Physics for Scientists and Engineers,
10th edition, Raymond A. Serway and John W. Jewett, Jr.

Chapter 3: Vectors

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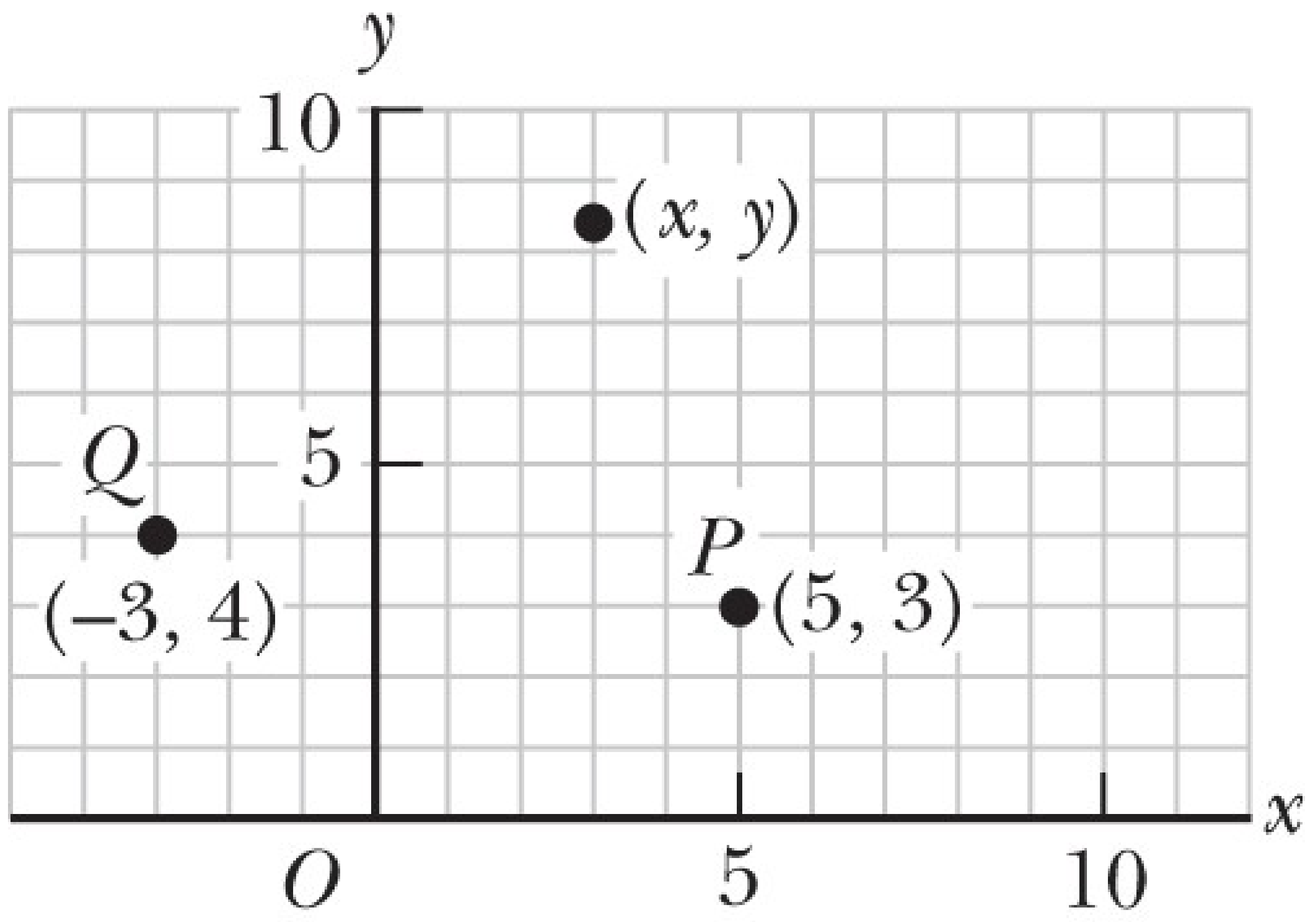
CENGAGE Learning

Customized by **Dr. H. Merabet, 2020**

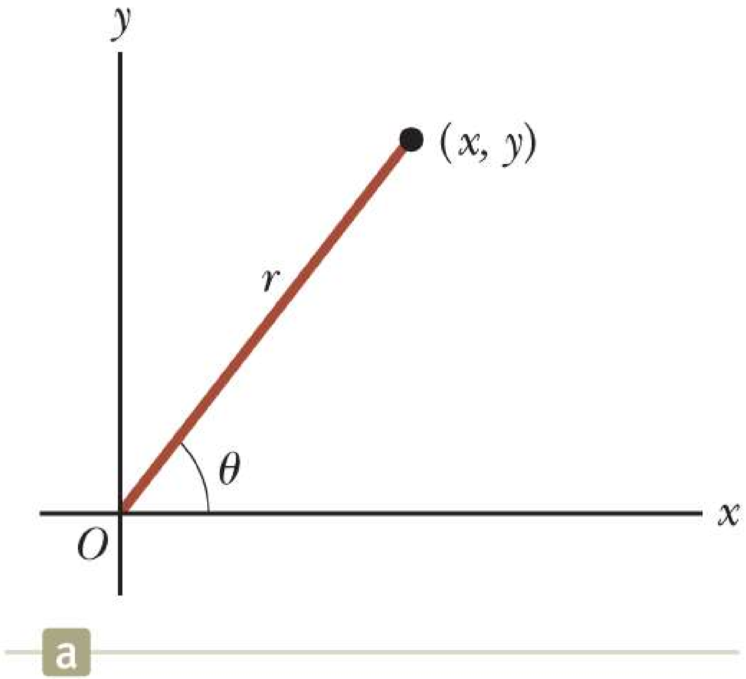
Cartesian Coordinate System

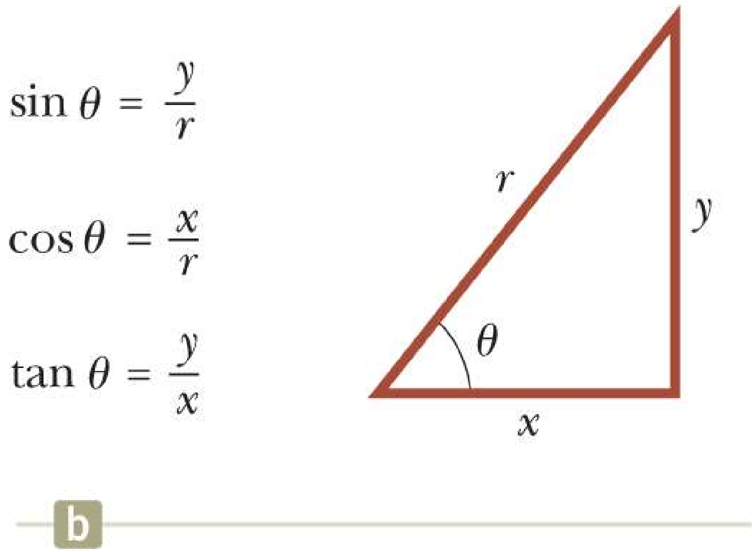
- Often need description of location in space
- Two dimensions: can use **Cartesian coordinate** system:
  - Perpendicular axes intersect at point defined as origin *O* (figure)
- Cartesian coordinates of point in space: ***rectangular coordinates***



Polar Coordinate System

- Also represent point in plane by its *plane polar coordinates* (r, θ) (figure (a))
- In *polar coordinate system:*
  - r = distance from origin to point having Cartesian coordinates (x, y)
  - θ = angle between fixed axis (usually $+x$ axis) and line drawn from origin to point
    - θ usually measured counterclockwise from $+x$ axis





Polar Coordinate System

- Starting with plane polar coordinates of a point, we can obtain Cartesian coordinates using:

$$x = r\cos\theta$$

$$y = r\sin\theta$$

- Starting with Cartesian coordinates, we can obtain polar coordinates using:

$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

  - r equation: Pythagorean theorem

- Four equations relating (x, y) to (r, θ) apply only when θ defined as shown in the previous slide.

Example 3.1: Polar Coordinates

The Cartesian coordinates of a point in the *xy* plane are $(x, y) = (-3.50, -2.50)$ m as shown in the figure. Find the polar coordinates of this point.

Conceptualize The drawing helps us conceptualize the problem. We wish to find r and θ. Based on the figure and the data given in the problem statement, we expect r to be a few meters and θ to be between 180° and 270°. We have:

- ***x*** and ***y*** are **positive**, the angle θ is between **0°** and **90°**
- ***x*** is **negative** and ***y*** is **positive**, the angle θ is between **90°** and **180°**
- ***x*** and *y* are **negative**, the angle θ is between **180°** and **270°**
- ***x*** is **positive** and *y* are **negative**, the angle θ is between **270°** and **0°**

Categorize Based on the statement of the problem, we recognize that we are simply converting from Cartesian coordinates to polar coordinates. We therefore categorize this example as a substitution problem.

