Computational study of far-field acoustic emission by collapsing bubbles

Magu Raam Prasaad R

August 31, 2022

Indian Institute of Science, Bangalore

Computational approach

- In this work we compute the far-field acoustic waves emitted from bubble collapse process using the Kirchhoff integral formulation.

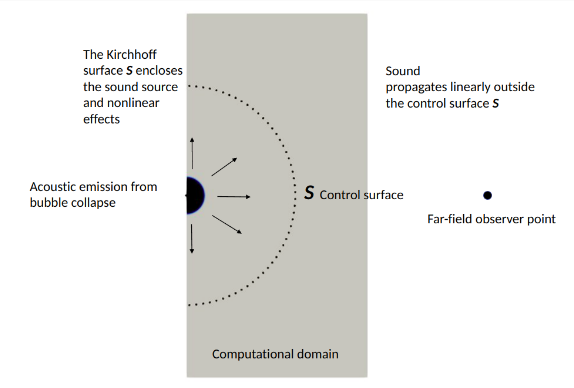

Figure 1: Schematic of far-field acoustic data computed from CFD solver using the Kirchhoff integral method.

Kirchhoff integral formulation

- The Kirchhoff integral equation for a stationary control surface is given by

$$p'(\mathbf{x},t) = \frac{1}{4\pi}\int_S \left[\frac{p'}{r'^2}\frac{\partial r'}{\partial n} - \frac{1}{r'}\frac{\partial p'}{\partial n} + \frac{1}{cr'}\frac{\partial r'}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau dS. \tag{1}$$

  p' is the acoustic pressure satisfying the wave equation outside the control surface **S**, c is the speed of sound at ambient conditions and n is the normal. The integrands are evaluated at the emission time $\tau = t - \mathbf{r}'/c$ and $\mathbf{r}' = |\mathbf{x} - \mathbf{y}|$ is the distance between observer and source.
- The p', $\partial p'/\partial t$ and $\partial p'/\partial n$ are computed from flow solver.

Interpolation of pressure data

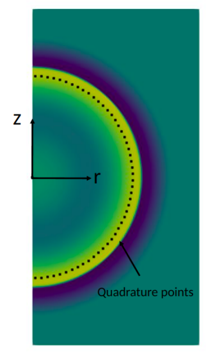

Figure 2: The fourth-order WENO polynomial is used to interpolate the data at quadrature points and the four-point Lagrange polynomial is used to interpolate the data at the emission time.

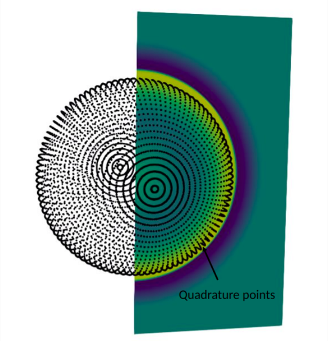

Figure 3: Using the axi-symmetry, the pressure data is copied along the azimuthal direction. We can use the Kirchhoff intergal on a spherical surface to compute the far-field pressure.

Kirchhoff integral on a sphere

- We compute the Kirchhoff integral on a spherical surface

$$p'(r,\theta,\phi,t) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left[\frac{p'}{r'^2}\frac{\partial r'}{\partial n} - \frac{1}{r'}\frac{\partial p'}{\partial n} + \frac{1}{cr'}\frac{\partial r'}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau R^2 \sin\theta d\theta d\phi \tag{2}$$

- The above integral is numerically computed using the mid-point rule

$$p'(r,\theta,\phi,t) = \frac{1}{4\pi}\sum_i^{N_\theta}\sum_j^{N_\phi}\left[\frac{p'}{r'^2}\frac{\partial r'}{\partial n} - \frac{1}{r'}\frac{\partial p'}{\partial n} + \frac{1}{cr'}\frac{\partial r'}{\partial n}\frac{\partial p'}{\partial \tau}\right]_\tau R^2 \sin\theta\Big|_{i,j}\Delta\theta\Delta\phi \tag{3}$$

Results: Computation of acoustic waves from Rayleigh collapse

- air bubble in water
- Axi-symmetric domain
  $\Omega = [-10R, 10R] \times [0, 10R]$
- Radius of the bubble $R = 38$
- Discretization 500×250 cells
- Boundary conditions:
  - top, bottom, left and right boundary – transmissive

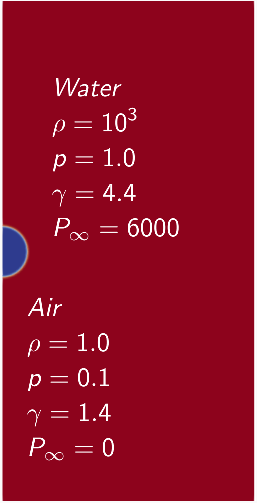

Figure 4: Initial condition

Results: Computation of acoustic waves from Rayleigh collapse

- The radius of the spherical Kirchhoff surface is $6R$
- The far-field observer point is at $(0, 9R)$
- The number of quadrature points $N_\theta = 500$ and $N_\phi = 1000$
- The speed of sound in the far-field acoustic medium is given by $\sqrt{\gamma(P + P_\infty)/\rho} = 66.5$

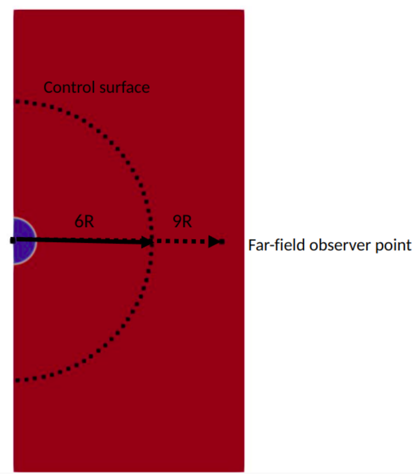

Figure 5: Location of control surface and observer point

Results: Computation of acoustic waves from Rayleigh collapse

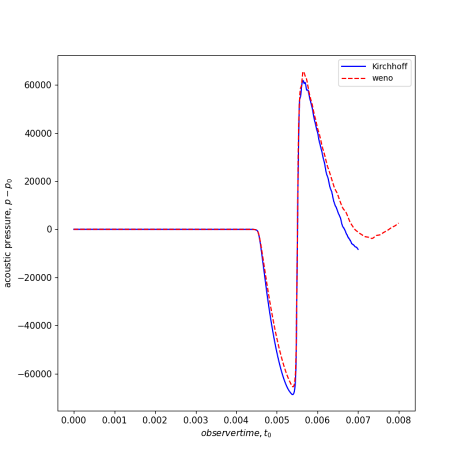

Figure 6: Comparison of acoustic pressure at observer point from flow solver and Kirchhoff solver.

observer time, t_0	relative error, $\left\lvert \frac{p'_{kirchhoff} - p'_{weno}}{p'_{weno}} \right\rvert$
5.00e-03	1.29e-01
6.00e-03	3.99e-02
7.00e-03	6.03e+00

Conclusion

- In this work we have compared the far-field acoustic pressure computed from the flow solver and the Kirchhoff solver for the collapse of a single bubble.
- At observer point $(0, 9R)$ and the observer time, $t_0 = .007$ the relative error is 6.03.
- This shows the Kirchhoff solver does not predict the far-field pressure accurately. This might be due to the boundary condition as the observer point is located near the domain boundary.

Thank you!