NEW HORIZON
COLLEGE OF ENGINEERING

DEPARTMENT OF HRD

QUANTITATIVE ABILITY

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EXERCISE NO 1
NUMBERS - 1

- **Natural Numbers : N = { 1,2,3,4,5……..}**
- **Whole Numbers : W = { 0,1,2,3,4,5……. }**
- **Integers : Z = {…,-3,-2,-1,0,1,2,3,.....}**
- **Rational Numbers**

  Any number which can be expressed in the form

  "p/q" where p & q are integers and q ≠ 0 is called a rational number.

Any natural Number is a rational Number.

Any whole Number is a rational Number.

Any integer is a rational Number.

Any terminating decimal is a rational number.

For Eg., 0.25 = 25/100

Any recurring decimal can be converted in to the form "p/q" where p & q are integers and q≠0. Therefore, any recurring decimal is a rational Number

- **Irrational Numbers**

Any non-terminating decimal and non-recurring decimal is an irrational number.

π and "e" are irrational numbers

Odd numbers: Any number of the form 2n+1, where n is an Integer .e.g. 1,3,5,7

Even numbers: Any number of the form 2n where n is an integer e.g. 2,4,6,8

Prime Numbers: A natural number larger than unity is a prime number if it does not have other divisors except for itself and unity.

Note:-Unity i e,1 is not a prime number.

Composite Numbers: The numbers which are not prime are known as composite numbers.

Co-Primes:

- Two numbers a and b are said to be co-primes, if their H.C.F is 1.
- Example (2,3), (4,5), (7,9), (8,11) ….

- Place value or Local value of a digit in a Number. be two types of natural numbers. They are **Prime and composite.**

Rule of Simplification: In simplifying an expression various operation must be performed as per the following order. VBODMAS

V → Vinculum

B → Remove Brackets - in the order (), {}, []

O → Of

D → Division

M→ Multiplication

A → Addition

S → Subtraction

Divisibility Rules:

No	Rule Should be Applied
2	unit's digit is any of 0,2,4,6,8.
3	if the sum of its digits is divisible by 3
4	if the number formed by last two digits is divisible by 4.
5	if its unit's digit is either 0 or 5.
6	If the number is divisible by both 2 and 3.
7	Multiply the last digit by "2" and subtract the resultant from the leading truncated number (number made by the remaining digits). If the result is divisible by 7, then the original number is also divisible by 7. Repeat this process till you find a number which you know is divisible by 7.
8	if the last 3 digits of the number are divisible by 8.
9	if the sum of its digits is a multiple of 9.
11	If the difference of the sum of the digits in the odd places and the sum of the digits in the even places is zero or divisible by 11.
12	All numbers divisible by 3 and 4 are divisible by 12.
13	Multiply the last digit of the number by 4 and then add the resultant to the remaining leading truncated number. If the result is divisible by 13, then the original number is also divisible by 13.

Additional Divisibility Rules using Co-Primes:

- If a number should be divisible by 6, then it should be divisible by both 2 & 3.
- If a number should be divisible by 12, then it should be divisible by both 3 & 4.
- If a number should be divisible by 18, then it should be divisible by both 2 & 9.
- If a number should be divisible by 36, then the number should be divisible by 4 & 9.
- If a number should be divisible by 44, then the number should be divisible by 4 & 11.
- If a number should be divisible by 72, then the number should be divisible by 8 & 9.
- If a number should be divisible by 99, then the number should be divisible by 9 & 11.
- If a number should be divisible by 132, then the number should be divisible by 11 & 12.
- If a number should be divisible by 144, then the number should be divisible by 9 & 16.

