Exercise 1

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~~Input DFA⟨M⟩ (Assuming total transition function)~~
→ ~~Mark the starting state.~~
→ ~~Mar~~

Input DFA ⟨M⟩

→ Check all the transition from any state a to state b where, a can be equal to b.

→ For each of those transition
check if it is possible with all symbol.

→ If all transition is possible and all are accept states, then accept. Else reject.

Here, the transition is assumed to be total transition, i.e transition of one state to all the states.
Possible number of such transitions $= n^2$.
For each of those transition, we check for all 'K' symbols (K is a constant)
Hence we obtain a complexity of $Kn^2 \in O(n^2)$.
$\Rightarrow ALL_{DFA} \in P$

Exercise 2

Input graph ⟨G⟩

- Enumerate all possible triplets of the nodes of graph G.
- For each triplet:
  check the adjacency matrix to find if they are all connected
  If found, accept.
- If non such triplet found, reject.

Here,
Enumerating nodes :- Choosing 3 nodes from n nodes
$\Rightarrow {}^nC_3 \in O(n^3)$.

Checking for each triplet

$\Rightarrow$ Worst case scenario, we look for all n nodes in each of the ~~node~~ 3 nodes $\Rightarrow 3n \in O(n)$.

Hence the total complexity is $O(n) \cdot O(n^3) \in O(n^4)$. //

Exercise 3

$$n^2 + 3 \cdot n^{3/2} \in O(n^2)$$

We have to show,

$$n^2 + 3n^{3/2} \leq cn^2,$$

for $n > n_0$ and a c.

Let $c = 2$

$$n^2 + 3n^{3/2} \leq 2n^2$$

or, $3n^{3/2} \leq n^2$

or $3 \leq n^{0.5}$

or $\log(3) \leq 0.5 \log(n)$

or $\frac{\log(3)}{0.5} \leq \log(n)$

or $10^{\frac{\log(3)}{0.5}} \leq n$

or $n \geq 9$

Hence, we have found that for $n \geq 9$ and $c = 2$

$$n^2 + 3n^{3/2} \leq cn^2.$$

$\Rightarrow n^2 + 3n^{3/2} \in O(n^2)$ //

(b) $c \cdot n^{1-\varepsilon} \in o(n)$, for all $\varepsilon > 0$ and constant c.

Here, we evaluate the limit,

$$\lim_{n \to \infty} \frac{cn^{1-\varepsilon}}{n}$$

$$= \lim_{n \to \infty} \frac{c}{\frac{n}{n^{1-\varepsilon}}}$$

$$= \lim_{n \to \infty} \frac{c}{n^{\varepsilon}}$$

$\varepsilon > 0$,

$= 0.$

Hence, $cn^{1-\varepsilon} \in o(n)$ //.

Exercise 4

On input n

(1) If n is 2, accept

(2) For all ~~number~~ x from 2 to $n/2$

(2a) Divide the number n by x
If remainder is 0, reject

(3) If remainer was never 0 accept.

Here (1) is in constant time $O(1)$

(2) is $O(n)$ [$\because$ looping till $n/2 \in O(n)$]

(2a) Constant operation $O(1)$

(3) Constant operation.

ence the complexity is $O(n)$.

(b) If ~~10~~ m is a decimal number, then the length of binary number, (n) :-

$$n \approx \log_2 m$$

$$\therefore m \approx 2^n .$$

We had obtained our optimal solution to be $O(n)$.

So, for a binary number, we get it to be $O(2^n)$.

$\Rightarrow$ L prime $\notin$ P. //