Computational study of far-field acoustic emission by collapsing bubbles

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Computational approach

- In this work we compute the far-field acoustic waves emitted from bubble collapse process using the Kirchhoff integral formulation.

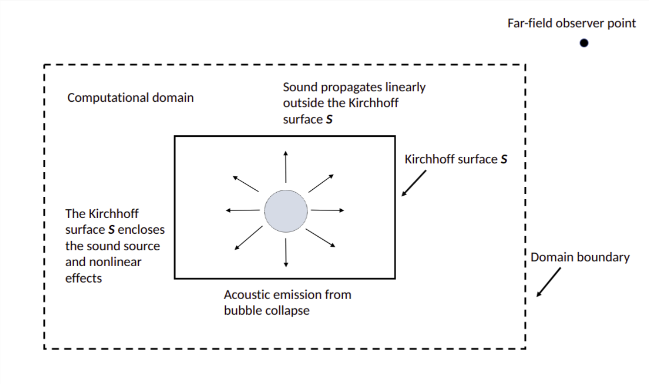

Figure 1: Schematic of far-field acoustic data computed from CFD solver using the Kirchhoff integral method.

Kirchhoff integral formulation

- We chose a control surface S that encloses all the acoustic sources, and the pressure perturbations p' satisfies the homogeneous wave equation.

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = 0 \qquad \text{in } V. \qquad (1)$$

- The control surface S is defined by $f(\mathrm{x}) = 0$, $f(\mathrm{x}) > 0$ for x in V and $f(\mathrm{x}) < 0$ for x inside surface S.

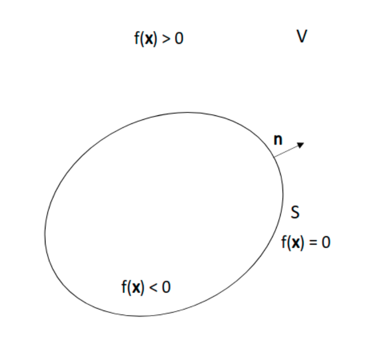

Figure 2: Stationary Kirchhoff surface S encloses sound source

Kirchhoff integral formulation

- We define the pressure p' as a generalized function $pH(f)$ where

$$p'H(f) = \begin{cases} p', & \text{for x in V.} \\ 0, & \text{for x inside S.} \end{cases}$$

- The acoustic wave equation in generalised pressure

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p'H = -\frac{\partial p'}{\partial n}\delta(f) - \nabla.(p'\mathrm{n}\delta(f)).$$

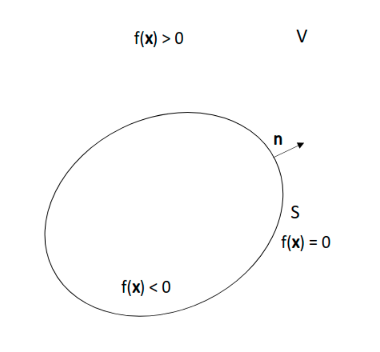

Figure 3: Stationary Kirchhoff surface S encloses sound source

Kirchhoff integral formulation

- The acoustic wave equation in generalized variables is valid in the entire unbounded space. Therefore we can use the free-space Green's function to solve the equation.

$$p'(\mathrm{x}, t) = \int s(\mathrm{y}, \tau) G(\mathrm{x}, t; \mathrm{y}, \tau) d\mathrm{y} d\tau. \tag{2}$$

where,

$$G(\mathrm{x}, t; \mathrm{y}, \tau) = \frac{\delta\left(t - \tau - \frac{|\mathrm{x}-\mathrm{y}|}{c_0}\right)}{4\pi|\mathrm{x} - \mathrm{y}|} \tag{3}$$

and

$$s(\mathrm{y}, \tau) = -\frac{\partial p'}{\partial n}\delta(f) - \nabla.(p'\mathrm{n}\delta(f)). \tag{4}$$

Kirchhoff integral formulation

- Simplifying the equation further, we obtain the Kirchhoff integral equation for a stationary control surface

$$p'(\mathrm{x}, t) = -\frac{1}{4\pi}\int_S \left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau dS. \tag{5}$$

  p' is the acoustic pressure satisfying the wave equation outside the control surface $\mathbf{S}$, c is the speed of sound at ambient conditions and n is the normal. The integrands are evaluated at the emission time $\tau = t - \mathrm{r}/c$ and $\mathrm{r} = |\mathrm{x} - \mathrm{y}|$ is the distance between observer and source.
- The p', $\partial p'/\partial t$ and $\partial p'/\partial n$ are computed from flow solver.

Kirchhoff integral on a cylinder

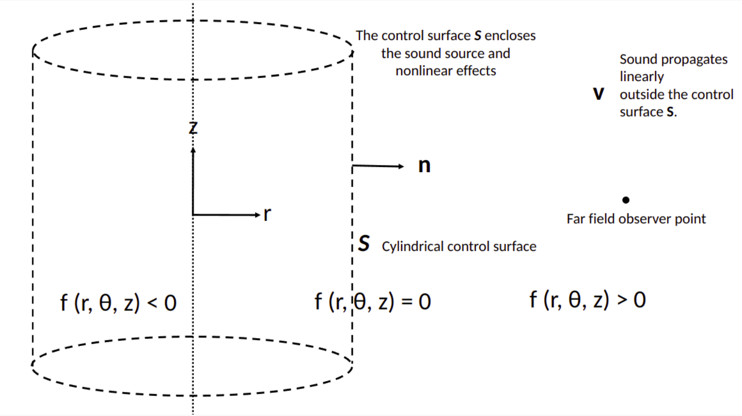

Figure 4: Cylindrical Kirchhoff surface

Kirchhoff integral on a cylinder

- We compute the Kirchhoff integral on a cylindrical surface

$$\begin{aligned} p'(r',z',t) = &-\frac{1}{4\pi}\int_{S_{top,bottom}} \left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau r'dr'd\theta' \\ &-\frac{1}{4\pi}\int_{S_{curved}} \left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau R d\theta' dz' \end{aligned} \tag{6}$$

