Physics for Scientists and Engineers,

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Chapter 6: Circular Motion and Other Applications of Newton's Laws

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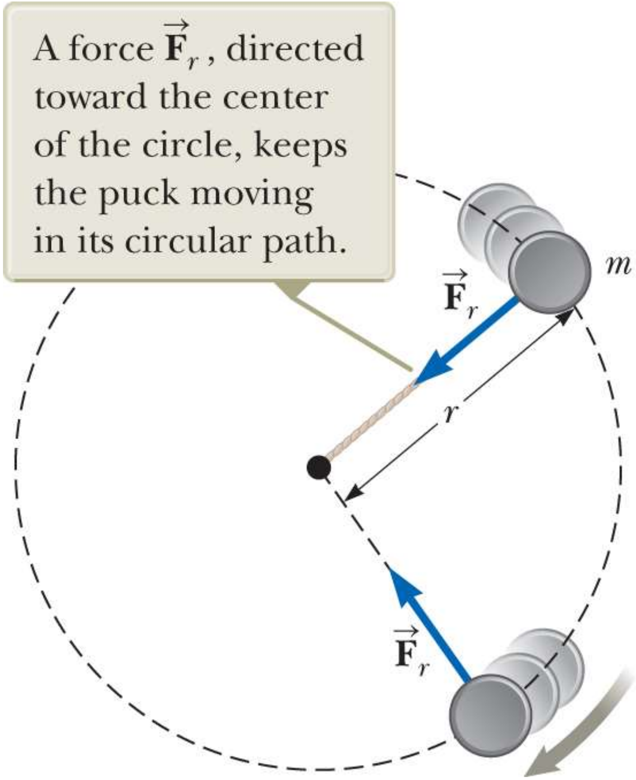
6.1. Extending the Particle in Uniform Circular Motion Model

A particle in uniform circular motion: particle moves with constant speed v in circular path of radius $r \rightarrow$ acceleration is:

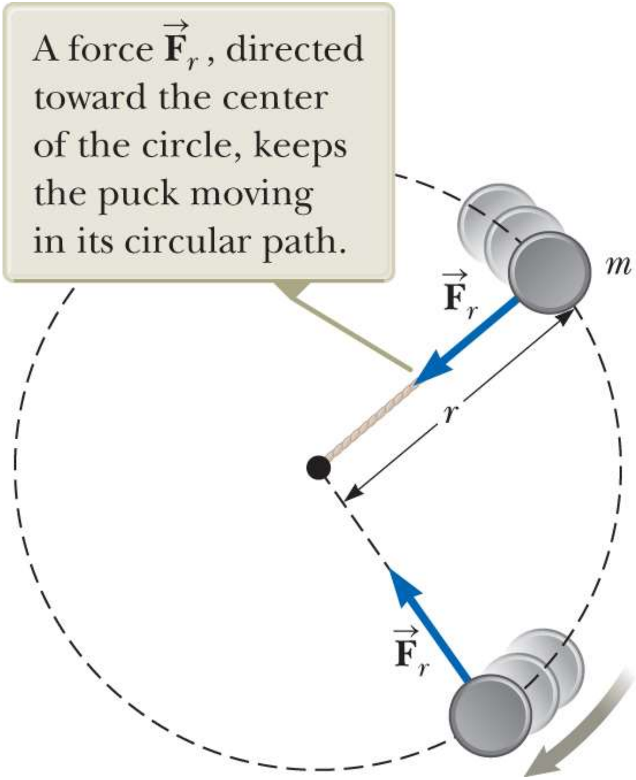
$$a_c = \frac{v^2}{r}$$

- *Centripetal acceleration* $\rightarrow \mathbf{a}_c$ directed toward center of circle
  - Note: $\mathbf{a}_c$ *always* perpendicular to $\mathbf{v}$. (If there were a component of acceleration parallel to $\mathbf{v}$: particle's speed would be changing.)
- Extend particle in uniform circular motion model $\rightarrow$ incorporate concept of force
- Consider puck of mass m tied to string of length r, moving at constant speed in horizontal, circular path (figure)
  - Weight supported by normal force from frictionless table, string anchored to peg at center of circular path



6.1. Extending the Particle in Uniform Circular Motion Model

- Why does the puck move in a circle?
  - Newton's first law → puck would move in straight line if no force acting on it →
    - string prevents motion along a straight line by exerting radial force $\mathbf{F}_r$ → circular path
  - Force directed along string toward center of circle



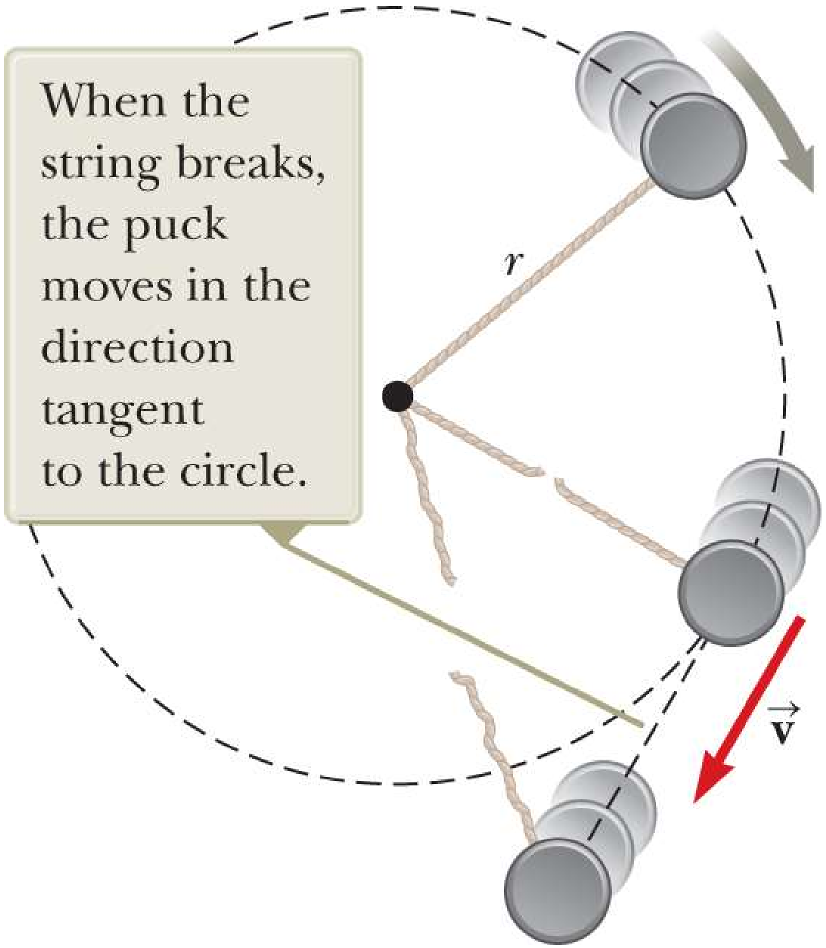
- Apply Newton's second law along radial direction:

$$\boxed{\sum F = ma_c = m\frac{v^2}{r}}$$

- A force causing centripetal acceleration acts toward center of circular path →
  - causes change in direction of velocity vector

