Let $f(x)$ be a function of a single variable and let x_i be the location of the i-th grid point. We can use Lagrange polynomial interpolation to approximate the function near this point:

$$f(x) \approx L_i(x) = \sum_{j=0}^{n} f(x_j) \prod_{k \neq j} \frac{x - x_k}{x_j - x_k} \tag{1}$$

where n is the number of grid points.

We can use this approximation to find the second derivative of the function at the i-th grid point by approximating $f(x_{i-1})$ and $f(x_{i+1})$:

$$f(x_{i-1}) \approx L_{i-1}(x_{i-1}) = \sum_{j=0}^{n} f(x_j) \prod_{k \neq j} \frac{x_{i-1} - x_k}{x_j - x_k} \tag{2}$$

$$f(x_{i+1}) \approx L_{i+1}(x_{i+1}) = \sum_{j=0}^{n} f(x_j) \prod_{k \neq j} \frac{x_{i+1} - x_k}{x_j - x_k} \tag{3}$$

Taking the derivative of the Lagrange polynomial approximation, we have:

$$f'(x) \approx \sum_{j=0}^{n} f(x_j) \sum_{k \neq j} \frac{1}{x_j - x_k} \prod_{\substack{l=0 \\ l \neq j,k}}^{n} \frac{x - x_l}{x_j - x_l} \tag{4}$$

Evaluating the derivative at x_{i-1} and x_{i+1}, we have: