**Physics for Scientists and Engineers,**
**10th edition, Raymond A. Serway and John W. Jewett, Jr.**

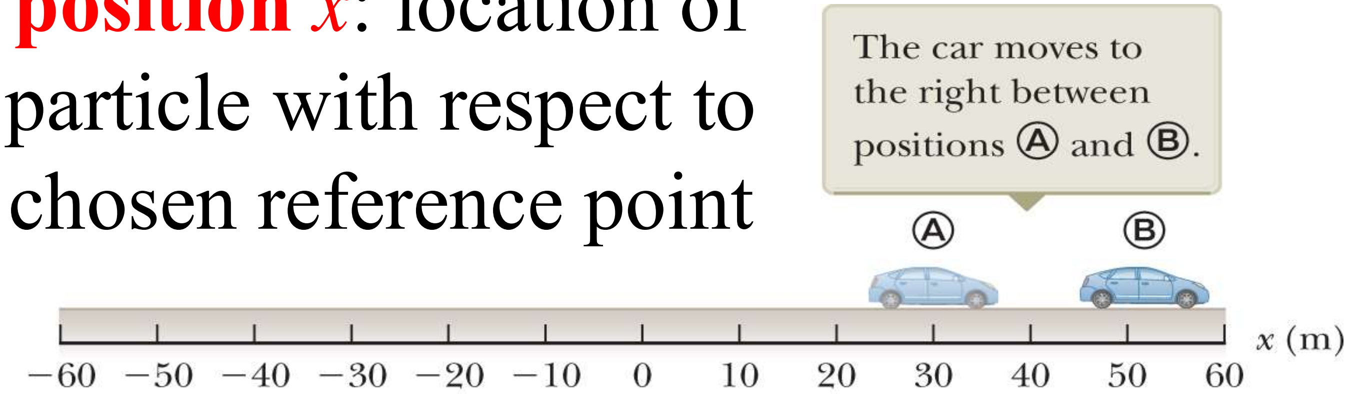
# Chapter 2: Motion in One Dimension

BROOKS/COLE
CENGAGE Learning

Customized by **Dr. H. Merabet, 2020**

# 2.1. Position

**position $x$**: location of particle with respect to chosen reference point




- Point A: car is 30 m to the right of the reference position $x = 0$

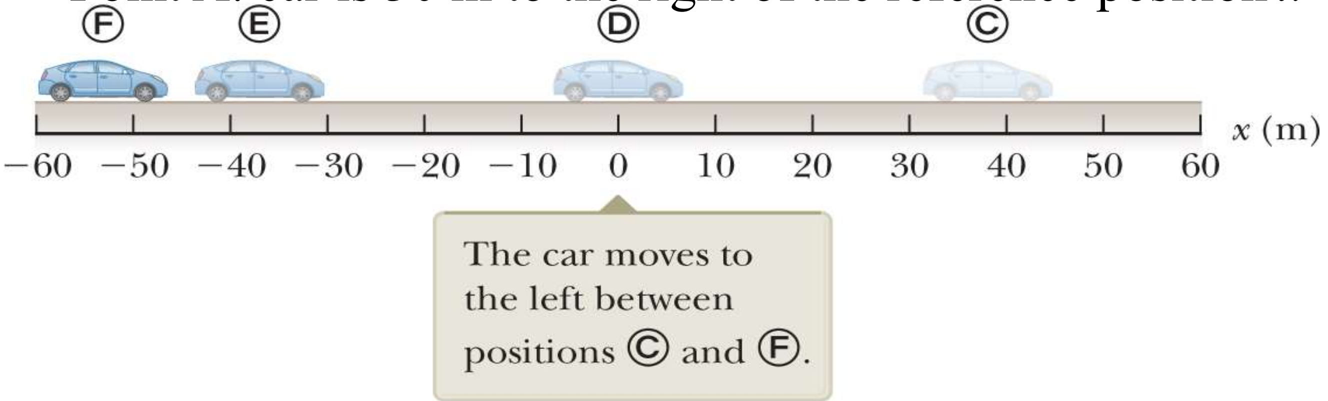
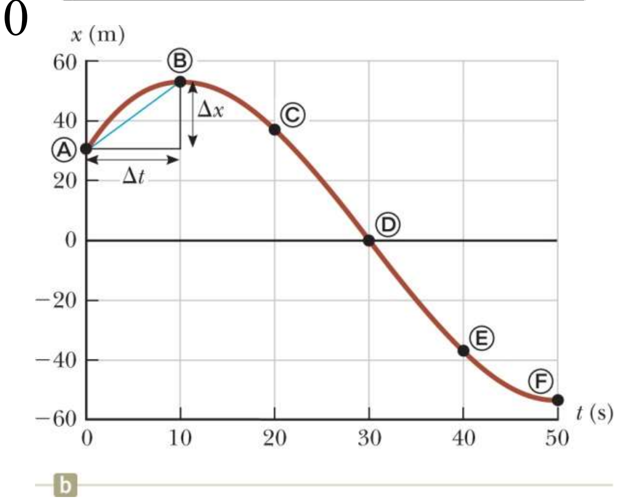



Figure (a): shows car moving back and forth along $x$ axis

**TABLE 2.1** Position of the Car at Various Times

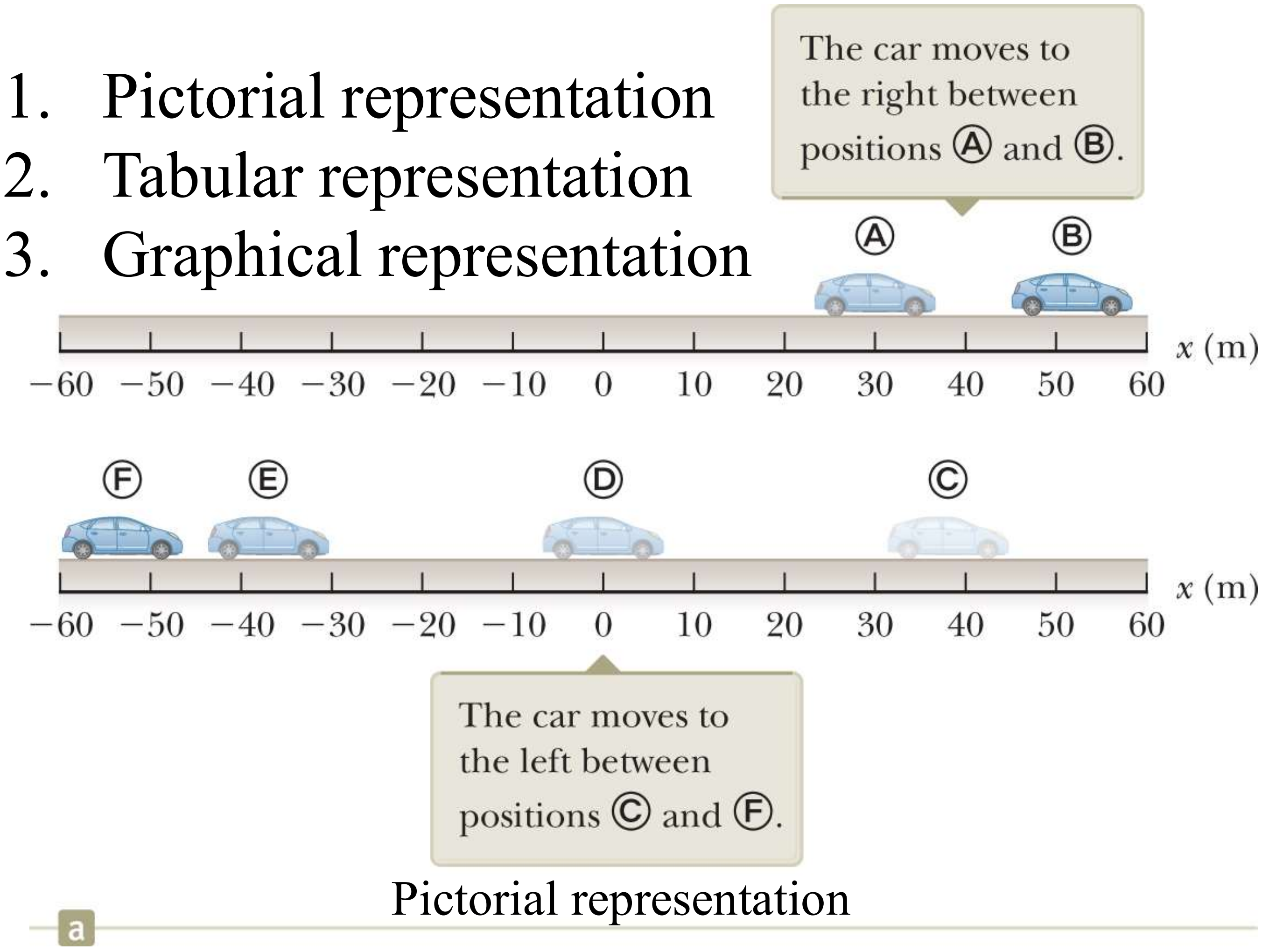
| Position | $t$ (s) | $x$ (m) |
|---|---|---|
| Ⓐ | 0 | 30 |
| Ⓑ | 10 | 52 |
| Ⓒ | 20 | 38 |
| Ⓓ | 30 | 0 |
| Ⓔ | 40 | −37 |
| Ⓕ | 50 | −53 |




- Car moves to right (positive direction) from A to B during first 10 s of motion
- After B, position values decrease → car backing up from B through F
- At D, 30 s after we start measuring, car at origin of coordinates
- Car continues moving left → more than 50 m to left of $x = 0$ when we stop recording

# 2.1. Alternative Representations

1. Pictorial representation
2. Tabular representation
3. Graphical representation




Pictorial representation

**TABLE 2.1** Position of the Car at Various Times

| Position | $t$ (s) | $x$ (m) |
|---|---|---|
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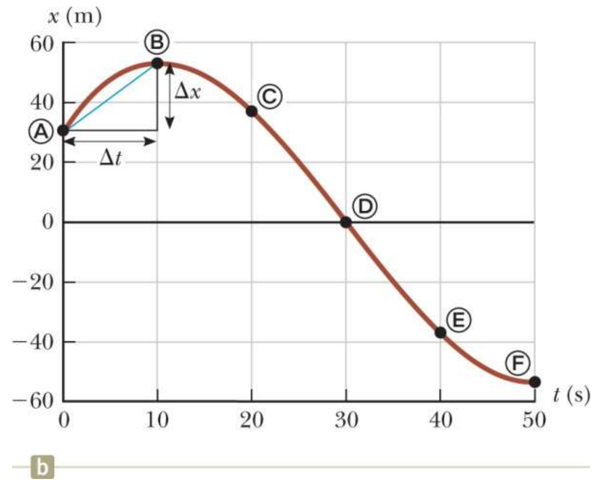
| Ⓓ | 30 | 0 |
| Ⓔ | 40 | −37 |
| Ⓕ | 50 | −53 |

Tabular representation




Graphical representation

- Different representations can help in understanding physics of situation
- Usually, the ultimate goal is mathematical representation

# 2.1. Displacement

**TABLE 2.1** Position of the Car at Various Times

| Position | $t$ (s) | $x$ (m) |
|---|---|---|
| Ⓐ | 0 | 30 |
| Ⓑ | 10 | 52 |
| Ⓒ | 20 | 38 |
| Ⓓ | 30 | 0 |
| Ⓔ | 40 | −37 |
| Ⓕ | 50 | −53 |

**Displacement $\Delta x$** of particle: change in position in a given time interval

As particle moves from initial position $x_i$ to final position $x_f \rightarrow$ displacement is given by:

$$\boxed{\Delta x \equiv x_f - x_i}$$

- $\Delta$ denotes *change* in a quantity
- From equation, $\Delta x > 0$ if $x_f$ is $>$ than $x_i$ and $\Delta x < 0$ if $x_f < x_i$

# 2.1. Distance and Displacement

- Displacement and distance different:

**Distance:** length of path followed by particle Example: basketball players in figure: if player runs from his own team's basket down court to other team's basket, then returns to his own basket, *displacement* = 0 because $x_f = x_i$, so $\Delta x = 0$ During this time interval: player moved through *distance* twice length of basketball court

  - Distance: always positive
  - Displacement: positive or negative

- **Displacement**: vector quantity
  - Other vectors: position, velocity, acceleration
- Example: How far did position of car change (magnitude) and in what direction (forward or backward)?

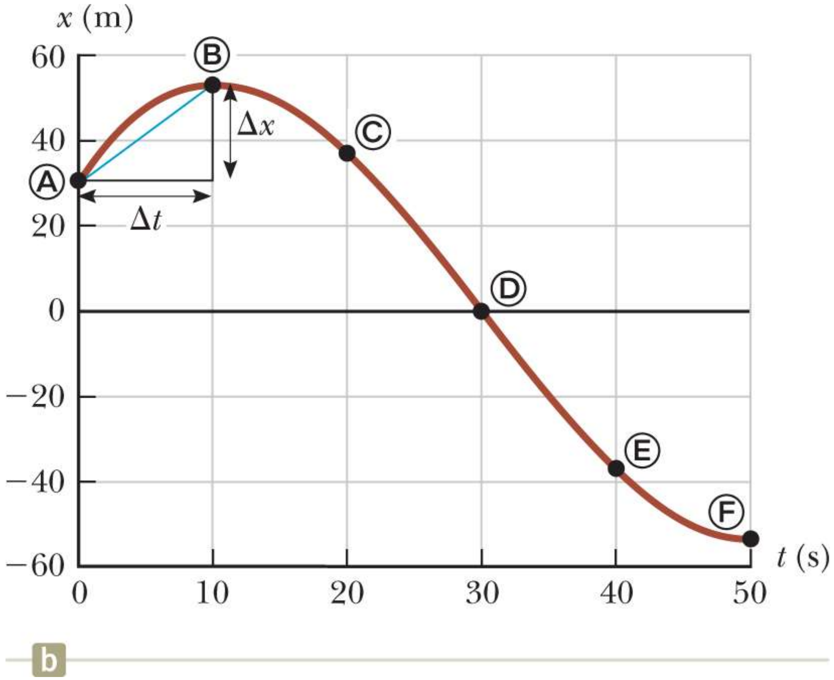
**Vector quantity** requires specification of both direction and magnitude
**Scalar quantity** has numerical value and no direction

# 2.1. Position, Velocity, and Speed of a Particle

- **Average velocity** $v_{x,\text{avg}}$ of particle: particle's displacement $\Delta x$ divided by time interval $\Delta t$ during which displacement occurs:

$$v_{x,\text{avg}} \equiv \frac{\Delta x}{\Delta t}$$

  - Subscript $x \rightarrow$ motion along the $x$ axis
  - Average velocity: dimensions of length divided by time (L/T)
    - Meters per second in SI units