6.1. Extending the Particle in Uniform Circular Motion Model

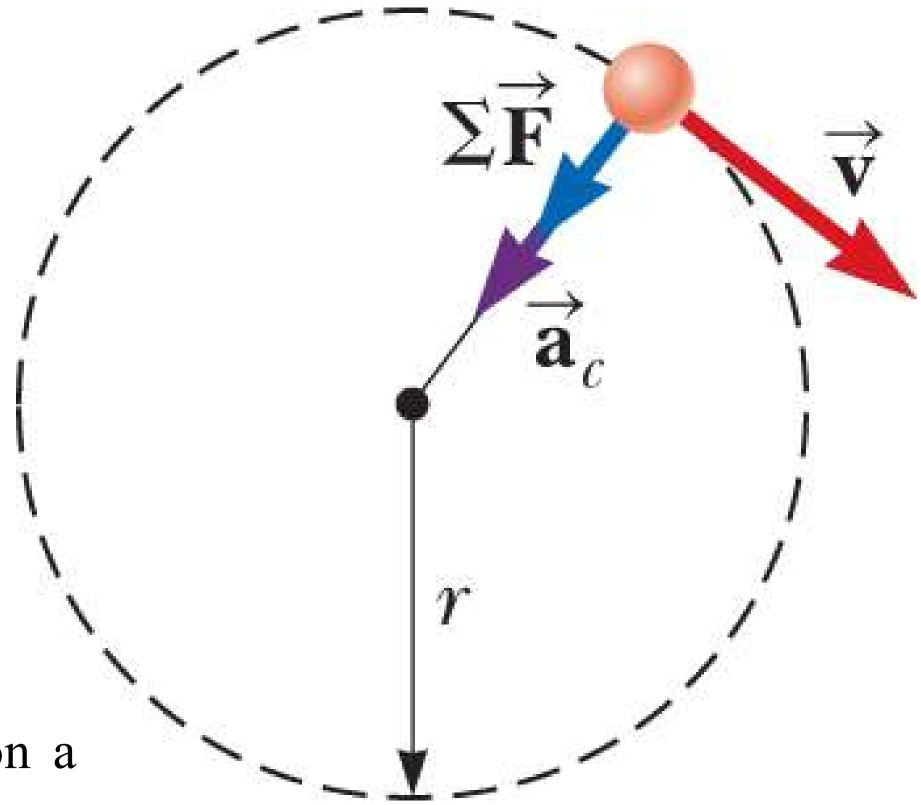
- Force vanishes (string breaks) →
  - puck moves along straight-line path tangent to the circle



Analysis Model: Particle in Uniform Circular Motion (Extension)

Imagine a moving object that can be modeled as particle. If it moves in a circular path of radius r at a constant speed v, it experiences a centripetal acceleration. Because the particle is accelerating, there must be a *net force acting on the particle*. That force is directed *toward the center of the circular path and is given by*:

$$\sum F = ma_c = m\frac{v^2}{r}$$



Examples

• The tension in a string of constant length acting on a rock twirled in a circle

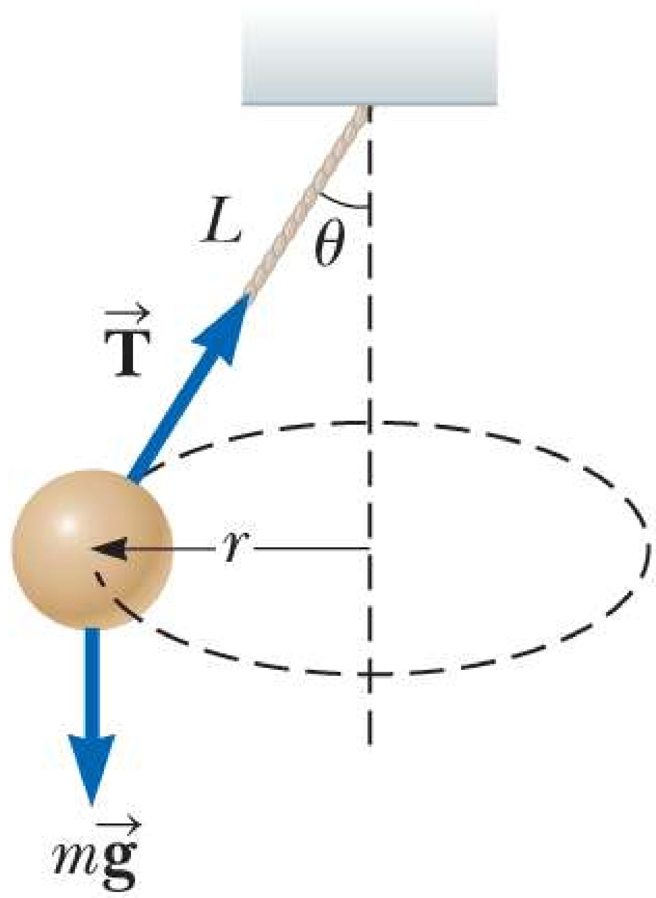
• The gravitational force acting on a planet traveling around the Sun in a perfectly circular orbit

• The magnetic force acting on a charged particle moving in a uniform magnetic field

• The electric force acting on an electron in orbit around a nucleus in the Bohr model of the hydrogen atom

Example 6.1:
The Conical Pendulum

A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown in the figure. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find an expression for v in terms of the length of the string and the angle it makes with the vertical in the figure.



Conceptualize Imagine the motion of the ball in the figure and convince yourself that the string sweeps out a cone and that the ball moves in a horizontal circle.

Categorize The ball in the figure does not accelerate vertically. Therefore, we model it as a *particle in equilibrium* in the vertical direction. It experiences a centripetal acceleration in the horizontal direction, so it is modeled as a *particle in uniform circular motion* in this direction.

Example 6.1:
The Conical Pendulum

Analyze Let θ represent the angle between the string and the vertical. In the diagram of forces acting on the ball, the force **T** exerted by the string on the ball is resolved into a vertical component $T\cos\theta$ and a horizontal component $T\sin\theta$ acting toward the center of the circular path. Apply the particle in equilibrium model in the vertical direction:

$$\sum F_y = T\cos\theta - mg = 0 \rightarrow T\cos\theta = mg$$

Use the equation from the particle in uniform circular motion model in the horizontal direction:

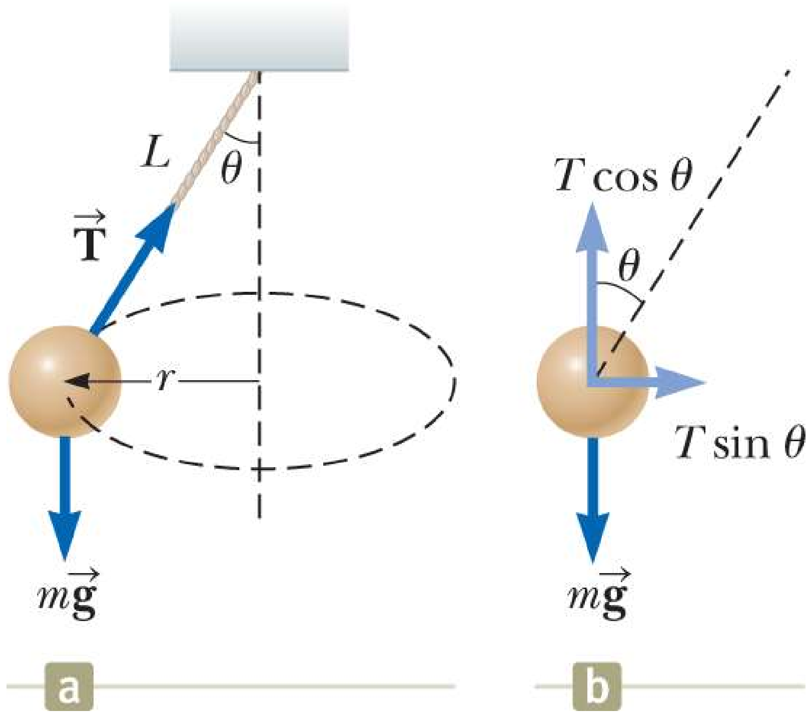
$$\sum F_x = T\sin\theta = ma_c = \frac{mv^2}{r}$$