DIVISION ALGORITHM

If we divide a number by another number, then Dividend = (Divisor * quotient) + Remainder

FACTORS OF A NUMBER

Any number 'N' can be expressed as $a^p.b^q.c^r$…. where a, b and c are prime factors on the number 'N' The number of factors of 'N' will be (a+1) (b+1) (c+1) …

- N = $a^p \times b^q \times c^r$ …….
- Number of factors is (p+1) (q+1) (r+1) ……..

  provided a, b, c are distinct prime numbers

Example:

1. Simplify: 197 - [1/9{42 + (56 - 8 + 9)} +108]

Solution:

197 - [1/9 {42 + (56 - 8 + 9)} +108]
= 197 – [1/9 {42 + (56 – 17)} + 108] [Removing vinculum]
= 197 - [1/9 {42 + 39} + 108] [Removing parentheses]
= 197 - [(81/9) + 108] [Removing braces]
= 197 - [9 + 108]
= 197 - 117
= 80

2. **Simplify: 15 - (-5) {4 - 7 - 3} ÷ [3{5 + (-3) x (-6)}]**

Solution:

15 - (-5) {4 - 7 - 3} ÷ [3{5 + (-3) x (-6)}]
= 15 - (-5) {4 - 4} ÷ [3 {5 + 18}]
= 15 - (-5) x 0 ÷ 3 x 23
= 15 - (-5) x 0 ÷ 69
= 15 - (-5) x 0 [Performing division 0 ÷ 69 = 01]
= 15

3. The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and the 15 as remainder. What is the smaller number?

Solution:

Let the smaller number be x. Then larger number = (x + 1365).
x + 1365 = 6x + 15
5x = 1350
x = 270
Smaller number = 270.

4. If one-third of one-fourth of a number is 15, then three-tenth of that number is:

Solution:

Let the number be x.
Then, 1/3 of ¼ of x = 15
x = 15 * 12 = 180.So, required number = ((3/10) * 180) = 54.

5. Three times the first of three consecutive odd integers are 3 more than twice the third. The third integer is:

Solution:

Let the three integers be x, x + 2 and x + 4.
Then, 3x = 2(x + 2) + 3.
x = 7
Third integer = x + 4 = 11.

PRACTICE PROBLEMS

1. If A, B, C, D are four consecutive positive integers in ascending order and if $(A+D)^2 = 169$, what is the value of A x B x C x D?

   (1) 1690 (2) 1680 (3) 1780 (4) 1790

2. Simplify: 172 + 4/7 of (448 / (13 + 24 - 39)).

   (1) 44 (2) - 22 (3) - 44 (4) 34

3. If 5278X4Y is divisible by 36, what is the maximum value for X+Y?

   (1) 1 (2) 4 (3) 8 (4) 10

4. I have a book with 400 pages. How many times do I have to press the numeric keys of the computer to type the numbers from 1 to 400?
   (1) 1086 (2) 1089 (3) 1092 (4) 1200

5. A student was asked to add the first few natural numbers and after addition, the student gave the answer as 600. The teacher told the student that the answer is wrong and that he has missed out one number. Can you help the student to find the number missed out?
   (1) 30 (2) 35 (3) 25 (4) 20

6. The value of 101 + 102 + 103 + ... + 200 is
   (1) 15050 (2) 2 0 2 0 0 (3) 10909 (4) 16500 (5) None of these

7. Find the sum of all odd numbers up to 100.
   (1) 99 x 50 (2) 2050 (3) 2501 (4) 2500

8. A teacher asked the students to write numbers from 1 to 100 continuously one after another on the board. What will be the reminder if the first 125 digits from the left is divided by 16?
   (1) 11 (2) 10 (3) 13 (4) 15

9. If all 4's are replaced by 6, then the algebraic sum of the numbers from 1 to 100 (both inclusive) is equal to
   (1) 5160 (2) 5270 (3) 5380 (4) 5490

10. The sum of the squares of three numbers is 138, whereas the sum of their products taken two at a time is 131. Their sum is....
    (1) 23 (2) 20 (3) 57 (4) 59

11. The sum of three numbers is 40. Sum of the products of the numbers taken two at a time out of the three numbers is 521. What is the sum of the squares of the numbers?
    (1) 558 (2) 458 (3) 642 (4) 647

12. The product of 10 consecutive even natural numbers is always divisible by
(1) 2^{10} x 11! (2) 2^{10} x 10! (3) 2^{10} x 12! (4) 2^{10} x 21!

