Wave equation solved using WENO4 and SSPRK54 on a periodic domain

7 Jan 2021

1 Plane wave solution of wave equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0 \qquad \mathbf{x} \in \Omega \tag{1}$$

The plane wave solution of the wave equation can be written as,

$$u(\mathbf{x}, t) = sin(k_x x + k_y y - \omega t) \tag{2}$$

where, $c^2 = \frac{\omega^2}{k_x^2 + k_y^2}$.

2 Grid generation

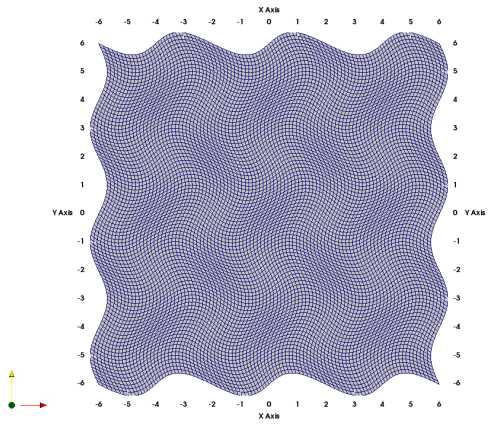

Figure 1: unstructured grid

We use the following deformation function to transform the structured grid to unstructured grid.

$$x = X + 0.3sin\Big(\frac{\pi Y}{2}\Big) \tag{3}$$

$$y = Y + 0.4sin\Big(\frac{\pi X}{2}\Big) \tag{4}$$

$(X, Y) \in [-2, 2] \times [-2, 2]$ are the coordinates of structured grid vertices and (x, y) are the coordinates of deformed grid vertices.

3 Numerical parameters

domain	$[-2, 2] \times [-2, 2]$
cfl	0.5
k_x	$2\pi/4$
k_y	$2\pi/4$
ω	2π
c	2.828427

We have used the fourth-order central WENO reconstruction and SSPRK54 for temporal discretization.

3.1 L_2 and L_∞ error

The L_2 and L_∞ error and the rate of convergence are given at the end of one time period.

N	L_∞error	L_2 error	L_2 convergence rate	L_∞convergence rate
16	1.134483e-02	4.413436e-03	-	-
32	1.125525e-03	4.148342e-04	3.411295	3.333364
64	7.935688e-05	2.926878e-05	3.825100	3.826098
128	5.105060e-06	1.888073e-06	3.954376	3.958355
256	3.214497e-07	1.189517e-07	3.988466	3.989263

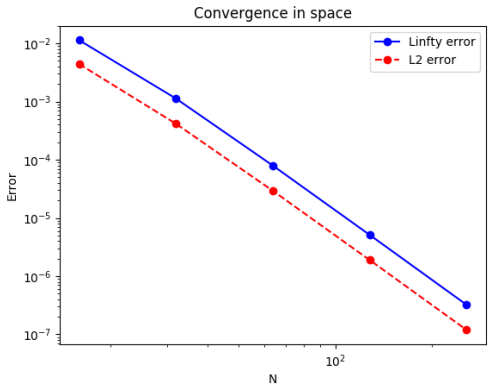

Figure 2: L_2 and L_∞ error plot at the end of one time period.

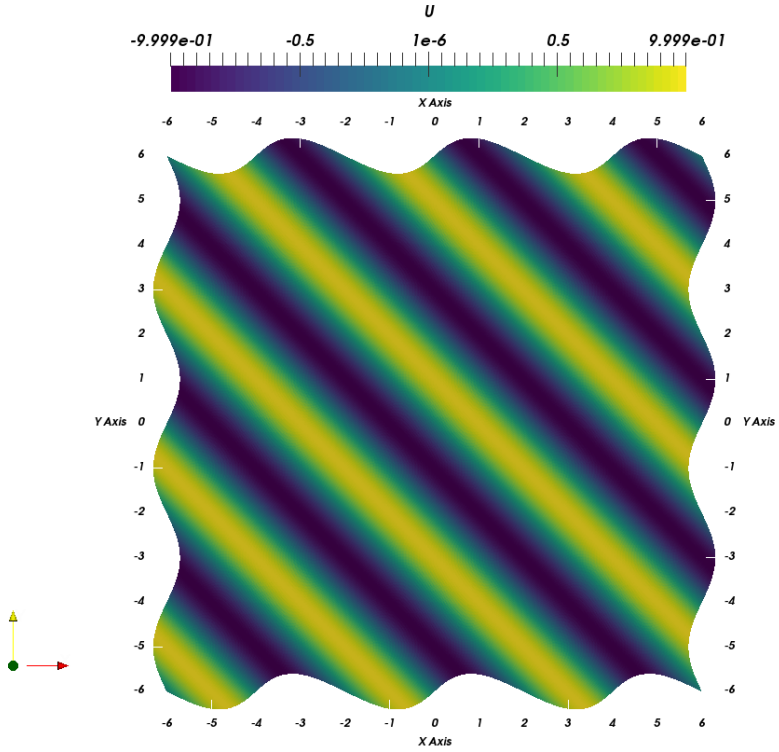

Figure 3: plane wave solution at the end of one time period