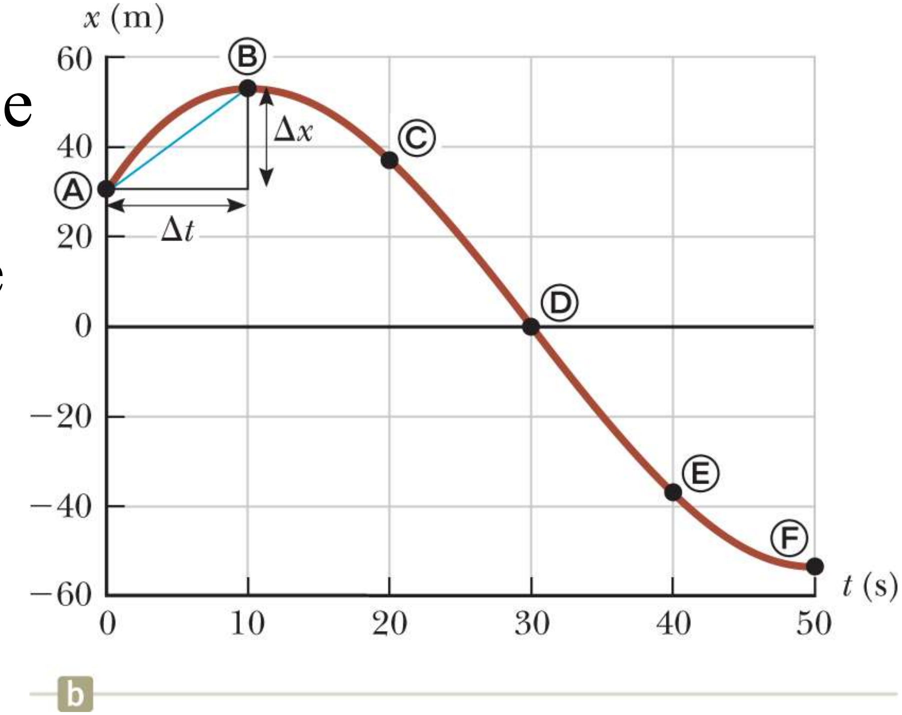



- Average velocity of particle moving in one dimension: positive or negative
  - Depending on sign of displacement
  - Time interval $\Delta t$ always positive
- If coordinate of particle increases in time (if $x_f > x_i$ ) $\rightarrow$ $\Delta x$ positive
  - $v_{x,\text{avg}} = \Delta x/\Delta t$ positive
    - Corresponds to particle moving in positive $x$ direction $\rightarrow$ toward larger values of $x$
- If coordinate decreases in time (if $x_f < x_i$ ) $\rightarrow$ $\Delta x$ negative
  - $v_{x,\text{avg}}$ negative
    - Corresponds to particle moving in negative $x$ direction

# Average Velocity

- Geometric interpretation of average velocity:
- Draw straight line between any two points on position–time graph

  - Line forms hypotenuse of right triangle of height $\Delta x$ and base $\Delta t$
  - *S*lope of line ratio $\Delta x/\Delta t$ is the average velocity




- Example: line between positions A and B (figure) has slope = average velocity of car between those two times:

$$\text{Example: } \frac{52\text{ m} - 30\text{ m}}{10\text{ s} - 0} = 2.2\text{ m/s}$$

# Average Speed

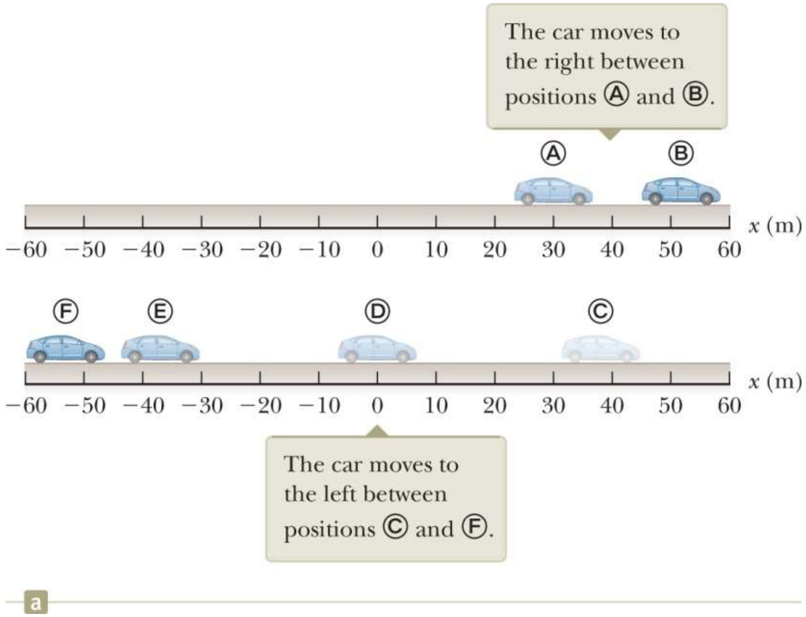
**Average speed** of particle (scalar quantity): total distance $d$ traveled divided by elapsed time $\Delta t$:

$$\boldsymbol{v_{\text{avg}} \equiv \frac{d}{\Delta t}}$$

- SI unit of average speed: same as unit of average velocity $\rightarrow$ meters per second
- Average speed:
  - No direction
  - Always expressed as positive number
- Example: It takes you 45.0 s to travel 100 m down a long, straight hallway at an airport. At 100-m mark, you return back 25.0 m along same hallway, taking 10.0 s to make return trip
  - Magnitude of average *velocity* is: $v_{\text{avg}} = \dfrac{75\text{ m}}{55.0\text{ s}} = +1.36\text{ m/s}$
  - Average *speed* for trip: $\text{average speed} = \dfrac{125\text{ m}}{55.0\text{ s}} = 2.27\text{ m/s}$

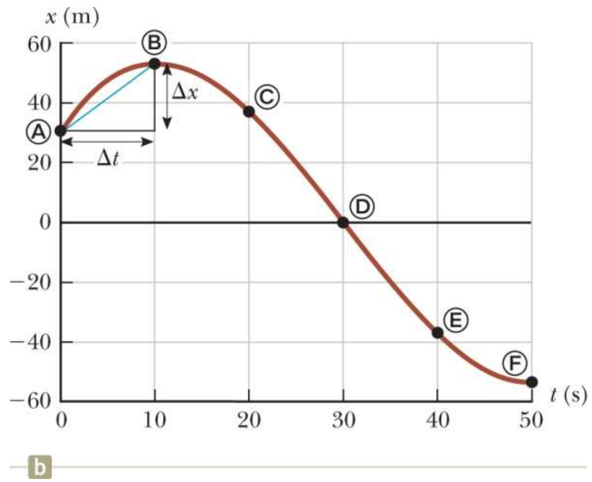
# Example 2.1: Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in the figure between positions A and F.




Consult the figure to form a mental image of the car and its motion. We model the car as a particle. From the position–time graph, notice that $x_A = 30$ m at $t_A = 0$ s and that $x_F = -53$ m at $t_F = 50$ s.

$$\begin{aligned}\Delta x &= x_F - x_A \\ &= -53\text{ m} - 30\text{ m} \\ &= -83\text{ m}\end{aligned}$$




This result means that the car ends up 83 m *in the negative direction (to the left, in this case) from where it started*. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at the figure indicates that it is the correct answer.

# Example 2.1: Calculating the Average Velocity and Speed

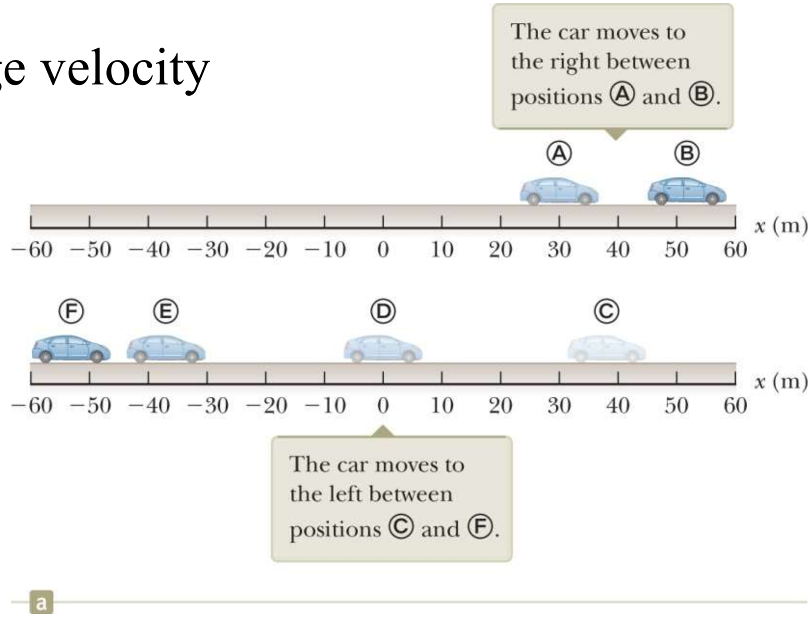
## Average velocity:

Use the equation below to find the car's average velocity

$$v_{x,\text{avg}} = \frac{x_F - x_A}{t_F - t_A}$$

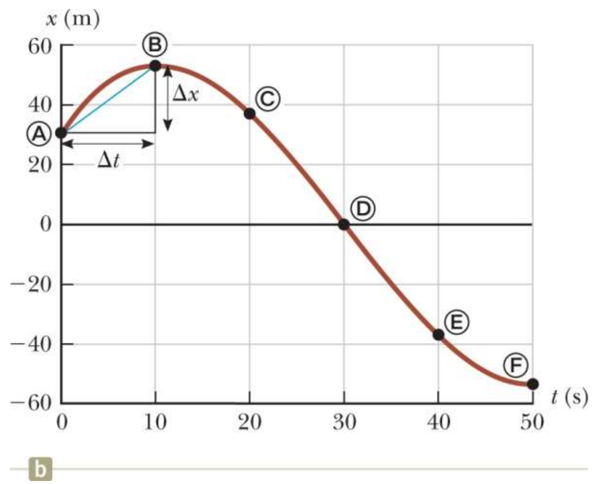
$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}}$$

$$= \frac{-83\text{ m}}{50\text{ s}} = \boxed{-1.7\text{ m/s}}$$




## Average speed:

$$v_{\text{avg}} = \frac{127\text{ m}}{50.0\text{ s}} = \boxed{2.54\text{ m/s}}$$

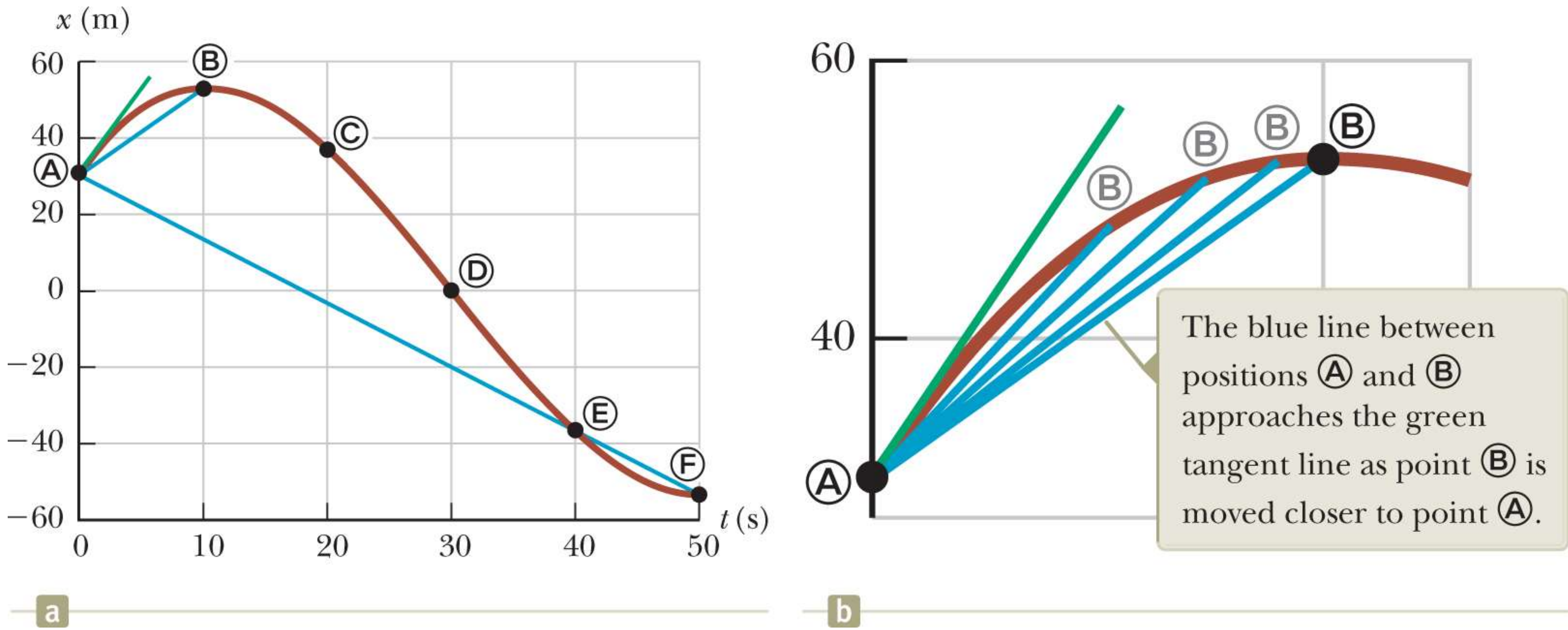


# 2.2. Instantaneous Velocity and Speed

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Consider figure (a):

- What is particle's velocity at t = 0?
- Average velocity from A to B (slope of blue line)
- Average velocity from A to F (slope of longer blue line)
- Focus on short blue line →imagine sliding point B to left along curve, toward point A, figure (b)
- Line between points becomes steeper and steeper
- As the two points become extremely close together → line becomes tangent line to curve (green line): Slope of tangent line represents velocity of car at point A



# 2.2. Instantaneous Velocity and Speed

- **Instantaneous velocity** $v_x$ = limiting value of ratio $\Delta x/\Delta t$ as $\Delta t \to 0$: $$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$
- In calculus notation: this limit called *derivative* of $x$ with respect to $t$, written $dx/dt$:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Instantaneous velocity: can be positive, negative, or zero
  - When slope of position–time graph positive (first 10 seconds) → $v_x$ positive and car moving toward larger values of $x$
  - After point B, $v_x$ negative because slope negative and car moving toward smaller values of $x$
  - At point B, slope and instantaneous velocity zero → car momentarily at rest

- ***Velocity*** designates instantaneous velocity
  - For *average velocity,* use "*average*"

- **Instantaneous speed** of particle: magnitude of its instantaneous velocity
  - No direction

- Example: one particle has instantaneous velocity of +25 m/s, another particle has instantaneous velocity of –25 m/s along same line, both have speed of 25 m/s

