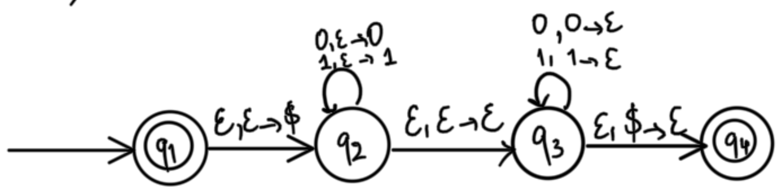
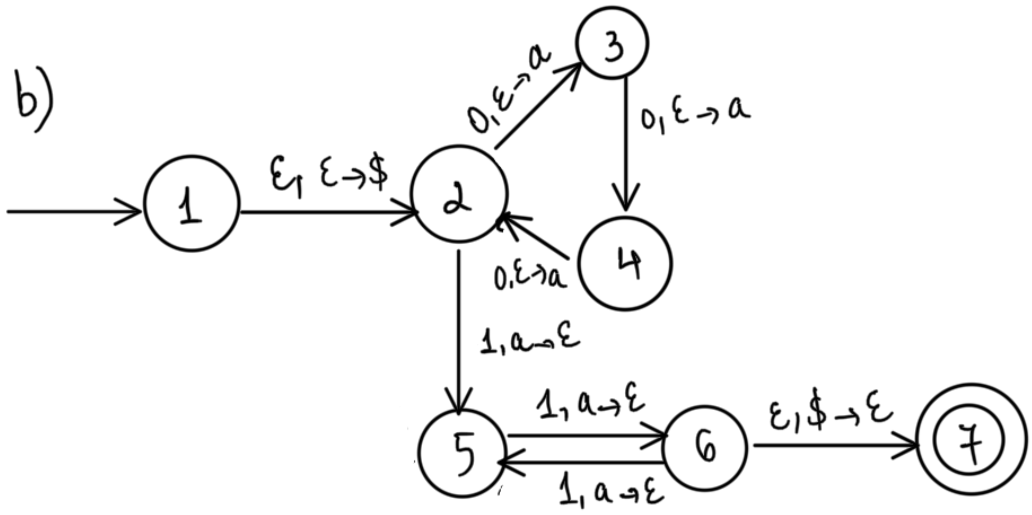
ACC HW 5

Problem 1:

a)



b)



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Exercise 2:

- $L(M_1) = \{ w = (a)^m (b)^n (c)^{m+n} \mid m, n \in \mathbb{N} \}$
  $= \{ w$: w has a consecutive sequence of a, then a consecutive sequence of b. finally a consecutive sequence of c: length of sequence of c = sum of length of sequences of a and b (including empty string $\}$
- $L(M_2) = \{ w$: empty string or each prefix of w has number of 0s ⩾ number of 1s and having the same number of 0 and 1 $\}$

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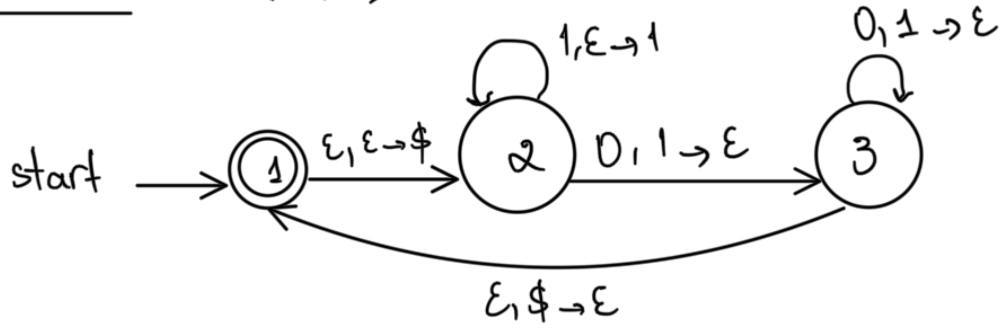
Exercise 3: mPDA $\to$ CFG

M:



- V = $\{A_{11}, A_{12}, A_{13}, A_{21}, A_{22}, A_{23}, A_{31}, A_{32}, A_{33}\}$
- Σ = $\Sigma = \{0, 1\}$
- S = A_{11}
- R. (i) $\begin{cases} (r,t) \in \delta(p,a,\varepsilon) \\ (q,\varepsilon) \in \delta(s,b,t) \end{cases}$ ‖ From STD, we have:

