Question #1:-

Goods	Services	P.B
.2	.7	Goods
.8	.3	Services.

P_G and P_s are the total annual amounts

$$P_G = .2P_G + .7P_S$$
$$P_S = .8P_G + .3P_S$$

Shifting all to the left side

$$.8P_G - .7P_S = 0$$
$$-.8P_G + .7P_S = 0$$

The augmented matrix becomes,

$$\begin{bmatrix} .8 & -.7 & 0 \\ -.8 & .7 & 0 \end{bmatrix}$$

Applying row reduction,

$$\begin{bmatrix} .8 & -.7 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 + R_1$$

$$\begin{bmatrix} 1 & -.875 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So,

The general solution is $P_G = .875P_S$ while P_s is free.

Question #2 :-

No, the ratios of the prices remain same for whatever currency you want. The only thing that changes is the equilibrium prices as they are being multiplied by a constant value.

Question # 3 :-

(a)

Columns describe where the output goes and rows describe where the input comes from.

Chemicals	Fuels	Machinery	P.B
.2	.8	.4	Chemicals
.3	.1	.4	Fuels
.5	.1	.2	Machinery

(b)

P_C, P_F and P_M are the total annual output.
Now,
developing a system of equations

$$P_C = .2p_C + .8p_F + .4p_M$$
$$P_F = .3p_C + .1p_F + .4p_M$$
$$P_M = .5p_C + .1p_F + .2p_M$$

Shifting all to the left side,

$.8p_C - .8p_F - .4p_M = 0$
$-.3p_C + .9p_F - .4p_M = 0$
$-.5p_C - .1p_F + .8p_M = 0$

(C)

The reduced echelon form is,

$$\begin{bmatrix} .8 & -.8 & -.4 & 0 \\ -.3 & .9 & -.4 & 0 \\ -.5 & -.1 & .8 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -.5 & 0 \\ -.3 & .9 & -.4 & 0 \\ -.5 & -.1 & .8 & 0 \end{bmatrix} \quad R_1/.8$$

$$\begin{bmatrix} 1 & -1 & -.5 & 0 \\ 0 & .6 & -5.5 & 0 \\ 0 & -.6 & 5.5 & 0 \end{bmatrix} \quad \begin{matrix} 0.3R_1 + R_2 \\ 0.5R_1 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -1.417 & 0 \\ 0 & 1 & -.917 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_1 + R_2 \\ R_2/.6 \\ R_3 + R_2 \end{matrix}$$

The general solution is $p_C = -1.417p_m$, $p_F = .917p_M$ while p_M is free.

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Question # 4:-

(a)

A	E	M	T	P.B
.65	.30	.30	.20	A
.10	.10	.15	.10	E
.25	.35	.15	.30	M
0	.25	.40	.40	T

(b)

P_A, P_E, P_M, P_T are the total annual amounts.

$$P_A = .65P_A + .30P_E + .30P_M + .20P_T$$
$$P_E = .10P_A + .10P_E + .15P_M + .10P_T$$
$$P_M = .25P_A + .35P_E + .15P_M + .30P_T$$
$$P_T = .25P_E + .40P_M + .40P_T$$

Shifting all to the left side.

$$.35P_A - .30P_E - .30P_M - .20P_T = 0$$
$$-.10P_A + .90P_E - .15P_M - .10P_T = 0$$
$$-.25P_A - .35P_E + .85P_M - .30P_T = 0$$
$$-.25P_E - .40P_M + .60P_T = 0$$

Augmented matrix becomes

$$\begin{bmatrix} .35 & -.30 & -.30 & -.20 & 0 \\ -.10 & +.90 & -.15 & -.10 & 0 \\ -.25 & -.35 & 0.85 & -.30 & 0 \\ 0 & -.25 & 0.40 & .60 & 0 \end{bmatrix}$$