Divide the two equations and use $\sin\theta/\cos\theta = \tan\theta$:

$$\tan\theta = \frac{v^2}{rg}$$

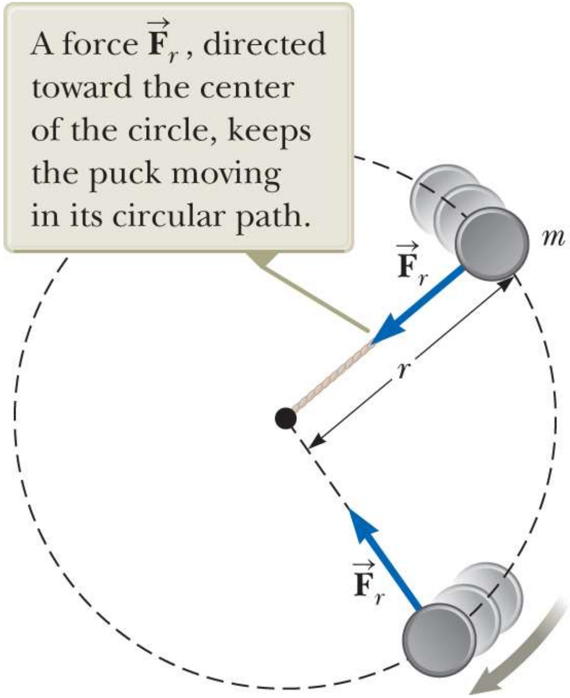
Solve for v: $$v = \sqrt{rg\tan\theta}$$

Incorporate $r = L\sin\theta$ from the geometry in the figure: $$v = \boxed{\sqrt{Lg\sin\theta\tan\theta}}$$



Example 6.2:
How Fast Can It Spin?

A puck of mass 0.500 kg is attached to the end of a cord 1.50 m long. The puck moves in a horizontal circle as shown in the figure. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the puck can move before the cord breaks? Assume the string remains horizontal during the motion.



Conceptualize It makes sense that the stronger the cord, the faster the puck can move before the cord breaks. Also, we expect a more massive puck to break the cord at a lower speed. (Imagine whirling a bowling ball on the cord!)

Categorize Because the puck moves in a circular path, we model it as a *particle in uniform circular motion.*

Example 6.2:
How Fast Can It Spin?

Analyze Incorporate the tension and the centripetal acceleration into Newton's second law. Solve for *v*:

$$T = m\frac{v^2}{r}$$

Find the maximum speed the puck can have, which corresponds to the maximum tension the string can withstand:

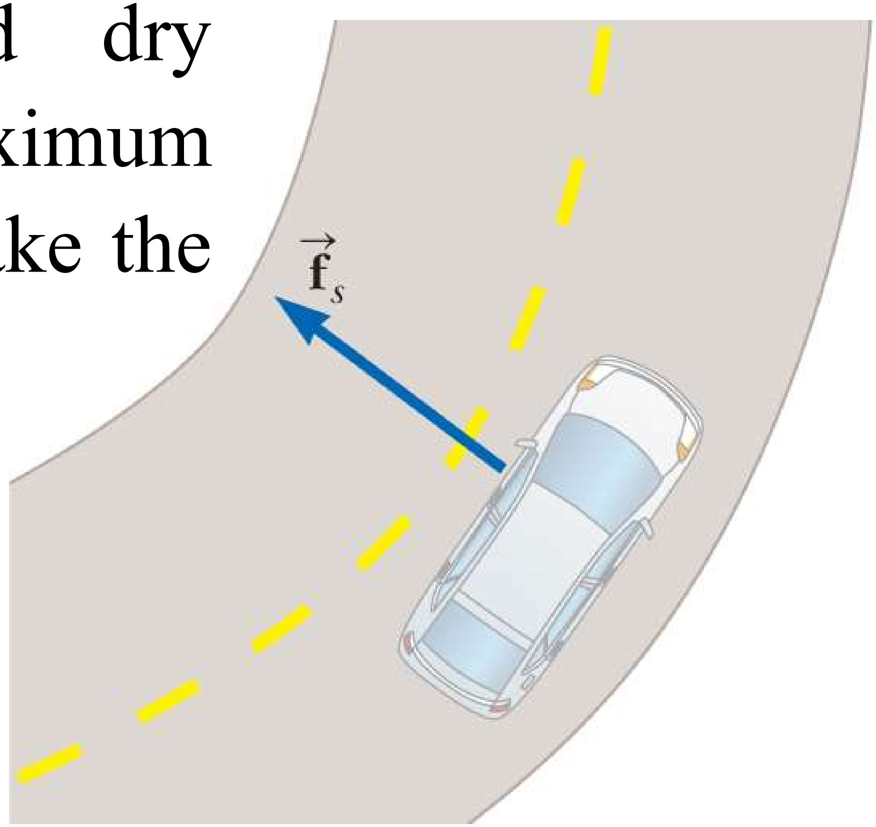
$$v = \sqrt{\frac{Tr}{m}}$$

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\ \mathrm{N})(1.50\ \mathrm{m})}{0.500\ \mathrm{kg}}} = \boxed{12.2\ \mathrm{m/s}}$$

Finalize The equation for v shows that v increases with T and decreases with larger m, as we expected from our conceptualization of the problem.

Example 6.3: What Is the Maximum Speed of the Car?

A 1 500-kg car moving on a flat, horizontal road negotiates a curve as shown in the overhead view in the figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.



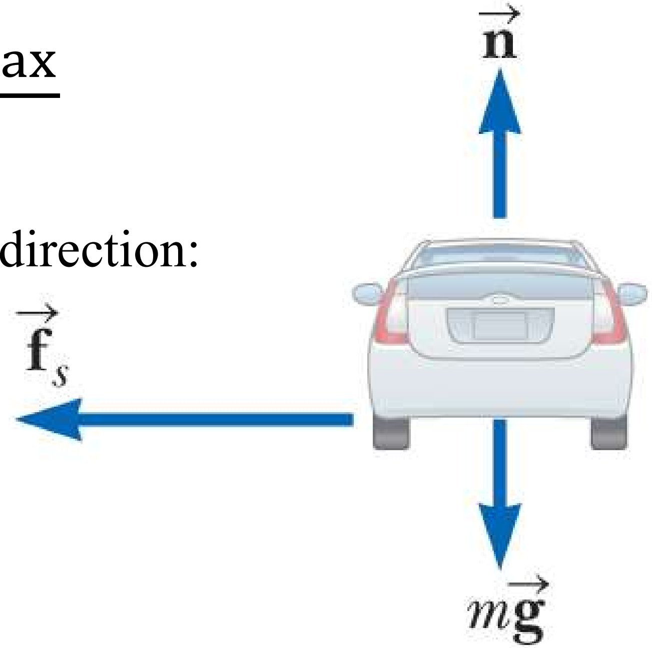
Conceptualize Imagine that the curved roadway is part of a large circle so that the car is moving in a circular path.

