University of Science and Technology at Zewail City

Finite Element Analysis SPC 402

Submitted to

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From:

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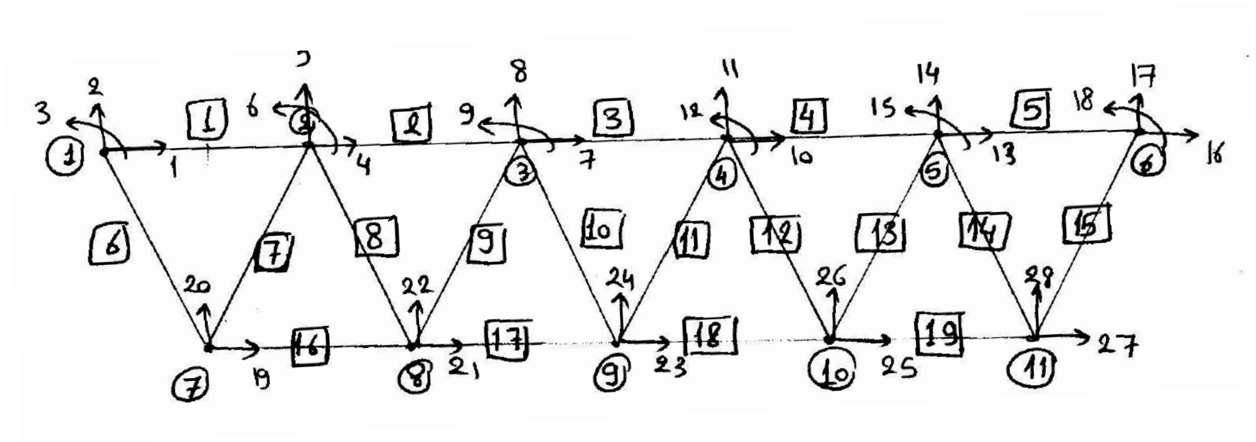
Analytical Solution

- The resulted K is 28*28 and the load vector is 28*1: as there are 28 degrees of freedom according to the following sketch.



Where the number degrees of freedom for each node in the upper nodes (from 1 till 6) is three, and two for the nodes from the rest (from 7 till 11).

Total degrees of freedom = (6*3) + (5*2) = 28

- **K locals**

1. **For the beam members**

$$
k=\left[\begin{array}{ccc|ccc}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
\hline
-\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{array}\right]
$$

2. **For the truss members**

$$[\mathbf{k}] = \frac{EA}{L}\left[\begin{array}{cc|cc} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ \hline -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{array}\right]$$

- **Load vector**

1. **For the beam members**

The local load vector is from two contributions

a) **Distributed load.**

$$L_i = \begin{bmatrix} 0 \\ L_2 \\ L_3 \end{bmatrix} \qquad L_j = \begin{bmatrix} 0 \\ L_4 \\ L_5 \end{bmatrix} \qquad L_e = \begin{bmatrix} L_i \\ L_j \end{bmatrix}$$

$$L_2 = \int_{-1}^{1} \omega_0 \cdot H_1(\xi) \cdot \frac{\partial x}{\partial \xi} \cdot \partial \xi$$

$$H_1(\xi) = \frac{1}{2} - \frac{3}{4}\xi + \frac{1}{4}\xi^3$$

$$\therefore L_2 = \int_{-1}^{1} \omega_0 \left(\frac{1}{2} - \frac{3}{4}\xi + \frac{1}{4}\xi^3\right)\left(\frac{l}{2}\right) d\xi$$

$$= \frac{\omega_0 l}{2}\left[\frac{1}{2}\xi - \frac{3}{8}\xi^2 + \frac{1}{16}\xi^4\right]_{-1}^{1} = \frac{\omega_0 l}{2}$$

$$L_3 = \int_{-1}^{1} \omega_0 \cdot H_2(\xi) \cdot \frac{\partial x}{\partial \xi} \cdot \partial x$$

$$H_2(\xi) = l\left[-\frac{1}{8}\xi^2 + \frac{1}{8} + \frac{1}{8}\xi^3 - \frac{1}{8}\xi\right]$$

$$\therefore L_3 = \int_{-1}^{1} \frac{\omega_0 l^2}{2}\left[-\frac{1}{8}\xi^2 + \frac{1}{8} + \frac{1}{8}\xi^3 - \frac{1}{8}\xi\right] \partial \xi$$