Substitution problems generally do not have an extensive Analyze step other than the substitution of numbers into a given equation. Similarly, the Finalize step consists primarily of checking the units and making sure that the answer is reasonable and consistent with our expectations. Therefore, for substitution problems, we will not label Analyze or Finalize steps.

Example 3.1:
Polar Coordinates

The Cartesian coordinates of a point in the *xy* plane are $(x, y) = (-3.50, -2.50)$ m as shown in the figure. Find the polar coordinates of this point.

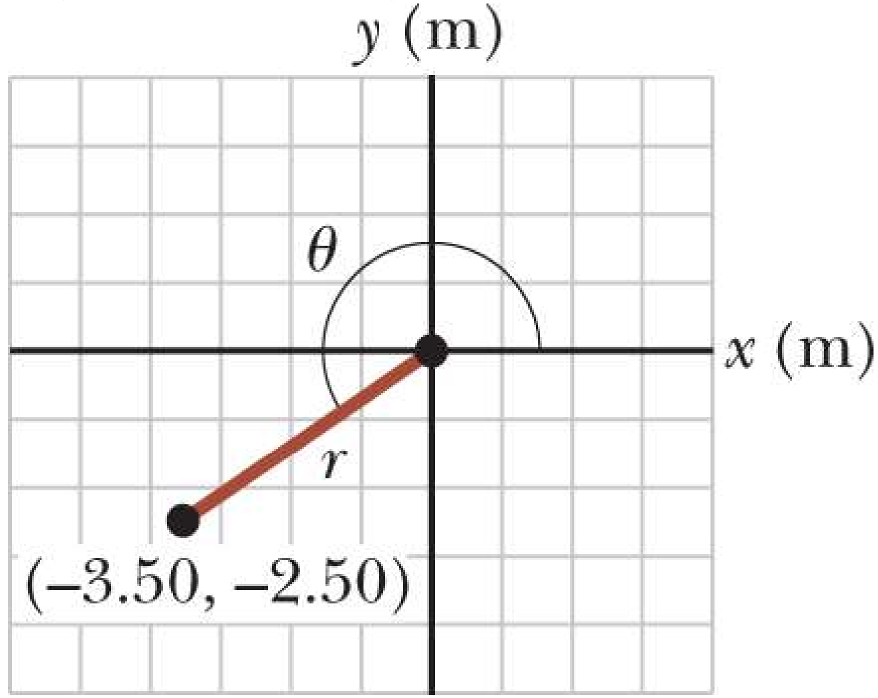
Use the Pythagorean theorem to find *r*:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = \boxed{4.30\text{ m}}$$

$$\tan\theta = \frac{y}{x}$$

$$= \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = 216°$$



3.2. Vector and Scalar Quantities

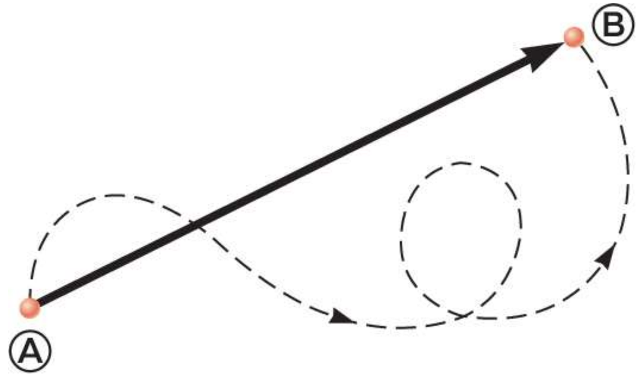
A **scalar quantity** is completely specified by a single value with an appropriate unit and has no direction.

A **vector quantity** is completely specified by a number with an appropriate unit (the *magnitude* of the vector) plus a direction.

- When you want to know the temperature outside, only need a number and unit “degrees C” or “degrees F”
    - Temperature: *scalar quantity*
        - Other examples: volume, mass, speed, time, and time intervals
    - Some scalars always positive (mass and speed)
    - Some scalars (temperature): positive or negative values
- Rules of ordinary arithmetic used to manipulate scalar quantities

- Pilot of small plane needs to know wind velocity (both speed of wind and direction)
    - Velocity: *vector quantity*

Displacement Vector



- In figure, a particle moves from A to B along a straight path
  - Represent displacement by drawing arrow from A to B
    - Tip of arrow pointing away from starting point
  - **Direction of arrowhead** = direction of displacement
  - **Length of arrow** = magnitude of displacement

- If particle travels along different path from A to B (broken line):
  - Displacement still an arrow drawn from A to B
- Displacement depends only on initial and final positions
  - Displacement vector independent of path taken between points

Quick Quiz 3.1

Which of the following are vector quantities and which are scalar quantities?

(a) your age

(b) acceleration

(c) velocity

(d) speed

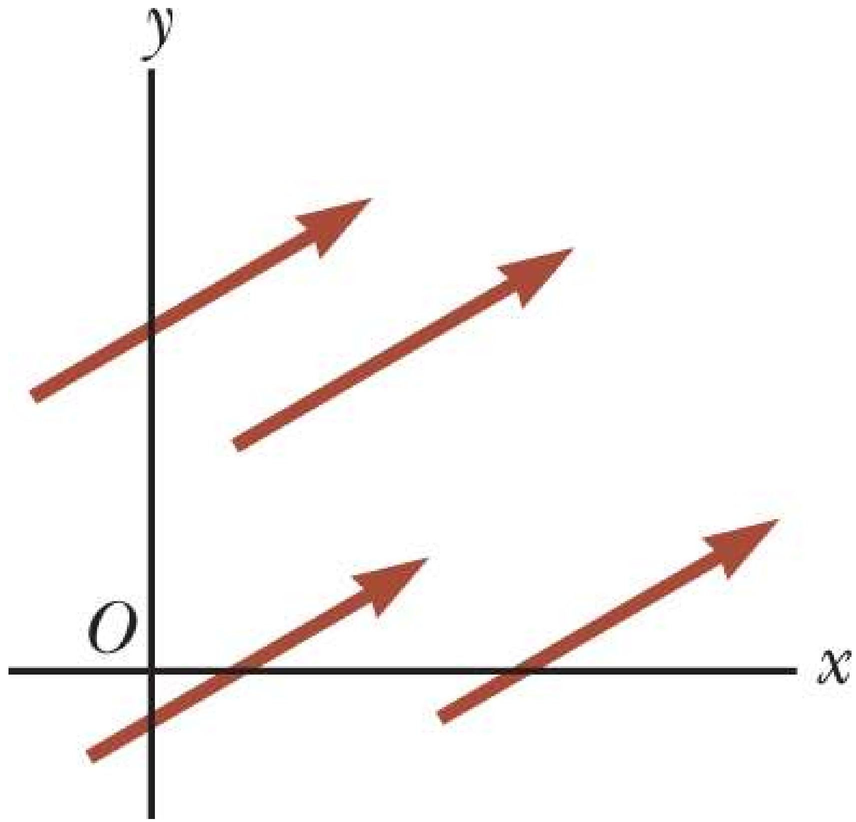
(e) mass

3.3. Basic Vector Arithmetic

$$\vec{\mathrm{A}} = \vec{\mathrm{B}} \text{ only if } A = B$$

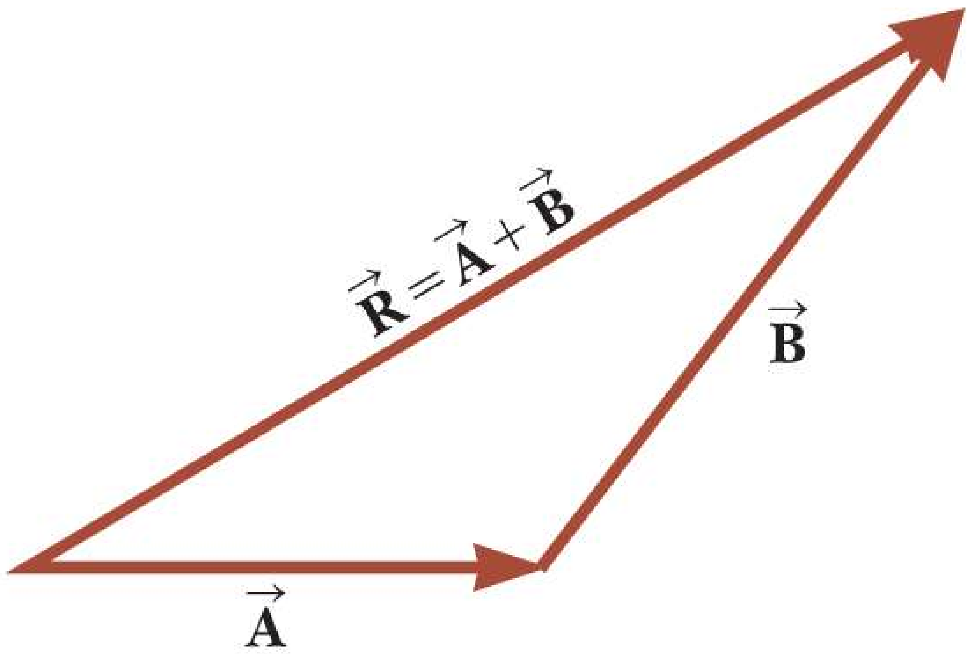
Both point in the same direction along parallel lines

- Figure: all vectors equal even though have different starting points
  - We can move vector to position parallel to itself in diagram without affecting the vector

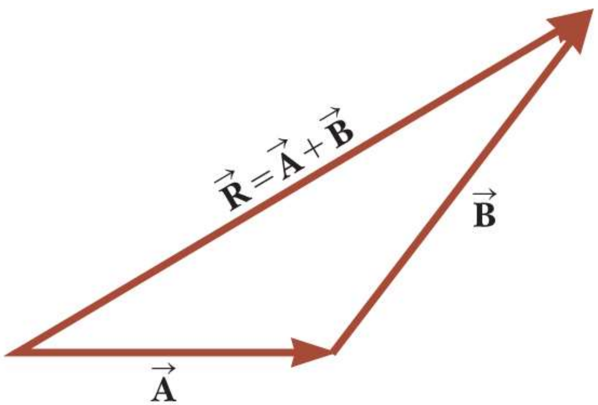


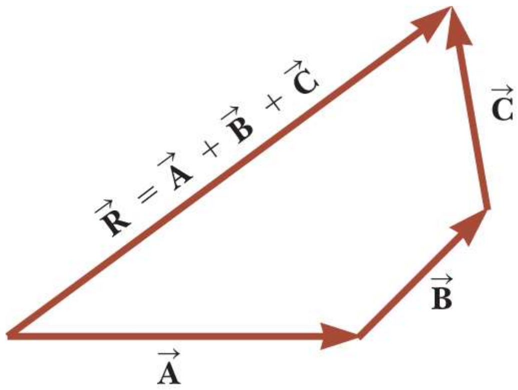
Vector Addition

- **Vector addition** → graphical method
- Add vector **B** to vector **A** :
  - First draw vector **A** on graph paper → magnitude represented by convenient length scale
  - Then draw vector **B** to same scale → tail starting from tip of **A** (figure below)
  - **Resultant vector R** = vector drawn from tail of **A** to tip of **B**



Vector Addition





- Technique called "*head to tail method*"

- When two vectors added: sum independent of order of addition → **commutative law of addition**:

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}} \;\; (\textbf{commutative law of addition})$$

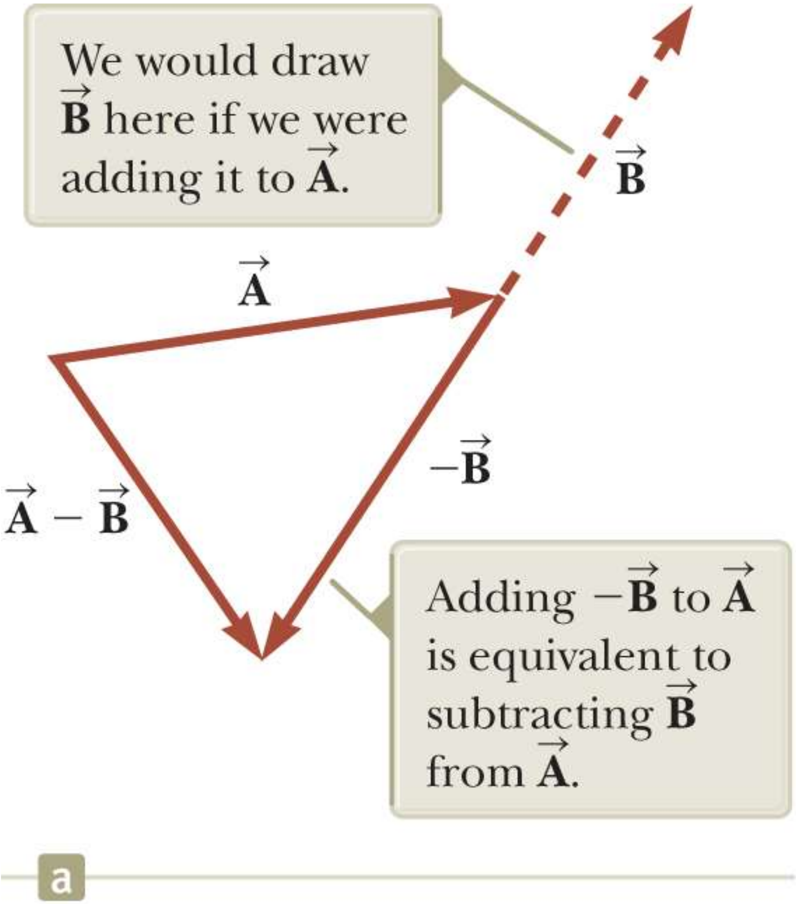
- When three or more vectors added: sum independent of way in which individual vectors grouped together → **associative law of addition**:

$$\vec{\mathbf{A}} + \left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) + \vec{\mathbf{C}} \;\; (\textbf{associative law of addition})$$

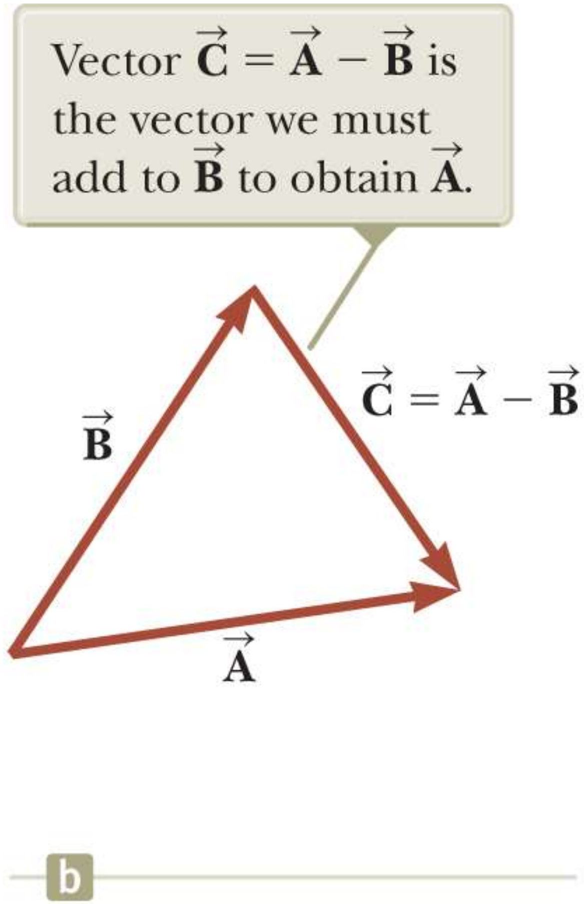
Vector Subtraction

$$\vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$$

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$$



Negative of vector **A** defined as vector that when added to **A** gives zero for vector sum



Define **A – B** as vector **–B** added to vector **A**

Scalar Multiplication

$$\text{scalar multiplication: } m\vec{\mathbf{A}}$$

- **Scalar multiplication** of vectors: If vector **A** multiplied by positive scalar quantity m: product $m\mathbf{A}$ = vector with same direction as **A** and magnitude mA
- If vector **A** multiplied by negative scalar quantity $-m$: product $-m\mathbf{A}$ directed opposite **A**
  - Examples:
    - Vector $5\mathbf{A}$: five times as long as **A** and points in same direction as **A**
    - Vector $-1/3\ \mathbf{A}$: one-third length of **A** and points in direction opposite **A**

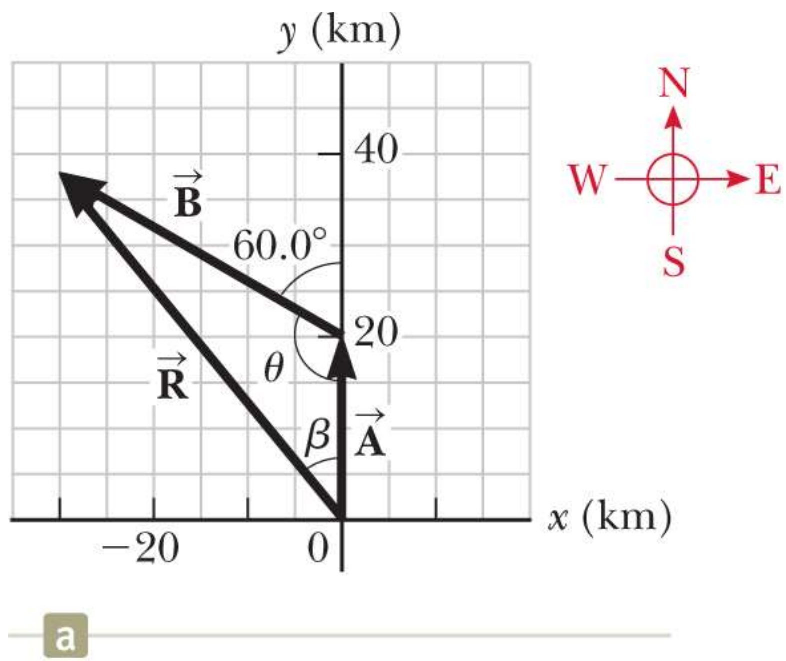
Quick Quiz 3.2

The magnitudes of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are $A = 12$ units and $B = 8$ units. Which pair of numbers represents the largest and smallest possible values for the magnitude of the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$?

