Exercises
Introduction to
Robotics and Intelligent Systems
Spring 2020

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General remarks

These exercises come from the final exam and make-up from General Intelligent Mobile Systems I. For Intro to RIS, the exam will be computer-based, so the exam question types may differ.

1 Linear Algebra and Quaternions

1. Consider a parallelogram with total area equal to 40, built by vectors v_1 and v_2. Considering that the magnitude of v_1 is equal to 10, and the angle between the two is equal to $\pi/6$:
   (a) calculate the length of v_2
   (b) calculate the dot product between the two vectors
   (c) calculate the cross-product between the two vectors
   (d) show graphically the sum and the difference of the two vectors
2. Consider a parallelogram with total area equal to 40, built by vectors v_1 and v_2. Considering that the magnitude of v_1 is equal to $10\sqrt{3}$, and the angle between the two is equal to $\pi/3$:
   (a) calculate the length of v_2
   (b) calculate the dot product between the two vectors
   (c) calculate the cross-product between the two vectors
   (d) show graphically the sum and the difference of the two vectors
3. Construct a quaternion to rotate 60deg about the y-axis. Calculate the conjugate of the quaternion. Calculate the rotation of the point $P = (1, 1, 1)^T$.
4. Construct a quaternion to rotate 30deg about the y-axis. Calculate the conjugate of the quaternion. Calculate the rotation of the point $P = (1, 1, 1)^T$.
5. Construct a quaternion to rotate 90deg about the z-axis. Calculate the rotation of the point $P = (1, 0, 1)^T$.
6. Construct a quaternion to rotate 90deg about the x-axis. Calculate the rotation of the point $P = (0, 1, 1)^T$.
7. Multiply the two following quaternions: $q_1 = 4 + 3i + 2j - k$ and $q_2 = i - k$.
8. Multiply the two following quaternions: $q_1 = 4 + 4i + 2j + k$ and $q_2 = j - k$.

2 URDF

1. Make appropriate modifications to the following lines to represent a correct URDF and draw the correspondent tree:

```
<robot name="Robot1">
 <link name="link1">
 <link name="link2">
 <link name="link3">
 <link name="link4">

 <joint name="joint1">
   <parent link="link1"/>
   <child link="link3"/>
 </joint>

 <joint name="joint2">
   <parent link="link1"/>
   <child link="link2"/>
 </joint>

 <joint name="joint3">
   <parent link="link3"/>
   <child link="link3"/>
 </joint>
</robot>
```

2. Make appropriate modifications to the following lines to represent a correct URDF and draw the correspondent tree:

```
<robot name="Robot1">
 <link name="link1"/>
 <link name="link2"/>
 <link name="link3"/>
 <link name="link4"/>

 <joint name="joint1">
   <parent link="link1"/>
   <child link="link3"/>

 <joint name="joint2">
   <parent link="link1"/>
   <child link="link2"/>

 <joint name="joint3">
   <parent link="link2"/>
   <child link="link2"/>
```

3. Make appropriate modifications to the following lines to represent a correct URDF and draw the correspondent tree:

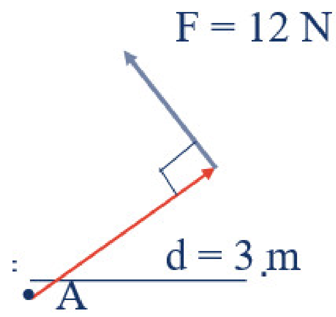
```
<robot name="Robot1">
 <link name="link1"/>
 <link name="link2"/>
 <link name="link3"/>
 <link name="link4"/>

 <joint name="joint1">
   <parent link="link1"/>
   <child link="link3"/>

 <joint name="joint2">
   <parent link="link1"/>
   <child link="link2"/>

 <joint name="joint3">
   <parent link="link3"/>
   <child link="link3"/>
</robot>
```