$$L_3 = \frac{\omega_0 l^2}{12}$$

$$L_4 = \int_{-1}^{1} \omega_0 \, H_3(\xi) \cdot \frac{\partial x}{\partial \xi} \, \partial \xi$$

$$H_3(\xi) = \frac{1}{2} + \frac{3}{4}\xi - \frac{1}{4}\xi^3$$

$$\therefore L_4 = \frac{\omega_0 l}{2} \int_{-1}^{1} \left[\frac{1}{2} + \frac{3}{4}\xi - \frac{1}{4}\xi^3\right] \partial \xi$$

$$L_4 = \frac{\omega_0 l}{2}$$

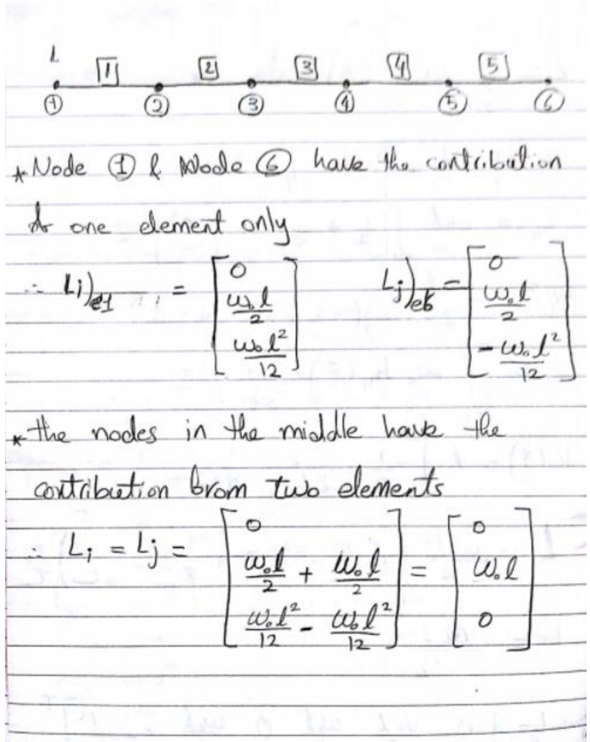
$$L_5 = \int_{-1}^{1} \omega_0 \, H_4(\xi) \cdot \frac{\partial x}{\partial \xi} \, \partial \xi$$

$$H_4(\xi) = l\left[-\frac{1}{8} - \frac{1}{8}\xi + \frac{1}{8}\xi^2 + \frac{1}{8}\xi^3\right]$$

$$\therefore L_5 = \frac{\omega_0 l^2}{2} \int_{-1}^{1} \left(-\frac{1}{8} - \frac{1}{8}\xi + \frac{1}{8}\xi^2 + \frac{1}{8}\xi^3\right) d\xi$$

$$L_5 = -\frac{\omega_0 l^2}{12}$$

$$\therefore L_e = \left[0 \quad \frac{\omega_0 l}{2} \quad \frac{\omega_0 l^2}{12} \quad 0 \quad \frac{\omega_0 l}{2} \quad -\frac{\omega_0 l^2}{12}\right]^T$$



* Node ① & Node ⑥ have the contribution of one element only

$$\therefore L_i)_{e1} = \begin{bmatrix} 0 \\ \frac{w_0 l}{2} \\ \frac{w_0 l^2}{12} \end{bmatrix} \qquad L_j)_{e6} = \begin{bmatrix} 0 \\ \frac{w_0 l}{2} \\ -\frac{w_0 l^2}{12} \end{bmatrix}$$

* the nodes in the middle have the contribution from two elements

$$\therefore L_i = L_j = \begin{bmatrix} 0 \\ \frac{w_0 l}{2} + \frac{w_0 l}{2} \\ \frac{w_0 l^2}{12} - \frac{w_0 l^2}{12} \end{bmatrix} = \begin{bmatrix} 0 \\ w_0 l \\ 0 \end{bmatrix}$$

b) External forces.

- ❑ There is only external forces at node 1 and node 6, then; the load vector due to the external forces is $[R1\ R2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ R17\ 0]^T$
- ❑ Load Vector for the beam elements= Load Vector due to distributed load + due to external force.

2. For the Truss members

- ❑ there is no external forces then the truss load vector is a zero vector of length = 10
- ❑ Global Load vector is [Load vector of beam members, Load vector of truss members]^T.