(a) 14.4 units, 4 units
(b) 12 units, 8 units
(c) 20 units, 4 units
(d) none of these answers

Example 3.2:
A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in the figure. Find the magnitude and direction of the car's resultant displacement.



Conceptualize The vectors **A** and **B** that appear in the figure help us conceptualize the problem. The resultant vector **R** has also been drawn. We expect its magnitude to be a few tens of kilometers. The angle β that the resultant vector makes with the *y* axis is expected to be less than 60°, the angle that vector **B** makes with the *y* axis.

Categorize We can categorize this example as a simple analysis problem in vector addition. The displacement **R** is the resultant when the two individual displacements **A** and **B** are added. We can further categorize it as a problem about the analysis of triangles, so we appeal to our expertise in geometry and trigonometry.

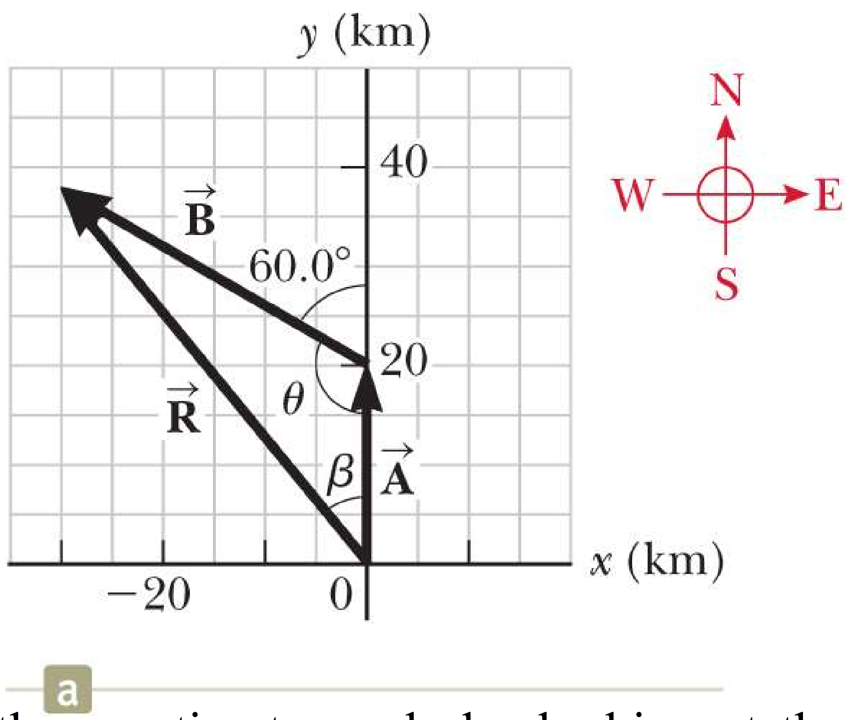
Example 3.2:
A Vacation Trip

Analyze In this example, we show two ways to analyze the problem of finding the resultant of two vectors. The first way is to solve the problem geometrically, using graph paper and a protractor to measure the magnitude of **R** and its direction. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) With an ordinary ruler and protractor, a large diagram typically gives answers to two-digit but not to three-digit precision.

The second way to solve the problem using algebra and trigonometry. The magnitude of **R** can be obtained from the law of cosines as applied to the triangle in the figure.

Use $R^2 = A^2 + B^2 - 2AB\cos\theta$ from the law of cosines to find R.

Use the law of sines to find the direction of **R** measured from the northerly direction.



Finalize Does the angle β that we calculated agree with an estimate made by looking at the figure or with an actual angle measured from the diagram using the graphical method? Is it reasonable that the magnitude of **R** is larger than that of both **A** and **B**? Are the units of **R** correct?

Example 3.2:
A Vacation Trip

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$R = \sqrt{(20.2\text{ km})^2 + (35.0\text{ km})^2 - 2(20.2\text{ km})(35.0\text{ km})\cos 120°}$$

$$= \boxed{48.2\text{ km}}$$

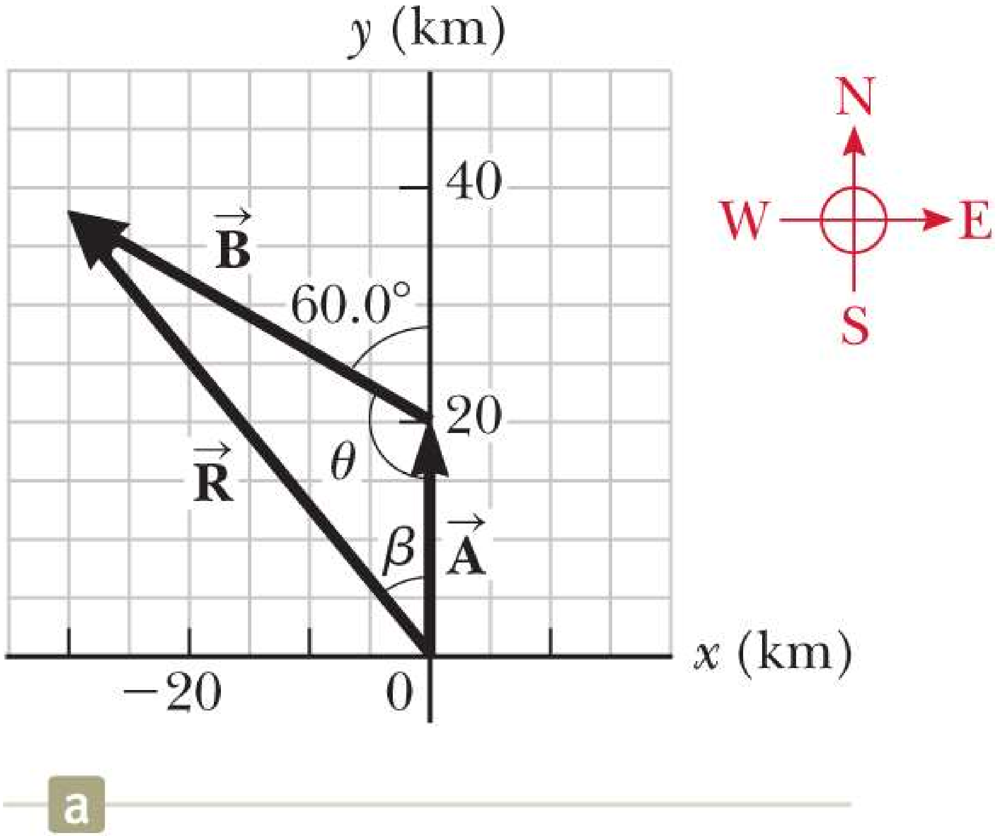
$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta$$

$$= \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = \boxed{38.9°}$$ West of North

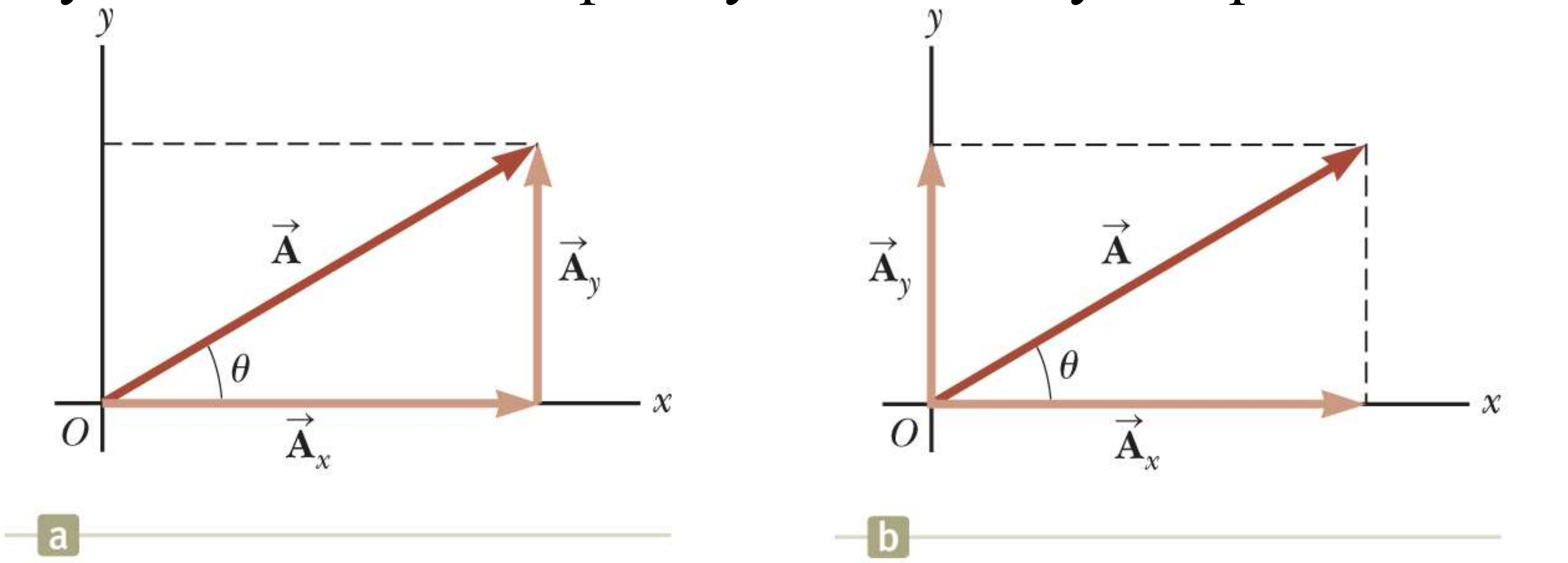
Use the law of sines to find the direction of **R** measured from the northerly direction