Categorize Based on the Conceptualize step of the problem, we model the car as a *particle in uniform circular motion* in the horizontal direction. The car is not accelerating vertically, so it is modeled as a *particle in equilibrium* in the vertical direction.

Example 6.3: What Is the Maximum Speed of the Car?

Analyze The back view in the figure shows the forces on the car. The force that enables the car to remain in its circular path is the *force of static friction*. (It is *static* because no slipping occurs at the point of contact between road and tires. If this force of static friction were zero—for example, if the car were on an icy road—the car would continue in a straight line and slide off the curved road.) The maximum speed v_{max} the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,max} = \mu_s n$. Apply the equation from the particle in uniform circular motion model in the radial direction for the maximum speed condition:

$$f_{s,\max} = \mu_s n = m\frac{v^2{}_{\max}}{r}$$



Apply the particle in equilibrium model to the car in the vertical direction:

$$\sum F_y = 0 \Rightarrow n - mg = 0 \Rightarrow n = mg$$

Solve the first equation for the maximum speed and substitute for n:

$$v_{\max} = \sqrt{\frac{\mu_s nr}{m}} = \sqrt{\frac{\mu_s mgr}{m}} = \sqrt{\mu_s gr}$$

Substitute numerical values: $$v_{\max} = \sqrt{(0.523)(9.80\ \text{m/s}^2)(35.0\ \text{m})} = 13.4\ \text{m/s}$$

Example 6.3: What Is the Maximum Speed of the Car?

Suppose a car travels this curve on a wet day and begins to skid on the curve when its speed reaches only 8.00 m/s. What can we say about the coefficient of static friction in this case?

The coefficient of static friction between the tires and a wet road should be smaller than that between the tires and a dry road. This expectation is consistent with experience with driving because a skid is more likely on a wet road than a dry road.

To check our suspicion, we can solve our equation for the coefficient of static friction:

$$v_{\text{max}} = \sqrt{\mu_s g r} \Rightarrow \mu_s = \frac{v_{\text{max}}^2}{gr}$$

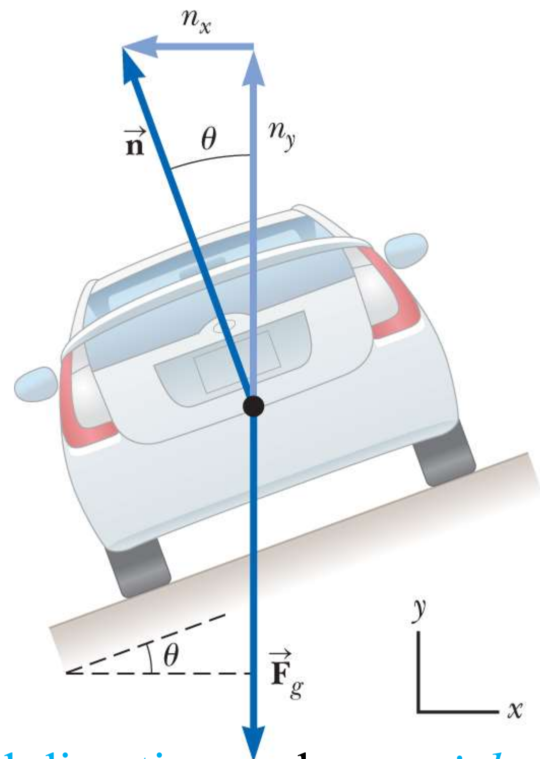
Substituting the numerical values gives:

$$\mu_s = \frac{(8.00 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(35.0 \text{ m})} = 0.187$$

which is indeed smaller than the coefficient of 0.523 for the dry road.

Example 6.4:
The Banked Roadway

You are a civil engineer who has been given the assignment to design a curved roadway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a road is usually *banked,* which means that the roadway is tilted toward the inside of the curve. Suppose the designated speed for the road is to be 13.4 m/s (30.0 mi/h) and the radius of the curve is 35.0 m. You need to determine the angle at which the roadway on the curve should be banked.



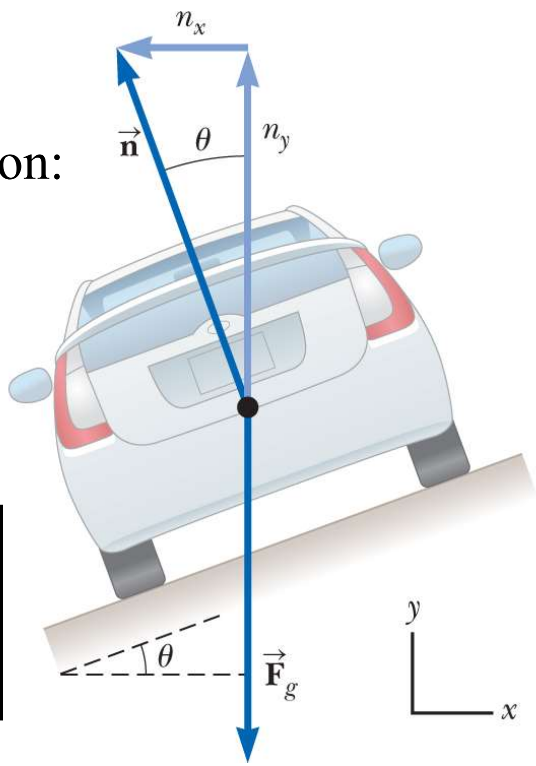
Categorize The car is modeled as a *particle in equilibrium* in the vertical direction and a *particle in uniform circular motion* in the horizontal direction.

Example 6.4:
The Banked Roadway

Analyze On a level (unbanked) road, the force that causes the centripetal acceleration is the force of static friction between tires and the road. If the road is banked at an angle θ, however, the normal force **n** has a horizontal component toward the center of the curve. Because the road is to be designed so that the force of static friction is zero, the component $n_x = n \sin\theta$ is the only force that causes the centripetal acceleration.

Write Newton's second law for the car in the radial direction, which is the $-x$ direction:

$$\sum F_r = n\sin\theta = \frac{mv^2}{r}$$



Apply the particle in equilibrium model to the car in the vertical direction:

$$\sum F_y = n\cos\theta - mg = 0 \Rightarrow n\cos\theta = mg$$

Divide the first equation by the second: $\tan\theta = \dfrac{v^2}{rg}$

Solve for the angle θ and substitute numerical values:

$$\theta = \tan^{-1}\left[\frac{(13.4\text{ m/s})^2}{(35.0\text{ m})(9.80\text{ m/s}^2)}\right]$$

$$= \boxed{27.6°}$$

Example 6.4:
The Banked Roadway

Imagine that this same roadway were built on Mars in the future to connect different colony centers. Could it be traveled at the same speed?

The reduced gravitational force on Mars would mean that the car is not pressed as tightly to the roadway. The reduced normal force results in a smaller component of the normal force toward the center of the circle. This smaller component would not be sufficient to provide the centripetal acceleration associated with the original speed. The centripetal acceleration must be reduced, which can be done by reducing the speed v.