3 Motion

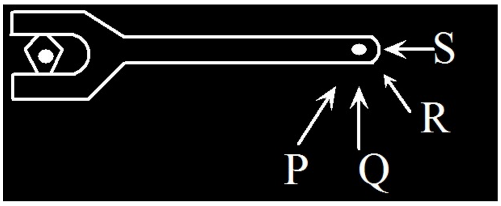
1. Consider a mobile robot that moves in the 2D space. It starts at (0,0). It will move along the y axis for 6 meters. Then it will rotate about the origin of the coordinate system by 45 degrees. Then it will move by a vector $(3\sqrt{2}, -3\sqrt{2})^T$.
   (a) Calculate the final transformation matrix as a combination of the three individual transformations, and show the robot path in a graph.
   (b) Considering a bang-bang motion law in the first segment, and considering that the robot took exactly 8 seconds to complete it, calculate the acceleration and maximum velocity, and show them on a graph.
2. Consider a mobile robot that moves in the 2D space. It starts at (0,0). It will move along the y axis for -6 meters. Then it will rotate about the origin of the coordinate system by 45 degrees. Then it will move by a vector $(-3\sqrt{2}, -3\sqrt{2})^T$.
   (a) Calculate the final transformation matrix as a combination of the three individual transformations, and show the robot path in a graph.
   (b) Considering a bang-bang motion law in the first segment, and considering that the robot took exactly 10 seconds to complete it, calculate the acceleration and maximum velocity, and show them on a graph.
3. Consider a mobile robot that moves in the 2D space. It starts at (0,0). It will move along the y axis for 8 meters. Then it will rotate about the origin of the coordinate system by 45 degrees. Then it will move by a vector $(4\sqrt{2}, -4\sqrt{2})^T$.
   (a) Calculate the final transformation matrix as a combination of the three individual transformations, and show the robot path in a graph.
   (b) Considering a bang-bang motion law in the first segment, and considering that the robot took exactly 8 seconds to complete it, calculate the acceleration and maximum velocity, and show them on a graph.
4. Consider a mobile robot that moves in the 2D space. Axis units are meters. It starts at (0,0), denoted as O. It will move along the y axis for 12 meters, arriving to point A. Then it will rotate about the origin of the coordinate system by 45 degrees, arriving to point B. Then it will move by a vector $(0, -6\sqrt{2})^T$ arriving to point C and then by a vector $(6\sqrt{2}, 0)^T$ arriving to point D.
   (a) Considering the vectors $\vec{BC}$ and $\vec{CD}$, calculate the dot product and the cross product
   (b) Considering the vectors $\vec{OA}$ and $\vec{CD}$, calculate the sum $\vec{OA} + \vec{CD}$ and the difference $\vec{OA} - \vec{CD}$
   (c) Discuss if the four operations above are commutative
   (d) Calculate the final transformation matrix as a combination of the four individual transformations, and show the robot path in a graph.
   (e) Considering a motion with uniform acceleration $a = 1m/s^2$ during the first segment $\vec{OA}$, calculate the time needed to arrive to A and the velocity.
   (f) Considering that the robot will continue in the path AB with the same velocity magnitude reached at point A, calculate the angular velocity and the tangential and centripetal accelerations.
   (g) Considering that the robot will then move on BC and CD with a velocity of magnitude $1m/s$, considering that it will smooth out the trajectory, without passing by C (over-fly), considering that the transition time between BC and CD is 3 seconds, calculate the point where the robot leaves the segment BC and the point where it arrives to the segment CD.
   (h) Express in polar and spherical coordinates the points O, A, and B.
5. Consider a mobile robot that moves in the 2D space. Axis units are meters. It starts at (0,0), denoted as O. It will move along the x axis for 12 meters, arriving to point A. Then it will rotate about the origin of the coordinate system by 45 degrees, arriving to point B. Then it will move by a vector $(-6\sqrt{2}, 0)^T$ arriving to point C and then by a vector $(0, -6\sqrt{2})^T$ arriving to point D.
   (a) Considering the vectors $\vec{BC}$ and $\vec{CD}$, calculate the dot product and the cross product
   (b) Considering the vectors $\vec{OA}$ and $\vec{CD}$, calculate the sum $\vec{OA} + \vec{CD}$ and the difference $\vec{OA} - \vec{CD}$
   (c) Discuss if the four operations above are commutative
   (d) Calculate the final transformation matrix as a combination of the four individual transformations, and show the robot path in a graph.
   (e) Considering a motion with uniform acceleration $a = 1m/s^2$ during the first segment $\vec{OA}$, calculate the time needed to arrive to A and the velocity.
   (f) Considering that the robot will continue in the path AB with the same velocity magnitude reached at point A, calculate the angular velocity and the tangential and centripetal accelerations.
   (g) Considering that the robot will then move on BC and CD with a velocity of magnitude $1m/s$, considering that it will smooth out the trajectory, without passing by C (over-fly), considering that the transition time between BC and CD is 3 seconds, calculate the point where the robot leaves the segment BC and the point where it arrives to the segment CD.
   (h) Express in polar and spherical coordinates the points O, A, and B.