$\theta = 180° - 60° = 120°$

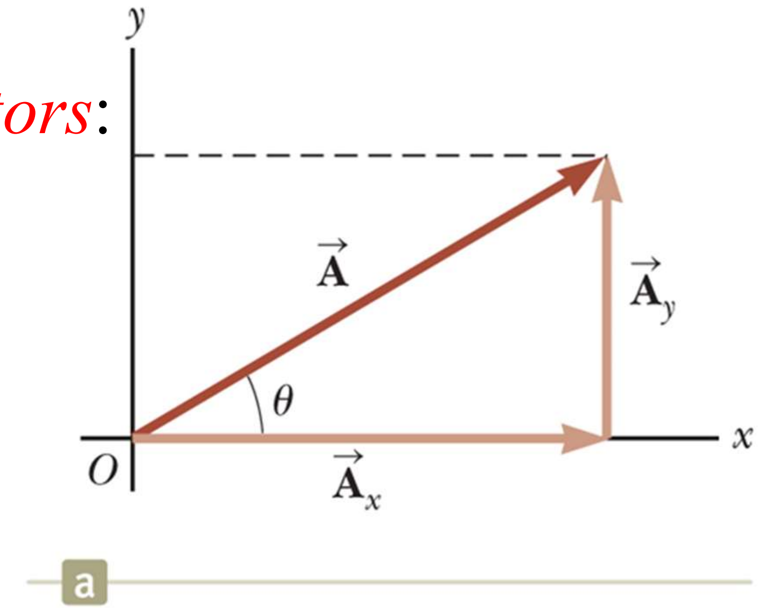
3.4. Components of a Vector

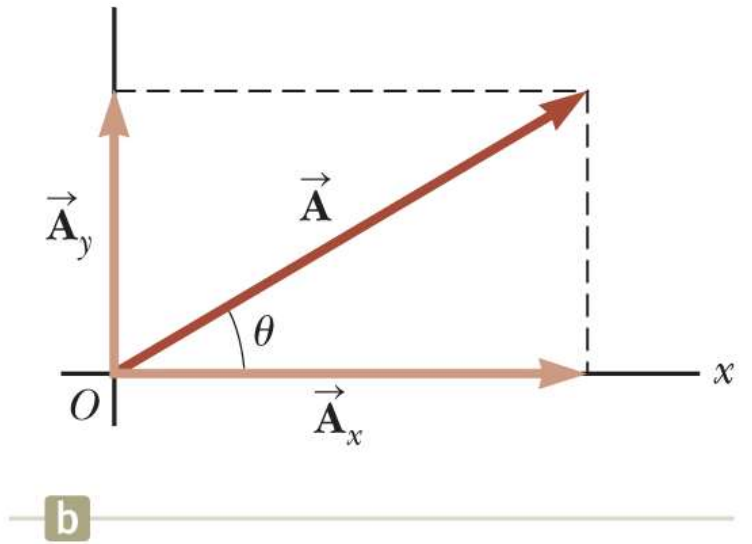
- Graphical method of adding vectors:
  - Not highly accurate and Not recommended for three-dimensional problems. At times very complicated!
- New way of adding vectors → uses projections of vectors along coordinate axes
  - These projections: **components** of the vector or **rectangular components**
  - Any vector can be completely described by components



3.4. Components of a Vector

- Consider vector **A** lying in the *xy* plane and making arbitrary angle θ with positive *x* axis (figure (a))
- Express vector as sum of two *component vectors*:
  - $\mathbf{A}_x$: parallel to *x* axis
  - $\mathbf{A}_y$: parallel to *y* axis
- Right figure:
  - Three vectors form right triangle
  - $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$
- Components of vector **A**: A_x and A_y
  - Note: figure (b) component vector $\mathbf{A}_y$ moved to left so it lies along *y* axis
  - Component A_x = projection of **A** along *x* axis
  - Component A_y = projection of **A** along *y* axis





3.4. Components of a Vector

- Components positive or negative
  - A_x positive if component vector $\mathbf{A}_x$ points in positive x direction
  - A_x negative if $\mathbf{A}_x$ points in negative x direction
  - Similarly for A_y
- From figures: $\cos\theta = A_x/A$ *and* $\sin\theta = A_y/A$
- *C*omponents of **A** are:

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

- Magnitudes of components = lengths of two sides of right triangle with hypotenuse of length A
- Magnitude and direction of **A** related to its components through:

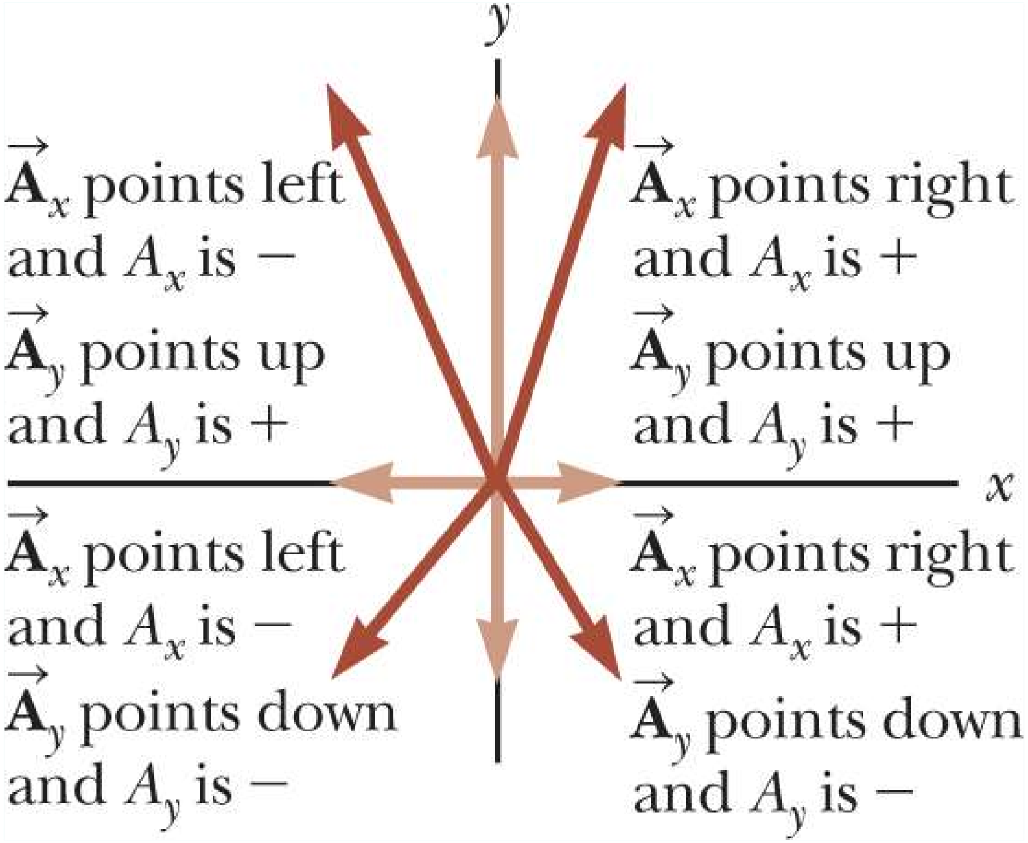
$$A = \sqrt{{A_x}^2 + {A_y}^2} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

- Note: signs of components A_x and A_y depend on angle θ:
  - If $\theta = 120°$ $\rightarrow$ A_x negative and A_y positive
  - If $\theta = -125°$ $\rightarrow$ both A_x and A_y negative

3.4. Components of a Vector

- Figure summarizes direction of component vectors and signs of components when **A** lies in various quadrants

- Specify vector **A** either with components A_x and A_y or with magnitude and direction A and θ



- Sometimes convenient to express components in coordinate system having axes not horizontal and vertical but still perpendicular to each other
  - Example: motion of objects sliding down inclined planes →
    - Orient x axis parallel to plane and y axis perpendicular to plane

