

Ics 2019 Program Sheet #3.

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### 3.1 Proof by Induction

Proof : Power set  $P(S)$  of finite set with  $n$  elements is  $2^n$ .

Base case. If  $|S| = n$  then,

Base case.

$P(\emptyset)$  has 1 element with it being an empty set.

$2^0$  or  $2^0 = 1$  therefore it is true for base set.

Now ~~when~~ Induction step.

Induction step  
 $P(N) \rightarrow P(n+1)$

$|P(n)| = n$  then according to inductive hypothesis then,

$$P(n) = \{e_1, e_2, \dots, e_n\}$$

when

$$|P(n+1)| = n+1 \text{ elements } \emptyset$$

$$P(n+1) = P(n) \cup \{e_{n+1}\}$$

Now, ~~from this we need to prove~~  
From this we know  $P(n)$  set ~~is~~ is actually a subset of the set ~~&~~  $P(n+1)$ .

Therefore if  $P(n)$  has  $2^n$  number of subsets in it then  $P(n+1)$  will have

$2^n$  number of subset when  $e_{n+1}$  is included  
 $2^n$  number of subset when  $e_{n+1}$  is NOT included

$$\begin{aligned} \therefore \text{Total subset} &= 2^n + 2^n + 1 \{ \text{Removing } \{ \} \} - 1 \{ \text{Taking out } \{ \} \} \\ &= 2^{n+1} = 2^n + 2^n + 1 - 1 \quad \{ \text{Repetitive } \{ \} \} \\ &= 2^{n+1} \end{aligned}$$

// prove.

Therefore proved.

3.3.2  $R = \{(a, b) \mid a, b \in \mathbb{Z} \wedge a \neq b\}$

a)

Reflexive property.

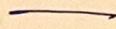
$$a \neq a$$

False.

Symmetric property.

$$a \neq b \text{ and } b \neq a$$

True



Transitive.

$$a \neq b \text{ and } b \neq c \quad \cancel{\text{so}} \quad a \neq c$$

False as this is not true all the time

$$3 \neq 4, \quad 4 \neq 5$$

however.

$$5 = 5$$

False.

b)  $R = \{(a, b) \mid a, b \in \mathbb{Z} \wedge |a - b| \leq 3\}$

Reflexive property

$$|a - a| \leq 3$$

$$|0| \leq 3$$

True.

Symmetric property

$$|a-b| \leq 3 \text{ and } |b-a| \leq 3$$

due to the absolute bar regardless whether a or b is negative positive it will hold True.

$$|a-b| = |b-a|$$

$$\therefore |a-b| \leq 3 \cancel{= |b-a|} \leq 3$$

True

Transitive.

$$|a-b| \leq 3 \text{ and } |b-c| \leq 3$$

False as when we look at  $a=4, b=2$  and  $c=1$

$$4 \Rightarrow |a-b| \leq 3$$

$$\text{or } |3-2| \leq 3 \text{ True.}$$

3 \leq 3 True

$$|b-c| \leq 3$$

$$|2-1| \leq 3$$

$$1 \leq 3 \text{ True}$$

~~However~~ However,

$$|a-c| \leq 3$$

$$|5-1| \leq 3$$

4 \leq 3 False it does not stand

$\therefore$  Transitive is not possible FALSE

$$c) R = \{(a, b) / a, b \in \mathbb{Z} \text{ and } (a \bmod 10) = (b \bmod 10)\}$$

Reflexive.

$$\text{To } (a \bmod 10) = (a \bmod 10)$$

True.

Symmetric.

$$(a \bmod 10) = (b \bmod 10)$$

and

$$(b \bmod 10) = (a \bmod 10)$$

True.

Transitive.

$$(a \bmod 10) = (b \bmod 10)$$

and

$$(b \bmod 10) = (c \bmod 10)$$

$$\therefore (a \bmod 10) = (c \bmod 10)$$

True.