4 Rigid Bodies

1. What is the moment of the force about point A (M_A)? Write the full formula.



2. If a force of magnitude F can be applied in 4 different -D configurations (P, Q, R, S), define the two cases (Max, Min) resulting in the maximum and minimum torque values on the nut, and explain why.



3. **If** $M = r \times F$, then what will be the value of $M \cdot r$ and why?

4. Using the counter-clockwise direction as positive, calculate the net moment of the two forces about point P.

5. If $r = 5j$ m and $F = 10k$ N, calculate the moment.

6. (3p) If $r = i + 2j$ m and $F = 10i + 20j + 30k$ N, calculate the moment of F about the y-axis

5 Sensors

Describe the difference between accuracy and repeatability.

6 Robot Units

1. Describe the functional units of a robot, with the concrete example of an autonomous car (i.e. don't write just the general description, but apply that to the specific case).

2. Describe the functional units of a robot, with the concrete example of an industrial arm in a pick-and-place scenario (i.e. don't write just the general description, but apply that to the specific case).

7 True or False?

Mark the following statements as True (T) or False (F)

*	The multiplication of two quaternions with zero scalar parts produces a quaternion with the cross-product as its scalar part, and the negative dot product as its vector part	T	F
*	The multiplication of two quaternions with zero scalar parts produces a quaternion with the cross-product as its vector part, and the negative dot product as its scalar part	T	F
*	The multiplication of two quaternions with zero scalar parts produces a quaternion with the cross-product as its vector part, and the dot product as its scalar part	T	F
*	Cylindrical coordinates are defined adding an additional coordinate (z) to the standard 2D polar coordinates (ρ, θ)	T	F
*	Spherical coordinates are defined adding an additional coordinate (z) to the standard 2D polar coordinates (ρ, θ)	T	F
*	Cartesian coordinates in 3D are defined adding an additional coordinate (z) to the standard 2D polar coordinates (ρ, θ)	T	F
*	Centripetal acceleration and tangent acceleration are orthogonal	T	F
*	Centripetal acceleration and tangent acceleration are parallel	T	F
*	Centripetal acceleration and tangent acceleration are two scalar values that can be summed up	T	F
*	Centripetal acceleration and tangent acceleration are the same concept expressed with two different names	T	F
*	The moment M of force F about point O can be expressed by the cross product of the vector r and the vector F	T	F
*	The moment M of force F about point O can be expressed by the cross product of the vector F and the vector r	T	F
*	The moment M of force F about point O can be expressed by the dot product of the vector F and the vector r	T	F
*	The moment M of force F about point O can be expressed by the dot product of the vector r and the vector F	T	F
*	If three couples act on a body, the overall result is that the net moment equals 0 but the net force is not necessarily equal to 0	T	F
*	If three couples act on a body, the overall result is that the net force and net moment are equal to 0	T	F
*	In statics, a couple is defined as two forces of equal magnitude acting in the opposite direction, separated by a perpendicular distance	T	F
*	In statics, a couple is defined as two forces of equal magnitude acting in the same direction, separated by a perpendicular distance	T	F