13. A number when divided by 221 gives a remainder 13. What will the remainder have obtained when the same number is divided by 17?
(1) 8 (2) 9 (3) 13 (4) 6

14. On dividing a number by 56, we get 29 as remainder. On dividing the same number by 8, what will be the remainder?
(1)4 (2) 5 (3)6 (4)7

15. On dividing a number by 357, we get 39 as remainder. On dividing the same number 17, what will be the remainder?
(1) 0 (2) 3 (3) 5 (4) 11

16. A teacher went to a class with a certain number of chocolates. He distributed the chocolates equally among the students, he was left with 7 chocolates. Next day he carried the same number of chocolates and found there were two times the number of students present on the first day. On dividing them equally among the students, he was left with 47 chocolates. Find the number of students present on the first day.
(1) 48 (2) 40 (3) 23 (4) 49

17. A teacher went to a class with a certain number of chocolates. He distributed the chocolates equally among the students, he was left with 5 chocolates. Next day he carried the same number of chocolates and found there were three times the number of students present on the first day. On dividing them equally among the students, he was left with 25 chocolates. Find the number of students present on the first day.
(1) 10 (2) 20 (3) 23 (4) either 10 or 20

18. Raman can make 1 cigarette out of 6 stubs. He buys 'x' cigarettes such that he smokes a maximum total of 259 cigarettes. Which of the following can be the value of 'x'?
(1) 216 (2) 240 (3) 220 (4) 210

19. How many numbers less than 100 will have exactly 10 factors?
(1) 4 (2) 3 (3) 2 (4) 1

20. There are 50 boxes and 50 persons, person 1 keeps 1 marble in every box. Person 2 keeps 2 marbles in every 2nd box. Person 3 keeps 3 marbles in every 3rd box. This process goes on till person 50 keeps 50 marbles in the 50th box. Find the total number of marbles kept in the 50th box.
(1) 6 (2) 93 (3) 78 (4) 43

21. Find the number of rectangles of area 144 square units that can be drawn if the sides may take only distinct integer values.
(1) 8 (2) 7 (3) 6 (4) 5

22. A total of 'x' students are studying in 'y' colleges such that in each college the number of students is the same as twice the total number of colleges. If $1200 < x < 1500$, find the value of y which is not possible.
(1) 25 (2) 28 (3) 27 (4)26

23. 120 apples were present with a total of seven persons. Each person had at least 13 apples. The number of apples with each is distinct. Which of the following cannot be the total number of apples with any 2 persons?
(1) 47 (2) 44 (3) 27 (4)33

24. 100 consecutive even numbers are equally spaced around a circle. If the least of them 2, which number is opposite to 48?

(1) 148 (2) 98 (3) 80 (4) 176

25. A certain supervisor had 120 tokens numbered from 1 to 120 in a box. To his surprise, he noticed that tokens which are multiples of either 2 or 3 or 5 are removed from the box. Totally how many tokens are removed from the box?

(1) 88 (2) 90 (3) 32 (4) 60

ASSESSMENT PROBLEMS

1. Simplify 3/8 of 880 / ((27 x 3) - 76)
   (1) 65 (2) 66 (3)56 (4)57

2. Which of the given number will make the decimal number 0.273 an integer if multiplied with 2?
   (1) 900 (2) 990 (3) 999 (4) 1000

3. What will be the reminder if the first 50 digits of the number 1112131415…… are divided by 125?
   (1) 25 (2) 60 (3) 75 (4) 100

4. If I have to number a note book of 200 pages from 1 to 200, how many digits will I write?
   (1) 450 (2) 489 (3) 492 (4) 600

