Assignment

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Question :-

What is matrix determinant? What are its properties? explain with example.

Determinant :-

The determinant is a scalar value that is a function of the entries of a matrix. It also characterizes some properties of the matrix and the linear map represented by it.

Properties:-

Reflection Property:-

The determinant remains the same when rows are changed into columns and columns are changed into rows.

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Now taking transpose of A and finding determinant

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$$|A^T| = \begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - bc$$

All-zero Property :-

If the elements of a row or column are zero, then determinant is also zero.

e.g:

$$|A| = \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} = (0)(2) - (0)(1)$$
$$= 0$$

Repetition Property :-

If the elements of a row or column are identical to another row or column, then the determinant is zero.

e.g:

$$|A| = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = (1)(2) - (2)(1)$$
$$= 0$$

Switching Property :-

If any two rows or columns are interchanged, the sign of the determinant also changes.

e.g :-

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3)$$
$$= -2$$

Interchanging row 1 and row 2

$$|A| = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$
$$= (3)(2) - (4)(1)$$
$$= +2$$

Scalar Multiple Property:-

If all elements of a row or column of a determinant are multiplied by a non-zero constant, the determinant gets multiplied by the same constant

e.g:-

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 4-6 = -2$$

now multiplying row 1 with 2

$$|A| = \begin{vmatrix} 2 & 6 \\ 2 & 4 \end{vmatrix} = 8-12 = -4$$

Sum Property:-

$$\begin{vmatrix} a_1+b_1 & c_1 \\ a_2+b_2 & c_2 \end{vmatrix} = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

e.g:-

$$\begin{vmatrix} 1+2 & 3 \\ 5+6 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 6 & 7 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 3 \\ 11 & 7 \end{vmatrix} = \left[(1)(7)-(3)(5)\right] + \left[(2)(7)-(3)(6)\right]$$

$$(3)(7)-(3)(11) = -8 + (-4)$$

$$21 - 33 = -12$$

$$-12 = -12$$

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Triangle Property:-

If all elements of a determinant above or below the main diagonal consist of zeros, the determinant is equal to the product of diagonal elements.

e.g.:-

$$\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} = (1)(3) - (2)(0) = 3$$

Factor Property:-

If a determinant becomes 0 on putting $x = \alpha$, then $(x - \alpha)$ is factor of the determinant.

e.g.:-

$$A = \begin{vmatrix} x & 1 \\ x+1 & 2 \end{vmatrix}$$

at $x = 1$

$$A = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = (1)(2) - (1)(2)$$

($\because$ This is also repetetion property)

$$= 0$$

Property of Invariance:-

If each entry of a row or column of determinant is multipilied by a real number and the resulting product is added to the corresponding entry in another ~~f~~ row or column in the determinant then the resulting determinant is equal to the original.

e.g

$$|A| = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$

$$= (3)(2) - (4)(1)$$

$$= 2$$

Now multiplying R_1 with '2' and adding with R_2

$$|A|' = \begin{vmatrix} 3 & 4 \\ 7 & 10 \end{vmatrix}$$

$$= (3)(10) - (4)(7)$$

$$= 30 - 28$$

$$= 2$$