(c)

$$\left[\begin{array}{ccccc} 1 & -6/7 & -6/7 & -4/7 & 0 \\ -1/10 & 9/10 & -3/20 & -1/10 & 0 \\ -1/4 & -7/20 & 17/20 & -3/10 & 0 \\ 0 & -1/4 & 2/5 & 3/5 & 0 \end{array}\right] \quad \frac{20}{7}R_1$$

$$\left[\begin{array}{ccccc} 1 & -6/7 & -6/7 & -4/7 & 0 \\ 0 & 57/70 & -33/140 & -11/70 & 0 \\ -1/4 & -7/20 & 17/20 & -3/10 & 0 \\ 0 & -1/4 & 2/5 & 3/5 & 0 \end{array}\right] \quad \frac{1}{10}R_1 + R_2$$

$$\left[\begin{array}{ccccc} 1 & -6/7 & -6/7 & -4/7 & 0 \\ 0 & 57/70 & -33/140 & -11/70 & 0 \\ 0 & -79/140 & 89/140 & -31/70 & 0 \\ 0 & -1/4 & 2/5 & 3/5 & 0 \end{array}\right] \quad R_3 + \frac{1}{4}R_1$$

$$\left[\begin{array}{ccccc} 1 & -6/7 & -6/7 & -4/7 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & -79/140 & 89/140 & -31/70 & 0 \\ 0 & -1/4 & 2/5 & 3/5 & 0 \end{array}\right] \quad \frac{70}{57}R_2$$

$$\left[\begin{array}{ccccc} 1 & 0 & -21/19 & -14/19 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & -79/140 & 89/140 & -31/70 & 0 \\ 0 & -1/4 & 2/5 & 3/5 & 0 \end{array}\right] \quad R_1 + \frac{6}{7}R_2$$

$$\left[\begin{array}{ccccc} 1 & 0 & -21/19 & -14/19 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & 0 & 359/760 & -629/1140 & 0 \\ 0 & -1/4 & 2/5 & 3/5 & 0 \end{array}\right] \quad \frac{79}{140}R_2 + R_3$$

$$\begin{bmatrix} 1 & 0 & -21/19 & -14/19 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & 0 & 359/760 & -629/1140 & 0 \\ 0 & 0 & 249/760 & 629/1140 & 0 \end{bmatrix} R_2 + \frac{1}{4} R_4$$

$$\begin{bmatrix} 1 & 0 & -21/19 & -728/359 & 0 \\ 0 & 1 & -11/38 & -11/57 & 0 \\ 0 & 0 & 1 & -1258/1077 & 0 \\ 0 & 0 & 249/760 & 629/1140 & 0 \end{bmatrix} R_1 + \frac{21}{19} R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -728/359 & 0 \\ 0 & 1 & 0 & -572/1077 & 0 \\ 0 & 0 & 1 & -1258/1077 & 0 \\ 0 & 0 & 249/760 & 629/1140 & 0 \end{bmatrix} R_2 + \frac{11}{38} R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -728/359 & 0 \\ 0 & 1 & 0 & -572/1077 & 0 \\ 0 & 0 & 1 & -1258/1077 & 0 \\ 0 & 0 & 0 & 5032/5385 & 0 \end{bmatrix} R_4 - \frac{249}{760} R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -728/359 & 0 \\ 0 & 1 & 0 & -572/1077 & 0 \\ 0 & 0 & 1 & -1258/1077 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

So, the general solution is

$$O_A = \frac{728}{359} O_T$$

$$O_E = \frac{572}{359} O_T$$

$$O_M = \frac{1258}{1077} O_T$$

If $O_T = 100$,

then $O_A = 202.7$, $O_E = 53.11$, $O_M = 1.16$

Question #5:-

Total compounds = 4

For each compound the vector is

$$B_2S_3 \begin{array}{cl} \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} & \begin{array}{l} \text{Boron} \\ \text{Sulphur} \\ \text{Hydrogen} \\ \text{Oxygen} \end{array} \end{array} \qquad H_2O \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} \qquad H_3BO_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}$$

$$H_2S \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

The augmented matrix becomes

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_1/2 \\ \frac{3R_1}{2} - R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \quad \text{Swap } R_2 \text{ and } R_3$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & -3/2 & -1 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \quad R_2/2$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & -3/2 & -1 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 0 & -3/2 & 1 & 0 \end{bmatrix} \quad R_4 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & -3/2 & -1 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & -3/2 & 1 & 0 \end{bmatrix} \quad R_1 + \frac{R_3}{2}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & -3/2 & 1 & 0 \end{bmatrix} \quad \frac{3}{2}R_3 + R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \frac{3}{2}R_3 + R_4$$

$x_1 = \frac{1}{3}x_4$, $x_2 = 2x_4$, $x_3 = \frac{2}{3}x_4$, x_4 is free

When $x_4 = 3$, $x_1 = 1$, $x_2 = 6$, $x_3 = 2$

So the balanced eq. will be

$$B_2S_3 + 6H_2O \longrightarrow 2H_3BO_3 + 3H_2S$$