Exercise 4

a) If the pumping lemma holds for any Context Free Language L over Σ', it means that $\exists p>0$ s.th. $\forall \omega \in L$ with $|\omega| \geq p$, ω can be decomposed into xyz and the following are satisfied:

- $|xy| \leq p$
- $|y| > 0$
- $xy^iz \in L \ \forall i \in \mathbb{N}$.

Because $|\Sigma| = 1$, $\forall \omega \in L$ the decomposition can be xzy.

$$\boxed{\sigma^{|x|}\sigma^{|y|}\sigma^{|z|} = \sigma^{|x|}\sigma^{|z|}\sigma^{|y|}}$$

Show that $\exists\, r \geq 0$ and $s>0$ satisfying:

* $\omega = \sigma^r\sigma^s$. Let $r = |x|+|z|$ and $s = |y|$. From the pumping lemma we have that $|y|>0$ and $(|x|\geq 0 \vee |z| \geq 0) \Rightarrow s>0$ and $r \geq 0$. $\square$
* $\exists\, t$ s.th $0 \leq t \leq r$ and $0 < t+s \leq p$. If t is chosen in the interval $[0,|x|]$, we are sure that $t<r$ because $t \leq |x| \leq |xz| = r$. From the pumping lemma we have $|xy| \leq p$. Deducing from

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Show that $\exists\, r \geq 0$ and $s > 0$ satisfying:

* $\omega = \sigma^r \sigma^s$. Let $r = |x| + |z|$ and $s = |y|$. From the pumping lemma we have that $|y| > 0$ and $(|x| \geq 0 \vee |z| \geq 0) \Rightarrow s > 0$ and $r \geq 0$. $\square$

* $\exists\, t$ s.th $0 \leq t \leq r$ and $0 < t + s \leq p$. If t is chosen in the interval $[0, |x|]$, we are sure that $t < r$ because $t \leq |x| \leq |xz| = r$. From the pumping lemma we have $|xy| \leq p$. Deducing from