3D structured grid acoustic solver

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Results

We solve the acoustic wave equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0 \qquad .$$

on a 3d structured periodic domain $\Omega = [0, 2\pi] \times [0, 2\pi] \times [0, 2\pi]$ using the initial conditions

$$u(\mathbf{x}, 0) = \sin(k_x x + k_y y + k_z z)$$

and

$$\frac{\partial u(\mathbf{x}, 0)}{\partial t} = -\omega \cos(k_x x + k_y y + k_z z).$$

The plane wave solution for the above initial contion is

$$u(\mathbf{x}, t) = \sin(k_x x + k_y y + k_z z - \omega t). \tag{1}$$

And the wave speed in terms of wave number and frequency is given by the dispersion relation

$$c^2 = \frac{\omega^2}{k_x^2 + k_y^2 + k_z^2}.$$

We discretize the wave equation using the fourth-order central WENO polynomial in space and SSPRK54 scheme in time. We did the convergence study for wave number $k_x = k_y = k_z = 1$, wave speed $c = 1$ and $cfl = 0.5$. We obtain fourth-order convergence in space.

N	L_2norm	L_∞ norm	L_2 rate	L_∞ rate
16	7.23e-03	6.48e-04	-	-
32	5.06e-04	4.54e-05	3.83	3.83
64	3.15e-05	2.82e-06	4.00	4.00
128	1.94e-06	1.74e-07	4.02	4.01

Table 1: Convergence table for plane wave solution

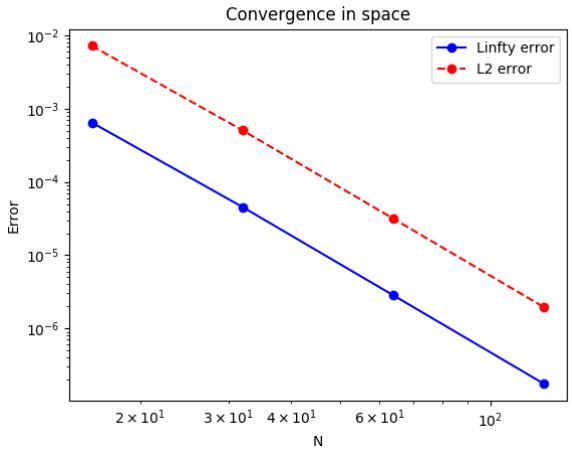

Figure 1: L_2 and L_∞ error plot