Homework 1, General IMS Fall 2018

Prof. Dr. Francesco Maurelli, Jacobs University Bremen

Handed out 17.09.2018, due 24.09.2018 23:59:59

Please use the moodle system to upload your homework as pdf (`moodle.jacobs-university.de`). The system will shut down at the deadline and homeworks will not be accepted at a later time. No other form of submission is allowed. Homeworks are not compulsory but highly recommended. You will be able to choose if counting them for your grade (up to 50%), or if taking a longer and more comprehensive final exam.
Use of LaTeXis recommended, though not compulsory. Scans of hand-written papers are accepted as long as the writing is clearly understandable.

1 Vectors

1. (8 points) Consider the following vectors:
   $u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}; v = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$
   (a) (1 point) Calculate the norm of u and of v
   (b) (1 point) Calculate the sum of the two vectors
   (c) (1 point) Show the result geometrically
   (d) (1 point) Calculate the difference $v - u$
   (e) (1 point) Calculate $7v - 5u$
   (f) (1 point) Calculate the dot product
   (g) (1 point) Calculate the cross product
   (h) (1 point) Are u and v linearly independent? Why?
2. (4 points) Suppose that $u \in \mathbb{R}^3$ is a vector which lies in the first quadrant of the xy$-$plane and has length 3 and that $v \in \mathbb{R}^3$ is a vector that lies along the positive z-axis and has length 5.
   (a) (1 point) Calculate $\|u \times v\|$
   (b) (1 point) The x-coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)
   (c) (1 point) The y-coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)
   (d) (1 point) The z-coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)
3. (4 points) Suppose that u and v are vectors in $\mathbb{R}^3$, both of length $2\sqrt{2}$ and that the length of $u - v$ is also $2\sqrt{2}$.
   (a) (2 points) Calculate $\|u + v\|$
   (b) (2 points) Calculate the angle between u and v

2 Matrices

1. (3 points) Consider the following matrices:

   $A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & -3 & 2 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 4 & 1 \end{bmatrix}; C = \begin{bmatrix} 4 & -1 \\ 1 & -2 \end{bmatrix}$

   Calculate, if possible:

   (a) (1 point) $A + B$

   (b) (1 point) $A + C$

   (c) (1 point) $2C + 3I_2$

2. (6 points) Consider the following matrices:

   $A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \\ -1 & 0 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & -2 & 2 & 1 \end{bmatrix}$

   Calculate, if possible:

   (a) (1 point) A^T

   (b) (1 point) $A + B$

   (c) (1 point) $A^T + B$

   (d) (1 point) AB

   (e) (1 point) AB

   (f) (1 point) BA

3. (6 points) Consider the following matrices:

   $A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \\ -1 & 0 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & -2 & 2 & 1 \end{bmatrix}$

   Calculate, if possible:

   (a) (1 point) A^T

   (b) (1 point) $A + B$

   (c) (1 point) $A^T + B$

   (d) (1 point) AB

   (e) (1 point) AB

   (f) (1 point) BA

4. (3 points) Consider the following matrix:

   $A = \begin{bmatrix} 1 & -1 & 3 \end{bmatrix}$

   Calculate, if possible:

   (a) (1 point) A^2

   (b) (1 point) $A^T A$

   (c) (1 point) AA^T

Homework 2, General IMS Fall 2018

Prof. Dr. Francesco Maurelli, Jacobs University Bremen

Handed out 2018.10.08, due 2018.10.15 23:59:59

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1 Matrices - 6 points

1. Using rotation matrices, prove that 2D rotation is commutative but 3D rotation is not.
2. Consider a 2D point P expressed in polar coordinates $[\rho, \theta]$. Define the rotation matrix M for a rotation of ϕ about the origin and the new position P' after the rotation.
3. Referring to the previous problem, use geometrical considerations to calculate the point P', instead of a Rotation Matrix
4. Write a matrix M that mirrors points about the yz-plane
5. Construct a matrix to rotate -30deg about the y-axis.
6. Calculate determinant and trace of the following matrix:
   $A = \begin{bmatrix} 3 & -1 \\ 2 & -3 \end{bmatrix}$

2 Quaternions - 6 points

1. Construct a quaternion to rotate 60deg about the z-axis
2. Calculate the magnitude of the quaternion from the previous question
3. Calculate the conjugate of the same quaternion
4. Calculate angle of rotation and axis vector for the following quaternion: $[0.5(0, 0, \sqrt{3}/2)]$
5. Express in both Rotation matrixes and quaternions the composition of two rotations on the z axis, the first of an angle θ and the second of an angle ϕ.
6. Calculate the multiplication between the quaternions $q' = (2 - i + j + 3k)$ and $q'' = (-1 + i + 4j - 2k)$

3 Robot motion - 2 points

Let's consider a mobile robot that moves in the 2D space. It starts at (0,0). It will move along the x axis for 5 meters. Then he/she/it will rotate about the origin of 45 degrees. Then he will move of a vector $(-2.5\sqrt{2}, -2.5\sqrt{2})^T$. Calculate the final transformation matrix as a combination of the three individual transformations, and show the robot path in a graph.

Homework 3, Introduction to RIS, Spring 2020

Prof. Dr. Francesco Maurelli, Jacobs University Bremen

13th March 2020

Please use the moodle system to upload your homework as pdf (`moodle.jacobs-university.de`). The system will shut down at the deadline and homeworks will **not** be accepted at a later time (submissions will be accepted until midnight). No other form of submission is allowed. Homeworks are not compulsory but highly recommended. You will be able to choose if counting them for your grade (up to 50%).
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1 Cylindrical coordinates

1. (2 points) Convert $(2, \frac{\pi}{6}, 4)$, $(\sqrt{18}, -\frac{\pi}{4}, -7)$ in cylindrical coordinates to rectangular coordinates
2. (4 points) Consider a cube centered at the origin, with edges parallel to the x, y, z axes and with edge length of 2, find out the cylindrical coordinates of the vertices
3. (1 point) M is a point with rectangular coordinates $x = y = 0, z = 1$, N is a point with cylindrical coordinates r, θ, z, find out the Euclidean distance from M to N
4. (1 point) Describe cylindrical equation $z = r$ geometrically

   (Hint: convert the equation to rectangular first)

2 Spherical coordinates

1. (3 points) Convert $(3, \pi/6, \pi/4)$ and $(3, \pi/6, 3\pi/4)$ in spherical coordinates to rectangular coordinates, and compute the Euclidean distance between these two points
2. (1 point) Convert $(1, \sqrt{3}, 2)$ in rectangular coordinates to spherical coordinates
3. (1 point) Convert $(\sqrt{2}, \frac{\pi}{4}, \sqrt{6})$ in cylindrical coordinates to spherical coordinates
4. (1 point) Describe spherical equation $r = 1$ geometrically
5. (2 points) Convert spherical equation $tan(\phi)sin(\theta) = 1$ to rectangular form and describe the meaning

3 System of Forces

1. (3 points) Consider $F_A = 20i - 10j$ kN applied on point A, whose vector from the origin is $r_A = 10i + 5k$ m and $F_B = -10i + 10k$ kN applied on point B, whose vector from the origin is $r_B = 20j$ m. Calculate the resultant moment about the origin.