Assignment 2

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Exercise 1

a) let Ω be the sample space

$\Omega = \{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTH, THTT, TTHH, TTHT, TTTH, TTTT\}$.

b) ~~let X be the random variable~~

~~de~~ let X denote the no. of heads

Y denote the no. of tails.

$X : \Omega = \{0, 1, 2, 4\}$

$Y : \Omega = \{0, 1, 2, 4\}$

Now we consider ~~Z or~~ Z as $Z = |X - Y|$

when we see,

when X is $\quad$ Y is $\quad$ $Z(|X-Y|)$

0	4	4
1	3	2
2	2	0
3	1	2
4	0	4.

c) PMF of (X, Z).

$$= \sum_{x=0}^{4} P(X=x, Z=0) + P(X=x, Z=2) + P(X=x, Z=4).$$

Instead of calculating for all we can just look at the above table and select cases.

$$= P(X=0, Z=4) + P(X=4, Z=4)$$
$$+ P(X=1, Z=2) + P(X=3, Z=2)$$
$$+ P(X=2, Z=0)$$

$$= \frac{1}{16} + \frac{1}{16} + \frac{4}{16} + \frac{4}{16} + \frac{6}{16}$$

$$= 1$$

c) ~~E(X) =~~

$E(Y) = \sum y_i \, p(y_i)$ → ~~Tail~~ Probability of getting 0 tail

$= 0 \times \frac{4C_0}{16} + 1 \times \frac{4C_1}{16} + 2 \times \frac{4C_2}{16}$

$+ 3 \times \frac{4C_3}{16} + 4 \times \frac{4C_4}{16}$

$= 0 \times \frac{1}{16} + 1 \times \frac{4}{16} + 2 \times \frac{6}{16} + 3 \times \frac{4}{16} + 4 \times \frac{1}{16}$

$= 2$ //.

Exercise 2

a) $\Omega =$ ~~x, y~~ $(x_i, y_i \in Z \mid x_i = \{1, \dots 6\}, z_i = \{1 \dots 6\})$

~~let X~~ $X = \{1, 2, 3, 4, 5, 6\}$

$Y = \{1, 2, 3, 4, 5, 6\}$

b) ~~$E(X_1 + X_2 \mid X_2 = x_2)$ =~~

~~$= \frac{1}{6}(1 + 2) + \dots$~~ $= (1 + x_2) \times \frac{1}{6} + (2 + x_2) \times \frac{1}{6}$ ....

~~$= \frac{1}{6}($~~

b) $E(X_1 + X_2 \mid X_2 = x_2)$

$= (1+x_2) \times \frac{1}{6} + (2+x_2)\frac{1}{6} + (3+x_2)\frac{1}{6} + (4+x_2)\frac{1}{6}$

$+ (5+x_2)\frac{1}{6} + (6+x_2)\frac{1}{6}$

$= \frac{21 + 6x_2}{6}$

$= 3.5 + x_2$ //.

c) $E(X_1 X_2 \mid X_2 = x_2)$

$= x_2 \frac{1}{6} + \frac{2x_2}{6} + \frac{3x_2}{6} + \frac{4x_2}{6} + \frac{5x}{6} + \frac{6x^2}{6}$

$= 3.5x^2$

d) $Var(X_1^2 . X_2 \mid X_2 = x_2)$.

Using the formula,

$Var(X/Y) = \underbrace{E(X^2/Y)}_{2} - \underbrace{(E(X|Y))}^2$

~~E(X~~

~~$\{E(X/Y)\}$~~

$= E(X_1^2 \cdot X_2 / X_2 = x^2)$

$= \frac{1x_2^2}{6} + \frac{4x_2^2}{6} + \frac{9x_2^2}{6} + \frac{16x_2^2}{6} + \frac{25x_2^2}{6} + \frac{36x_2^2}{6}$

$= 15.166x_2$.

