Analytical solution for axisymmetric acoustic wave equation

June 1, 2022

The acoustic wave equation in cylindrical coordinate system is given by

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\partial^2}{\partial z^2}\right)p = 0. \tag{1}$$

Assuming cylindrical wave solution, $\partial p/\partial\theta = 0$ and $\partial p/\partial z = 0$. Substituting it in the above equation we obtain

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right)p = 0. \tag{2}$$

Substituting $p(r,t) = R(r)e^{i\omega t}$ in the wave equation obtain

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + k^2R = 0. \tag{3}$$

Where $k^2 = \omega^2/c^2$. The general solution of equation (2) is given by Hankel function (Kinsler et al. 2000)

$$p(r,t) = AH_0^2(kr)e^{i\omega t}. \tag{4}$$

The above solution will be used to validate the axisymmetric Kirchhoff solver.

References

Kinsler, Lawrence E et al. (2000). *Fundamentals of acoustics.* John wiley & sons.