5. If 743X84Y is exactly divisible by 44, how many values can X and y take?
   (1) 3 (2) 2 (3) 1 (4) 0

6. What should be added to the product 10 x 12 x 14 x 16 to make it as a perfect square?
   (1) 2 (2) 4 (3) 8 (4) 16

7. What is the number to be multiplied to make 3528 as a perfect cube?
   (1) 6 (2) 14 (3) 21 (4) 42

8. The sum of three numbers is 30. The sum of the squares of the numbers is 314. What is the sum of the products of the numbers taken two at a time?
   (1) 293 (2) 586 (3) 360 (4) 485

9. 100 delegates participated in a computer conference and were given roll numbers from 1 to 100 in a serial order. Delegates with serial numbers which are multiples of 2 took part in C++ session. Delegates with serial numbers which are multiples of 5 attended session on networking. Delegates whose serial numbers are multiples of 10 spend time on cloud computing. The rest of the delegates participated in a debate on artificial intelligence. How many attended the debate on artificial intelligence?
   (1) 30 (2) 40 (3) 20 (4) None of these

10. How many natural numbers from 1 to 100 have even number of factors?(1) 25 (2) 10 (3) 75 (4) 90

11. There were N houses in a colony, numbered 1 to N. All but 5 of the houses were destroyed in an earthquake. The numbers on the houses which were

not destroyed are consecutive. If the sum of the numbers on the destroyed houses is 1085, then the least of the numbers on the houses which were not destroyed can be

(1) 36 (2) 26 (3) 25 (4) either 36 or 2

12. Find the total number of factors of number N = $12^3 \times 13^2 \times 14$?(1)

24 (2) 64 (3) 192 (4) 216

13. Find the sum of the first 50 common terms of 12, 16, 20,.... and 18, 24, 30,....

(1) 15900 (2) 12700 (3) 19990 (4) 18400

14. The number 673 and 865 is divisible by which of the following leaving a remainder 1?

(1)1 (2) 2 (3) 3 (4) 4 (5) 2, 3,4

15. Four children have small toys. The first child has 1/10 of the toys, the second child has 12 more toys than the first, the third child has one more toy of what the first child has and the fourth child has double the third child. How many toys are there?

16. If a - b = 3 and a2 + b2 = 29, find the value of ab.

(1) 10 (2) 12 (3) 15 (4) 18

17. Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally the entire cost of the car, then the share of each of the remaining persons increased by:

(1) 1/7 (2) 1/8 (3) 1/9 (4) 7/8

18. In a regular week, there are 5 working days and for each day, the working hours are 8. A man gets Rs. 2.40 per hour for regular work and Rs. 3.20 per hours for overtime. If he earns Rs. 432 in 4 weeks, then how many hours does he work for?

19. How many of the following numbers are divisible by 132 ?

264, 396, 462 792, 968, 2178, 5184, 6336

(1)4 (2)5 (3) 6 (4)7

20. The largest 4 digit number exactly divisible by 88 is:

(1) 9944 (2) 9768 (3) 9988 (4) 8888

21. Phaneesh's motorbike broke the traffic regulations. Policeman did not remember the vehicle's license number, but noticed that the first two

digits were same and last two digits were also alike. He also noticed that the number was a perfect square. The number was...

(1) 5588 (2) 1122 (3) 7744 (4) 2299

22. A devotee keeps a count of her prayers by using a rosary having 108 beads. (When the rosary is twirled once, the count is 108). It is known, that over the years her count has come to 5 A 1 9 6 B where A and B stand for single digit integers. She has twirled the rosary a certain number of complete rounds. The difference between A and B is 1. Then, the value of A is:

(1) 7 (2) 1 (3) 5 (4) 3

23. Let A be a decimal of the form, A = o.ab cd cd cd cd cd cd ..., Where digits a, b, c, and d are integers lying between 0 and 9, At most one of c and d is zero. By what number A be multiplied so that result is a natural number?

(1) 1980 (2) 198 x102 (3) 9999 (4) Any of above

24. A number when divided by 72 leaves a remainder of 15. Which amongthe following would be the remainder when the number is divided by 18?

