Spherically symmetric bubble collapse problem

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Objective

The present work aims to compute the suitable distance to place the Kirchhoff control surface from the bubble center where the linear acoustic wave equation is valid. For this, we solve the bubble collapse problem in one dimension, assuming spherical symmetry, and obtain the distance at which the ratio of acoustic pressure and ambient pressure of the fluid $p'/p_0 << 1$.

Governing equation

The Compressible Euler equations assuming spherical symmetry for the mulitphase system can be written as

$$(\rho)_t + (\rho u)_r = \frac{-2\rho u}{r}, \tag{1}$$

$$(\rho u)_t + (\rho u^2 + p)_r = \frac{-2\rho u^2}{r}, \tag{2}$$

$$(E)_t + ((E+p)u)_r = \frac{-2(E+p)u}{r}, \tag{3}$$

$$(\phi)_t + (\phi u)_r = \phi u_r, \tag{4}$$

where the equation of state for the mixture is given by

$$p = (\gamma - 1)(E - \rho u^2/2) - \gamma P^{\infty}, \tag{5}$$

$$\gamma = 1 + \frac{(\gamma_w - 1)(\gamma_a - 1)}{(1-\phi)(\gamma_w - 1) + \phi(\gamma_a - 1)}, \tag{6}$$

$$P^{\infty} = \frac{\gamma - 1}{\gamma}\Big(\phi \frac{\gamma_w P_w^{\infty}}{\gamma_w - 1} + (1-\phi)\frac{\gamma_a P_a^{\infty}}{\gamma_a - 1}\Big). \tag{7}$$

Initial condition

An air bubble of radius R is placed in a water medium, assuming the fluid to be stationary. The initial condition for the primitive variables is given by

$$\rho = \frac{(\rho_a + \rho_w)}{2} + \frac{(\rho_w - \rho_a)}{2}\tanh\left(\frac{r-R}{\epsilon h}\right), \tag{8}$$

$$u = 0, \tag{9}$$

$$p = \frac{(p_a + p_w)}{2} + \frac{(p_w - p_a)}{2}\tanh\left(\frac{r-R}{\epsilon h}\right), \tag{10}$$

$$\phi = \frac{(\phi_a + \phi_w)}{2} + \frac{(\phi_w - \phi_a)}{2}\tanh\left(\frac{r-R}{\epsilon h}\right). \tag{11}$$

$$\begin{aligned} \rho_a &= 1.0kgm^{-3} & \rho_w &= 10^3 kgm^{-3} \\ p_a &= 0.1MPa & p_w &= 1.0MPa \\ \gamma_a &= 1.4 & \gamma_w &= 4.4 \\ P_a^{\infty} &= 0.0MPa & P_w^{\infty} &= 600MPa \end{aligned}$$

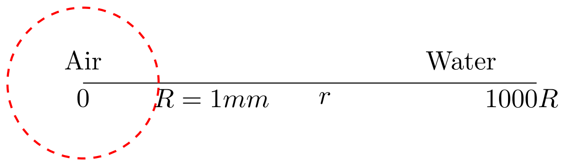

Figure 1: Schematic of air bubble in water medium with the initial conditions.

Boundary condition

We enforce homogeneous Neumann boundary condition at both ends of the domain $\frac{\partial(*)}{\partial r}\Big|_0 = \frac{\partial(*)}{\partial r}\Big|_L = 0$.

Scaling

Let $\bar{p}$ be the pressure scale, $\bar{\rho}$ be the density scale and $\bar{x}$ be the length scale. Then

$$p = \bar{p} p_s, \quad \rho = \bar{\rho}\rho_s, \quad x = \bar{l} x_s. \tag{12}$$

Where p_s, ρ_s and x_s are the scaled pressure, density and length. We can obtain the velocity and time scale from the above scales.

$$\frac{M}{LT^2} = \bar{p}\frac{M_s}{L_s T_s^2}, \quad \frac{M}{L^3} = \bar{\rho}\frac{M_s}{L_s^3}. \tag{13}$$

From the above relations we can obtain,

$$\frac{L}{T} = \sqrt{\frac{\bar{p}}{\bar{\rho}}}\frac{L_s}{T_s} \tag{14}$$

Therefore the velocity and time scale is given by

$$v = \sqrt{\frac{\bar{p}}{\bar{\rho}}} v_s, \quad t = \bar{l}\sqrt{\frac{\bar{\rho}}{\bar{p}}} t_s. \tag{15}$$

For the bubble problem we use the following scales,

$$\bar{p} = 10^6 Pa, \quad \bar{\rho} = 1000 kgm^{-3}, \quad \bar{l} = 10^{-3} m. \tag{16}$$

The scaled initial conditions are

$$\begin{aligned} \rho_a^s &= 0.001 & \rho_w^s &= 1 \\ p_a^s &= 0.1 & p_w^s &= 1.0 \\ \gamma_a &= 1.4 & \gamma_w &= 4.4 \\ P_a^\infty &= 0.0 & P_w^\infty &= 600 \end{aligned}$$

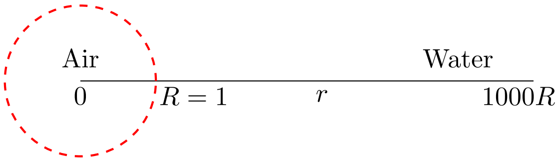

Figure 2: Schematic of air bubble in water medium with the initial conditions.