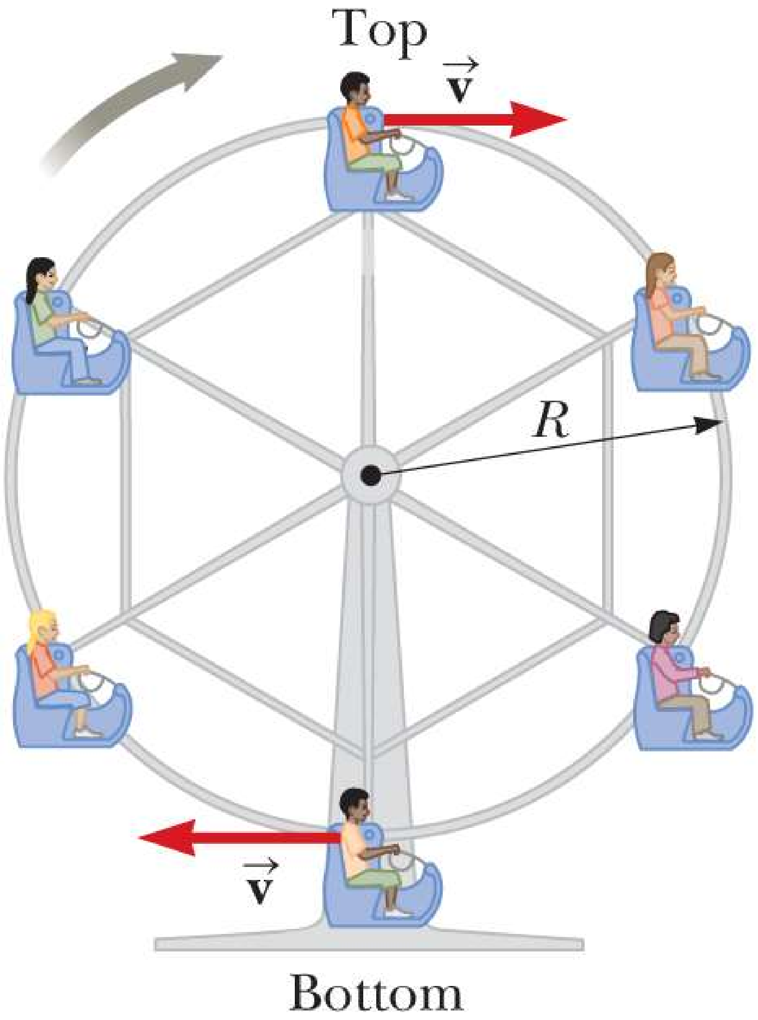
No, v would be reduced.

Mathematically, notice that if we solve our equation for v, we see that the speed v is proportional to the square root of g for a roadway of fixed radius r banked at a fixed angle θ. Therefore, if g is smaller, as it is on Mars, the speed v with which the roadway can be safely traveled is also smaller:

$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$$

Example 6.5:
Riding the Ferris Wheel

A child of mass m rides on a Ferris wheel as shown in the figure. The child moves in a vertical circle of radius 10.0 m at a constant speed of 3.00 m/s.



(A) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child, mg.

Conceptualize Look carefully at the figure. At both the bottom of the path and the top, the normal and gravitational forces on the child act in *opposite* directions. The vector sum of these two forces gives a force of constant magnitude that keeps the child moving in a circular path at a constant speed. To yield net force vectors with the same magnitude, the normal force at the bottom must be greater than that at the top.

Example 6.5: Riding the Ferris Wheel

Categorize Because the speed of the child is constant, we can categorize this problem as one involving a *particle* (the child) *in uniform circular motion,* complicated by the gravitational force acting at all times on the child.

Analyze We draw a diagram of forces acting on the child at the bottom of the ride as shown in the figure. The only forces acting on him are the downward gravitational force $\mathbf{F}_g = m\mathbf{g}$ and the upward force $\mathbf{n}_{\text{bot}}$ exerted by the seat. The centripetal acceleration of the child at this point is upward and the nhe net upward force on the child has a magnitude $n_{\text{bot}} - mg$.
Using the particle in uniform circular motion model, apply Newton's second law to the child in the radial direction when he is at the bottom of the ride:

$$\sum F = n_{\text{bot}} - mg = m\frac{v^2}{r}$$

Solve for the force exerted by the seat on the child:

$$n_{\text{bot}} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

Example 6.5: Riding the Ferris Wheel

Substitute numerical values given for the speed and radius:

$$n_{\text{bot}} = mg\left(1+\frac{(3.00\text{ m/s})^2}{(10.0\text{ m})(9.80\text{ m/s}^2)}\right)$$

$$= \boxed{1.09mg}$$

Hence, the magnitude of the force $\mathbf{n}_{\text{bot}}$ exerted by the seat on the child is *greater* than the weight of the child by a factor of 1.09. So, the child experiences an apparent weight that is greater than his true weight by a factor of 1.09.

Example 6.5: Riding the Ferris Wheel

(B) Determine the force exerted by the seat on the child at the top of the ride.

Analyze The diagram of forces acting on the child at the top of the ride is shown in the figure. The centripetal acceleration of the child at this point is downward and the net downward force has a magnitude $mg - n_{top}$.

Apply Newton's second law to the child at this position:

$$\sum F = mg - n_{\text{top}} = m\frac{v^2}{r}$$

Solve for the force exerted by the seat on the child:

$$n_{top} = mg - m\frac{v^2}{r} = mg\left(1 - \frac{v^2}{rg}\right)$$

Substitute numerical values:

$$n_{top} = mg\left(1 - \frac{(3.00\ \text{m/s})^2}{(10.0\ \text{m})(9.80\ \text{m/s}^2)}\right)$$

$$= \boxed{0.908mg}$$

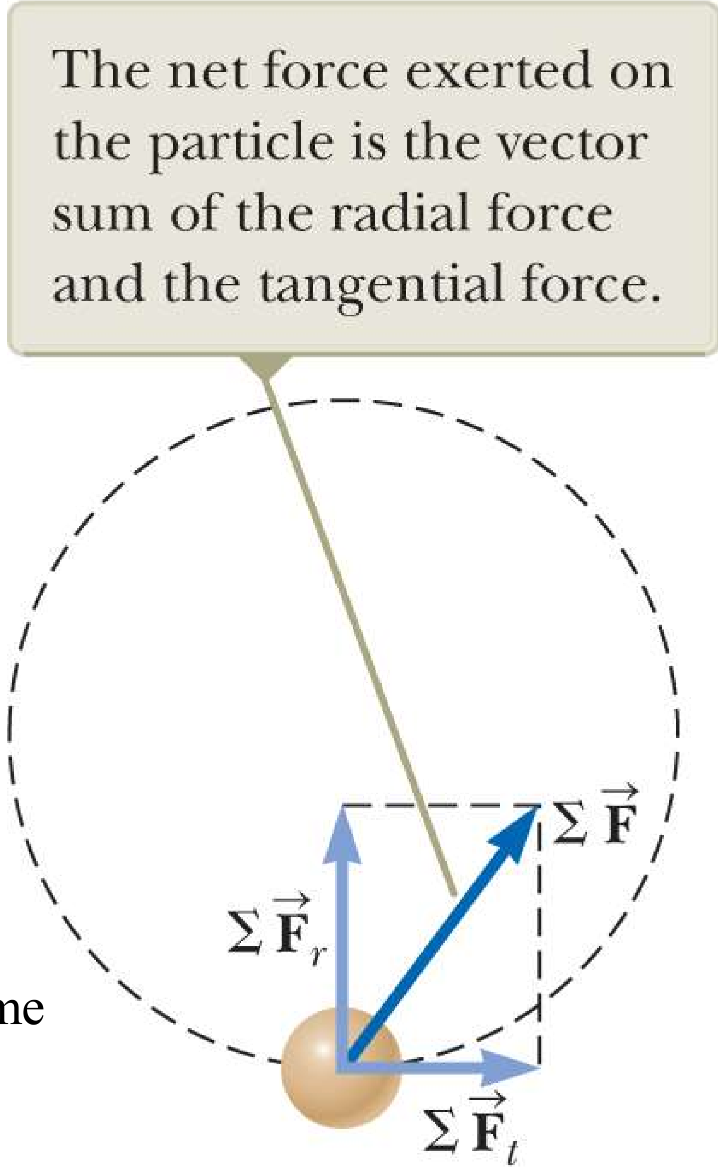
In this case, the magnitude of the force exerted by the seat on the child is *less* than his true weight by a factor of 0.908, and the child feels lighter.

