Computational study of far-field acoustic emission by collapsing bubbles

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Introduction

- High-speed flows lead to the formation of cavitation bubbles.
- Collapsing cavitation bubbles form blast and shock waves.
- Blast wave emission process are the strong source of acoustic waves.

Figure 1: Cavitation bubbles around rotating propeller[1]

Computational approach

- In this work we compute the far-field acoustic waves emitted from bubble collapse process using the Kirchhoff integral formulation.

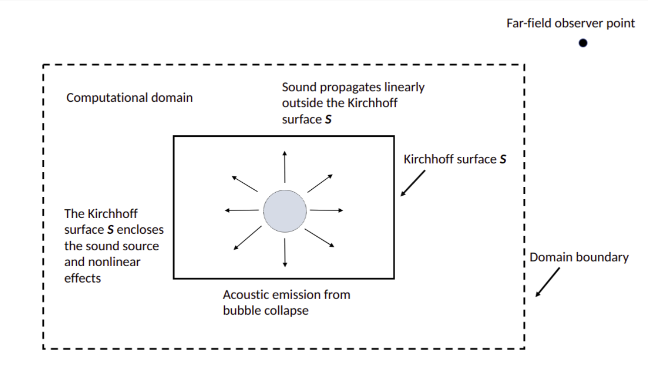

Figure 2: Schematic of far-field acoustic data computed from CFD solver using the Kirchhoff integral method.

Kirchhoff integral formulation

- We chose a control surface S that encloses all the acoustic sources, and the pressure perturbations p' satisfies the homogeneous wave equation.

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = 0 \qquad \text{in } V. \qquad (1)$$

- The control surface S is defined by $f(\mathrm{x}) = 0$, $f(\mathrm{x}) > 0$ for x in V and $f(\mathrm{x}) < 0$ for x inside surface S.

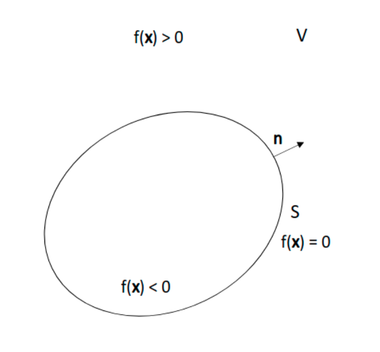

Figure 3: Stationary Kirchhoff surface S encloses sound source

Kirchhoff integral formulation

- We define the pressure p' as a generalized function $pH(f)$ where

$$p'H(f) = \begin{cases} p', & \text{for x in V.} \\ 0, & \text{for x inside S.} \end{cases}$$

- The acoustic wave equation in generalised pressure

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p'H = -\frac{\partial p'}{\partial n}\delta(f) - \nabla.(p'\mathrm{n}\delta(f)).$$

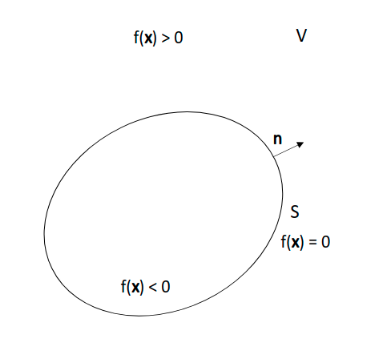

Figure 4: Stationary Kirchhoff surface S encloses sound source

Kirchhoff integral formulation

- The acoustic wave equation in generalized variables is valid in the entire unbounded space. Therefore we can use the free-space Green's function to solve the equation.

$$p'(\mathrm{x}, t) = \int s(\mathrm{y}, \tau) G(\mathrm{x}, t; \mathrm{y}, \tau) d\mathrm{y} d\tau. \tag{2}$$

where,

$$G(\mathrm{x}, t; \mathrm{y}, \tau) = \frac{\delta\left(t - \tau - \frac{|\mathrm{x}-\mathrm{y}|}{c_0}\right)}{4\pi|\mathrm{x} - \mathrm{y}|} \tag{3}$$

and

$$s(\mathrm{y}, \tau) = -\frac{\partial p'}{\partial n}\delta(f) - \nabla.(p'\mathrm{n}\delta(f)). \tag{4}$$

