Computational study of acoustic waves emitted by collapsing bubbles

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Kirchhoff integral formulation

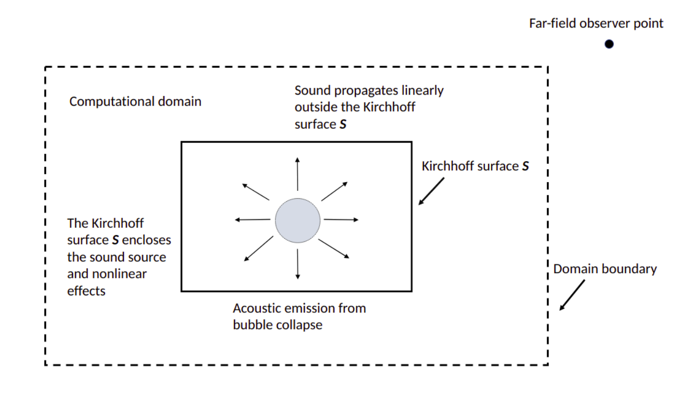

Figure 1: Schematic of far-field acoustic data computed from CFD solver using the Kirchhoff integral method.

Kirchhoff integral formulation

- Given a stationary control surface **S** that encloses the sound sources and nonlinear fluid flow effects, then the far-field acoustic pressure is computed from the Kirchhoff integral

$$p'(\mathrm{x}, t) = -\frac{1}{4\pi}\int_S \left[\frac{p'}{r^2}\frac{\partial r}{\partial n} - \frac{1}{r}\frac{\partial p'}{\partial n} + \frac{1}{cr}\frac{\partial r}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau dS. \tag{1}$$

  p' is the acoustic pressure satisfying the wave equation outside the control surface **S**, c is the speed of sound at ambient conditions and n is the normal. The integrands are evaluated at the emission time $\tau = t - \mathrm{r}/c$ and $\mathrm{r} = |\mathrm{x} - \mathrm{y}|$ is the distance between observer and source.
- The p', $\partial p'/\partial t$ and $\partial p'/\partial n$ are computed from flow solver.

Validation of Kirchhoff solver - Acoustic monopole

- We solve the acoustic wave equation

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p'(\mathrm{x}, t) = -q(t)\delta(\mathrm{x}), \tag{2}$$

for a monopole source using the **Kirchhoff** method.

- And compare it with the exact solution $p'(\mathrm{x}, t) = -\frac{1}{4\pi}\frac{q(t-\frac{r}{c_0})}{r}$.
- We chose a monopole source

$$q(t) = 2(t - t0)f_0^2 \exp(-f_0^2(t - t_0)^2), \tag{3}$$

where $f_0 = 100$ is the dominant frequency and $t_0 = \frac{4}{f_0}$.

Validation of Kirchhoff solver - Acoustic monopole

- We discretize a cuboidal domain of size $[-5.0, 5.0] \times [-5.0, 5.0] \times [-5.0, 5.0]$ using $200 \times 200 \times 200$ cells.
- We enclose the monopole source using a cuboidal Kirchhoff surface whose diagonally opposite points are $p_1 = (-1.0, -1.0, -1.0)$ and $p_2 = (1.0, 1.0, 1.0)$.
- The Kirchhoff surface is discretized into square cells of size $h = 0.1$.
- We store the analytical pressure at cell centers and interpolate it to the Kirchhoff surface using the fourth-order WENO polynomial.
- We interpolate the pressure data at emission time using the Linear polynomial.
- The Kirchhoff Integral is computed using the two-point Gauss quadrature formula.

Validation of Kirchhoff solver - Acoustic monopole

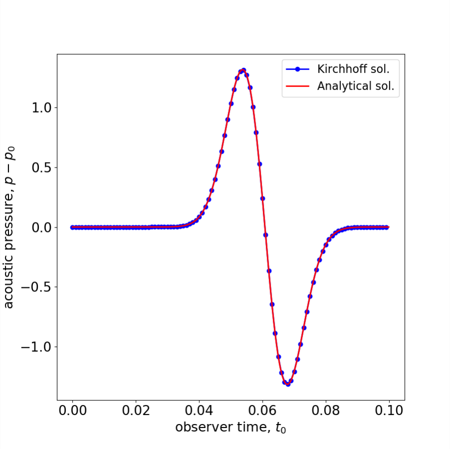

Figure 2: Acoustic pressure is computed at observer point $x_0 = (3, 3, 3)$ using the Kirchhoff method and compared with the analytical solution.

Validation of Kirchhoff solver - Euler equation

- We solve the compressible Euler equation

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathrm{u}) = 0, \tag{4}$$

$$\frac{\partial \rho \mathrm{u}}{\partial t} + \nabla.(\rho \mathrm{u}\mathrm{u}) + \nabla P = 0, \tag{5}$$

$$\frac{\partial E}{\partial t} + \nabla.((E+P)\mathrm{u}) = 0, \tag{6}$$

$$p = (\gamma - 1)(E - \frac{1}{2}\rho \mathrm{u}^2). \tag{7}$$

- for an initial density and pressure perturbation from the ambient state

$$\rho = \rho_0 + \rho',$$

$$\mathrm{u} = 0,$$

$$p = p_0 + c_0^2 \rho'.$$

Validation of Kirchhoff solver - Euler equation

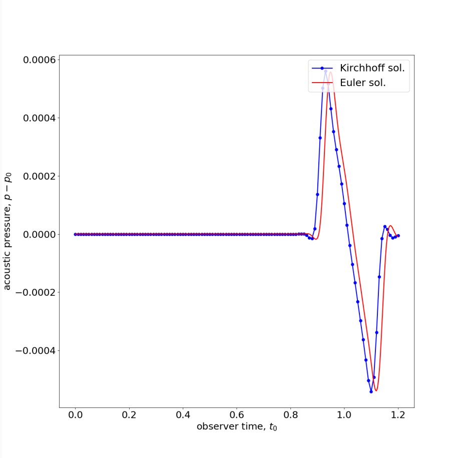

Figure 3: Acoustic pressure is computed at observer point $x_0 = (0.8975, 0.8795, 0.8795)$ using the Kirchhoff method and compared with the Euler equation solution.

Acoustic emission from Rayleigh collapse of a bubble

Thank you!