(1) 15 (2) 12 (3) 7 (4) Cannot be determined

25. A number when divided by 48 leaves a remainder of 25. What will be the remainder when the same number is divided by 12?

(1) 7 (2) 1 (3) 5 (4) 2

MODULE -2

NUMBERS - 2

Introduction:

LCM and HCF

Factors:

They are also called as Sub-Multiples or Divisors. If one number divides a second number exactly, then the first number is said to be a factor of the second number.

E.g.: 6 is a factor of 30.

Multiples:

If one number is divisible exactly by a second number, then the first number is said to be a Multiple of the second number.

E.g.: 20 is a Multiple of 4.

Least Common Multiple (L.C.M):

L.C.M. is the least non-zero number in common multiples of two or more numbers.

Multiple of 6 = 6, 12, 18, 24, 30, ...

Multiple of 8 = 8, 16, 24, 32, 40, ...

Common Multiple of 6 and 8 = 24, 48 .....

Least Common Multiple = 24

L.C.M is a multiple of H.C.F.

Highest Common Factor (H.C.F):

The highest common factor of two or more numbers is the greatest number which divides each of them exactly.

E.g. Find the H.C.F. of 24 and 56

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 56 = 1, 2, 4, 7, 8, 14, 28, 56

Common factors of 24 and 56 are 1, 2, 4, 8

H.C.F. of 24 and 56 = 8

$N_1 = H*K_1$ $N_2 = H*K_2$

- HCF = H
- LCM = HK_1K_2
- N_1 ,N_2 & LCM should be multiples of HCF.
- LCM should be multiples of N_1& N_2
- K_1& K_2 should be co primes

- Product of two numbers = L.C.M x H.C.F

Formulae Required:

- ✓ H.C.F of Fractions = $\frac{\text{H.C.F of Numerators}}{\text{L.C.M of Denominators}}$
- ✓ L.C.M of Fractions = $\frac{\text{L.C.M of Numerators}}{\text{H.C.F of Denominators}}$

Examples

1. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.

   Solution:

   H.C.F. of (91 - 43), (183 - 91) and (183 - 43)

   H.C.F. of 48, 92 and 140

   =4.

2. The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The largest of the two numbers is:

   Solution:

   The numbers are (23 x 13) and (23 x 14).

   Larger number = (23 x 14) = 322.

3. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?

   Solution:

   L.C.M. of 2, 4, 6, 8, 10, 12 is 120.

   So, the bells will toll together after every 120 seconds (2 minutes).

   In 30 minutes, they will toll together (30/ 2) + 1 = 16 times.

4. Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same

   remainder in each case. Then sum of the digits in N is:

   Solution:

   N = H.C.F. of (4665 - 1305), (6905 - 4665) and (6905 - 1305)

= H.C.F. of 3360, 2240 and 5600
= 1120.
Sum of digits in N = (1 + 1 + 2 + 0) = 4

5. The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:

Solution:

Greatest number of 4-digits is 9999.
L.C.M. of 15, 25, 40 and 75 is 600.
On dividing 9999 by 600, the remainder is 399.
Required number (9999 - 399) = 9600.

6. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:

Solution:

Let the numbers be 37a and 37b.
Then, 37a x 37b = 4107
ab = 3.
Now, co-primes with product 3 are (1, 3).
So, the required numbers are (37 x 1, 37 x 3) *i.e.,* (37, 111). Greater number = 111.

7. Three number are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:

Solution:

Let the numbers be 3x, 4x and 5x.
Then, their L.C.M. = 60x.
So, 60x = 2400 or x = 40.
The numbers are (3 x 40), (4 x 40) and (5 x 40).
Hence, required H.C.F. = 40.

8. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:

Solution:

Let the numbers 13a and 13b.
Then, 13a x 13b = 2028
ab = 12.
Now, the co-primes with product 12 are (1, 12) and (3, 4).
[Note: Two integers a and b are said to be **co-prime** or relatively prime if they have no common positive factor other than 1 or, equivalently, if their greatest common divisor is1]
So, the required numbers are (13 x 1, 13 x 12) and (13 x 3, 13 x 4).
Clearly, there are 2 such pairs.