6.2. Nonuniform Circular Motion

- Particle moves with varying speed in circular path → acceleration has radial and tangential components →
    - force must also have tangential and radial component
- Total acceleration: $\vec{\mathbf{a}} = \vec{\mathbf{a}}_r + \vec{\mathbf{a}}_r$
- Total force exerted on the particle:

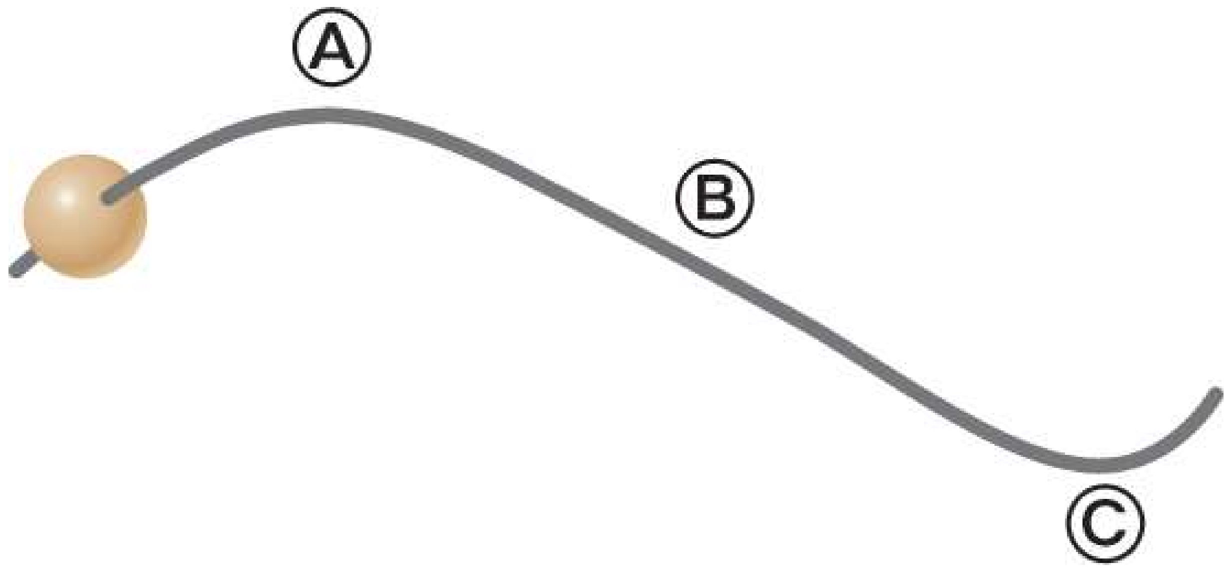
$$\sum \vec{\mathbf{F}} = \sum \vec{\mathbf{F}}_r + \sum \vec{\mathbf{F}}_t$$

- $\Sigma \mathbf{F}_r$ directed toward center of circle:
    - responsible for centripetal acceleration
- $\Sigma \mathbf{F}_t$ tangent to circle:
    - responsible for tangential acceleration →
        - represents change in particle's speed with time



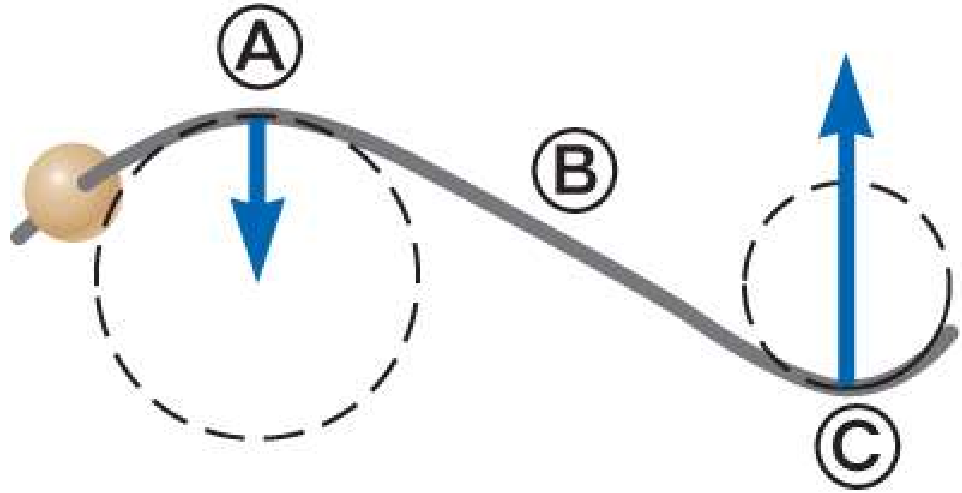
Quick Quiz 6.2 Part I

A bead slides at constant speed along a curved wire lying on a horizontal surface as shown in the figure.



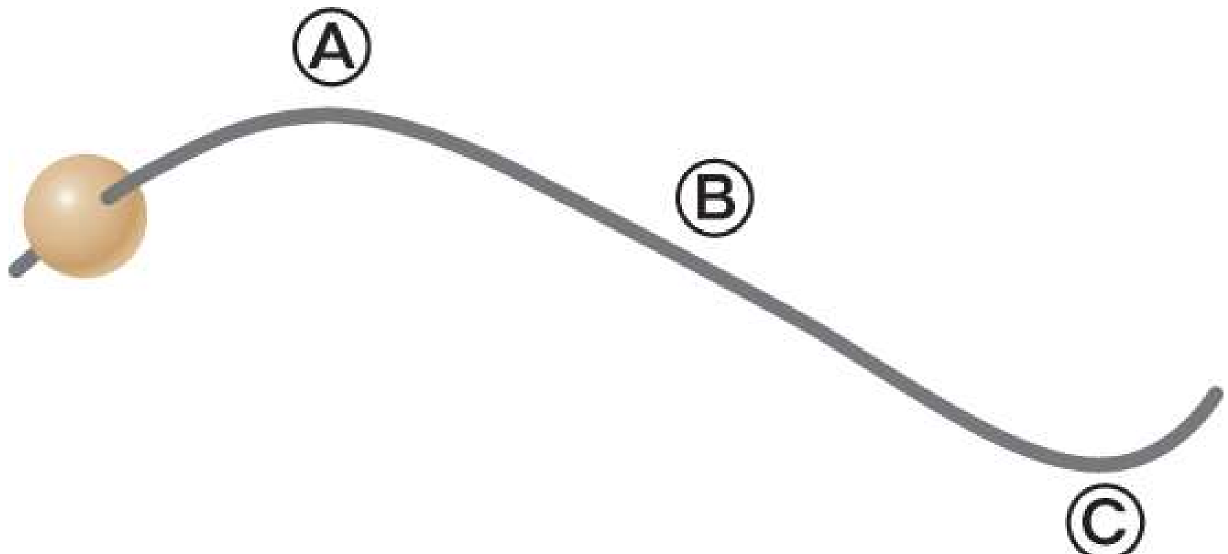
Draw the vectors representing the force exerted by the wire on the bead at points A, B, and C.

