

Computational study of far-field acoustic emission by collapsing bubbles

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Introduction

- High-speed flows lead to the formation of cavitation bubbles.
- Collapsing cavitation bubbles form blast and shock waves.
- Blast wave emission process are the strong source of acoustic waves.



Figure 1: Cavitation bubbles around rotating propeller[1]

Computational approach

- In this work we compute the far-field acoustic waves emitted from bubble collapse process using the Kirchhoff integral formulation.

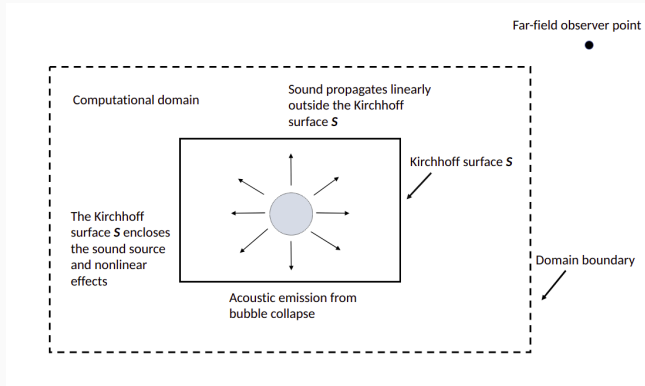


Figure 2: Schematic of far-field acoustic data computed from CFD solver using the Kirchhoff integral method.

Kirchhoff integral formulation

- We chose a control surface S that encloses all the acoustic sources, and the pressure perturbations p' satisfies the homogeneous wave equation.

$$\left(\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p' = 0 \quad \text{in } V. \quad (1)$$

- The control surface S is defined by $f(\mathbf{x}) = 0$, $f(\mathbf{x}) > 0$ for \mathbf{x} in V and $f(\mathbf{x}) < 0$ for \mathbf{x} inside surface S .

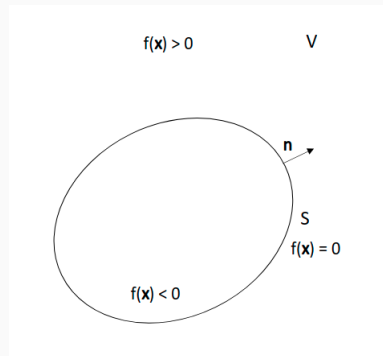


Figure 3: Stationary Kirchhoff surface S encloses sound source

Kirchhoff integral formulation

- We define the pressure p' as a generalized function $p'H(f)$ where

$$p'H(f) = \begin{cases} p', & \text{for } x \text{ in } V. \\ 0, & \text{for } x \text{ inside } S. \end{cases}$$

- The acoustic wave equation in generalised pressure

$$\left(\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p'H = -\frac{\partial p'}{\partial n} \delta(f) - \nabla \cdot (p'n \delta(f)).$$

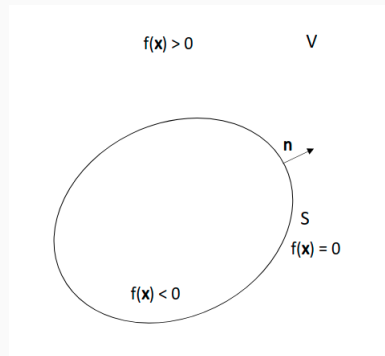


Figure 4: Stationary Kirchhoff surface S encloses sound source

Kirchhoff integral formulation

- The acoustic wave equation in generalized variables is valid in the entire unbounded space. Therefore we can use the free-space Green's function to solve the equation.

$$p'(x, t) = \int s(y, \tau) G(x, t; y, \tau) dy d\tau. \quad (2)$$

where,

$$G(x, t; y, \tau) = \frac{\delta\left(t - \tau - \frac{|x-y|}{c_0}\right)}{4\pi|x-y|} \quad (3)$$

and

$$s(y, \tau) = -\frac{\partial p'}{\partial n} \delta(f) - \nabla \cdot (p' n \delta(f)). \quad (4)$$