Quick Quiz 3.4

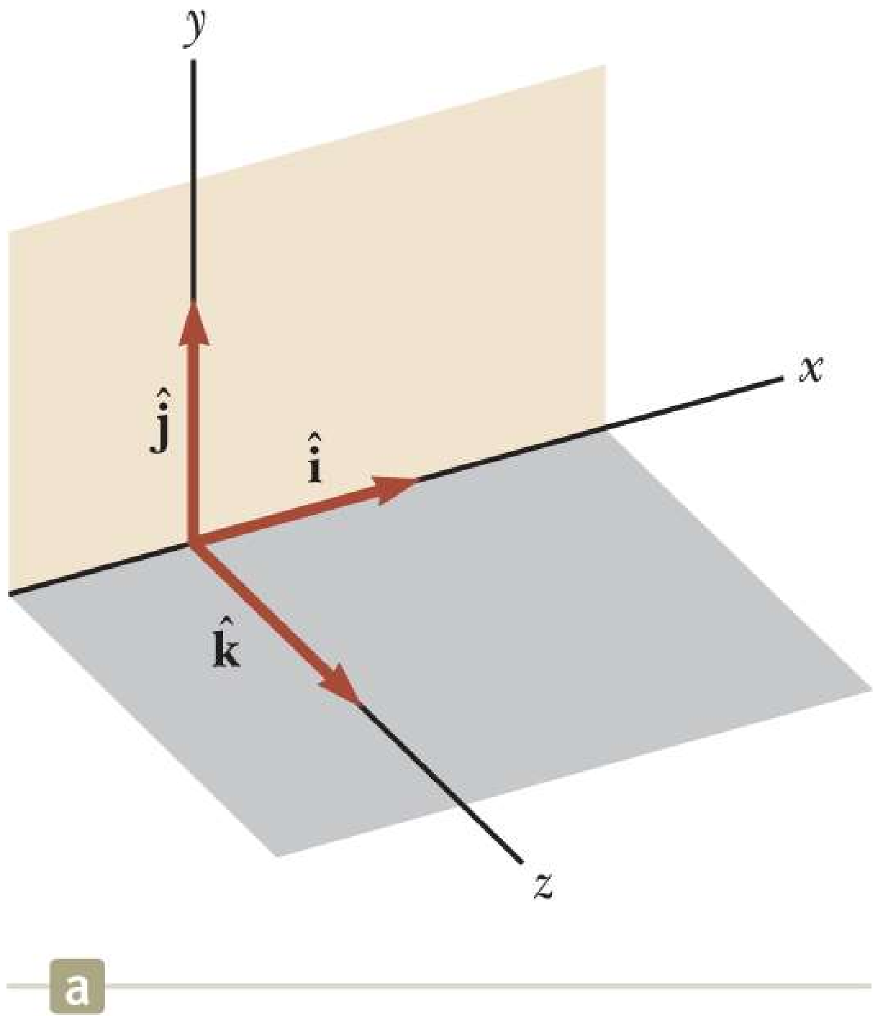
Choose the correct response to make the sentence true:
A component of a vector is
(a) always
(b) never
(c) sometimes
larger than the magnitude of the vector.

3.4. Unit Vectors

- Vector quantities often expressed in terms of unit vectors
  - **Unit vector:** dimensionless vector having magnitude of exactly 1

- Unit vectors used to specify given direction and have no other physical significance
  - Used in describing direction in space

- Symbols **i**, **j**, and **k** represent unit vectors pointing in positive *x*, *y*, and *z* directions, respectively
  - The "hats," or circumflexes, on symbols standard notation for unit vectors
- Unit vectors **i**, **j**, and **k** form set of mutually perpendicular vectors in right-handed coordinate system (figure)



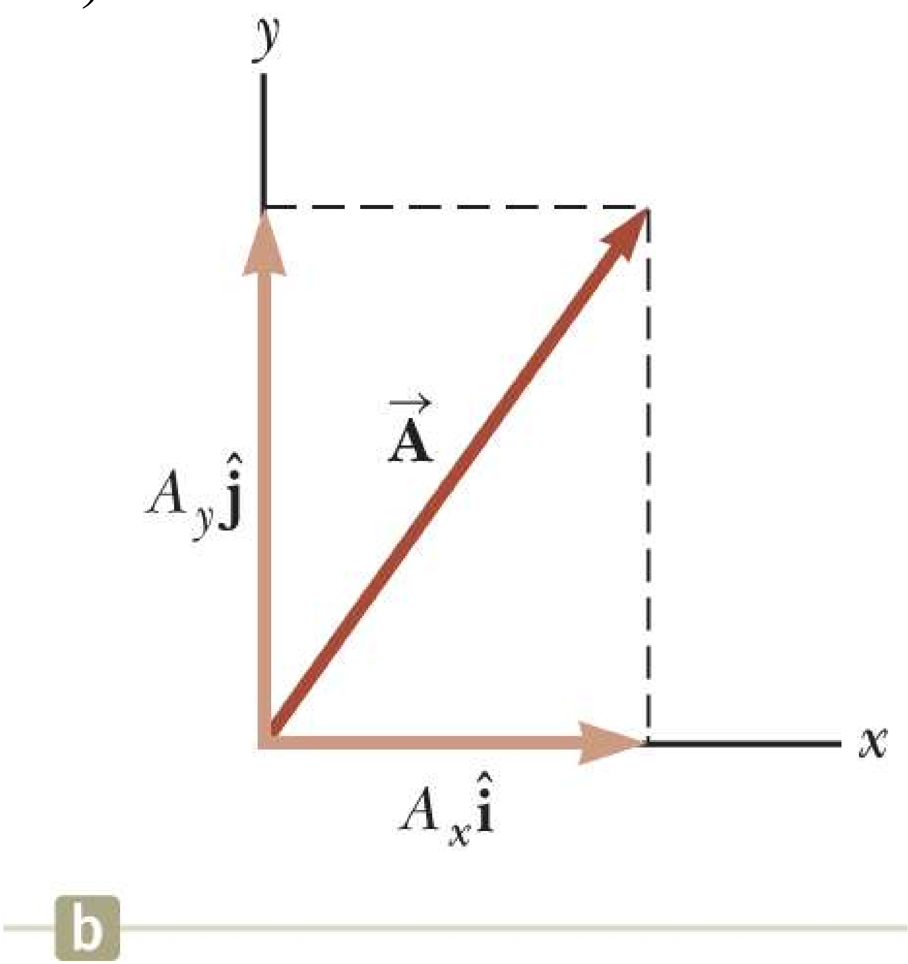
3.4. Components of a Vector and Unit Vectors

- Consider vector **A** lying in *xy* plane (figure)

- Product of component A_x and unit vector **i** = component vector $\mathbf{A}_x = A_x\,\mathbf{i}$:
    - Lies on *x* axis
    - Magnitude $|A_x|$
    - Likewise, $\mathbf{A}_y = A_y\,\mathbf{j}$ = component vector of magnitude $|A_y|$ lying on *y* axis

- Unit-vector notation for vector **A**:

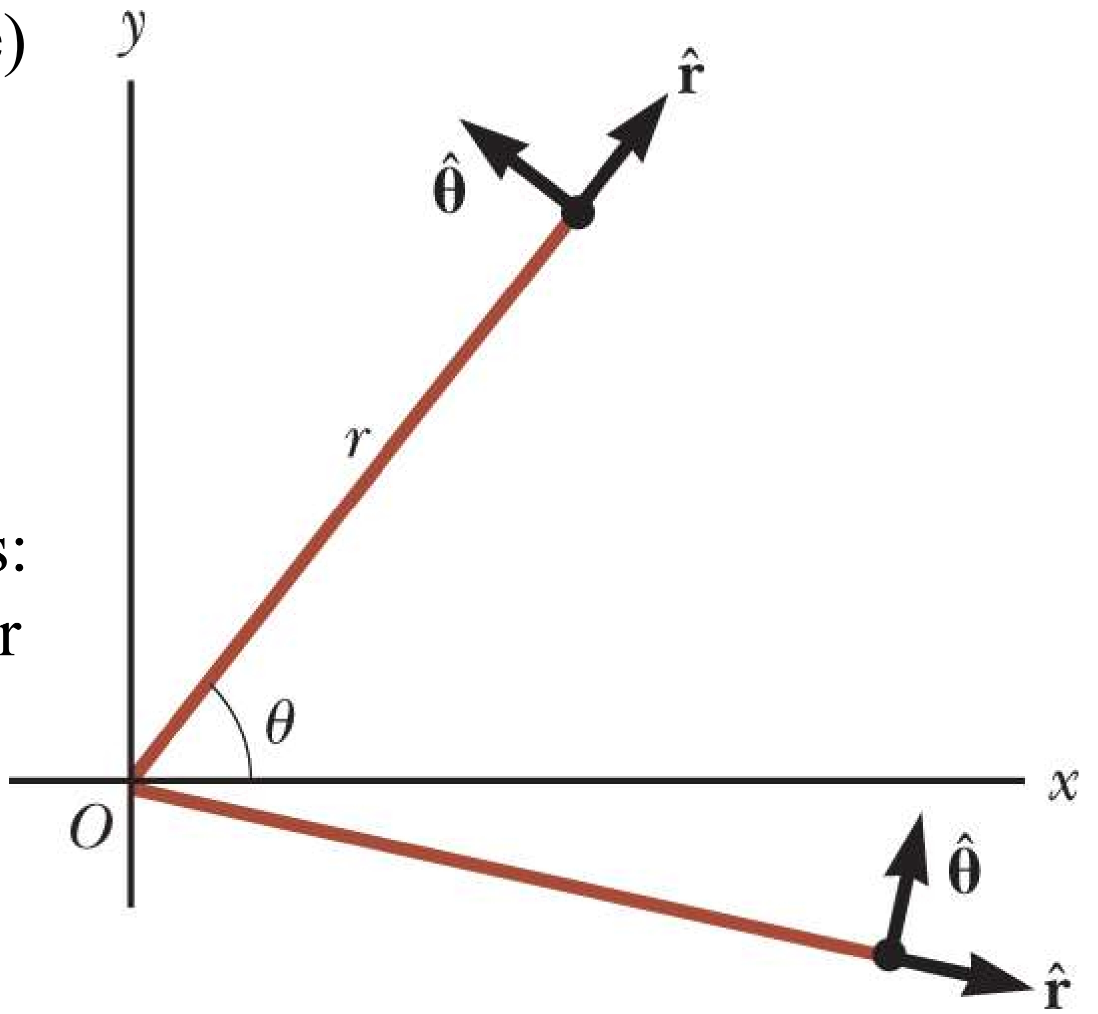
$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$