Kirchhoff integral formulation

- Simplifying the equation further, we obtain the Kirchhoff integral equation for a stationary control surface

$$p'(\mathrm{x}, t) = -\frac{1}{4\pi} \int_S \left[\frac{p'}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial p'}{\partial n} + \frac{1}{cr} \frac{\partial r}{\partial n} \frac{\partial p'}{\partial \tau} \right]_\tau dS. \tag{5}$$

  p' is the acoustic pressure satisfying the wave equation outside the control surface **S**, c is the speed of sound at ambient conditions and n is the normal. The integrands are evaluated at the emission time $\tau = t - \mathrm{r}/c$ and $\mathrm{r} = |\mathrm{x} - \mathrm{y}|$ is the distance between observer and source.

- The p', $\partial p'/\partial t$ and $\partial p'/\partial n$ are computed from flow solver.

Axisymmetric test case - Cylindrical wave

- The axisymmetric acoustic wave equation in a cylindrical coordinate system, assuming cylindrical wave solution, $\partial p'/\partial\theta = 0$ and $\partial p'/\partial z = 0$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right)p' = 0. \tag{6}$$

- Substituting $p'(r,t) = R(r)e^{i\omega t}$ in the wave equation we obtain

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + k^2R = 0. \tag{7}$$

- Where $k^2 = \omega^2/c^2$. The general solution of equation is given by Hankel function

$$p'(r,t) = AH_0^{(1)}(kr)e^{i\omega t}. \tag{8}$$

Axisymmetric test case - Cylindrical wave

- The solution $p'(r,t) = AH_0^{(1)}(kr)e^{i\omega t}$ is singular at $r = 0$.
- For $r \to 0$ The function has a logarithmic singularity

$$p'(r,t) \cong (2iA/\pi)\log(kr)e^{-i\omega t}. \tag{9}$$

Axisymmetric test case - Cylindrical wave

- We use the cylindrical wave solution $p'(r,t) = AH_0^2(kr)e^{i\omega t}$ to validate the Kirchhoff solver.
- Where the amplitude, wave number and angular frequency are $A = 1.0, k = 1.0, \omega = 1.0$.

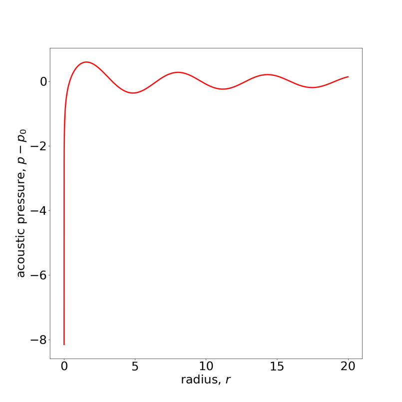

Figure 5: Cylindrical wave solution at $t = 0.0$. The solution is singular at $r = 0.0$