$E(X^2/Y)$

$= E(X_1^4 \cdot X_2^2 / X_2 = x^2)$

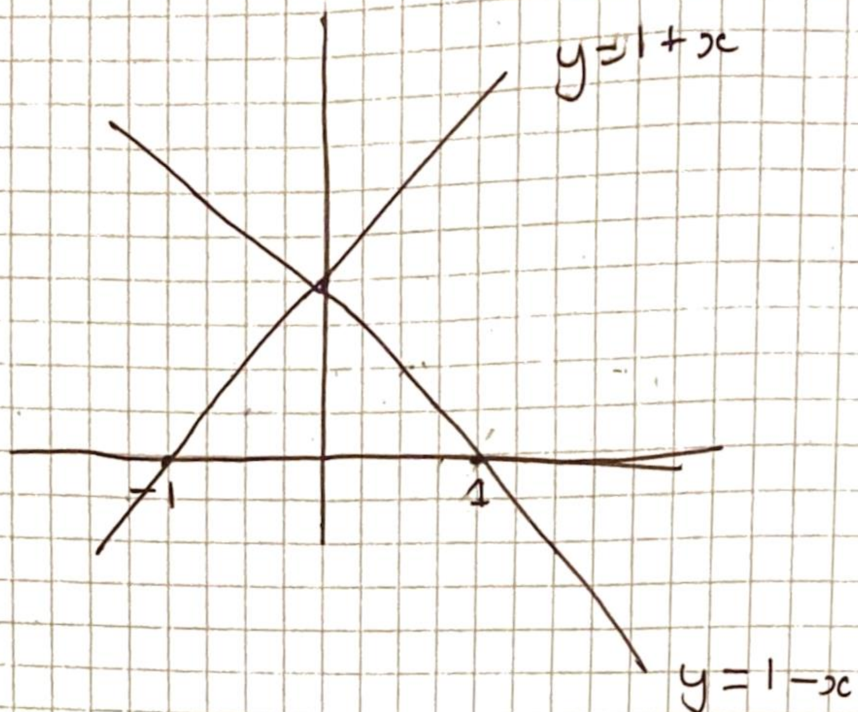
$= \frac{1x_2^2}{6} + \frac{16}{6}x_2^2 + \frac{81x_2^2}{6} + \frac{256}{6}x_2^2 + \frac{625}{6}x_2^2 + \frac{1296}{6}x_2^2$

$= 379.166x_2^2$

$\therefore 379.166x_2^2 + (15.166x_2)^2$

$= 149.15x_2^2$ //.

Exercise 3



$$\int_{y=0}^{y=1}\int_{x=y-1}^{x=1-y} c \, dx \, dy$$

$$= c\int_0^1 [x]_{y-1}^{1-y} \, dy$$

$$= c\int_0^1 (1-y) - (y-1)$$

$$= c\int_0^1 1-y-y+1$$

$$= c\int_0^1 2-2y$$

$$= c \left[2y - \frac{2y^2}{2} \right]_0^1$$

$= \cancel{x}$

$= C$ //.

Integrating them should have total probability 1 anyway so.

$\therefore C = 1$ //.

~~Seen~~ looking at figure,

~~y range~~ Range of y is $0 \to 0.5$

~~$0 \le y$~~

Range of x is $y-1$ to -0.5

a) $P(X < 0.5) = \int_0^{0.5} \int_{y-1}^{-0.5} c$

$= \int_0^{0.5} \int_{y-1}^{-0.5} 1$

$= \int_0^{0.5} [x]_{y-1}^{-0.5}$

$= \int_0^{0.5} \frac{1}{2} - y \; dy$

$= \left[\frac{y}{2} - \frac{y^2}{2} \right]_0^{0.5}$

$= \frac{1}{8} - 0$

$P(X \geq 0.5) = 1 - P(X < 0.5)$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

b) $$P(Y \geq 0.5) = \int_{0.5}^{1} \int_{y-1}^{1-y} 1 \, dx \, dy$$

$$= \int_{0.5}^{1} [x]_{y-1}^{1-y} \, dy$$

$$= \int_{0.5}^{1} 2 - 2y$$

$$= \left[2y - \frac{2y^2}{2} \right]_{0.5}^{1}$$

$$= 1 - 3/4$$

$$= 1/4$$

c) $$f_X(x) = \int_{0}^{1+x} 1 \, dy \qquad \text{for } -1 \leq x \leq 0$$

$$= 1 + x$$

And

$$f_X(x) = \int_{0}^{1-x} 1 \, dy \qquad \text{for } 0 \leq x \leq 1$$

$$= 1 - x$$

$$E(X]) = \int_{-1}^{0} x\,(1+x) + \int_{0}^{1} x(1-x)$$

$$= \int_{-1}^{0} (x + x^2 + \int_{0}^{1} x - x^2$$

$$= \left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{0}^{1}$$

$$= -\left[\frac{1}{2} - \frac{1}{3}\right] + \left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= \cancel{1 - \frac{2}{3}}\ 0$$

$$= \cancel{1/3}$$

Now for y.

$$f_Y(y) = \int_{y-1}^{y+1} 1 \, dx \quad \text{for} \quad 0 \leq y \leq 1$$

$$= \left[x\right]_{y-1}^{y+1}$$

$$= 2 - 2y$$

$$E[Y] = \int_0^1 y(2-2y)$$

$$= \int_0^1 2y - 2y^2$$

$$= \left[\frac{2y^2}{2} - \frac{2y^3}{3}\right]_0^1$$

$$= 1/3$$