MATLAB Solution

- **Procedures**

1. **Solution Phase**

a) **local K's calculation**

Local K for the beam members

```
 m=Eb*Ib/le^3;
 n=Ab*Eb/le;
K{i}=[n 0 0 -n 0 0 ;...
    0 m*12 m*6*le 0 -12*m 6*le*m;...
    0  6*m*le 4*m*le^2 0 -6*m*le 2*m*le^2;...
    -n 0 0 n 0 0 ; ...
    0  -12*m -6*m*le 0  12*m -6*m*le;...
    0 6*m*le 2*m*le^2 0  -6*m*le 4*m*le^2];%
```

Local K for the truss members

```
 C=cosd(theta(i));
 S=sind(theta(i));
 K{i}=(A*Et/le)*[C*C C*S -C*C -C*S;C*S S*S -C*S -S*S;...
-C*C -C*S C*C C*S; -C*S -S*S C*S S*S];
```

b) K_global assembly

- **Formulating the K_vector**

```
iii=1;
for ii=1:19
for jj=1:length(K{ii}) % raw loop
for iij=1:length(K{ii}) %col. loop

    p(iii,1)=K{ii}(jj,iij);
iii=iii+1;
end
end
end
Kvec=p;
```

c) Calculating the global L vector

```
le=2; %meter
w=-0.9*9.81*1000; %N/m
```

```
%% L global
lb =[0 w*le/2 w*le^2/12 ]';
lb2=[0 w*le/2 -w*le^2/12 ]';
R=[R1;R2;0;0;0;0;0;0;0;0;0;0;0;0;0;0;R17;0];
Lb=[lb;lb2+lb;lb2+lb;lb2+lb;lb2+lb;lb2]+R;
Lt=[0 0 0 0 0 0 0 0 0 0]';
L=[Lb;Lt];
```

d) Calculating the Deltas

```
k_global(1,:)=[]; k_global(:,1)=[]; L(1)=[];
k_global(1,:)=[]; k_global(:,1)=[]; L(1)=[];
k_global(15,:)=[]; k_global(:,15)=[]; L(15)=[];
%% Deltas

u=k_global\L;
u = [0; 0; u];
u1=u(1:16);
u2=u(17:27);
u=[u1;0;u2];
```

E) Calculating the Internal Forces and weights

F = k*T*u

2. Optimization Phase

a) Observed from solution

- ❑ Axial force of elements (17 18) are the same = F3, let their area are A3
- ❑ Axial force of elements (8 9 12 13) are the same = F2, let their area are A2
- ❑ Axial force of elements (6 7 14 15 16 19) are the same = F1, let their area are A1
- ❑ Axial force of elements (14 15) are zero force members, let their area are A4 is approximate zero
- ❑ From Results F2/F3 = 0.31621, so A2/A3 roughly is the same ratio
- ❑ From Results F1/F3 = 0.6838, so A1/A3 roughly is the same ratio
- ❑ Based on solution observations, weight is mainly function of h and A3

b) Procedures

- ❑ Assume interval for h and A3, where h belongs to [0.0001 0.0068], and A3 belongs to [0.00001 0.001]
- ❑ Looping for each combination of those two parameters to get the minimum bridge weight after excluding failure cases
- ❑ Factor of safety was not taken into consideration

