (This string is contained by L_2)

$|s| = 2p$

By pumping lemma

$|xy| \leq p$, hence xy contains only 1.

Pumping y, eg. with $i=2$, we get $xyyz$.

In $xyyz$, since $|y|>0$ and only contains 1, it has more 1 than -1. $\therefore \sum_{i=1}^{n} \omega_{i_{xyyz}} \neq 0$. $\lightning$ $\square$

Ex 2

(a) $L_3 = \{w \in \Sigma^* : N(w,0) = 2N(w,1)\}$.

Assume that L_3 is regular. By pumping lemma, $\exists$ pumping length p, such that, $s = xy^iz \ \forall i \geq 0$
$|xy| \leq p$
$|y| > 0$.

Assume,

$s = 0^{2p}1^p$

This has twice the number of 0's than 1's, hence is contained in L_3

$|xy| \leq p$, xy contains only zeros.

Pumping ~~the~~ y by $i=2$, we get $xyyz$.

~~This contains 0's~~

The number of ~~zero~~ 0's in $xyyz$ is greater than $2p$.

Hence, $N(xyyz,0) \neq 2N(xyyz,1)$ $\}$ $\square$

By contradiction, we prove it is not regular.

(b) $L_4 = \{w \in \Sigma^* : N(w,0) = 0, N(w,1) = p, p \text{ is prime}\}$.

Assume L_4 is regular. By pumping lemma, $\exists$ pumping length p of s, such that $s = xy^iz \ \forall i \geq 0$
$|xy| \leq p$
$|y| > 0$.

$s = 1^p$

where p is an arbitrary prime number.

$s = xyz$

$|xyz| = p$.

let $|x| = a$

$|y| = b$

$|z| = c$

$a+b+c = p$.

Pumbing y by $(a+c)$, i.e $i = a+c$

$a + (a+c)b + c$

$= a + ab + cb + c$

$= a(1+b) + c(b+1)$

$= (a+c)(1+b)$

since we have factored the length of the pumped string into $(a+c)(1+b)$, it is not a prime $\exists$ $\square$

Hence, we have proved that L_4 is not regular by contradiction.

Ex 3

$G_1 = (\{A, B\}, \{0, 1, 2\}, R, A)$

$A \to 00A1 \mid B$

$B \to \varepsilon \mid 22B$

(a)

$L(G_1) = \{0^{2n} 2^{2m} 1^n \mid m \geq 0, n \geq 0\}$.

(b) $X \to XZ \mid Ym$

$Y \to YY \mid m \mid n$

$Z \to n \mid mm$

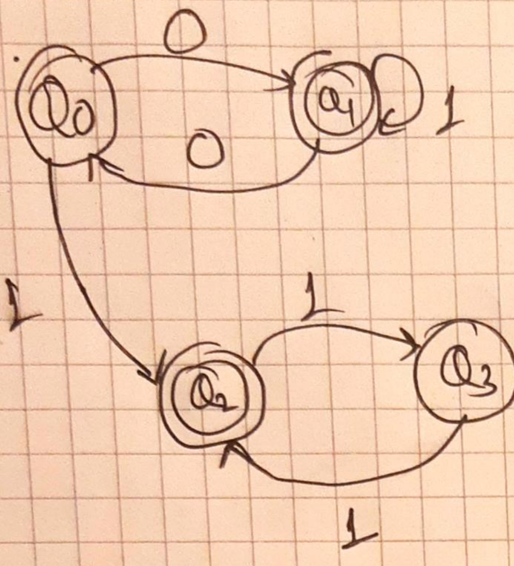
$L(G_2) = \{$ w over $\{0, 1\}$ | the first part of w has at least two symbols ending with m, concatenated with second part that has a string with zero m or even number of m $\}$.

$[m n]^+ m \; n^* (mm)^*$ //

Ex 2

Ex 4

a



~~$G = (Q, \Sigma, R, Q_0)$~~

$G = (\{Q_0, Q_1, Q_2, Q_3\}, \{0, 1\}, R, Q_0)$

R :-

$Q_0 \to 0Q_1 \mid 1Q_2$

$Q_1 \to 1Q_1 \mid 0Q_0 \mid \varepsilon$

$Q_2 \to 1Q_3 \mid \varepsilon$

$Q_3 \to 1Q_2$.

(6) Convert $G_3 = (\{X, Y, Z\}, \{m, n\}, R, X)$

$X \to XZ \mid Ym$

$Y = YY \mid nn$

$Z \to \varepsilon \mid mm$

into Chomsky normal form

Step 1. Add a starting S'

$S' \to X$

$X \to XZ \mid Ym$

$Y \to YY \mid nn$

$Z \to \varepsilon \mid mm$

Step 2 Remove null productions.

Null production: $Z \to \varepsilon$

$X \to XZ \mid Ym \mid X$

$Y \to YY \mid nn$

$Z \to mm$

$S' \to X$

Step 3 Remove unit production.

$X \to X$, $S' \to X$

Removing $X \to X$: $X \to XZ \mid Ym$, $Y \to YY \mid nn$, $Z \to mm$, $S' \to X$

Removing $S' \to X$; $X \to XZ \mid Ym$, $Y \to YY \mid nn$, $Z \to mm$, $S' \to XZ \mid Ym$

Step 4 : fix $S' \to Ym$, $X \to Ym$.

R^c : $X \to XZ \mid YL$

$Y \to YY \mid nn$

$Z \to mm$

$S' \to XZ \mid YL$

$L \to m$

Chomsky form: $G_3^{\text{Chomsky}} = (\{X, Y, Z, L\}, \{m, n\}, R^c, S')$