+ For $p = 1 \to a = \varepsilon \Rightarrow r = 2, \underline{t = \$} \Rightarrow s = 3, b = \varepsilon, q = 1$

  $\Rightarrow$ Add $\boxed{A_{11} \to A_{23}}$

+ For $p = 2 \to a = 1, r = 2, t = 1 \Rightarrow \begin{cases} s = 2, b = 0, q = 3 \\ s = 3, b = 0, q = 3 \end{cases}$

  $\Rightarrow$ Add $\boxed{A_{23} \to 1A_{22}0 \mid 1A_{23}0}$

+ For $p = 3 \Rightarrow$ No satisfying edge for (i)

Alltogether R:

+ $A_{11} \to \varepsilon \mid A_{11}A_{11} \mid A_{12}A_{21} \mid A_{13}A_{31} \mid A_{23}$
+ $A_{22} \to \varepsilon \mid A_{21}A_{12} \mid A_{22}A_{22} \mid A_{23}A_{32}$
+ $A_{33} \to \varepsilon \mid A_{31}A_{13} \mid A_{32}A_{23} \mid A_{33}A_{33}$
+ $A_{12} \to A_{11}A_{12} \mid A_{12}A_{22} \mid A_{13}A_{32}$
+ $A_{13} \to A_{11}A_{13} \mid A_{12}A_{23} \mid A_{13}A_{33}$
+ $A_{21} \to A_{21}A_{11} \mid A_{22}A_{21} \mid A_{23}A_{31}$
+ $A_{23} \to A_{21}A_{13} \mid A_{22}A_{23} \mid A_{23}A_{33} \mid 1A_{22}0 \mid 1A_{23}0$
+ $A_{31} \to A_{31}A_{11} \mid A_{32}A_{21} \mid A_{33}A_{31}$
+ $A_{32} \to A_{31}A_{12} \mid A_{32}A_{22} \mid A_{33}A_{32}$

b) 1100 111000

$$A_{11} \to A_{11}A_{11} \to A_{23}A_{23} \to 1A_{23}0\ 1A_{23}0$$
$$\to 11A_{22}00\ 11A_{23}00 \to 11\varepsilon 00\ 111A_{22}000$$
$$\to 11\,00\,111\varepsilon 000 \to 1100\ 111000$$

$\square$

Exercise 4:

$$\Phi_{\Sigma}: \Sigma^* \to \mathbb{N}^n$$
$$\omega \to (|\omega|_{\sigma_1}, \dots, |\omega|_{\sigma_n})$$

Lemma 1: Any context free language over $\Sigma = \{\sigma\}$ is regular

Proof: Let $\Sigma = \{\sigma\}$: fixed alphabet & L: any context free language over Σ.

a) Pumping lemma $\Rightarrow \exists q$: $|s| \geq q$, then
$\omega = uvxyz$ satisfying the conditions ($u, v, x, y, z \in \{\sigma\}^*$)
(i) $\forall i \geq 0$, $uv^ixy^iz \in L$
(ii) $|vy| > 0 \Rightarrow$ at least one of them is not empty
(iii) $|vxy| \leq q$

Choose $p = q$, $r = |uxz|$, $s = |vy| \Rightarrow r \geq 0, s > 0$ (by ii)
$|\omega| = r + s$ and since $u, v, x, y, z \in \{\sigma\}^*$
$\Rightarrow \omega = \sigma^{r+s} = \sigma^r . \sigma^s$ (satisfy (a)) ✓
Choose $t = |x| \Rightarrow t \leq r$ ($|uxz| = r$).
By (iii) $\Rightarrow 0 < t + s \leq q = p$ (satisfy (b)) ✓
By (i) $uv^ixy^iz = \sigma^r . \sigma^{si} \in L$ (satisfy (c)) ✓

b) $L_p = \{\omega \in L : |\omega| \geq p\} \subseteq L$.

$L'_p = L \setminus L_p$: <u>regular</u> & express L in terms of L_p & L'_p

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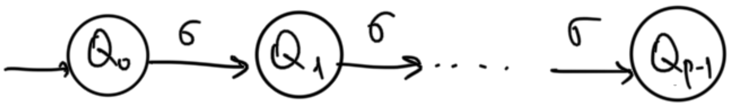
Consider the NFA N



$\forall\, \omega \in L'_p \Rightarrow$ we will make each state $Q_{|\omega|}$ accepted

Hence $L'_p = L(N) \Rightarrow L'_p$: regular.

Moreover $L'_p = L \setminus L_p \Rightarrow L = L_p \cup L'_p$

$\blacksquare$