# Conceptual Example 2.2: The Velocity of Different Objects

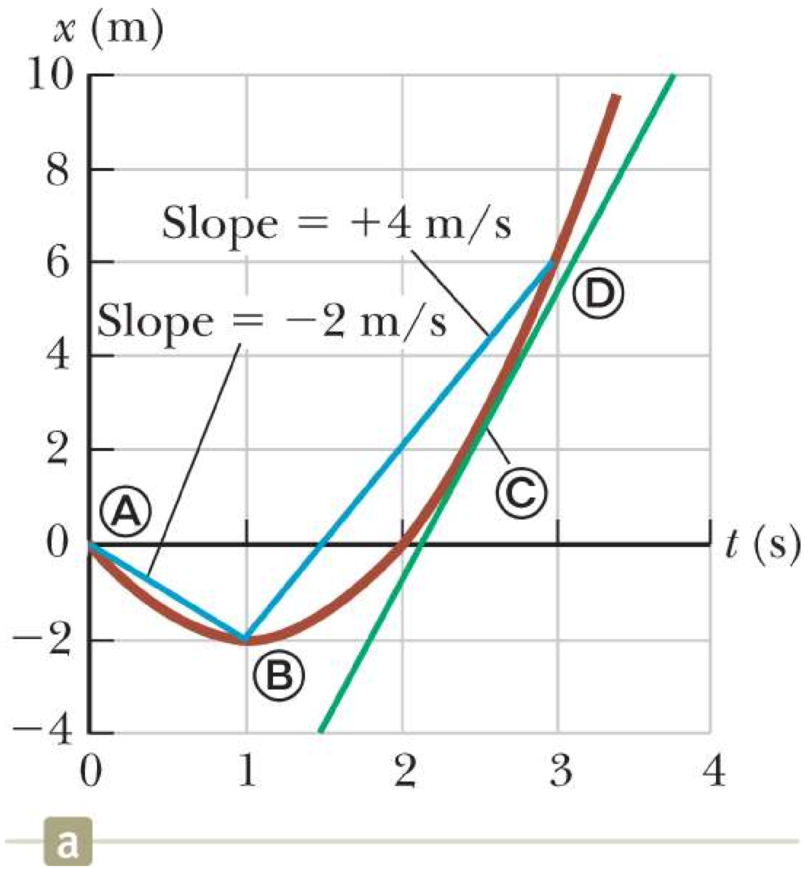
Consider the one-dimensional motion of a ball thrown directly upward, which rises to a highest point and falls back into the thrower's hand. Are there any points in the motion of this object at which the instantaneous velocity has the same value as the average velocity over the entire motion? If so, identify the point(s).

**One point at top of motion**

The average velocity for the thrown ball is zero because the ball returns to the starting point; therefore, its displacement is zero. There is one point at which the instantaneous velocity is zero: at the top of the motion.

# Example 2.3: Average and Instantaneous Velocity

A particle moves along the $x$ axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where $x$ is in meters and $t$ is in seconds. The position–time graph for this motion is shown in the figure. Because the position of the particle is given by a mathematical function, the motion of the particle is known at all times. Notice that the particle moves in the negative $x$ direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive $x$ direction at times $t > 1$ s.



# Example 2.3: Average and Instantaneous Velocity

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

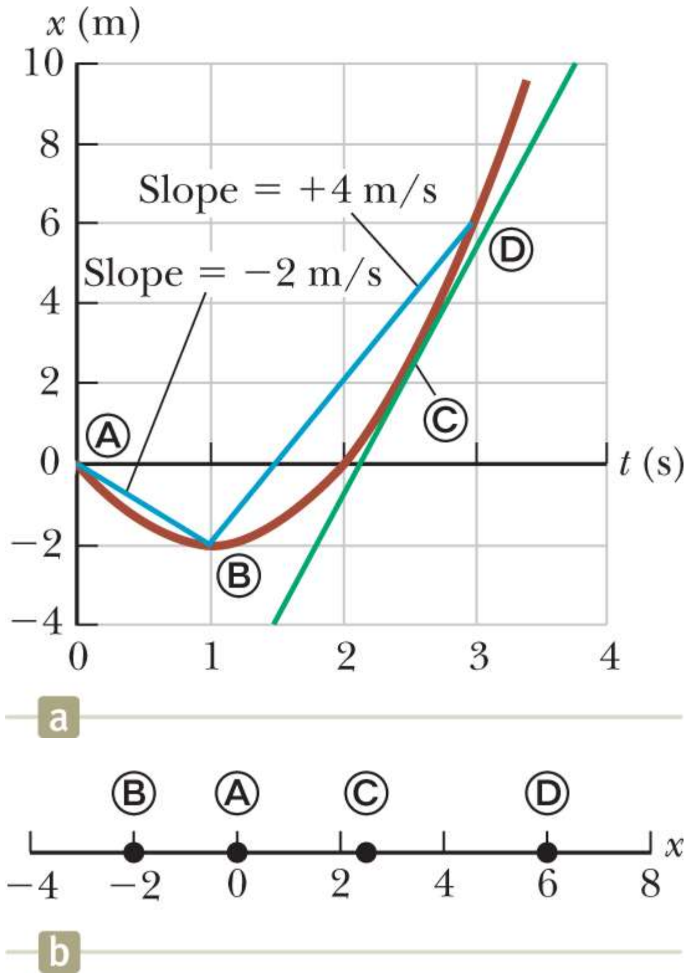
In the first time interval, set $t_i = t_A = 0$ and $t_f = t_B = 1$ s and use the equation $x = -4t + 2t^2$ to find the displacement:

$$\begin{aligned} \Delta x_{A\to B} &= x_f - x_i = x_B - x_A \\ &= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] \\ &= \boxed{-2\text{ m}} \end{aligned}$$

For the second time interval ($t = 1$ s to $t = 3$ s), set $t_i = t_B = 1$ s and $t_f = t_D = 3$ s

$$\begin{aligned} \Delta x_{B\to D} &= x_f - x_i = x_D - x_B \\ &= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] \\ &= \boxed{+8\text{ m}} \end{aligned}$$

These displacements can also be read directly from the position–time graph.



# Example 2.3:
# Average and Instantaneous Velocity

## (B) Calculate the average velocity during these two time intervals.

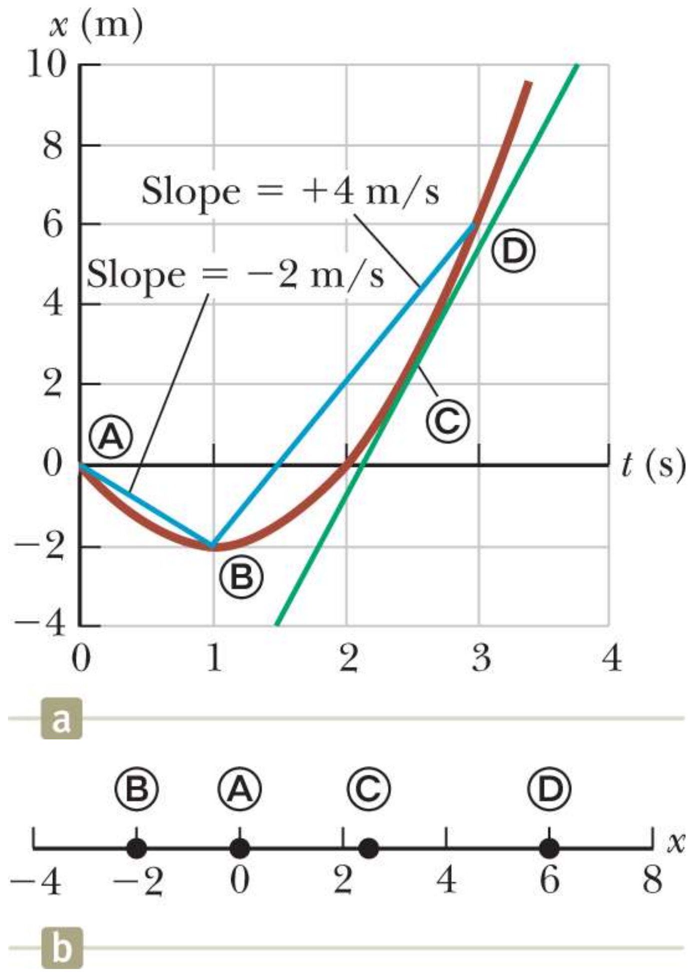
The first time interval, $\Delta t = t_f - t_i = t_B - t_A = 1$ s.

$$v_{x,\text{avg}(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = \boxed{-2\text{ m/s}}$$

The second time interval, $\Delta t = 2$ s

$$v_{x,\text{avg}(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = \boxed{+4\text{ m/s}}$$

These values are the same as the slopes of the blue lines joining these points in the graph.



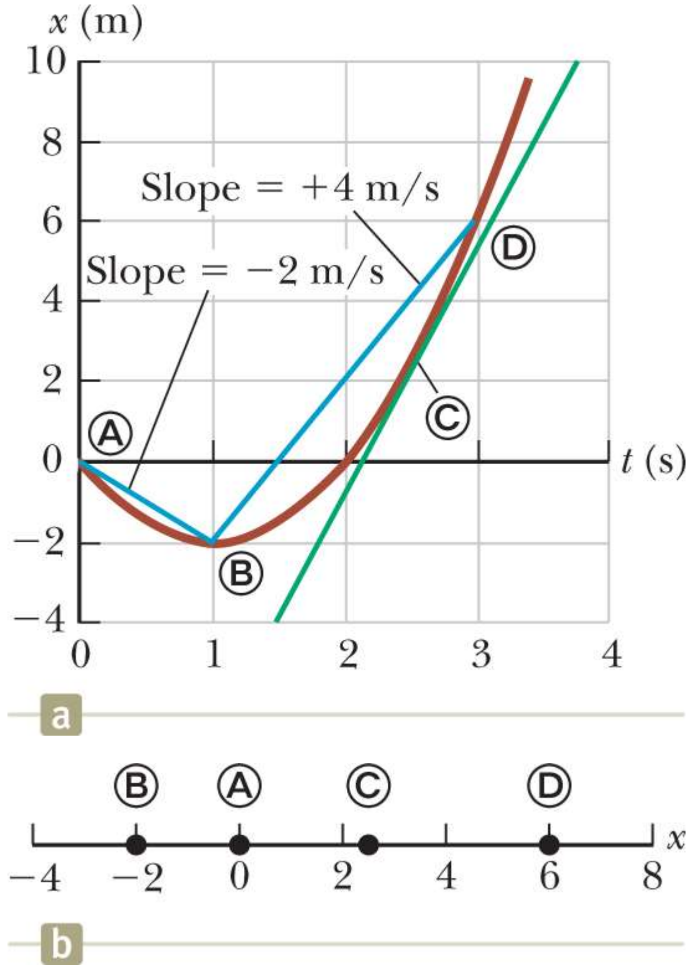
# Example 2.3: Average and Instantaneous Velocity

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

Calculate the slope of the green line at $t = 2.5$ s (point C) by reading position and time values for the ends of the green line from the graph.

$$v_x = \frac{10\text{ m} - (-4\text{ m})}{3.8\text{ s} - 1.5\text{ s}} = \boxed{+6\text{ m/s}}$$

Notice that this instantaneous velocity is on the same order of magnitude as our previous results, that is, a few meters per second

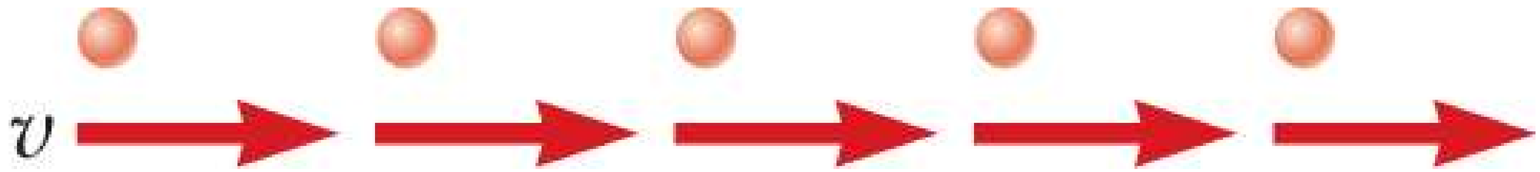


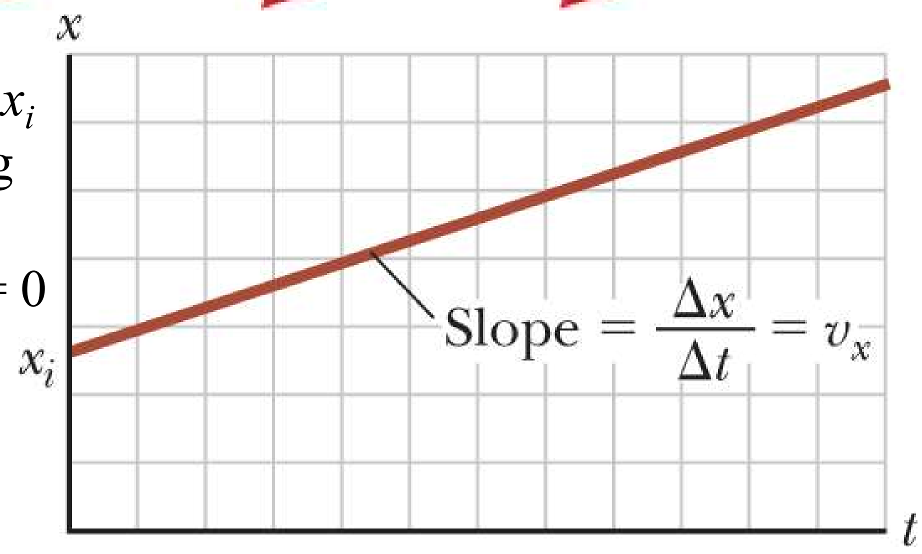
# 2.3. Analysis Model: Particle Under Constant Velocity

- Imagine particle moving with constant velocity
- Model of **particle under constant velocity** can be applied in *any* situation in which entity that can be modeled as particle moving with constant velocity : $v_x = v_{x,\,\text{avg}}$

$$v_{x,\text{avg}} \equiv \frac{\Delta x}{\Delta t} \quad \rightarrow v_x = \frac{\Delta x}{\Delta t} \qquad \Delta x = x_f - x_i \rightarrow v_x = \frac{x_f - x_i}{\Delta t} \qquad x_f = x_i + v_x \Delta t$$

$$\boxed{x_f = x_i + v_x t \;\; (\text{for constant } v_x)}$$




- Position of particle given by sum of original position $x_i$ at time $t = 0$ plus displacement $v_x \Delta t$ that occurs during time interval $\Delta t$
- Usually choose time at beginning of interval to be $t_i = 0$ and time at end of interval to be $t_f = t$

# Example 2.4: Modeling a Runner as a Particle

A kinesiologist is studying the biomechanics of the human body. (*Kinesiology* is the study of the movement of the human body. Notice the connection to the word *kinematics*.) She determines the velocity of an experimental subject while he runs along a straight line at a constant rate. The kinesiologist starts the stopwatch at the moment the runner passes a given point and stops it after the runner has passed another point 20 m away. The time interval indicated on the stopwatch is 4.0 s.

# Example 2.4:
# Modeling a Runner as a Particle

(A) What is the runner's velocity?