- **Solution**

Bridge design which gives minimum weight

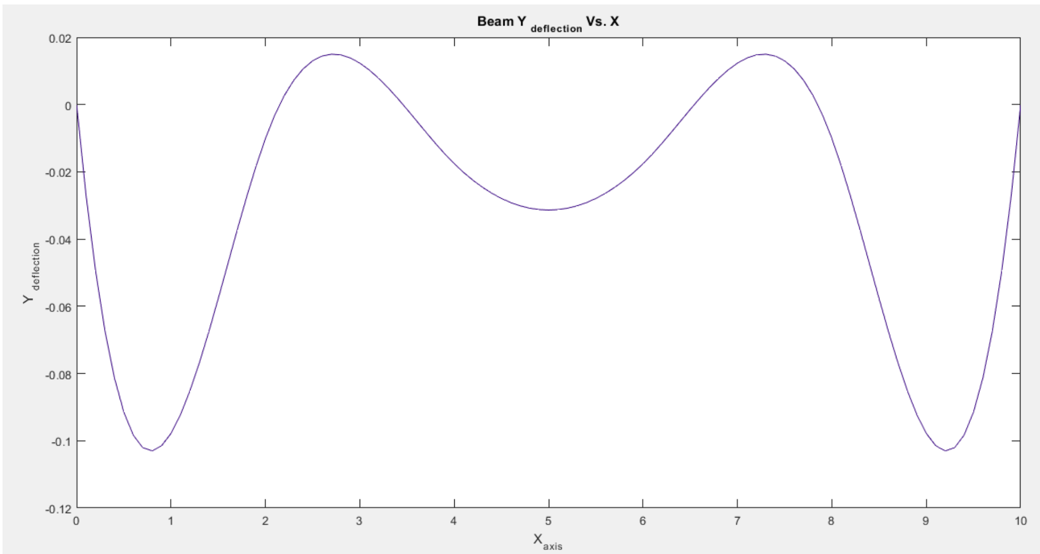
```
%% Assumptions
A3 = 2.853400e-04;  %Element (17 18) area in m^2
A1 = 0.6838*A3;     %Element (6 7 14 45 16 19) area in m^2
A2 = 0.31621*A3;    %Element (8 9 12 13) area in m^2
A4 = 0.00000001*A3; %Element (10 11) area Zero force members in m^2 (almost zero)
h = 0.0062532;      %Beam height in meter
```

```
%% Givens
Ab = 3.5*h;             %meter^2
Ib=(3.5*h^3)/12;        %meter^4
Beam_density = 2300;    %kg/m^3
Truss_density = 7800;   %kg/m^3

Eb=41*10^9;         %Beam Young's modulus
Et=207*10^9;        %Truss Young's modulus
le=2;               %Element length in meter
w=-0.9*9.81*1000;   %Distributed load in N/m
```