Axisymmetric test case

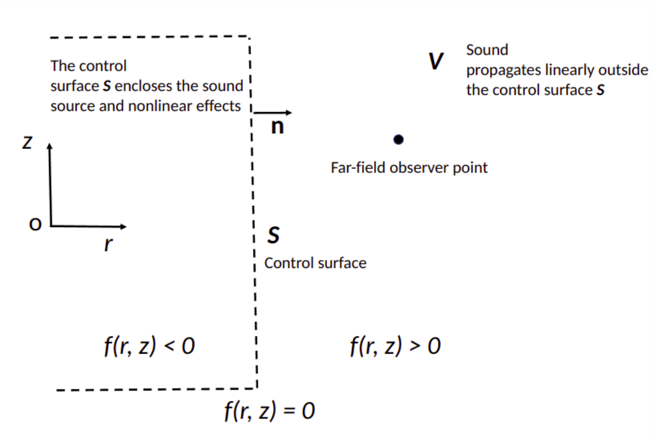

Figure 6: Axisymmetric Kirchhoff surface

Kirchhoff integral on a cylinder

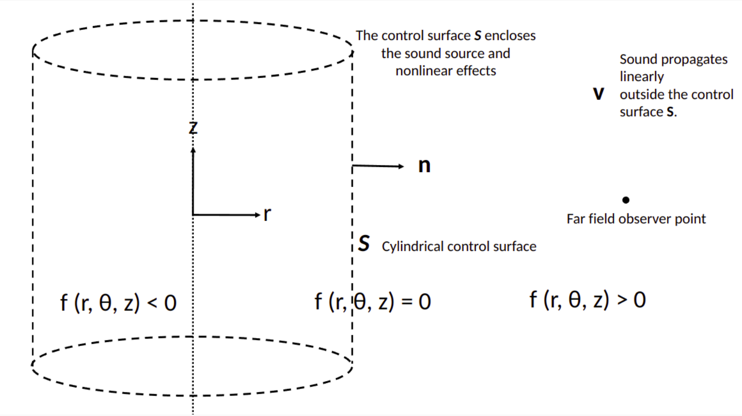

Figure 7: Cylindrical Kirchhoff surface

Kirchhoff integral on a cylinder

- We compute the Kirchhoff integral on a cylindrical surface

$$\begin{aligned} p'(r',z',t) = &-\frac{1}{4\pi}\int_{S_{top,bottom}}\left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau r'dr'd\theta' \\ &-\frac{1}{4\pi}\int_{S_{curved}}\left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau Rd\theta'dz' \end{aligned} \tag{10}$$

- The above integral is numerically computed using Gauss quadrature

$$\begin{aligned} p'(r',z',t) = &-\frac{1}{4\pi}\sum_i^{N_r}\sum_j^{N_\theta}\sum_q^{Nqpts}\left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau r'\Big|_q J_q w_q \\ &-\frac{1}{4\pi}\sum_j^{N_\theta}\sum_k^{N_z}\sum_q^{Nqpts}\left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau R\Big|_q J_q w_q \end{aligned} \tag{11}$$

- We use two-point Gauss quadrature on a reference cell $[-1,1]\times[-1,1]$. The quadrature points are mapped to computational cell using bi-linear interpolation.

Kirchhoff integral on a cylinder

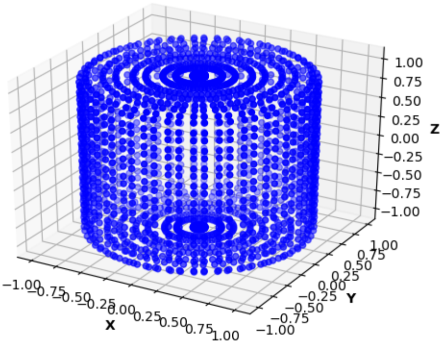

Figure 8: Schematic of quadrature points on cylindrical surface

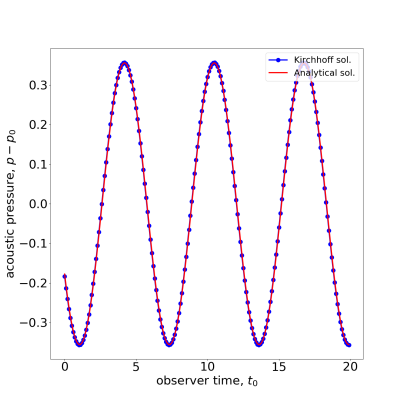

Figure 9: Acoustic pressure is computed at observer point $(r_0, \theta_0, z_0) = (5.0, 0, 0)$ using the Kirchhoff method and compared with the analytical solution. We chose a cylinder of radius $R = 0.5$ and height $H = 200$ centered at origin and $dr = d\theta = dz = 0.1$.