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{20\ \text{m} - 0}{4.0\ \text{s}} = 5.0\ \text{m/s}$$

(B) If the runner continues his motion after the stopwatch is stopped, what is his position after 10 s have passed?

$$x_f = x_i + v_x t = 0 = 0 + (5.0\ \text{m/s})(10\ \text{s}) = \boxed{50\ \text{m/s}}$$

**Conceptualize** We model the moving runner as a particle because the size of the runner and the movement of arms and legs are unnecessary details.

**Categorize** Because the problem states that the subject runs at a "*constant rate*," we can model him as a *particle under constant velocity.*

**Analyze** Having identified the model, we can use this equation to find the constant velocity of the runner.

# 2.4. Analysis Model Approach to Problem-Solving

**Conceptualize**

- *Think about* and *understand* situation
  - Carefully study any representations of information (diagrams, graphs, tables, or photographs)
- If pictorial representation not provided: make quick drawing (sketch) of situation
  - Indicate ***known values (given)*** , in a table (*first column*) or directly on sketch
- Focus on what algebraic or numerical information given in problem
  - Carefully read problem statement, look for key phrases:
    - "starts from rest" ($v_i = 0$)
    - "stops" ($v_f = 0$)
- Focus on expected result of solving problem:
  - Exactly what is question asking?
  - Indicate ***unknown values (wanted)*** , in a table (*second column*)
  - Will final result be numerical, algebraic, or verbal?
  - Do you know what units to expect?
- Incorporate information from your own experiences and common sense
  - What should a reasonable answer look like?
    - Example:
      - You wouldn't expect to calculate speed of automobile to be $5 \times 10^6$ m/s

# 2.4. Analysis Model Approach to Problem-Solving

**Categorize**

- *Simplify* problem
    - Use simplification model to remove details not important to solution
        - For example, model moving object as particle
        - If appropriate, ignore air resistance or friction between sliding object and surface
        - Add a ***list of all definitions and equations*** given for the physical concepts in a table (*third column*)
- *Categorize* problem in one of two ways:
    - Is it simple *substitution problem* such that numbers can be substituted into simple equation or definition?
        - If so, problem likely to be finished when this substitution is done
    - If not: *analysis problem:*
        - Situation must be analyzed more deeply to generate appropriate equation and reach solution
- If analysis problem → needs to be categorized further:
    - Have you seen this type of problem before?
    - Does it fall into the growing list of types of problems that you have solved previously?
        - If so, identify any *analysis model*(*s*) appropriate for problem

# 2.4. Analysis Model Approach to Problem-Solving

**Analyze**

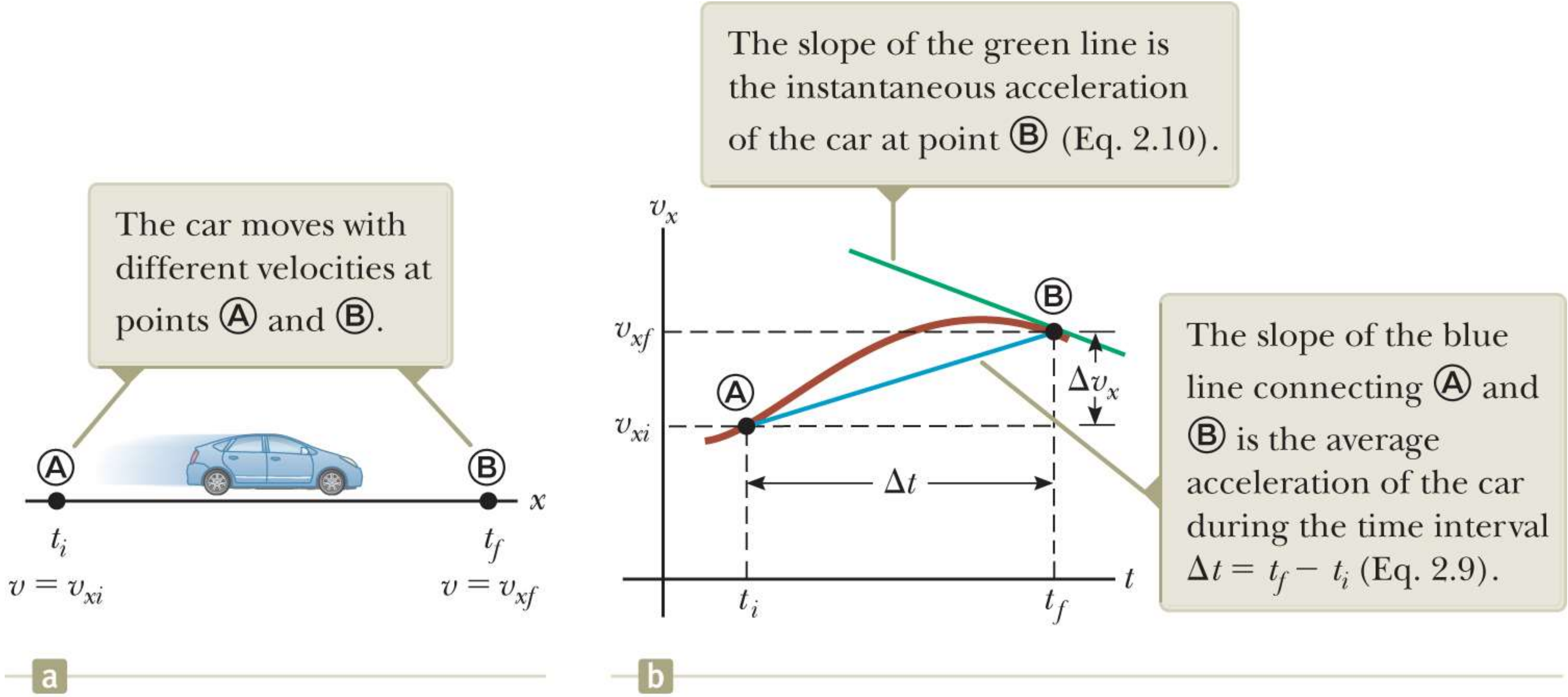
- Analyze problem and strive for mathematical solution
    - Select relevant equations that apply to type of situation in problem from the "*third* column"
- Use algebra (and calculus, if necessary) to solve symbolically for unknown variable (*wanted*) in terms of what is given
    - Finally, substitute in appropriate numbers, calculate result, and round it to proper number of significant figures

**Finalize**

- Examine numerical answer
    - Does it have the correct units?
    - What about the algebraic form of the result? Does it make sense?
- Think about how this problem compared with others you have solved
    - How was it similar? In what critical ways did it differ?
    - Why was this problem assigned?
    - If it is a new category of problem, be sure you understand it so that you can use it as a model for solving similar problems in the future

# 2.5. Average Acceleration

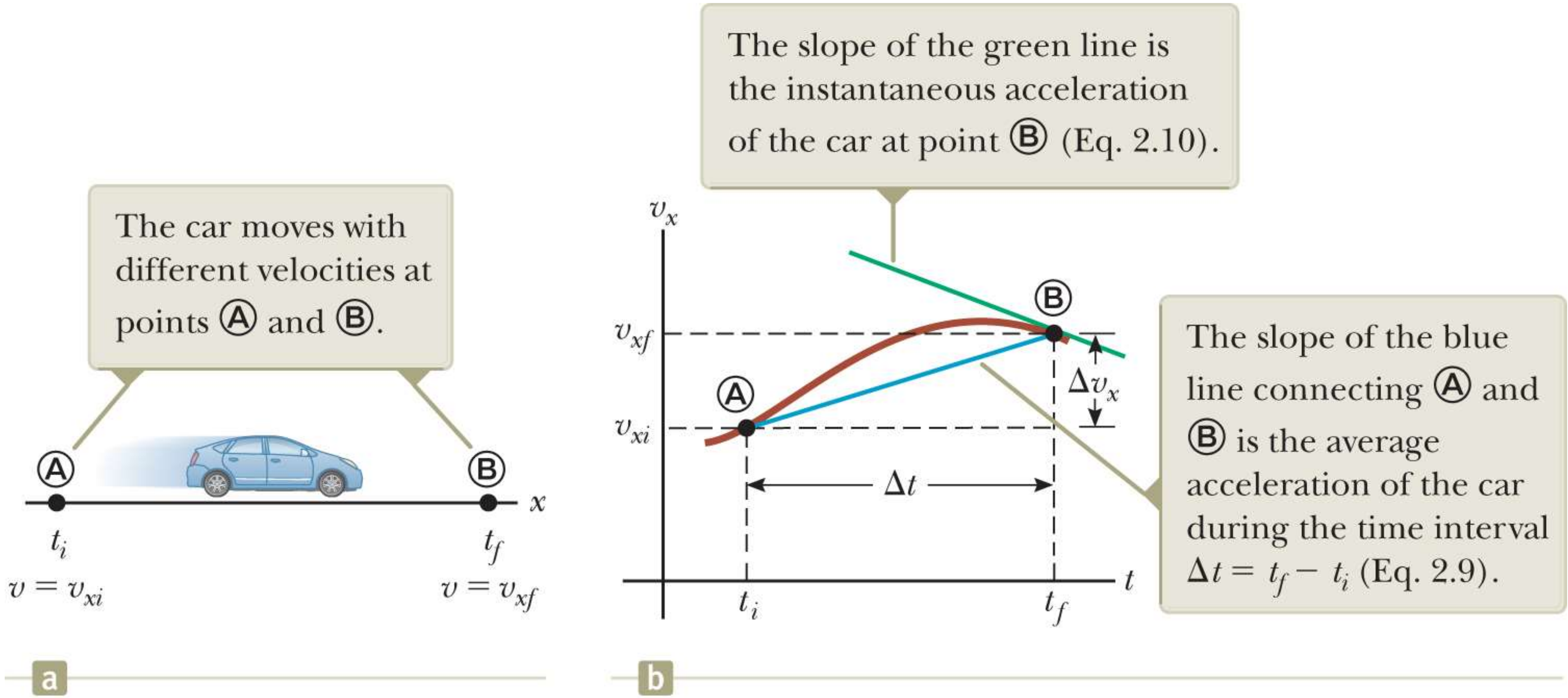
- Acceleration has dimensions of length divided by time squared, or $L/T^2$ , SI unit: $m/s^2$




**Average acceleration** $a_{x,\text{avg}}$ of particle defined as *change* in velocity $\Delta v_x$ divided by time interval $\Delta t$ during which that change occurs:

$$\boxed{a_{x,\text{avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}}$$

# 2.5. Instantaneous Acceleration




- **Instantaneous acceleration**: limit of average acceleration as $\Delta t \to$ zero

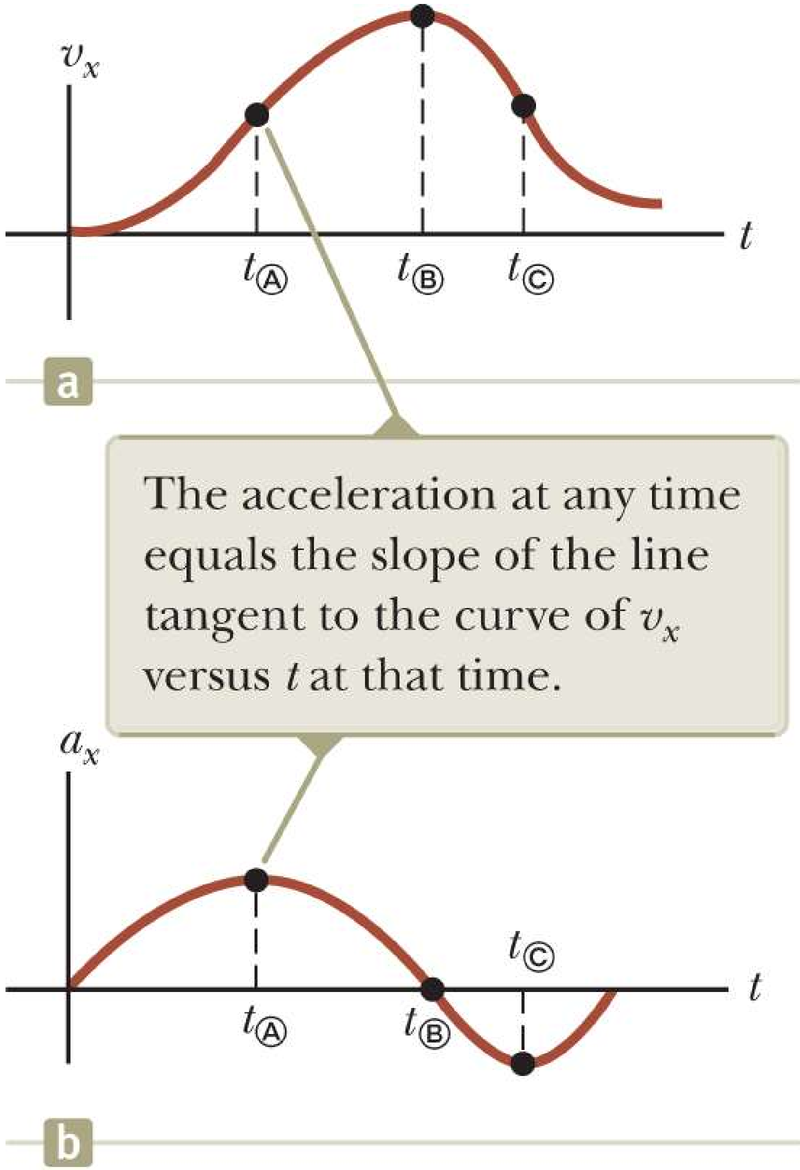
$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

# 2.5. Acceleration vs. Time Graph

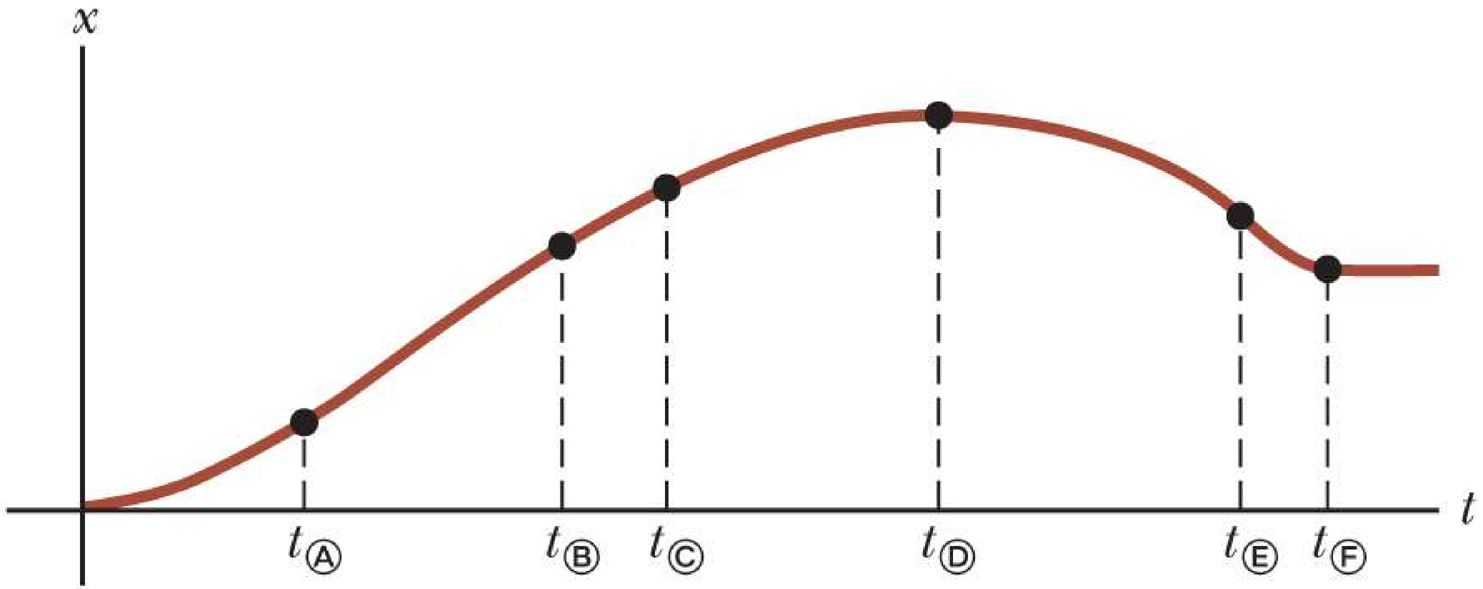
- Figure illustrates how acceleration–time graph related to velocity–time graph

- Acceleration at any time = slope of velocity–time graph at that time
  - Positive values of acceleration correspond to points where velocity increasing in positive $x$ direction
  - Acceleration reaches maximum at time $t_A$ → slope of velocity–time graph is maximum
  - Acceleration goes to zero at time $t_B$ → velocity maximum (when slope of $v_x$–$t$ graph = 0)
  - Acceleration negative when velocity decreasing in positive $x$ direction
    - Reaches most negative value at time $t_C$



# Conceptual Example 2.5: Graphical Relationships Between $x$, $v_x$, and $a_x$

The position of an object moving along the $x$ axis varies with time as in the figure. Graph the velocity versus time and the acceleration versus time for the object.