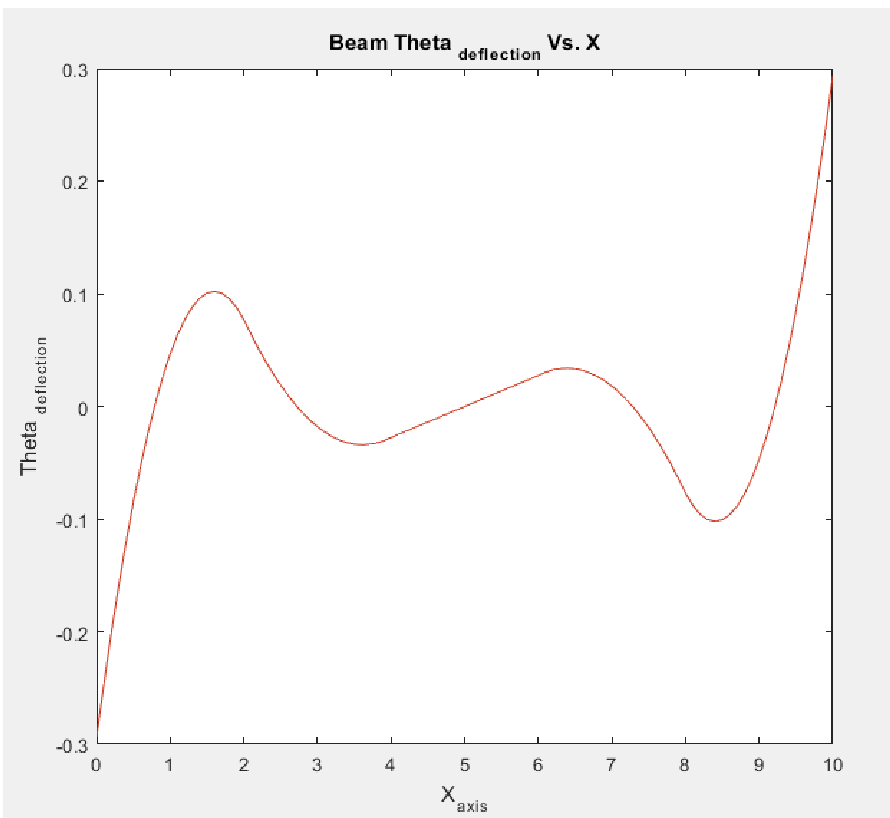
- **Results**

a) Solution Phase Results

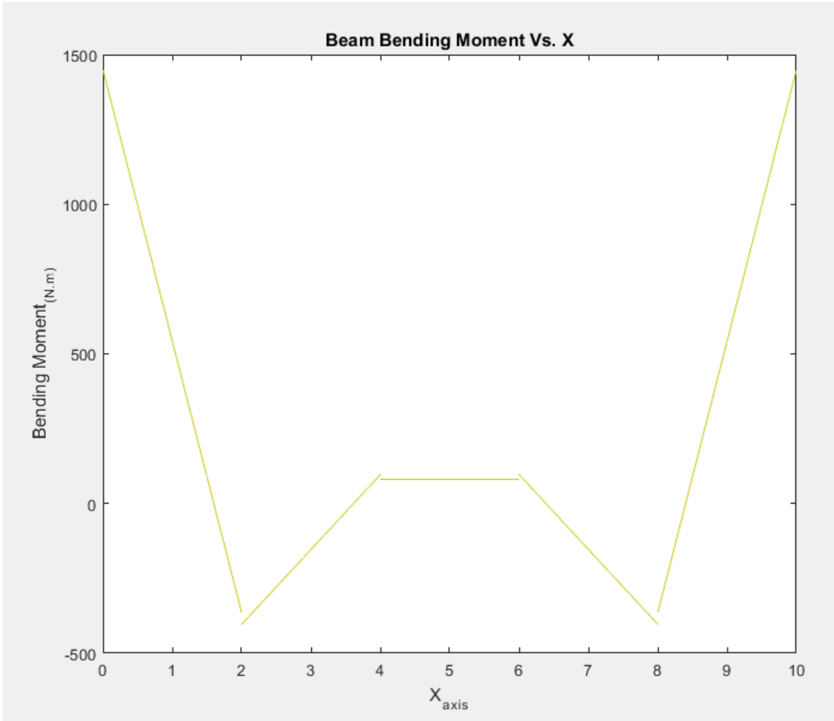
❑ Beam Deflections in y-direction



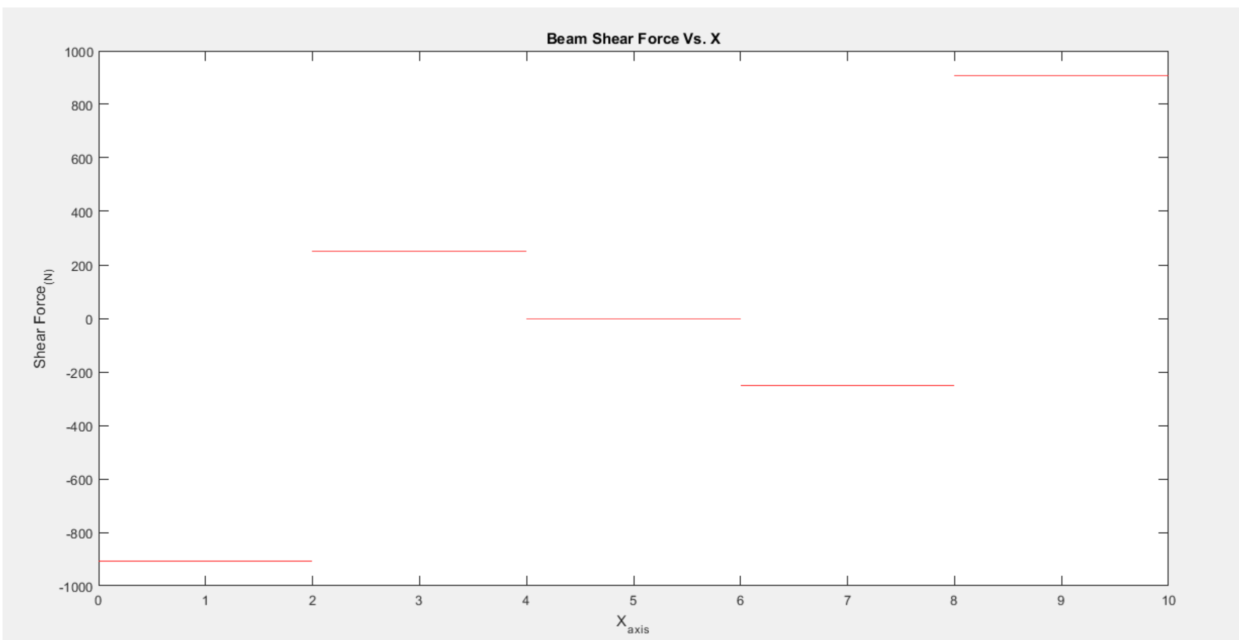
❑ Beam angular Deflections



❑ Beam Bending Moment Distribution



❑ Beam Shear Force Distribution



- **Internal forces of the five beam elements in N and N.m represented each in a column**