3.4. Components of a Vector and Unit Vectors

- Consider polar coordinates (figure)

- Note: we can identify radial and angular unit vectors **r** and θ
  - Like rectangular coordinates: these vectors of unit length
  - Unlike rectangular coordinates: directions of radial and angular unit vectors depend on point



Vector Addition using Components

- Adding vectors using components:
  - Add vector **B** to vector **A** →
    - Add x and y components separately

$$\vec{\mathrm{R}} = R_x\hat{\imath} + R_y\hat{\jmath} \Rightarrow R_x = A_x + B_x \quad and \quad R_y = A_y + B_y$$

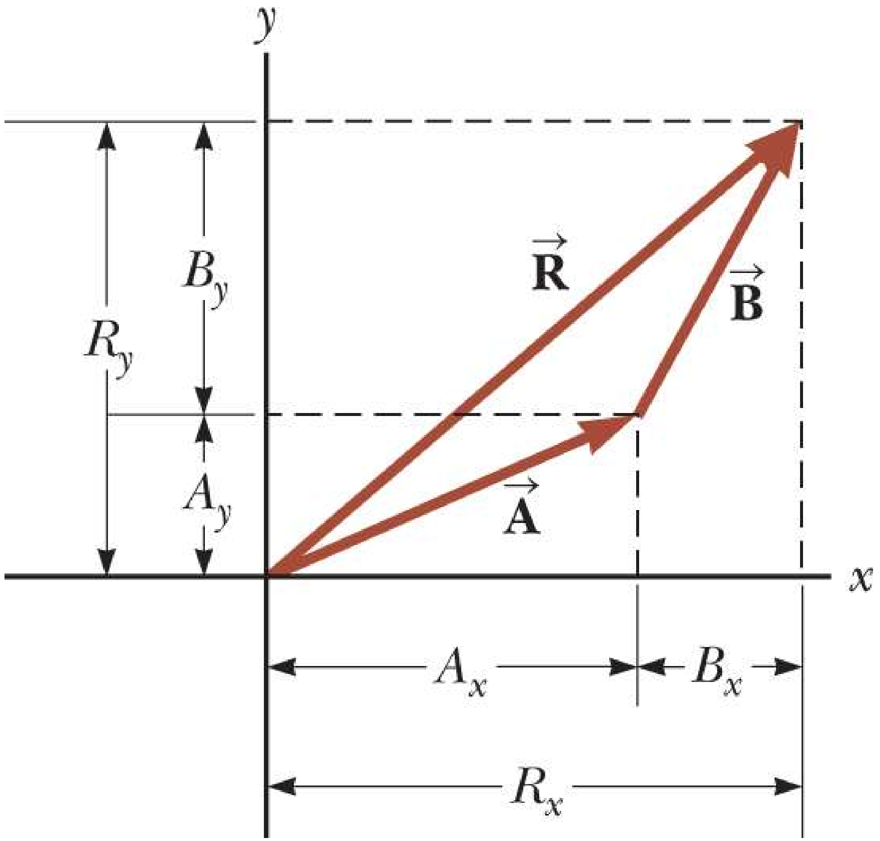
- Rearrange terms: $\vec{\mathrm{R}} = (A_x + B_x)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath}$

- Components of resultant vector:

$$\vec{\mathrm{R}} = \vec{\mathrm{A}} + \vec{\mathrm{B}} = \left(A_x\hat{\imath} + A_y\hat{\jmath}\right) + \left(B_x\hat{\imath} + B_y\hat{\jmath}\right)$$

Vector Addition using Components

- Component method of adding vectors:
  - Add all x components together to find x component of resultant vector
  - Use same process for y components
- Check addition by components with geometric construction (figure)



$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{(A_x + B_x)^2 + \left(A_y + B_y\right)^2}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x}$$

Vectors in Three Dimensions

- Extension to three-dimensional vectors:
- If **A** and **B** both have *x*, *y*, and *z* components → can be expressed in form shown

$$\vec{\mathrm{A}} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{\mathrm{k}}$$
$$\vec{\mathrm{B}} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{\mathrm{k}}$$

- Sum of **A** and **B** is

$$\vec{\mathrm{R}} = (A_x + B_x)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath} + (A_z + B_z)\hat{\mathrm{k}}$$

- If vector **R** has *x*, *y*, and *z* components → magnitude of vector is

$$R_z = A_z + B_z$$
$$R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2}$$

- Angle θ_x that **R** makes with *x* axis found from

$$\cos\theta_x = \frac{R_x}{R}$$

Similar expressions for angles with respect to *y* and *z* axes

$$\cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R}$$

Quick Quiz 3.5

For which of the following vectors is the magnitude of the vector equal to one of the components of the vector?

$\left(\mathrm{a}\right)\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$

$\left(\mathrm{b}\right)\vec{\mathbf{B}} = -3\hat{\mathbf{j}}$

$\left(\mathrm{c}\right)\vec{\mathbf{C}} = 5\hat{\mathbf{k}}$

Example 3.3:
The Sum of Two Vectors

Find the sum of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in the *xy* plane and given by

$$\vec{\mathbf{A}} = \left(2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}\right) \text{ and } \vec{\mathbf{B}} = \left(2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$A_x = 2.0,\ A_y = 2.0,\ A_z = 0$$

$$B_x = 2.0,\ B_y = -4.0,\ B_z = 0$$

Example 3.3:
The Sum of Two Vectors

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}}$$
$$= (2.0 + 2.0)\hat{\mathbf{i}} + (2.0 - 4.0)\hat{\mathbf{j}}$$
$$= 4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}}$$

$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2}} = \sqrt{(4.0)^2 + (2.0)^2}$$
$$= \sqrt{20} = \boxed{4.5}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0}{4.0} = -0.50 \rightarrow \theta = 333^\circ$$

Your calculator likely gives the answer 227° for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° *clockwise* from the x axis. But our standard form has been to quote the angles measured *counterclockwise* from the $+x$ axis, and that angle for this vector is $\theta = 333°$.

Example 3.4: The Resultant Displacement

A particle undergoes three consecutive displacements: $\Delta\mathbf{r}_1 = \left(15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}}\right)\text{cm}$, $\Delta\mathbf{r}_2 = \left(23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}}\right)\text{cm}$, and $\Delta\mathbf{r}_3 = \left(-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}}\right)\text{cm}$. Find unit-vector notation for the resultant displacement and magnitude.

Example 3.4: The Resultant Displacement

To find the resultant displacement, add the three vectors:

$$\Delta\vec{\mathbf{r}} = \Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3$$

$$= (15 + 23 - 13)\hat{\mathbf{i}}\text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}}\text{ cm}$$

$$+ (12 - 5.0 + 0)\hat{\mathbf{k}}\text{ cm}$$

$$= \left(25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}}\right)\text{cm}$$

Find the magnitude of the resultant vector using the following equation:

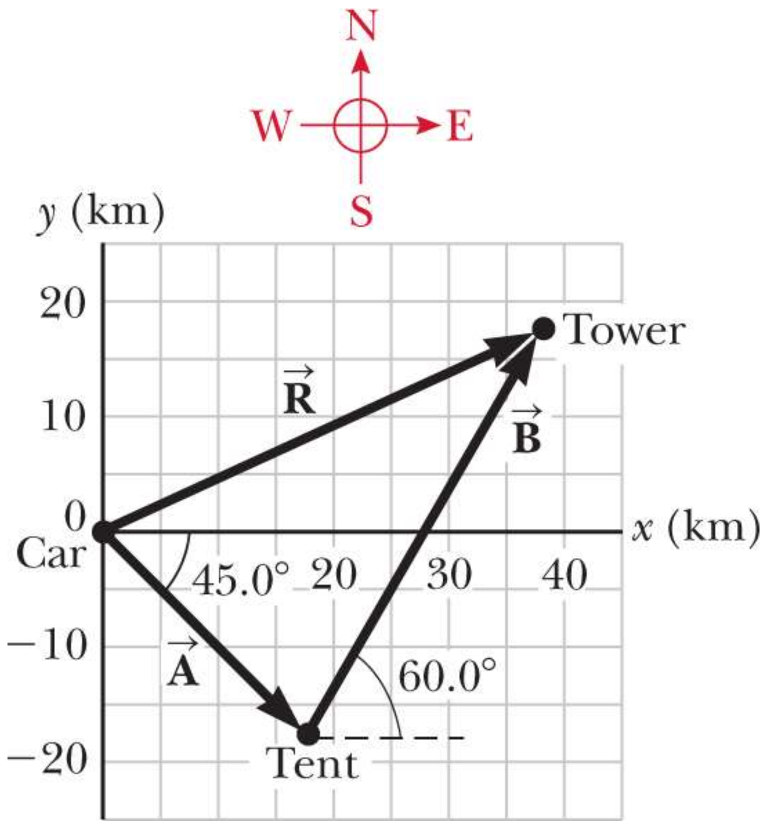
$$R = \sqrt{R_x{}^2 + R_y{}^2 + R_z{}^2}$$

$$= \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = \boxed{40\text{ cm}}$$

Example 3.5: Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night.
On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.