# Conceptual Example 2.5: Graphical Relationships Between $x$, $v_x$, and $a_x$

The velocity at any instant is the slope of the tangent to the $x$–$t$ graph at that instant.

Between $t = 0$ and $t = t_A$, the slope of the $x$–$t$ graph increases uniformly, so the velocity increases linearly as shown here.

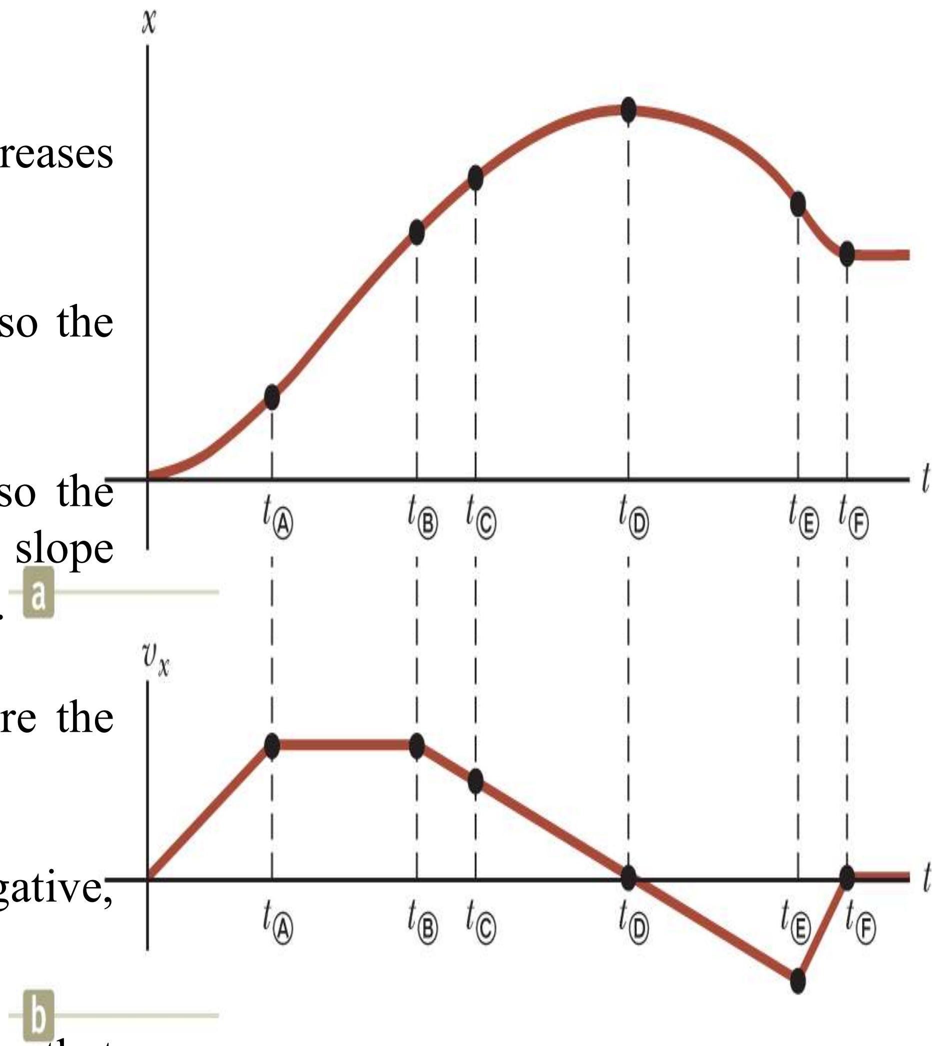
Between $t_A$ and $t_B$, the slope of the $x$–$t$ graph is constant, so the velocity remains constant.

Between $t_B$ and $t_D$, the slope of the $x$–$t$ graph decreases, so the value of the velocity in the $v_x$–$t$ graph decreases. At $t_D$, the slope of the $x$–$t$ graph is zero, so the velocity is zero at that instant.

Between $t_D$ and $t_E$, the slope of the $x$–$t$ graph and therefore the velocity are negative and decrease uniformly in this interval.

In the interval $t_E$ to $t_F$, the slope of the $x$–$t$ graph is still negative, and at $t_F$ it goes to zero.

Finally, after $t_F$, the slope of the $x$–$t$ graph is zero, meaning that the object is at rest for $t > t_F$.



# Conceptual Example 2.5: Graphical Relationships Between $x$, $v_x$, and $a_x$

The acceleration at any instant is the slope of the tangent to the $v_x$–$t$ graph at that instant.

The graph of acceleration versus time for this object is shown in the bottom figure.

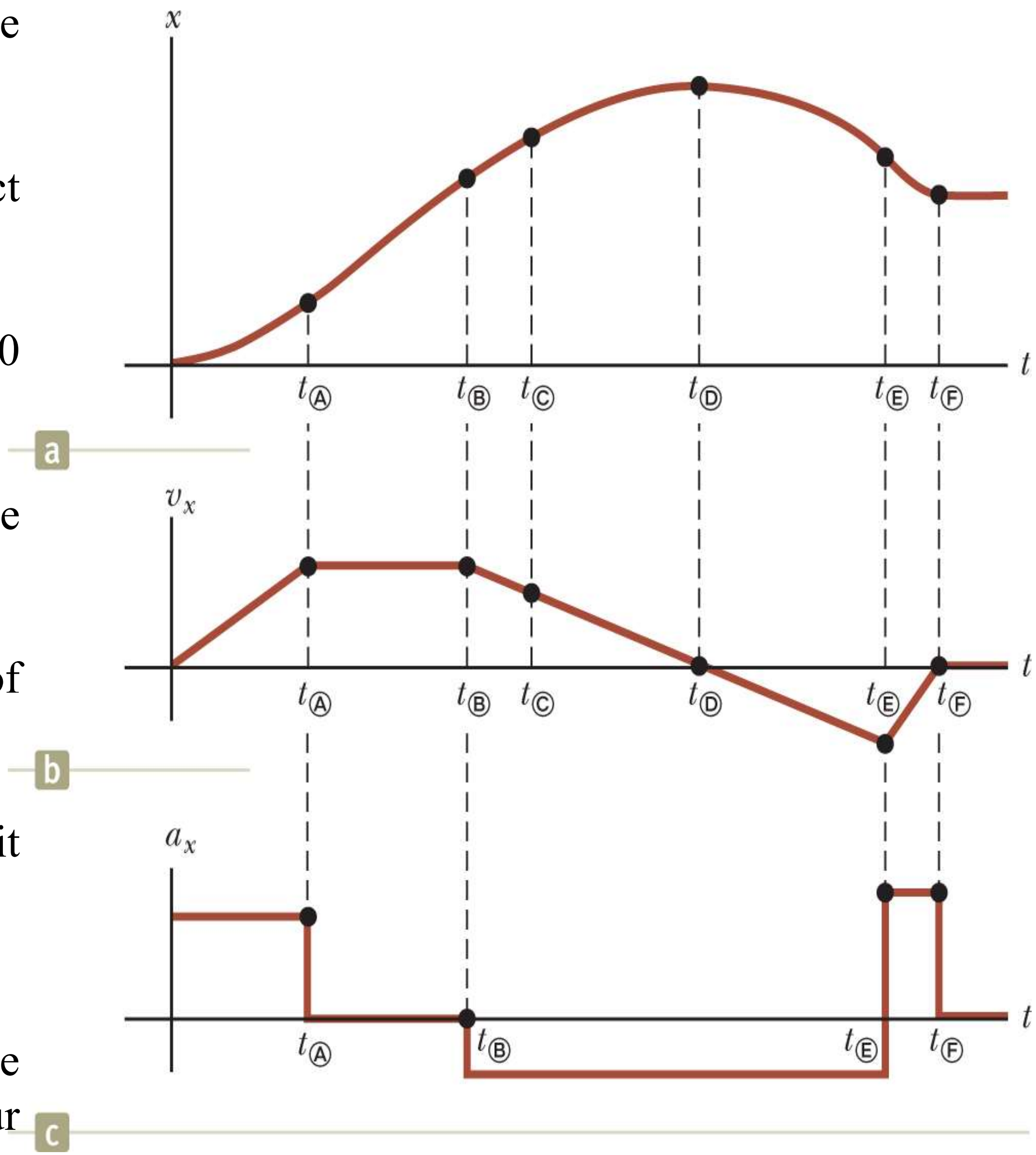
The acceleration is constant and positive between 0 and $t_A$, where the slope of the $v_x$–$t$ graph is positive.

It is zero between $t_A$ and $t_B$ and for $t > t_F$ because the slope of the $v_x$–$t$ graph is zero at these times.

It is negative between $t_B$ and $t_E$ because the slope of the $v_x$–$t$ graph is negative during this interval.

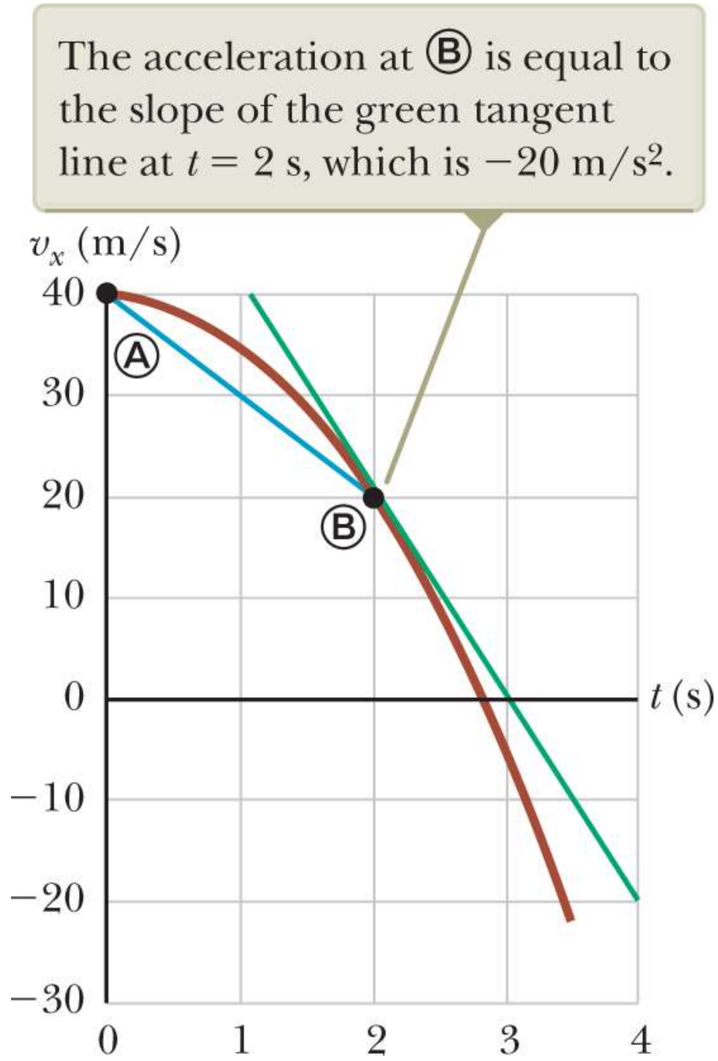
Between $t_E$ and $t_F$, the acceleration is positive like it is between 0 and $t_A$, but higher in value because the slope of the $v_x$–$t$ graph is steeper.

Notice that the sudden changes in acceleration are unphysical. Such instantaneous changes cannot occur in reality.



# Example 2.6: Average and Instantaneous Acceleration

The velocity of a particle moving along the $x$ axis varies according to the expression $v_x = 40 - 5t^2$, where $v_x$ is in meters per second and $t$ is in seconds.