Kirchhoff integral formulation

- Simplifying the equation further, we obtain the Kirchhoff integral equation for a stationary control surface

$$p'(x, t) = -\frac{1}{4\pi} \int_S \left[\frac{p'}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial p'}{\partial n} + \frac{1}{cr} \frac{\partial r}{\partial n} \frac{\partial p'}{\partial \tau} \right]_{\tau} dS. \quad (5)$$

p' is the acoustic pressure satisfying the wave equation outside the control surface \mathbf{S} , c is the speed of sound at ambient conditions and n is the normal. The integrands are evaluated at the emission time $\tau = t - r/c$ and $r = |x - y|$ is the distance between observer and source.

- The p' , $\partial p'/\partial t$ and $\partial p'/\partial n$ are computed from flow solver.

Axisymmetric test case - Cylindrical wave

- The axisymmetric acoustic wave equation in a cylindrical coordinate system, assuming cylindrical wave solution, $\partial p'/\partial\theta = 0$ and $\partial p'/\partial z = 0$

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) p' = 0. \quad (6)$$

- Substituting $p'(r, t) = R(r)e^{i\omega t}$ in the wave equation we obtain

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + k^2 R = 0. \quad (7)$$

- Where $k^2 = \omega^2/c^2$. The general solution of equation is given by Hankel function

$$p'(r, t) = AH_0^{(1)}(kr)e^{i\omega t}. \quad (8)$$

Axisymmetric test case - Cylindrical wave

- The solution $p'(r, t) = AH_0^{(1)}(kr)e^{i\omega t}$ is singular at $r = 0$.
- For $r \rightarrow 0$ The function has a logarithmic singularity

$$p'(r, t) \cong (2iA/\pi) \log(kr)e^{-i\omega t}. \quad (9)$$

Axisymmetric test case - Cylindrical wave

- We use the cylindrical wave solution $p'(r, t) = AH_0^2(kr)e^{i\omega t}$ to validate the Kirchhoff solver.
- Where the amplitude, wave number and angular frequency are $A = 1.0, k = 1.0, \omega = 1.0$.

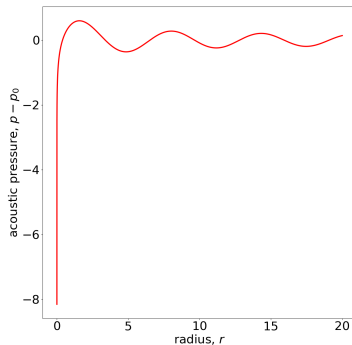


Figure 5: Cylindrical wave solution at $t = 0.0$. The solution is singular at $r = 0.0$