Quick Quiz 6.2 Part I



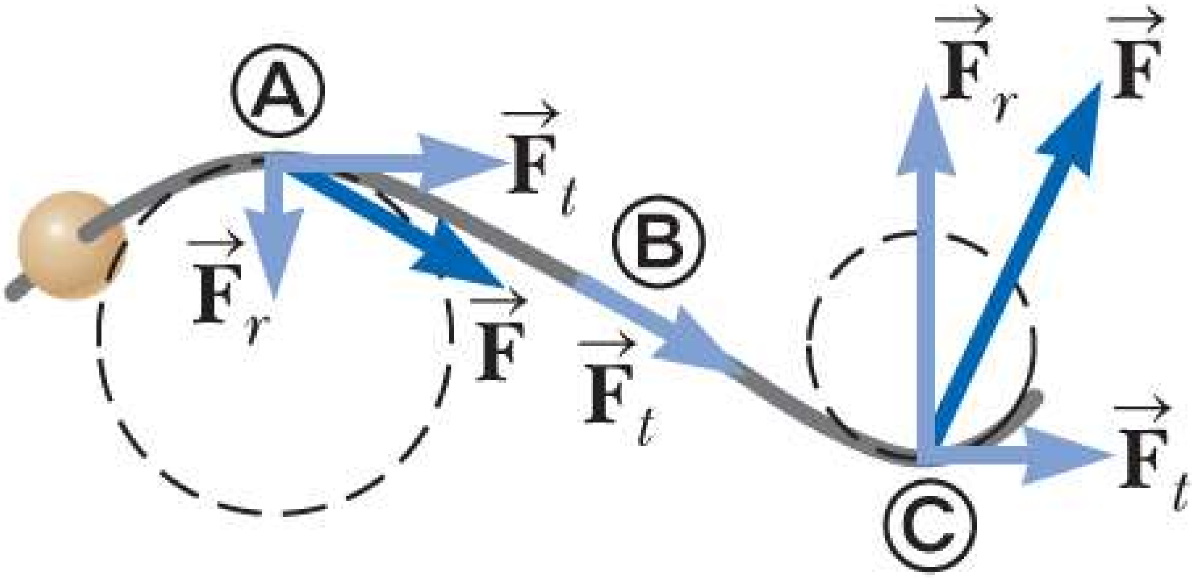
Quick Quiz 6.2 Part II

A bead slides at constant speed along a curved wire lying on a horizontal surface as shown in the figure.



Suppose the bead speeds up with constant tangential acceleration as it moves toward the right. Draw the vectors representing the force on the bead at points A, B, and C.

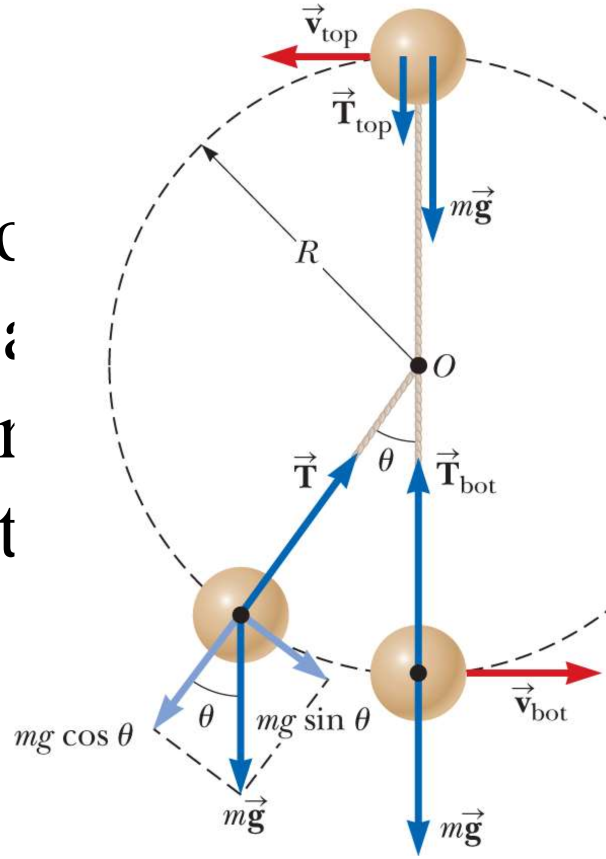
Quick Quiz 6.2 Part II



Example 6.6:
Keep Your Eye on the Ball

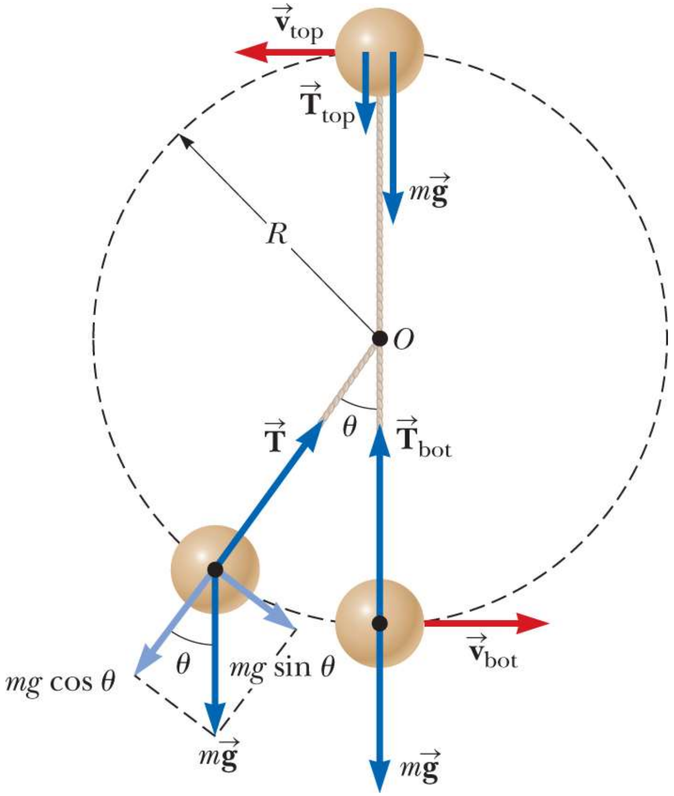
A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed point O as illustrated in the figure.