# Example 2.6: Average and Instantaneous Acceleration

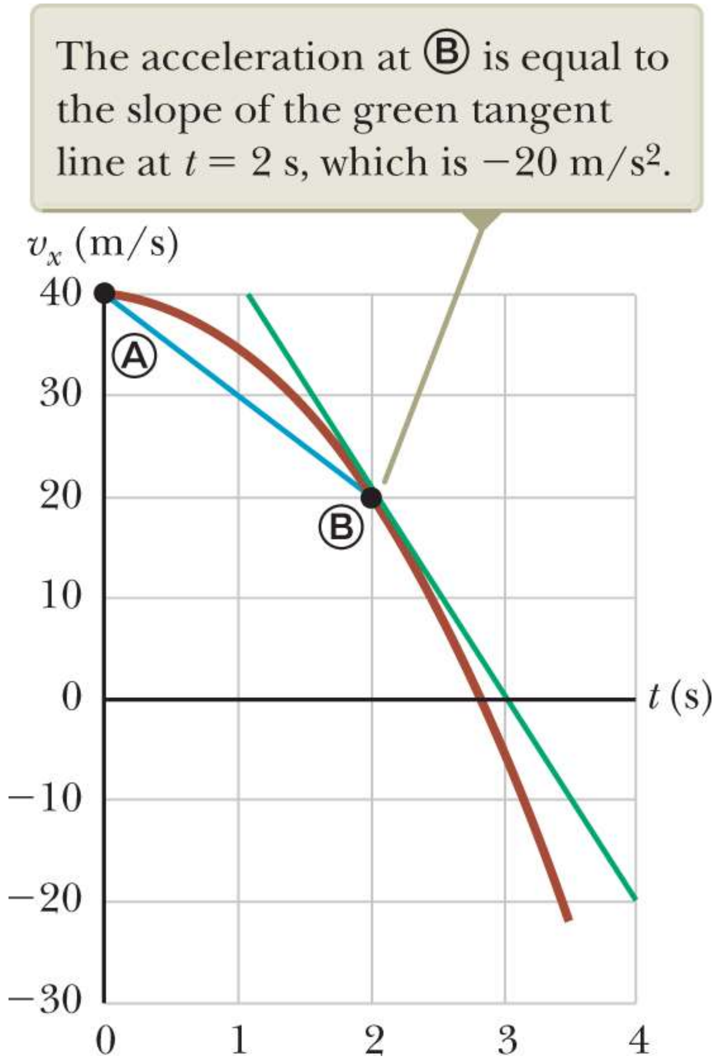
(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

**Conceptualize** Think about what the particle is doing from the mathematical representation. Is it moving at $t = 0$? In which direction? Does it speed up or slow down? Shown is a $v_x$–$t$ graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire $v_x$–$t$ curve is negative, we expect the acceleration to be negative.

**Categorize** The solution to this problem does not require either of the analysis models we have developed so far, and can be solved with simple mathematics. Therefore, we categorize the problem as a substitution problem.

$$v_{xA} = 40 - 5t_A^{\,2} = 40 - 5(0)^2 = +40 \text{ m/s}$$
$$v_{xB} = 40 - 5t_B^{\,2} = 40 - 5(2.0)^2 = +20 \text{ m/s}$$

$$a_{x,\text{avg}} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{20 \text{ m/s} - 40 \text{ m/s}}{2.0 \text{ s} - 0 \text{ s}}$$
$$= \boxed{-10 \text{ m/s}^2}$$



# Example 2.6: Average and Instantaneous Acceleration

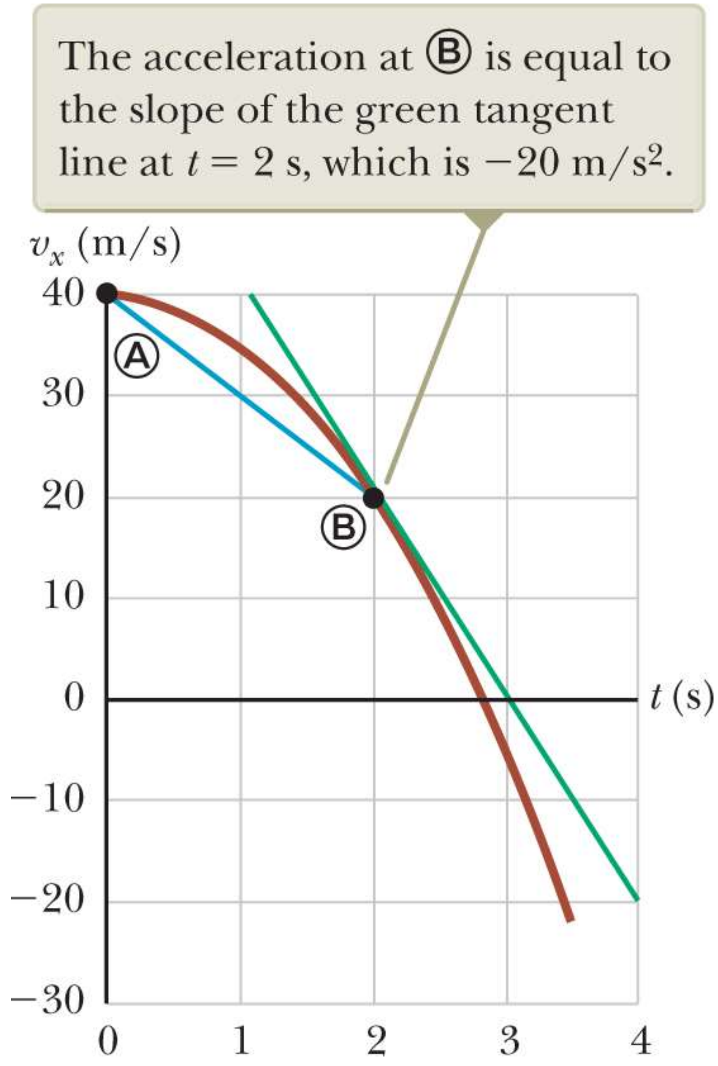
(B) Determine the acceleration at $t = 2.0$ s.

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\Delta t - 5(\Delta t)^2$$

$$\Delta v_x = v_{xf} - v_{xi} = -10t\Delta t - 5(\Delta t)^2$$

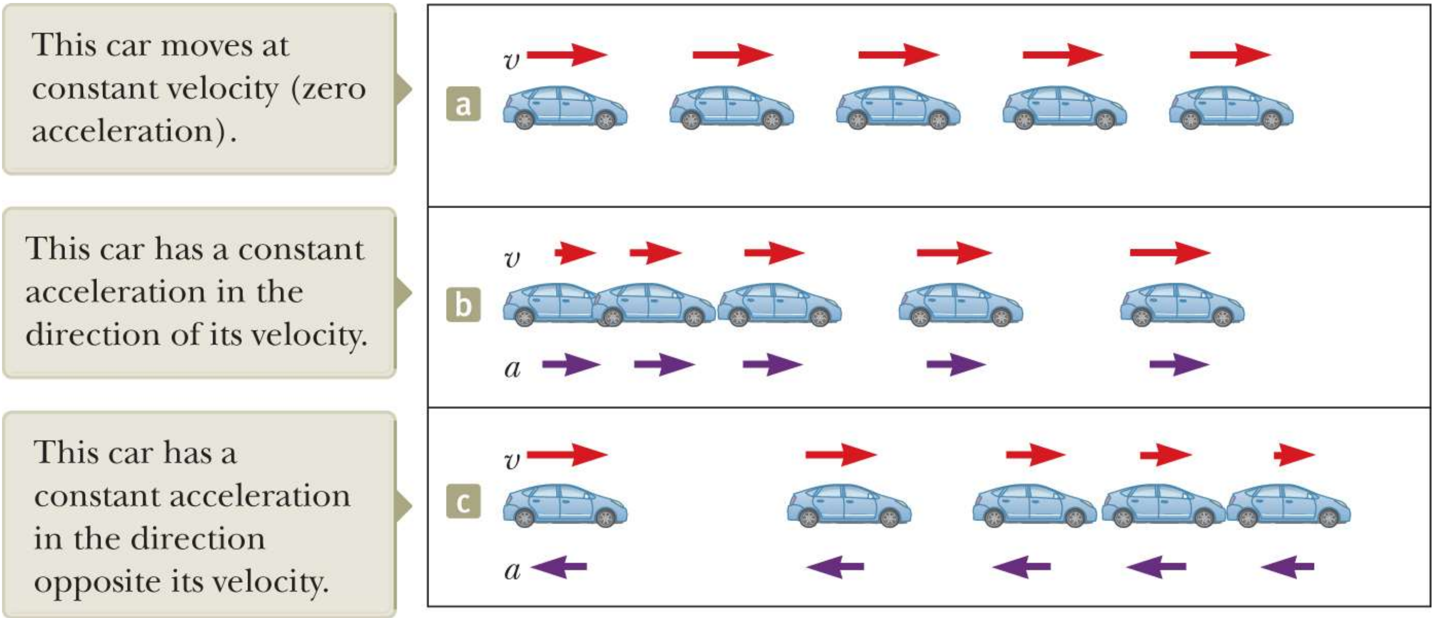
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t$$

$$a_x = (-10)(2.0)\ \text{m/s}^2 = \boxed{-20\ \text{m/s}^2}$$




**Finalize** Notice that the answers to parts (A) and (B) are different. The average acceleration in part (A) is the slope of the blue line in the graph connecting points A and B. The instantaneous acceleration in part (B) is the slope of the green line tangent to the curve at point B. Notice also that the acceleration is *not* constant in this example.

# 2.6. Motion Diagrams

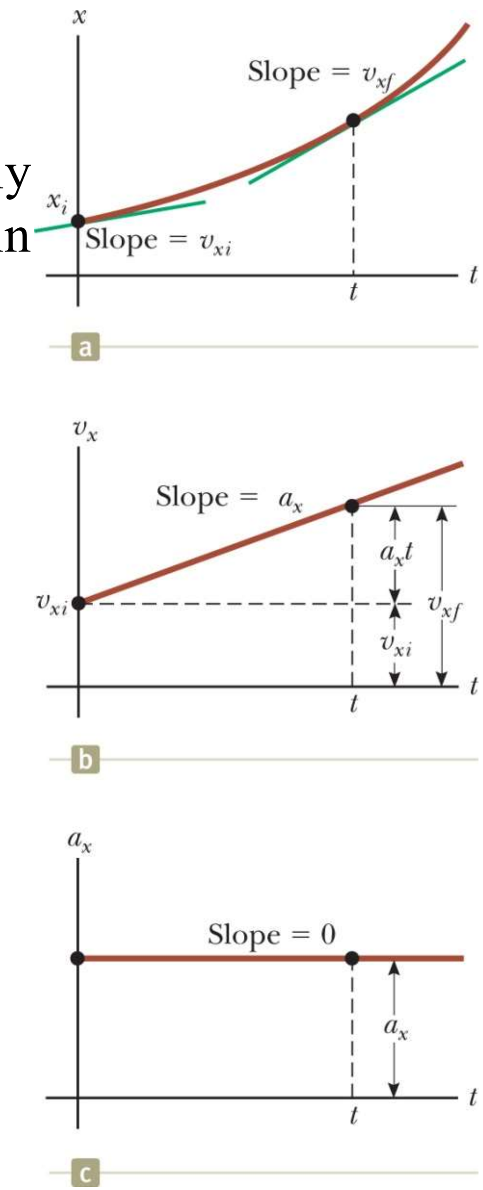



- Velocity and acceleration often confused with each other → different quantities
- Motion diagram useful in forming mental representation of moving object →
  - describes velocity and acceleration while object in motion

- Motion diagram: Imagine *a* photograph of moving object → shows several images of object taken as different time at constant rate
  - Figures: time intervals equal in each part of diagram
    - Red arrows = velocity
    - Purple arrows = acceleration

# 2.7. Analysis Model: Particle Under Constant Acceleration

- If acceleration of particle varies in time → motion can be complex and difficult to analyze
- Simple type of one-dimensional motion → acceleration is constant
  - Average acceleration $a_{x,\text{avg}}$ over any time interval numerically equal to instantaneous acceleration $a_x$ at any instant within interval
  - Velocity changes at same rate throughout motion
- Analysis model: ***particle under constant acceleration***




- Replace $a_{x,\text{avg}}$ by $a_x$ and take $t_i = 0$ and $t_f$ to be any later time $t$ to get:

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

$$\boxed{v_{xf} = v_{xi} + a_x t \text{ (for constant } a_x)}$$

Average velocity in any time interval arithmetic mean of initial velocity $v_{xi}$ and final velocity $v_{xf}$:

$$\boxed{v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2} \text{ (for constant } a_x)}$$

# 2.7. Analysis Model: Particle Under Constant Acceleration

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t}, \; v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2}$$

$$\Delta x = x_f - x_i, \; \Delta t = t_f - t_i = t - 0 = t$$

$$x_f - x_i = v_{x,\text{avg}} t = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

$$\boxed{x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \;\; (\text{for constant } a_x)}$$

- Equation provides final position of particle at time $t$ in terms of initial and final velocities

# 2.7. Analysis Model: Particle Under Constant Acceleration

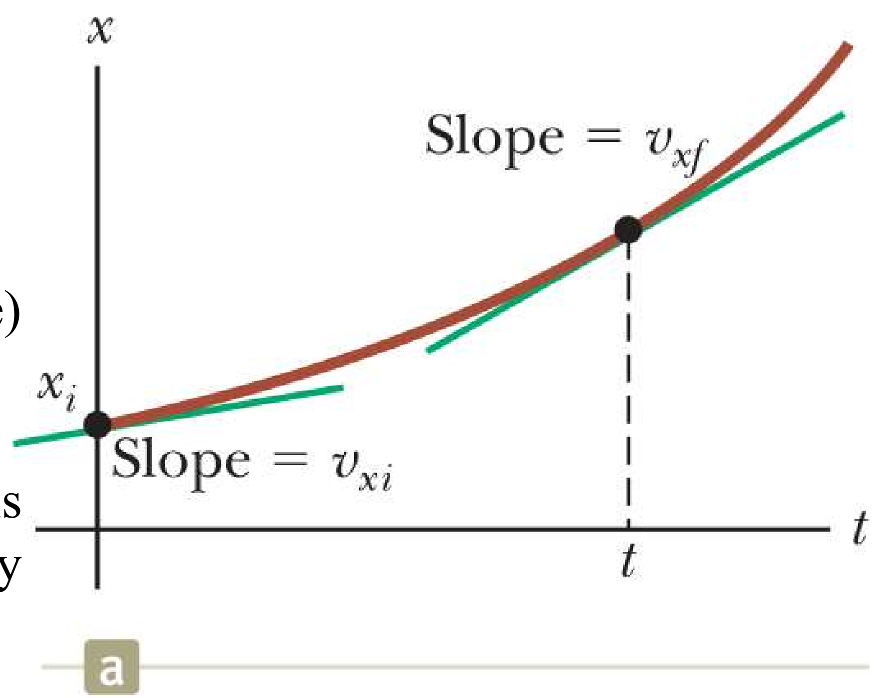
$v_{xf} = v_{xi} + a_x t$ substitute into $x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$

$$x_f = x_i + \frac{1}{2}[v_{xi} + (v_{xi} + a_x t)]t$$

- Equation: final position of particle at time $t$ in terms of initial position, initial velocity, and constant acceleration:

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad \text{(for constant } a_x\text{)}}$$

- Position–time graph for motion at constant (positive) acceleration obtained from this equation
    - Note: curve is parabola
    - Slope of tangent line to this curve at $t = 0$ equals initial velocity $v_{xi}$, and slope of tangent line at any later time $t$ equals velocity $v_{xf}$ at that time



# 2.7. Analysis Model: Particle Under Constant Acceleration

$$v_{xf} = v_{xi} + a_x t \rightarrow t = \frac{v_{xf} - v_{xi}}{a_x}$$

substitute into $x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^2 - v_{xi}^2}{2ax}$$

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \ \text{(for constant } a_x\text{)}}$$

- Result does not contain variable *t*
- Equation gives final velocity in terms of initial velocity, constant acceleration, and position of particle

# 2.7. Analysis Model: Particle Under Constant Acceleration

$$v_{xf} = v_{xi} + a_x t \quad x_f = x_i + v_x t + \frac{1}{2} a_x t^2$$

- When acceleration of particle is zero:
  - Velocity is constant
  - Position changes linearly with time
- Particle under constant acceleration model reduces to particle under constant velocity model

$$\boxed{\left.\begin{aligned} v_{xf} = v_{xi} = v_x \\ x_f = x_i + v_x t \end{aligned}\right\} \text{when } a_x = 0}$$

# Analysis Model: Particle Under Constant Acceleration

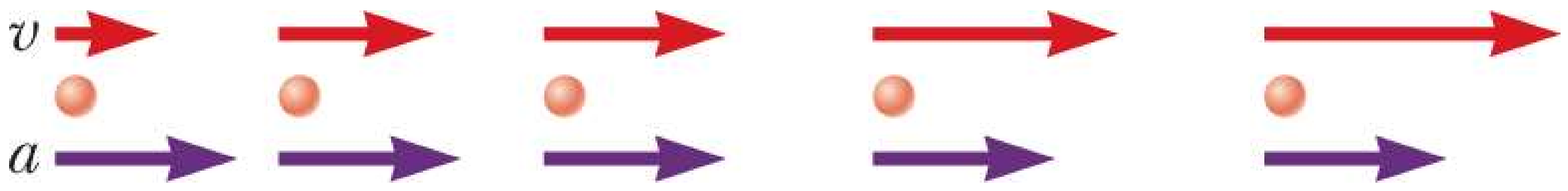
Imagine a moving object that can be modeled as a particle. If it begins from position $x_i$ and initial velocity $v_{xi}$ and moves in a straight line with a constant acceleration $a_x$, its subsequent position and velocity are described by the **kinematic equations**:

$$v_{xf} = v_{xi} + a_x t$$

$$v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2}$$

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x (x_f - x_i)$$



# Example 2.7: Carrier Landing

A jet lands on an aircraft carrier at a speed of 140 mi/h ( ≈ 63 m/s). (A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?