Axisymmetric test case

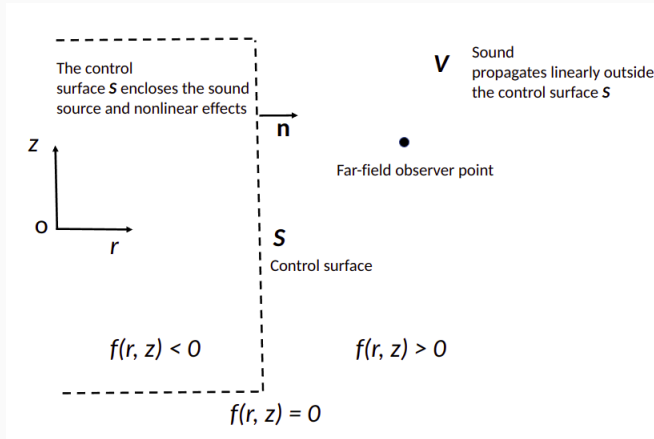


Figure 6: Axisymmetric Kirchhoff surface

Kirchhoff integral on a cylinder

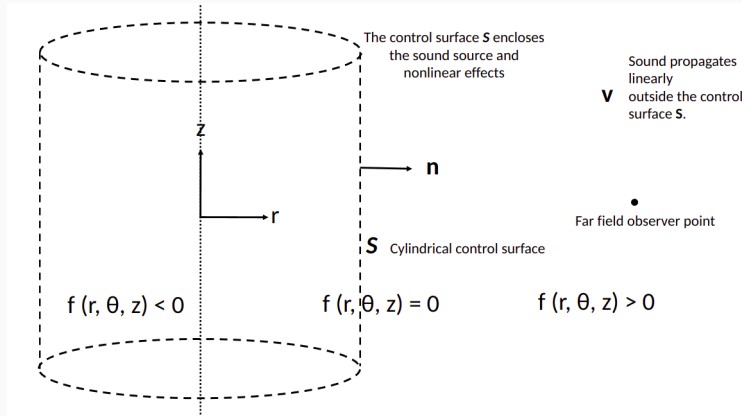


Figure 7: Cylindrical Kirchhoff surface

Kirchhoff integral on a cylinder

- We compute the Kirchhoff integral on a cylindrical surface

$$\begin{aligned} p'(r', z', t) = & -\frac{1}{4\pi} \int_{S_{top,bottom}} \left[\frac{p'}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial p'}{\partial n} + \frac{1}{cr} \frac{\partial r}{\partial n} \frac{\partial p'}{\partial \tau} \right]_{\tau} r' dr' d\theta' \\ & - \frac{1}{4\pi} \int_{S_{curved}} \left[\frac{p'}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial p'}{\partial n} + \frac{1}{cr} \frac{\partial r}{\partial n} \frac{\partial p'}{\partial \tau} \right]_{\tau} R d\theta' dz' \end{aligned} \quad (10)$$

- The above integral is numerically computed using Gauss quadrature

$$\begin{aligned} p'(r', z', t) = & -\frac{1}{4\pi} \sum_i^{N_r} \sum_j^{N_{\theta}} \sum_q^{N_{qpts}} \left[\frac{p'}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial p'}{\partial n} + \frac{1}{cr} \frac{\partial r}{\partial n} \frac{\partial p'}{\partial \tau} \right]_{\tau} r' \Big|_q J_q w_q \\ & - \frac{1}{4\pi} \sum_j^{N_{\theta}} \sum_k^{N_z} \sum_q^{N_{qpts}} \left[\frac{p'}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial p'}{\partial n} + \frac{1}{cr} \frac{\partial r}{\partial n} \frac{\partial p'}{\partial \tau} \right]_{\tau} R \Big|_q J_q w_q \end{aligned} \quad (11)$$

- We use two-point Gauss quadrature on a reference cell $[-1, 1] \times [-1, 1]$. The quadrature points are mapped to computational cell using bi-linear interpolation.

Kirchhoff integral on a cylinder

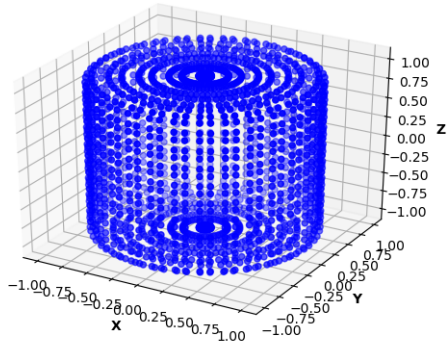


Figure 8: Schematic of quadrature points on cylindrical surface

Results - Cylindrical wave

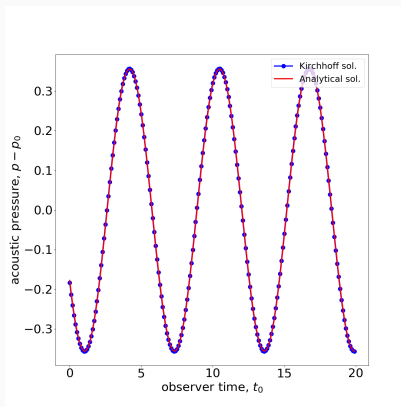


Figure 9: Acoustic pressure is computed at observer point $(r_0, \theta_0, z_0) = (5.0, 0, 0)$ using the Kirchhoff method and compared with the analytical solution. We chose a cylinder of radius $R = 0.5$ and height $H = 200$ centered at origin and $dr = d\theta = dz = 0.1$.

