Sound Emitted by Point Source

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The state of a fluid at any point $\mathbf{x}$ and time t is completely determined by specifying the velocity $\mathbf{v}$ and any two thermodynamic variables. The equation of motion for a compressible isentropic

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p(\mathbf{x},t) = s(\mathbf{x},t) \tag{1}$$

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)G(\mathbf{x},t;\mathbf{y},\tau) = \delta(\mathbf{x}-\mathbf{y})\delta(t-\tau) \tag{2}$$

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)s(\mathbf{y},\tau)G(\mathbf{x},t;\mathbf{y},\tau) = s(\mathbf{y},\tau)\delta(\mathbf{x}-\mathbf{y})\delta(t-\tau) \tag{3}$$

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\int s(\mathbf{y},\tau)G(\mathbf{x},t;\mathbf{y},\tau)d\mathbf{y}d\tau = \int s(\mathbf{y},\tau)\delta(\mathbf{x}-\mathbf{y})\delta(t-\tau)d\mathbf{y}d\tau \tag{4}$$

Using the properties of delta function we can simplify further

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\int s(\mathbf{y},\tau)G(\mathbf{x},t;\mathbf{y},\tau)d\mathbf{y}d\tau = s(\mathbf{x},t) \tag{5}$$

$$p(\mathbf{x},t) = \int s(\mathbf{y},\tau)G(\mathbf{x},t;\mathbf{y},\tau)d\mathbf{y}d\tau$$

Deriving Green's function for 3d acoutic wave equation. For simplicity we assume point source is located at $\mathbf{y}=0$ and time $\tau=0$

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)G(\mathbf{x},t) = \delta(\mathbf{x})\delta(t) \tag{6}$$

for $\mathbf{x} > 0$

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)G(\mathbf{x}, t) = 0 \tag{7}$$

Spherically symmetric Laplacian operator in radial coordinate system