**Conceptualize** A careful reading of the problem reveals that in addition to being given the initial speed of 63 m/s, we also know that the final speed is zero. We define our *x* axis as the direction of motion of the jet. Notice that we have no information about the change in position of the jet while it is slowing down.

**Categorize** Because the acceleration of the jet is assumed constant, we model it as a *particle under constant acceleration*.

**Analyze** This below equation is the only equation in the particle under constant acceleration model that does not involve position, so we use it to find the acceleration of the jet, modeled as a particle.

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\ \mathrm{m/s}}{2.0\ \mathrm{s}} = \boxed{-32\ \mathrm{m/s^2}}$$

# Example 2.7: Carrier Landing

(B) If the jet touches down at position $x_i = 0$, what is its final position?

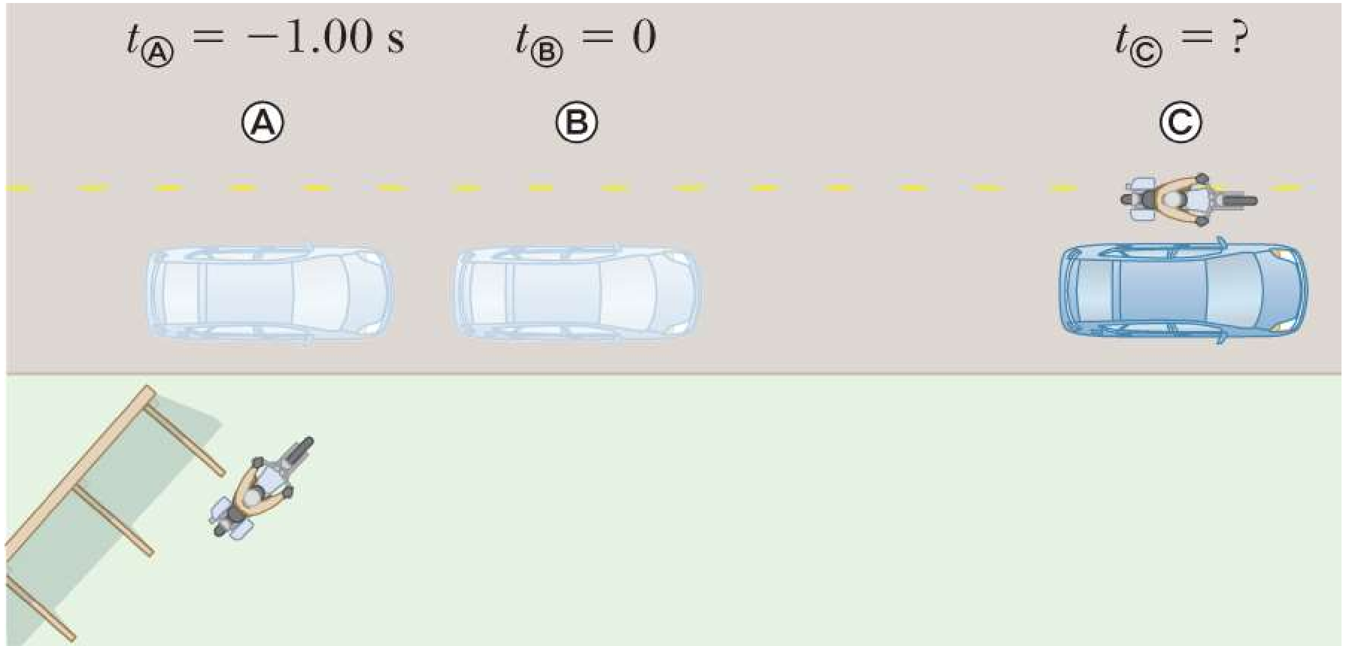
$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$$

$$= 0 + \frac{1}{2}(63\ \mathrm{m/s} + 0)(2.0\ \mathrm{s}) = \boxed{63\ \mathrm{m}}$$

**Finalize** Given the size of aircraft carriers, a length of 63 m seems reasonable for stopping the jet. The idea of using arresting cables to slow down landing aircraft and enable them to land safely on ships originated at about the time of World War I. The cables are still a vital part of the operation of modern aircraft carriers.

# Example 2.8: Watch Out for the Speed Limit!

You are driving at a constant speed of 45.0 m/s when you pass a trooper on a motorcycle hidden behind a billboard. One second after your car passes the billboard, the trooper sets out from the billboard to catch you, accelerating at a constant rate of 3.00 m/s$^2$. How long does it take the trooper to overtake your car?

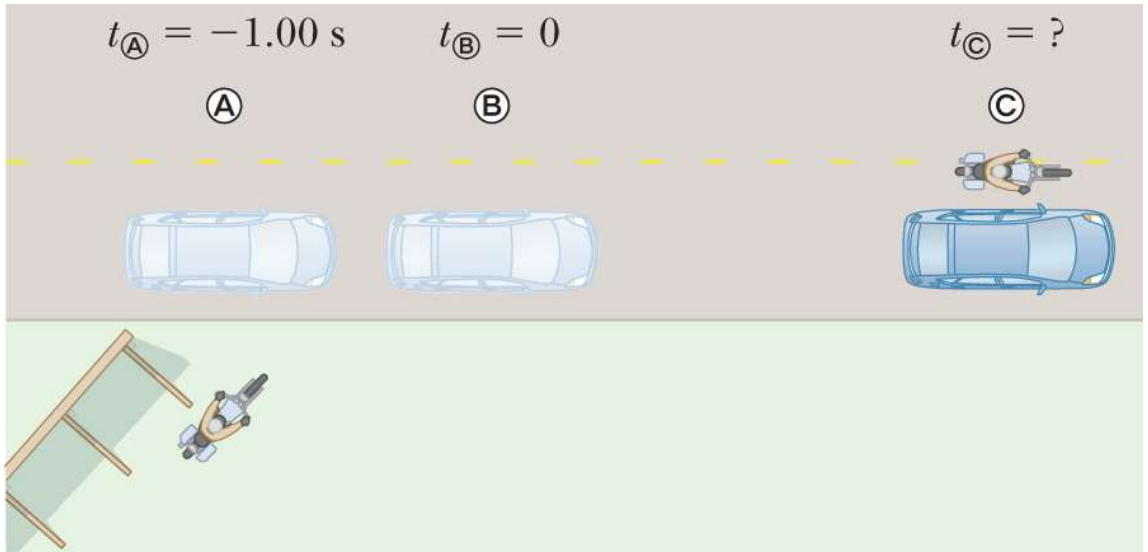


# Example 2.8: Watch Out for the Speed Limit!

**Conceptualize** This example represents a class of problems called *context-rich* problems. These problems involve real-world situations that one might encounter in one's daily life. These problems also involve "you" as opposed to an unspecified particle or object. With you as the character in the problem, *you* can make the connection between physics and everyday life!

**Categorize** A pictorial representation helps clarify the sequence of events. Your car is modeled as a *particle under constant velocity,* and the trooper is modeled as a *particle under constant acceleration.*

First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set $t_B = 0$ as the time the trooper begins moving. At that instant, the car has already traveled a distance of 45.0 m from the billboard because it has traveled at a constant speed of $v_x = 45.0$ m/s for 1 s. Therefore, the initial position of your car is $x_B = 45.0$ m.



# Example 2.8:
# Watch Out for the Speed Limit!

Using the particle under constant velocity model, apply the equation below to give your car's position at any time t:

$$x_{\text{car}} = x_{\text{B}} + v_{x\,\text{car}}t$$

A quick check shows that at $t = 0$, this expression gives your car's correct initial position when the trooper begins to move: $x_{\text{car}} = x_{\text{B}} = 45.0$ m.

The trooper starts from rest at $t_{\text{B}} = 0$ and accelerates at $a_x = 3.00\ \text{m/s}^2$ away from the origin. Use this equation to give her position at any time $t$:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$x_{\text{trooper}} = 0 + (0)t + \frac{1}{2}a_x t^2 = \frac{1}{2}a_x t^2$$

Set the positions of your car and trooper equal to represent the trooper overtaking your car at position C:

$$x_{\text{trooper}} = x_{\text{car}}$$

$$\frac{1}{2}a_x t^2 = x_{\text{B}} + v_{x\,\text{car}}t$$

# Example 2.8: Watch Out for the Speed Limit!

Rearrange to give a quadratic equation: $\frac{1}{2}a_x t^2 - v_{x\,\text{car}} t - x_B = 0$

Solve the quadratic equation for the time at which the trooper catches your car:

$$t = \frac{v_{x\,\text{car}} \pm \sqrt{v_{x\,\text{car}}^2 + 2a_x x_B}}{a_x} = \frac{v_{x\,\text{car}}}{a_x} \pm \sqrt{\frac{v_{x\,\text{car}}^2}{{a_x}^2} + \frac{2x_B}{a_x}}$$

Evaluate the solution, choosing the positive root because that is the only choice consistent with a time $t > 0$:

$$t = \frac{45.0\ \text{m/s}}{3.00\ \text{m/s}^2} + \sqrt{\frac{(45.0\ \text{m/s})^2}{(3.00\ \text{m/s}^2)^2} + \frac{2(45.0\ \text{m})}{3.00\ \text{m/s}^2}} = \boxed{31.0\ \text{s}}$$

Why didn't we choose $t = 0$ as the time at which your car passes the trooper? If we did so, we would not be able to use the particle under constant acceleration model for the trooper. His acceleration would be zero for the first second and then 3.00 m/s$^2$ for the remaining time. By defining the time $t = 0$ as when the trooper begins moving, we can use the particle under constant acceleration model for her movement for all positive times.

# 2.8. Freely Falling Objects

- In absence of air resistance: all objects dropped near Earth's surface fall toward Earth with *same constant acceleration under influence of Earth's gravity*, regardless of their masses

- Legend: Galileo observed that two different weights dropped simultaneously from Leaning Tower of Pisa hit ground at approximately same time
- Galileo performed experiments on objects moving on inclined planes
  - Rolled balls down slight incline and measured distances they covered in successive time intervals
    - Incline: reduced acceleration → easier to make accurate measurements of time intervals
    - By increasing slope of incline → drew conclusions about freely falling objects (freely falling ball equivalent to ball moving down vertical incline)

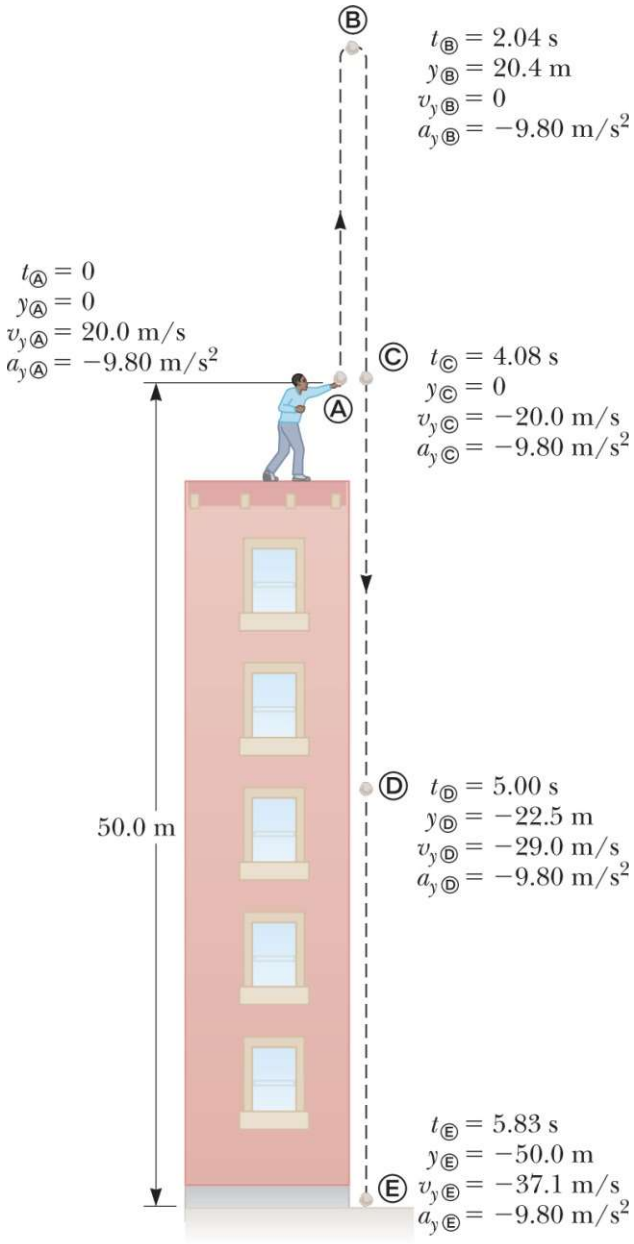
Leaning Tower of Pisa

# 2.8. Freely Falling Objects

- *Freely falling object* → not just for objects dropped from rest : Any object moving freely under influence of gravity alone, regardless of its initial motion
    - Objects thrown upward or downward and those released from rest all falling freely once released
    - Any freely falling object experiences *acceleration* directed *downward*, regardless of its initial motion
- Magnitude of *free-fall acceleration* (*acceleration due to gravity*) by symbol $\boldsymbol{g}$
    - Value of $\boldsymbol{g}$ decreases with increasing altitude above Earth's surface
    - Slight variations in $g$ occur with changes in latitude
- At Earth's surface**:** $\boldsymbol{g \approx 9.80\ \mathrm{m/s^2}}$
    - For making quick estimates, use $g \sim 10\ \mathrm{m/s^2}$
- Neglect air resistance and assume free-fall acceleration does not vary with altitude over short vertical distances:
    - Motion of freely falling object moving vertically equivalent to motion of particle under constant acceleration in one dimension
    - Can use equations for particle under constant acceleration model
- Modification for freely falling objects:
    - Motion in vertical direction ($y$ direction)
    - Acceleration downward with magnitude $9.80\ \mathrm{m/s^2}$
        - Choose $\boldsymbol{a_y = -g = -9.80\ \mathrm{m/s^2}}$