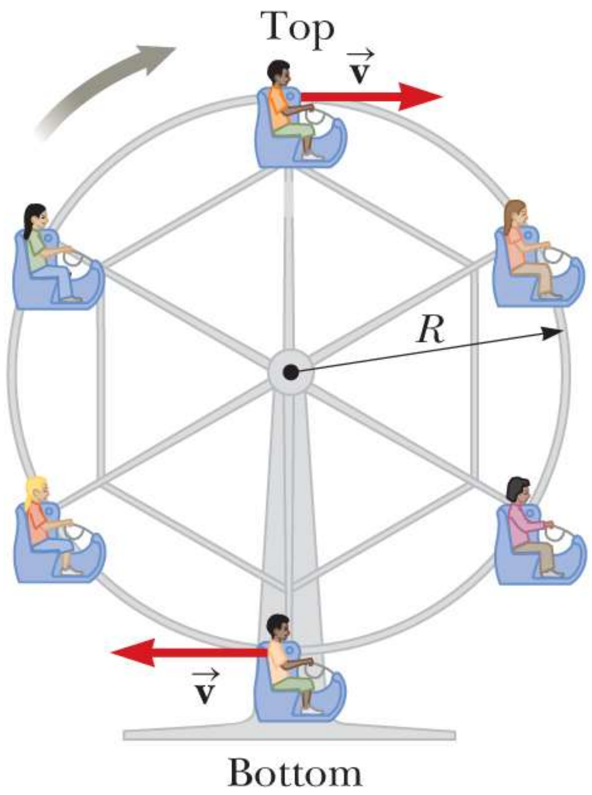
Determine the tangential acceleration
the sphere and the tension in the cord
any instant when the speed of the spher
is v and the cord makes an angle θ wit
the vertical.



Example 6.6: Keep Your Eye on the Ball

Conceptualize Compare the motion of the sphere with that of the child on the Ferris wheel. Both objects travel in a circular path. Unlike the child, however, the speed of the sphere is *not* uniform in this example because, at most points along the path, a tangential component of acceleration arises from the gravitational force exerted on the sphere.





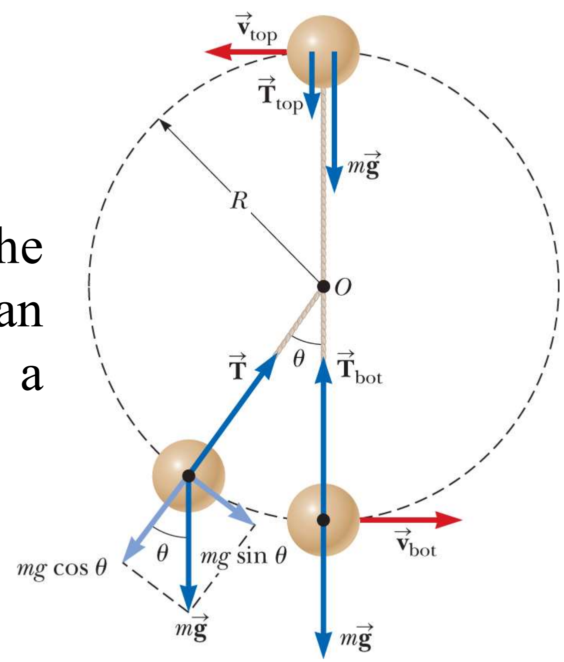
Categorize We model the sphere as a *particle under a net force* and moving in a circular path, but it is not a particle in *uniform* circular motion. We need to use the techniques discussed in this section on nonuniform circular motion.

Example 6.6: Keep Your Eye on the Ball

Analyze From the force diagram, we see that the only forces acting on the sphere are the gravitational force $\mathbf{F}_g = m\mathbf{g}$ exerted by the Earth and the force $\mathbf{T}$ exerted by the cord. We resolve $\mathbf{F}_g$ into a tangential component $mg\sin\theta$ and a radial component $mg\cos\theta$.
From the particle under a net force model, apply Newton's second law to the sphere in the tangential direction:

$$\sum F_t = mg\sin\theta = ma_t$$

$$a_t = \boxed{g\sin\theta}$$



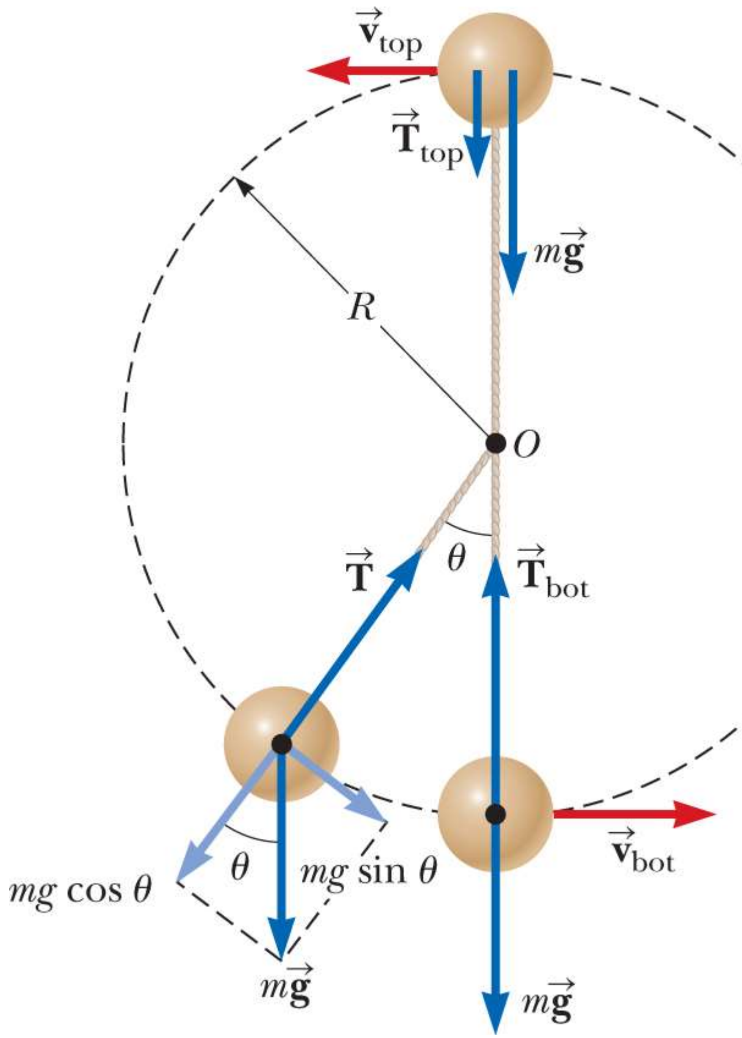
Apply Newton's second law to the forces acting on the sphere in the radial direction, noting that both $\mathbf{T}$ and $\mathbf{a}_r$ are directed toward O. We can use the equation for the instantaneous centripetal acceleration of a particle even when it moves in nonuniform circular motion:

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = \boxed{mg\left(\frac{v^2}{Rg} + \cos\theta\right)}$$

Example 6.6: Keep Your Eye on the Ball

Finalize Let us evaluate this result at the top and bottom of the circular path:



$$T_{top} = mg\left(\frac{v_{top}^{\ 2}}{Rg} - 1\right)$$

$$T_{bot} = mg\left(\frac{v_{bot}^{\ 2}}{Rg} + 1\right)$$

Example 6.6:
Keep Your Eye on the Ball

(A) What speed would the sphere have if it passes over the top of the circle if the tension in the cord goes to zero instantaneously at this point?

Let us set the tension equal to zero in the expression for T_{top}:

$$0 = mg\left(\frac{v_{top}{}^{2}}{Rg} - 1\right) \Rightarrow v_{\text{top}} = \sqrt{gR}$$

Example 6.6: Keep Your Eye on the Ball

(B) What if the sphere is set in motion such that the speed at the top is less than this value? What happens?

In this case, the sphere never reaches the top of the circle. At some point on the way up, the tension in the string goes to zero and the sphere becomes a projectile:

The ball enters free fall for part of its motion near the top, and the string becomes slack during that time.

It follows a segment of a parabolic path, with its peak below the topmost position of the sphere shown in the figure, rejoining the circular path on the other side when the tension becomes nonzero again.