Machine Learning

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Exercise 1

a) As they are disjoint ~~sets~~ events this cannot happen.
So probability is $P(\emptyset)$

b) So $P(A) + P(B) + P(C)$

$= P(A \cup B \cup C)$

Exercise 2

a)
$\Omega = \{HHHH, HHHT, HHTH, HH,TT, HTHH, HTHT, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTT, HTTH, TTTH\}$

b) Let X be a discrete random variable

$X : \Omega \to \{0, 1, 2, 3, 4\}$

where it denotes the number of heads.

c) <u>PMF</u>

$$P(X=0) = 1/16$$

$$P(X=1) = 4/16 = 1/4$$

$$P(X=2) = 6/16 = 3/8$$

$$\cancel{P(X=3) = 4/16 =}$$

$$\cancel{P(X=4) = 1/16}$$

$$P(X=3) = 4/16 = 1/4$$

$$P(X=4) = 1/16$$

<u>CDF</u>

$$F(0) = P(X \leq 0) = 1/16$$

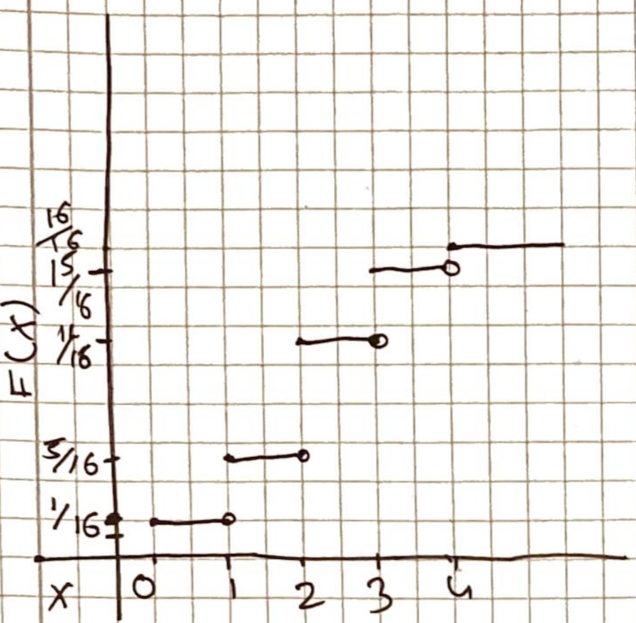
$$F(1) = P(X \leq 1) = 1/16 + 4/16 = \frac{5}{16}$$

$$F(2) = P(X \leq 2) = \frac{5}{16} + \frac{6}{16} = \frac{11}{16}$$

$$F(3) = P(X \leq 3) = \frac{11}{16} + \frac{4}{16} = \frac{15}{16}$$

$$F(4) = P(X \leq 4) = \frac{15}{16} + \frac{1}{16} = 1 \ //$$

Plotting this CDF



Exercise 3

~~a)~~

c) $P(X>1) = P(X=2) + P(X=3) + P(X=4)$

$= 1 - P(X \leq 1)$

$= 1 - F(1)$

$= 1 - \frac{5}{16}$

$= \frac{11}{16}$

d) $E(X) = 0 \times \frac{1}{16} + 1 \times \frac{4}{16} + 2 \times \frac{3}{8} + 3 \times \frac{4}{16} + 4 \times \frac{1}{16}$

$= 2 \%.$

Exercise 3

a). $p(x) = \begin{cases} c\,|x|(1+x)(1-x) & \text{if } -1 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$

OR.

$p(x) = \begin{cases} -cx(1-x^2) & \text{if } -1 \leq x \leq 0 \\ cx(1-x^2) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$

computing each side.

$$\int_{-1}^{0} -cx(1-x^2) \qquad\qquad \int_{0}^{1} cx(1-x^2)$$

$$= \int_{-1}^{0} -cx + cx^3 \qquad\qquad = \int_{0}^{1} cx - cx^3$$

$$= \left[\frac{-cx^2}{2} + \frac{cx^4}{4}\right]_{-1}^{0} \qquad\qquad = \left[\frac{cx^2}{2} - \frac{cx^4}{4}\right]_{0}^{1}$$

$$= 0 - \left[\frac{-c}{2} + \frac{c}{4}\right] \qquad\qquad = \left[\frac{c}{2} - \frac{c}{4}\right] - 0$$

$$= c/4 \qquad\qquad = c/4$$

$\frac{c}{4} + \frac{c}{4} = 1$ as it is ~~has~~ ~~to be~~ a PDF so,

$\frac{c}{2} = 1$

$c = 2$

b) For $-1 \leq x \leq 0$

we have

$$CDF = \int_{-1}^{x} -2x(1-x^2)$$

$$= 2\int_{-1}^{x} -x + x^3$$

$$= 2\left[\frac{-x^2}{2} + \frac{x^4}{4}\right]_{-1}^{x}$$

$$= -x^2 + \frac{x^4}{2} + \frac{1}{2}$$

For $0 \leq x \leq 1$

we have

$$CDF = \int_{-1}^{0} -2x(1-x^2) + \int_{0}^{x} 2x(1-x^2)$$

As we also have to consider the part less than 0.

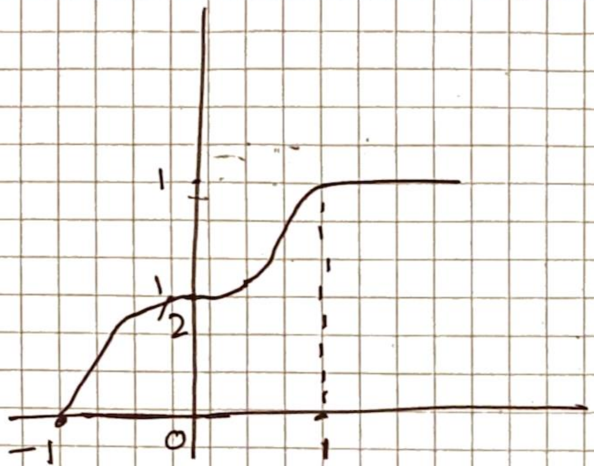
$$= 2x\left[\frac{-x^2}{2} + \frac{x^4}{4}\right]_{-1}^{0} + 2x\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{x}$$

$$= \frac{1}{2} + 0 + 2x\left[\frac{x^2}{2} - \frac{x^4}{4}\right]$$

$$= \frac{1}{2} + x^2 - \frac{x^4}{2}$$

$$= x^2 - \frac{x^4}{2} + \frac{1}{2}.$$

Plotting this CDF:



c) $P(X < -0.5)$

$$= \int_{-1}^{-0.5} -2x(1-x^2)$$

$$= 2 \times \left[\left(\frac{-(-0.5)^2}{2} + \frac{(-0.5)^4}{4} \right] - \left(\frac{-(-1)^2}{2} + \frac{(-1)^4}{4} \right) \right]$$

$$= 9/32$$

$P(X > 0.5)$

$$= \int_{0.5}^{1} x(1-x^2)$$

$$= 2 \times \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_{0.5}^{1} = 9/32.$$

a) $P(-0.5 < x < 0.5)$

$= (1 - P(x < -0.5)) - (P(x > 0.5))$

$= 1 - 9/32 - 9/32$

$= 7/16.$