# Example 2.10:
# Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in the figure.

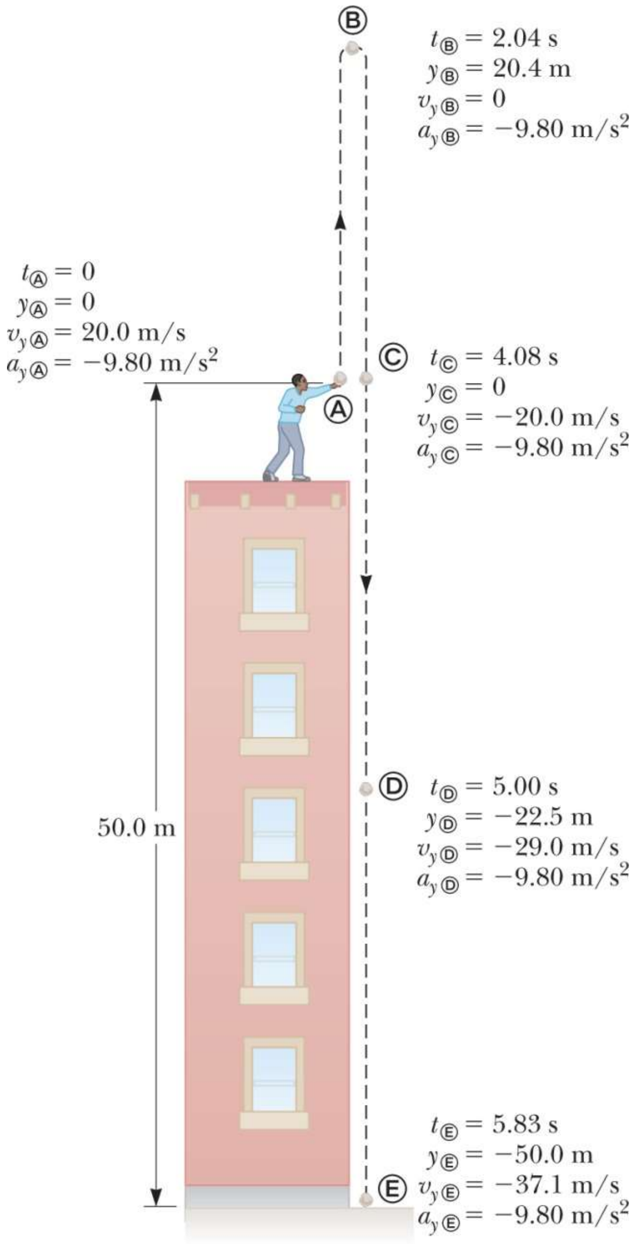


# Example 2.10: Not a Bad Throw for a Rookie!

**Conceptualize** You most likely have experience with dropping objects or throwing them upward and watching them fall, so this problem should describe a familiar experience. To simulate this situation, toss a small object upward and notice the time interval required for it to fall to the floor. Now imagine throwing that object upward from the roof of a building.

**Categorize** Because the stone is in free fall, it is modeled as a *particle under constant acceleration* due to gravity.

**Analyze** Recognize that the initial velocity is positive because the stone is launched upward. The velocity will change sign after the stone reaches its highest point, but the acceleration of the stone will *always* be downward so that it will always have a negative value. Choose an initial point just after the stone leaves the person's hand and a final point at the top of its flight.



# Example 2.10: Not a Bad Throw for a Rookie!

(A) Using $t_A = 0$ as the time the stone leaves the thrower's hand at position $A$, determine the time at which the stone reaches its maximum height.

Use the equation below to calculate the time at which the stone reaches its maximum height:

$$v_{yf} = v_{yi} + a_y t \Rightarrow t = \frac{v_{yf} - v_{yi}}{a_y} = \frac{v_{y\mathrm{B}} - v_{y\mathrm{A}}}{-g}$$

Substitute numerical values, recognizing that $v = 0$ at point B (maximum height):

$$t = t_{\mathrm{B}} = \frac{0 - 20.0\ \mathrm{m/s}}{-9.80\ \mathrm{m/s^2}} = \boxed{2.04\ \mathrm{s}}$$

# Example 2.10: Not a Bad Throw for a Rookie!

## (B) Find the maximum height of the stone (above its initial position).

As in part (A), choose the initial and final points at the beginning and the end of the upward flight:

$$y_{\text{max}} = y_B = y_A + v_{xA}t + \frac{1}{2}a_y t^2$$

Set $y_A = 0$ and substitute the time from part (A) into the equation below to find the maximum height:

$$\begin{aligned} y_B &= 0 + (20.0\ \mathrm{m/s})(2.04\ \mathrm{s}) \\ &\quad + \frac{1}{2}(-9.8\ \mathrm{m/s^2})(2.04\ \mathrm{s})^2 \\ &= \boxed{20.4\ \mathrm{m}} \end{aligned}$$

# Example 2.10: Not a Bad Throw for a Rookie!

## (C) Determine the velocity of the stone when it returns to the height from which it was thrown.

Choose the initial point where the stone is launched and the final point when it passes this position coming down. Substitute known values into the equation below:

$$v_{y\mathrm{C}}{}^2 = v_{y\mathrm{A}}{}^2 + 2a_y(y_\mathrm{C} - y_\mathrm{A})$$

$$v_{y\mathrm{C}}{}^2 = (20.0\ \mathrm{m/s})^2 + 2(-9.80\ \mathrm{m/s^2})(0 - 0)$$

$$= 400\ \mathrm{m^2/s^2}$$

$$v_{yC} = \boxed{-20.0\ \mathrm{m/s}}$$

When taking the square root, we could choose either a positive or a negative root. We choose the negative root because we know that the stone is moving downward at point C. The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but is opposite in direction.

# Example 2.10: Not a Bad Throw for a Rookie!

(D) Find the velocity and position of the stone at $t = 5.00$ s.

Choose the initial point just after the throw and the final point 5.00 s later. Calculate the velocity at D from the equation below:

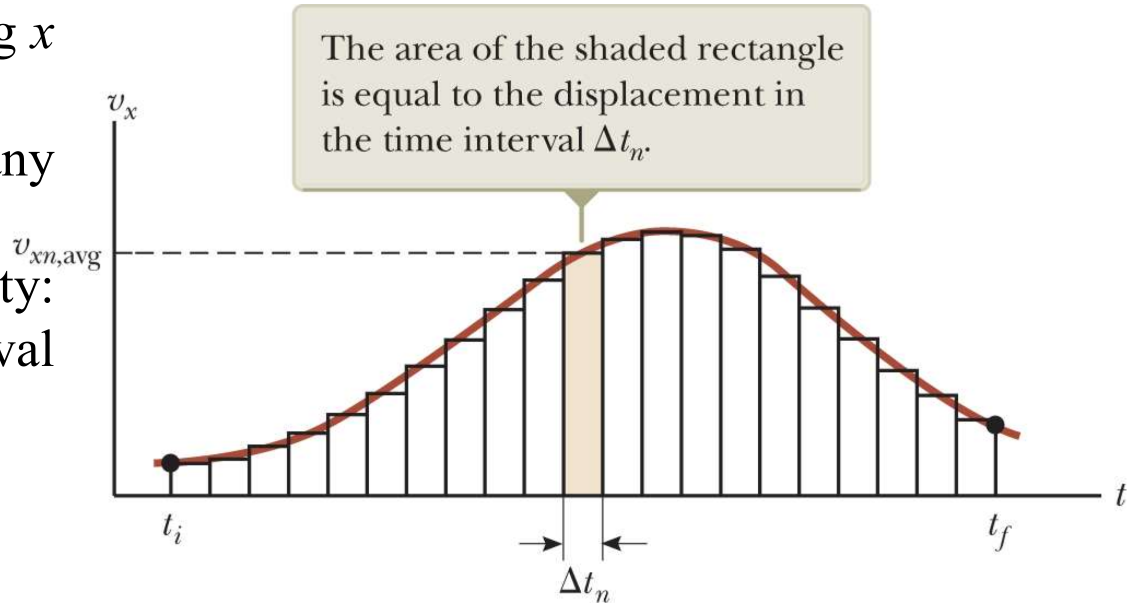
$$v_{y\mathrm{D}} = v_{y\mathrm{A}} + a_y t$$
$$= 20.0\ \mathrm{m/s} + (-9.80\ \mathrm{m/s^2})(5.00\ \mathrm{s}) = \boxed{-29.0\ \mathrm{m/s}}$$

Use the equation below to find the position of the stone at $t_\mathrm{D} = 5.00$ s:

$$y_\mathrm{D} = y_\mathrm{A} + v_{y\mathrm{A}} t + \frac{1}{2} a_y t^2$$
$$= 0 + (20.0\ \mathrm{m/s})(5.00\ \mathrm{s}) + \frac{1}{2}(-9.80\ \mathrm{m/s^2})(5.00\ \mathrm{s})^2$$
$$= \boxed{-22.5\ \mathrm{m}}$$

# 2.9. Kinematic Equations Derived from Calculus

- Figure: $v_x$–$t$ graph for particle moving along $x$ axis
  - Divide time interval $t_f - t_i$ into many small intervals $\Delta t_n$
- From definition of average velocity: displacement of particle during small interval (i.e., shaded area in figure) given by:




$$\Delta x_n = v_{xn,\text{avg}} \Delta t_n$$

  - Displacement during small interval = area of shaded rectangle

- Total displacement for interval $t_f - t_i$ = sum of the areas of all rectangles from $t_i$ to $t_f$ :

$$\Delta x = \sum_n v_{xn,\text{avg}} \Delta t_n$$

  - Sigma signifies sum over all values of $n$

# 2.9. Kinematic Equations Derived from Calculus

- *A*s intervals made **smaller** and **smaller** → **number of terms** in sum **increases**
    - Sum approaches value equal to area under curve in velocity–time graph

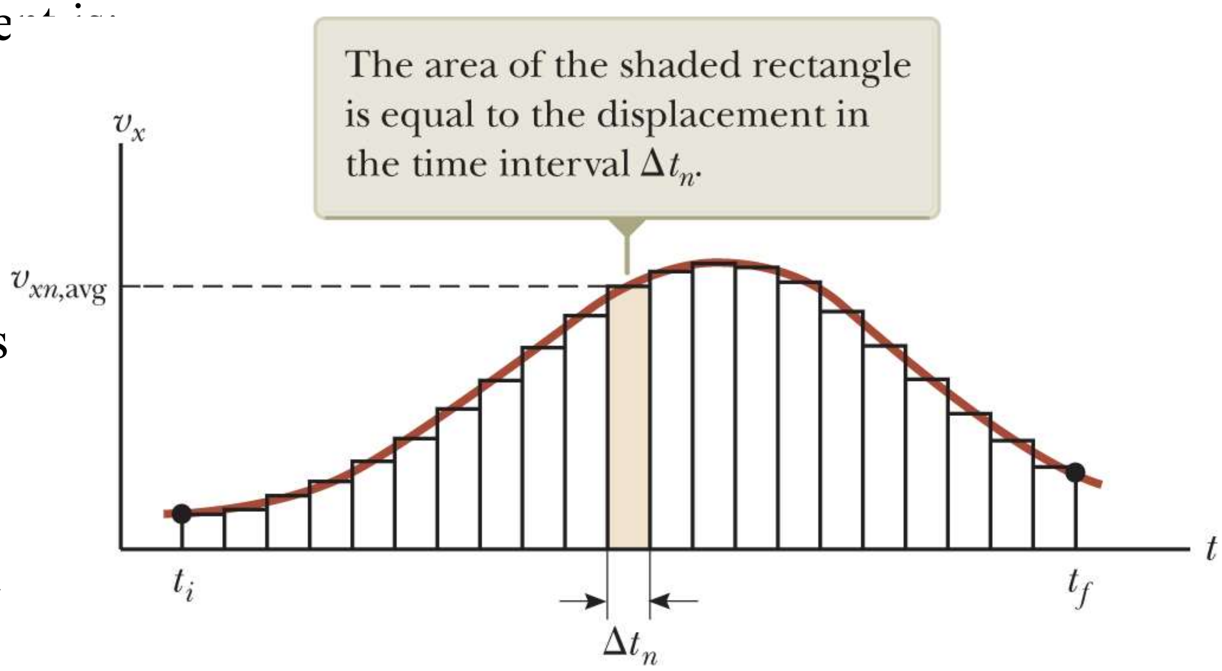
- In limit $n \to \infty$, or $\Delta t_n \to 0$: displaceme

$$\boxed{\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn,\text{avg}} \Delta t_n}$$

- Limit of sum = **definite integral** and displacement of particle can be written as

$$\boxed{\Delta x = \int_{t_i}^{t_f} v_x(t)\, dt}$$

    - $v_x(t)$ denotes velocity at any time $t$




- If explicit functional form of $v_x(t)$ known and limits given, the integral can be evaluated
- Velocity of particle moving in straight line → derivative of position with respect to time

- Can also find position of particle if its velocity known as function of time
    - In calculus: *integration* or find *antiderivative*

# 2.9. Kinematic Equations Derived from Calculus

- Defining equation for acceleration:

$$a_x = \frac{dv_x}{dt} \Rightarrow dv_x = a_x dt$$

- In terms of integral (or antiderivative):

$$\boxed{v_{xf} - v_{xi} = \int_0^t a_x dt}$$

- For special case where acceleration constant → $a_x$ can be removed from integral:

$$v_{xf} - v_{xi} = a_x \int_0^t dt = a_x(t - 0) = a_x t$$

  - Equation in particle under constant acceleration model

# 2.9. Kinematic Equations Derived from Calculus

- Defining equation for velocity: $v_x = \frac{dx}{dt} \Rightarrow dx = v_x dt$
- Can write in integral form as: $\boxed{x_f - x_i = \int_0^t v_x dt}$
- Because: $v_x = v_{xf} = v_{xi} + a_x t$ , this expression becomes:

$$x_f - x_i = \int_0^t (v_{xi} + a_x t)\, dt$$

$$= \int_0^t v_{xi}\, dt + a_x \int_0^t t\, dt = v_{xi}(t - 0) + a_x\left(\frac{t^2}{2} - 0\right)$$

$$\boxed{x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2}$$

- Equations in particle under constant acceleration model