ICS 2019 Program Sheet #3.

Maniem Agrawal
CS '22

3.1 Proof by Induction

Proof : Power set P(S) of finite set with n elements is 2^n.

~~Base case.~~ ~~If~~ If $|S| = n$ then,

Base case.

P(0) has 1element with it being an empty set.

2^n or $2^0 = 1$ therefore it is true for base set.

~~Now~~ ~~when~~ Induction step.

Induction step
$P(n) \rightarrow P(n+1)$

$|P(n)| = n$ ~~then~~ according to inductive hypothesis then,

$P(n) = \{e_1, e_2, \dots e_n)$

when
$|P(n+1)| = n+1$ elements.

$P(n+1) = P(N) \cup \{e_{n+1}\}$

Now, ~~from this we need to prove~~
From this we know $P(n)$ set ~~has~~ is actually a subset of the set $P(n+1)$.

Therefore if $P(n)$ has 2^n number of subsets in it then $P(n+1)$ will have

2^n number of subset when e_{n+1} is included
2^n number of subset when e_{n+1} is NOT included

$\therefore$ Total subset $= 2^n + 2^n + 1$ (Removing $\{3\}$) ~~and~~ -1 $\{$taking out repetive $\{0\}$

$= \cancel{2^{n+1}} = 2^n + 2^n + 1 - 1$

$= 2^{n+1}$

// prove.

Therefore proved.

3.3.2 $R = \{ (a,b) \mid a,b \in Z \wedge a \neq b)$

a)

Reflexive propety.

$a \neq a$.

False.

Symmetric propety.

$a \neq b$ and $b \neq a$

True

Transitive.

$a \neq b$ and $b \neq c$ ~~the~~ so $a \neq c$

False as this is not true all the time

$5 \neq 4$, $4 \neq 5$ however.

$5 = 5$

False.

b) $R = \{ (a,b) \mid a,b \in Z \wedge |a-b| \leq 3$

Reflexive propety

$|a-a| \leq 3$

$|0| \leq 3$

True.

Symetric propety :

$|a-b| \leq 3$ and $|b-a| \leq 3$

Due to the absolote bar regardless whether a or b is negative positive it will hold True.

$|a-b| = |b-a|$

$\therefore |a-b| \leq 3$ ~~and~~ $= |b-a| \leq 3$

True

Transitive.

$|a-b| \leq 3$ and $|b-c| \leq 3$

False as whe we loot at $a = \cancel{4}5, b=2$ and $c=1$

$\cancel{4\rightarrow}$ $|a-b| \leq 3$

or $|5-2| \leq 3$ True.

$3 \leq 3$ True

$|b-c| \leq 3$

$|2-1| \leq 3$

$1 \leq 3$ True

~~Howe~~ However,

$|a-c| \leq 3$

$|5-1| \leq 3$

$4 \leq 3$ False it does not string

$\therefore$ Transitive is not possible FALSE

c) $R = \{ (a,b) \mid a,b \in \mathbb{Z} \text{ and } (a \bmod 10) = (b \bmod 10) \}$

Reflexive.

~~ta~~ $(a \bmod 10) = (a \bmod 10)$

True.

Symetric.

$(a \bmod 10) = (b \bmod 10)$

and

$(b \bmod 10) = (a \bmod 10)$

True.

Transitive.

$(a \bmod 10) = (b \bmod 10)$

and

$(b \bmod 10) = (c \bmod 10)$

$\therefore (a \bmod 10) = (c \bmod 10)$

True.