- The above integral is numerically computed using Gauss quadrature

$$\begin{aligned} p'(r',z',t) = &-\frac{1}{4\pi}\sum_i^{N_r}\sum_j^{N_\theta}\sum_q^{Nqpts} \left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau r'\Big|_q J_q w_q \\ &-\frac{1}{4\pi}\sum_j^{N_\theta}\sum_k^{N_z}\sum_q^{Nqpts} \left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau R\Big|_q J_q w_q \end{aligned} \tag{7}$$

- We use two-point Gauss quadrature on a reference cell $[-1,1]\times[-1,1]$. The quadrature points are mapped to computational cell using bi-linear interpolation.

Kirchhoff integral on a cylinder

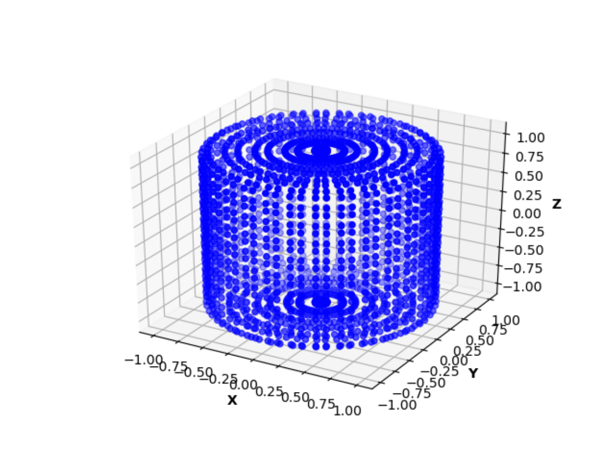

Figure 5: Schematic of quadrature points on cylindrical surface

Results - Acoustic monopole

- We solve the acoustic wave equation

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p'(\mathrm{x}, t) = -q(t)\delta(\mathrm{x}), \tag{8}$$

  for a monopole source using the **Kirchhoff** method.
- And compare it with the exact solution $p'(\mathrm{x}, t) = -\frac{1}{4\pi}\frac{q(t-\frac{r}{c_0})}{r}$.
- We chose a monopole source

$$q(t) = 2(t - t0)f_0^2 \exp(-f_0^2(t - t_0)^2), \tag{9}$$

  where $f_0 = 100$ is the dominant frequency and $t_0 = \frac{4}{f_0}$.

Results - Acoustic monopole

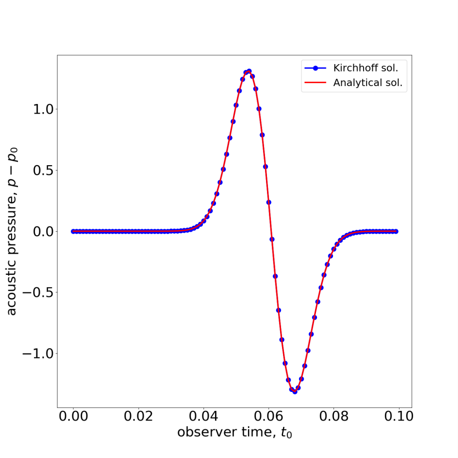

Figure 6: Acoustic pressure is computed at observer point $(x_0, y_0, z_0) = (3.0, 3.0, 3.0)$ using the Kirchhoff method and compared with the analytical solution. We chose a cylinder of radius $R = 1.0$ and height $H = 20$ centered at origin and $dr = d\theta = dz = 0.01$.

Results - Acoustic monopole

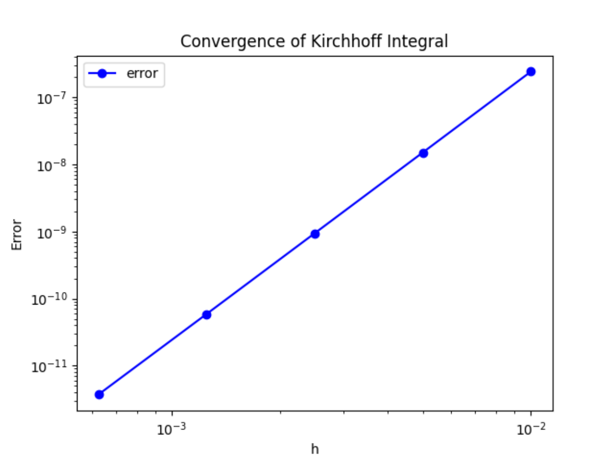

Figure 7: We have obtained fourth-order convergence for the Kirchhoff integral computed using the two-point Gauss quadrature

Conclusion

- In this work we have computed the acoustic waves emitted by a monopole source using the Kirchhoff method. And showed fourth-order convergence for the Kirchhoff integral computed using two-point Gauss quadrature.

Thank you!