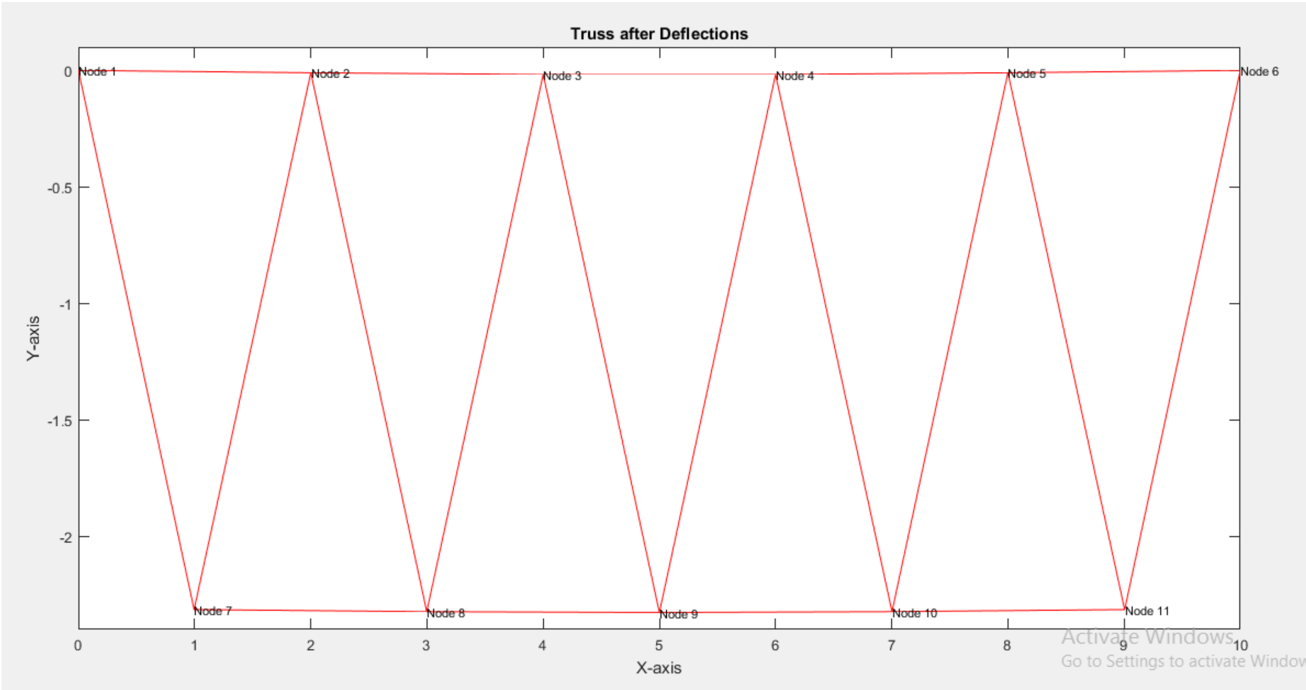
1.0e+04 *

Internal force along x-axis @ node i	2.1462	5.2850	6.2775	5.2850	2.1462
Internal force along y-axis @ node i	-0.1858	0.0467	-0.0000	-0.0467	0.1858
Internal moment along z-axis @ node i	-0.2943	0.0773	-0.0161	-0.0161	0.0773
Internal force along x-axis @ node j	-2.1462	-5.2850	-6.2775	-5.2850	-2.1462
Internal force along y-axis @ node j	0.1858	-0.0467	0.0000	0.0467	-0.1858
Internal moment along z-axis @ node j	-0.0773	0.0161	0.0161	-0.0773	0.2943