Results - Cylindrical wave

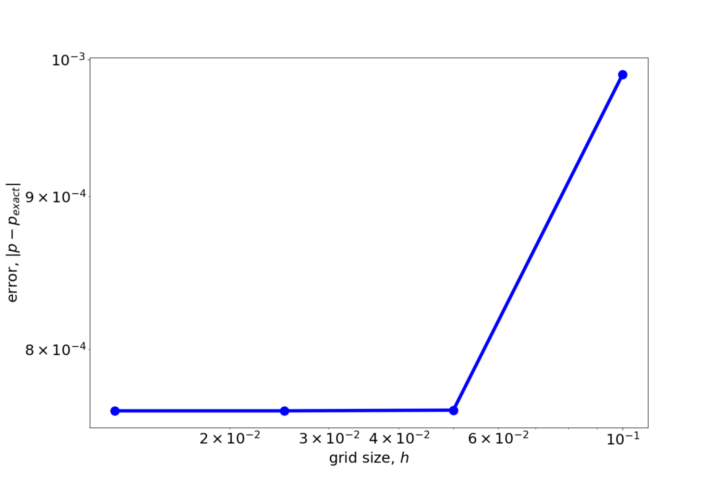

Figure 10: Convergence of Kirchhoff integral.

Three dimensional test case - Acoustic monopole

- We solve the acoustic wave equation

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p'(\mathrm{x}, t) = -q(t)\delta(\mathrm{x}), \tag{12}$$

  for a monopole source using the **Kirchhoff** method.
- And compare it with the exact solution $p'(\mathrm{x}, t) = -\frac{1}{4\pi}\frac{q(t-\frac{r}{c_0})}{r}$.
- We chose a monopole source

$$q(t) = 2(t - t0)f_0^2 \exp(-f_0^2(t - t_0)^2), \tag{13}$$

  where $f_0 = 100$ is the dominant frequency and $t_0 = \frac{4}{f_0}$.

Three dimensional test case - Acoustic monopole

- We discretize a cuboidal domain of size $[-5.0, 5.0] \times [-5.0, 5.0] \times [-5.0, 5.0]$ using $200 \times 200 \times 200$ cells.
- We enclose the monopole source using a cuboidal Kirchhoff surface whose diagonally opposite points are $p_1 = (-1.0, -1.0, -1.0)$ and $p_2 = (1.0, 1.0, 1.0)$.
- The Kirchhoff surface is discretized into square cells of size $h = 0.1$.
- We store the analytical pressure at cell centers and interpolate it to the Kirchhoff surface using the fourth-order WENO polynomial.
- We interpolate the pressure data at emission time using the Linear polynomial.
- The Kirchhoff Integral is computed using the two-point Gauss quadrature formula.

Results - Acoustic monopole

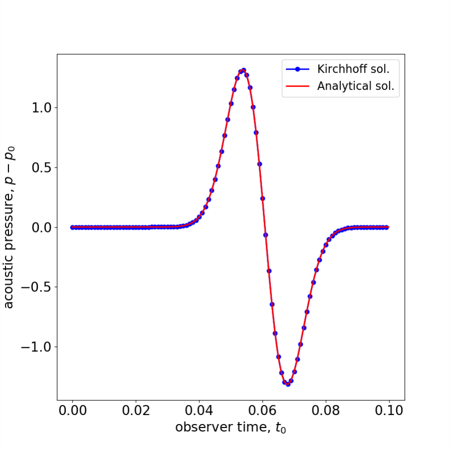

Figure 11: Acoustic pressure is computed at observer point $x_0 = (3, 3, 3)$ using the Kirchhoff method and compared with the analytical solution.

Conclusion

- We have written a Kirchhoff solver to compute the far-field pressure from CFD data.
- We have tested our solver with analytical solution for axisymmetric and three-dimensional wave equation.

Future work:

- To compute the far-field acoustic waves emitted from axisymmetric bubble collapse using the developed Kirchhoff solver.

Duttweiler M. E., Brennen C. E. Surge instability on a cavitating propeller [J]. Journal of Fluid Mechanics, 2002, 458: 133-152.

Thank you!