Conceptualize We conceptualize the problem by drawing a sketch. If we denote the displacement vectors on the first and second days by **A** and **B**, respectively, and use the car as the origin of coordinates, we obtain the vectors shown in the figure. The sketch allows us to estimate the resultant vector as shown.

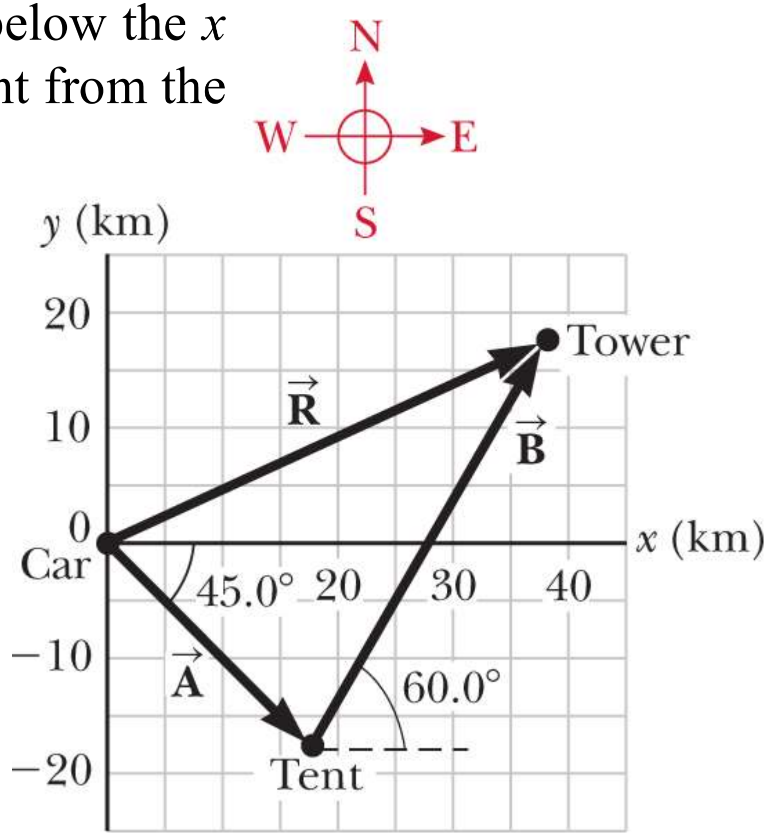
Example 3.5: Taking a Hike

Analyze Displacement **A** has a magnitude of 25.0 km and is directed 45.0° below the positive x axis:

$$A_x = A\cos(-45.0°) = (25.0 \text{ km})(0.707) = \boxed{17.7 \text{ km}}$$

$$A_y = A\sin(-45.0°) = (25.0 \text{ km})(-0.707) = \boxed{-17.7 \text{ km}}$$

The negative value of A_y indicates that the hiker ends up below the x axis on the first day. The signs of A_x and A_y also are evident from the figure.



$$B_x = A\cos(60.0°)$$

$$= (40.0 \text{ km})(0.500) = \boxed{20.0 \text{ km}}$$

$$B_y = A\sin(60.0°)$$

$$= (40.0 \text{ km})(0.866) = \boxed{34.6 \text{ km}}$$

Example 3.5: Taking a Hike

(B) Determine the components of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip. Find an expression for $\vec{\mathbf{R}}$ in terms of unit vectors.

First, we find the components of the resultant displacement **R** = **A** + **B**:

$$R_x = A_x + B_x = 17.7 \text{ km} + 20.0 \text{ km} = \boxed{37.3 \text{ km}}$$

$$R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = \boxed{17.0 \text{ km}}$$

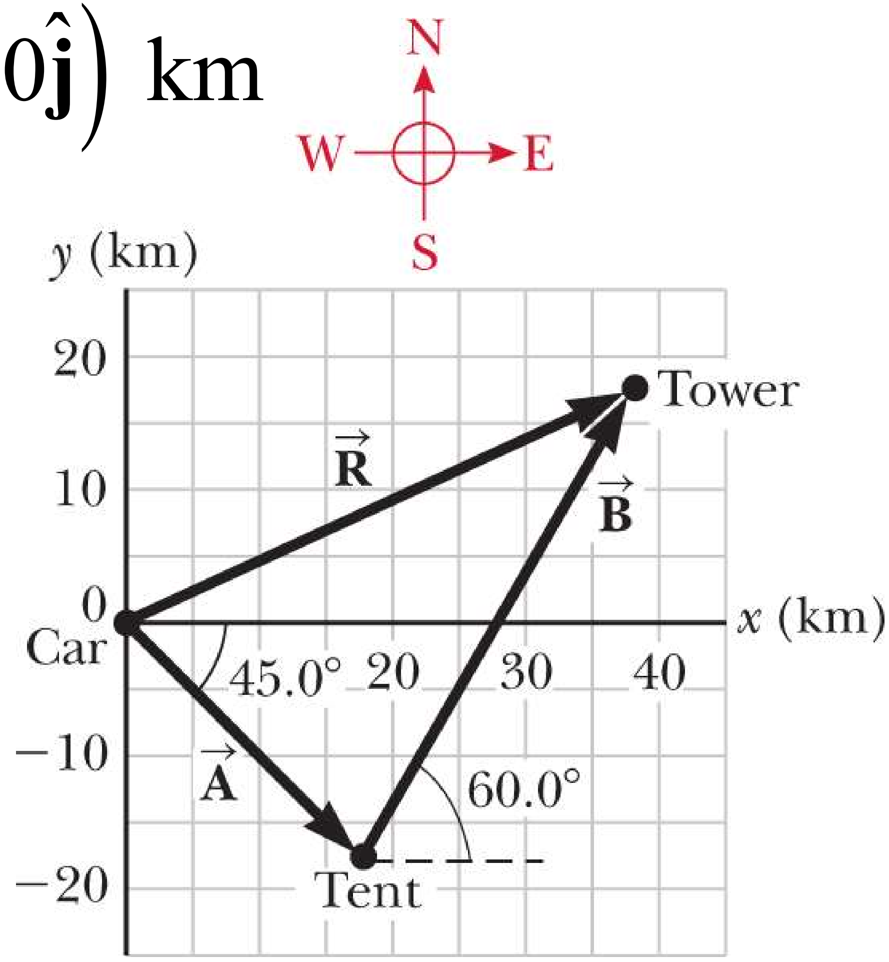
Then, we write the total displacement in unit-vector form:

$$\vec{\mathbf{R}} = \left(37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}}\right) \text{ km}$$

Example 3.5: Taking a Hike

$$\vec{\mathbf{R}} = \left(37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}}\right)\ \text{km}$$

Finalize Looking at the graphical representation, we estimate the position of the tower to be about (38 km, 17 km), which is consistent with the components of **R** in our result for the final position of the hiker. Also, both components of **R** are positive, putting the final position in the first quadrant of the coordinate system, which is also consistent with the figure



Example 3.5:
Taking a Hike

After reaching the tower, the hiker wishes to return to her car along a single straight line. What are the components of the vector representing this hike? What should the direction of the hike be?

The desired vector $\mathbf{R}_{\mathbf{car}}$ is the negative of vector **R:**

$$\vec{\mathbf{R}}_{\text{car}} = -\vec{\mathbf{R}} = \left(-37.7\hat{\mathbf{i}} - 17.0\hat{\mathbf{j}}\right) \text{ km}$$

The direction is found by calculating the angle that the vector makes with the x axis

$$\tan\theta = \frac{R_{\text{car},y}}{R_{\text{car},x}} = \frac{-17.0 \text{ km}}{-37.7 \text{ km}} = 0.450$$

$$\Rightarrow \theta = 204.2°, \text{ or } 24.2° \text{ south of west}$$