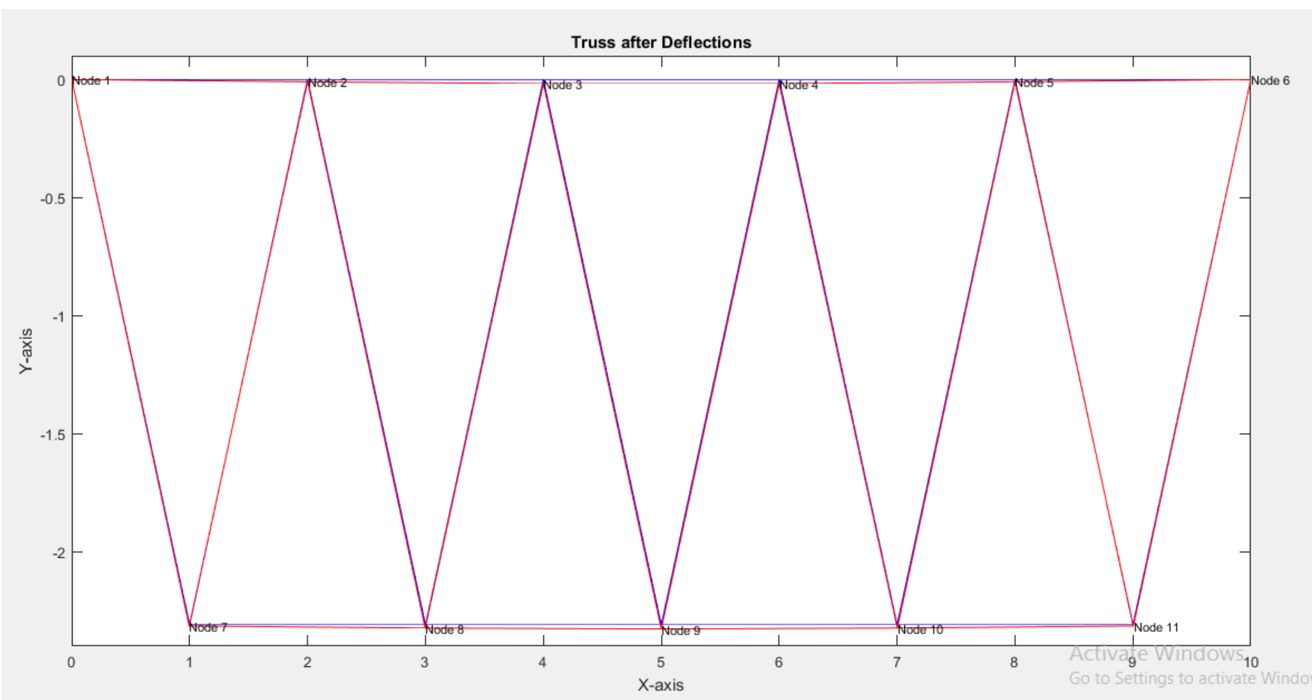
- Axial forces of the 14 truss elements in N and represented each in a column

```
f_axial_truss =

   1.0e+04 *

  Columns 1 through 12

   -4.2925    4.2925   -1.9850    1.9850    0.0000   -0.0000    1.9850   -1.9850    4.2925   -4.2925   -4.2925   -6.2775
   -0.0000         0    0.0000   -0.0000   -0.0000   -0.0000    0.0000         0         0         0         0         0
    4.2925   -4.2925    1.9850   -1.9850   -0.0000    0.0000   -1.9850    1.9850   -4.2925    4.2925    4.2925    6.2775
    0.0000         0   -0.0000    0.0000    0.0000    0.0000   -0.0000         0         0         0         0         0

  Columns 13 through 14

   -6.2775   -4.2925
         0         0
    6.2775    4.2925
         0         0
```

❑ Truss after deflections



❑ Truss after deflections relative to original position



b) Optimization Phase Results

❑ Maximum Stress value at each beam element

```
sigma_beam =

   1.0e+08 *

    1.3000    0.3632    0.0994    0.3632    1.3000
```

❑ Maximum Stress value at each truss member

```
sigma_truss =

   1.0e+08 *

  Columns 1 through 12

    2.2000    2.2000    2.2000    2.2000    0.0000    0.0000    2.2000    2.2000    2.2000    2.2000    2.2000    2.2000

  Columns 13 through 14

    2.2000    2.2000
```

❑ Weight of each member in kg

```
Weight =

  Columns 1 through 12

  100.6765  100.6765  100.6765  100.6765  100.6765    3.0438    3.0438    1.4075    1.4075    0.0000    0.0000    1.4075

  Columns 13 through 19

    1.4075    3.0438    3.0438    3.0438    4.4513    4.4513    3.0438
```

❑ Total bridge weight in kg

```
Total_Weight =

  536.1782
```