Results - Cylindrical wave

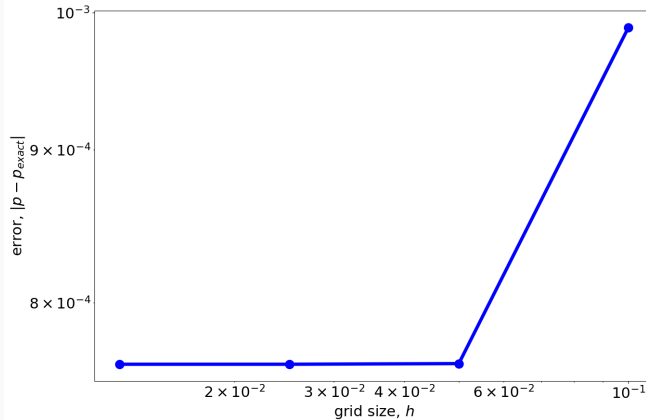


Figure 10: Convergence of Kirchhoff integral.

Three dimensional test case - Acoustic monopole

- We solve the acoustic wave equation

$$\left(\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p'(\mathbf{x}, t) = -q(t)\delta(\mathbf{x}), \quad (12)$$

for a monopole source using the **Kirchhoff** method.

- And compare it with the exact solution $p'(\mathbf{x}, t) = -\frac{1}{4\pi} \frac{q(t - \frac{r}{c_0})}{r}$.
- We chose a monopole source

$$q(t) = 2(t - t_0)f_0^2 \exp(-f_0^2(t - t_0)^2), \quad (13)$$

where $f_0 = 100$ is the dominant frequency and $t_0 = \frac{4}{f_0}$.

Three dimensional test case - Acoustic monopole

- We discretize a cuboidal domain of size $[-5.0, 5.0] \times [-5.0, 5.0] \times [-5.0, 5.0]$ using $200 \times 200 \times 200$ cells.
- We enclose the monopole source using a cuboidal Kirchhoff surface whose diagonally opposite points are $p_1 = (-1.0, -1.0, -1.0)$ and $p_2 = (1.0, 1.0, 1.0)$.
- The Kirchhoff surface is discretized into square cells of size $h = 0.1$.
- We store the analytical pressure at cell centers and interpolate it to the Kirchhoff surface using the fourth-order WENO polynomial.
- We interpolate the pressure data at emission time using the Linear polynomial.
- The Kirchhoff Integral is computed using the two-point Gauss quadrature formula.

Results - Acoustic monopole

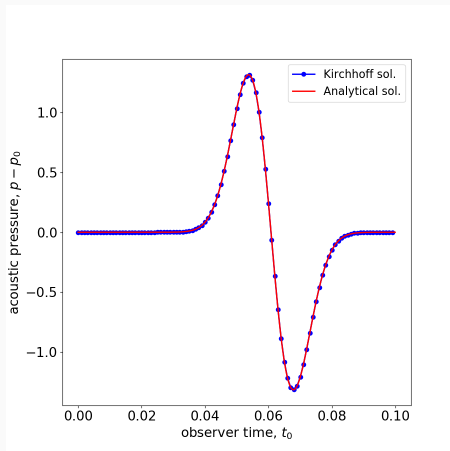


Figure 11: Acoustic pressure is computed at observer point $x_0 = (3, 3, 3)$ using the Kirchhoff method and compared with the analytical solution.

- We have written a Kirchhoff solver to compute the far-field pressure from CFD data.
- We have tested our solver with analytical solution for axisymmetric and three-dimensional wave equation.

Future work:

- To compute the far-field acoustic waves emitted from axisymmetric bubble collapse using the developed Kirchhoff solver.

-  Duttweiler M. E., Brennen C. E. Surge instability on a cavitating propeller [J]. Journal of Fluid Mechanics, 2002, 458: 133-152.

Thank you!