PRACTICE PROBLEMS

1. Find the smallest four-digit number which when increased by 3 is divisible by 4,5 and 6.
   (1) 1087 (2) 1023 (3) 1017 (4) 1003

2. A certain number of apples can be distributed equally amongst 4, 5, 6, 7, 8, 9 and 10 children. Find the minimum number of apples.
   (1) 720 (2) 2520 (3) 840 (4) 420

3. A number when divided by 3, 4, 5, 6 and 7 leaves 2, 3, 4, 5 and 6 as the respective remainders. Find the smallest number.
   (1) 421 (2) 419 (3) 839 (4) 841

4. A rice merchant has 14 kg of Ponni rice, 42 kg of Sambha rice and 70 kg of Kuruvai rice. He wants to pack the rice in bags such that each bag contains the same quantity of rice and there is no mixing of rice of different varieties. What is the least number of bags required?
   (1) 5 (2) 20 (3) 9 (4) 14

5. I have three measuring tapes of lengths 15 cm, 20 cm and 25 cm. What is the minimum length of cable of integral length that I can measure using any of the three tapes?
   (1) 180 (2) 300 (3) 200 (4) 150

6. Given the fractions 3/7, 2/5 and 4/9, how many times the H.C.F of these fractions is their L.C.M?
   (1) 3780 (2) 3600 (3) 3545 (4) 3549

7. Three pieces of wood 65 m, 104 m and 169 m long have to be divided into wood pieces of the same length. What is the minimum number of equal lengths possible?
   (1) 20 (2) 26 (3) 14 (4) 17

8. In a bus depot, three category of buses viz, ordinary, deluxe and super deluxe to city X operates with a frequency of 18 minutes, 24 minutes and 36 minutes. All the buses start together at 6 A.M. In a 12-hour period, how many times the three buses start simultaneously including the one at 6 A.M?
   (1) 8 (2) 9 (3) 10 (4) 11

9. How many pairs of numbers exist such that their LCM is 240 and their HCF is 16?
   (1) 4 (2) 3 (3) 2 (4) 1

10. LCM of two numbers is 120. Their HCF is 10. Which of the following can be the sum of those two numbers?

(1) 70 (2) 80 (3) 140 (4) 60

11. The LCM of two numbers is 15 times their HCF. The sum of LCM and HCF is 800. If one of the numbers is 250, what is the other number?
(1) 175 (2) 200 (3) 150 (4) 225

12. When n.is divided by 2, it leaves a remainder of 1. When n is divided by 5 it leaves a remainder of 2. Which of the following can be the value of n?
(1) 1007 (2) 1009 (3) 853 (4) None of these

13. What is the remainder of the division of 2^{48} by 7?
(1) 1 (2) 6 (3) 5 (4) 4

14. Find the remainder when 37^{137} is divided by 38.
(1) 1 (2) 37 (3) 2 (4) 3

15. What is the remainder of the division of (1! + 2! + 3! + …. + 10!) by 5?
(1) 0 (2) 3 (3) 2 (4) 1

16. How many zeroes does 150! ends with?
(1) 16 (2) 37 (3) 53 (4) 38

17. How many zeros are there at the end of 125!?
(1) 30 (2) 31 (3) 29 (4) 34

18. If AB + CD = AAA and A, B, C, D are distinct natural numbers, how many pairs of numbers (AB, CD) exist?
(1) 6 (2) 7 (3) 8 (4) 9

19. A + B + C + D = D + E + F + G = G + H + I = 17. If A to I are 1 to 9 in any order and A = 4, find D + G.
(1) 5 (2) 6 (3) 7 (4) 8

20. a, b, c, d, e, f, g and h are 8 single digit distinct natural numbers. They satisfy the following conditions

```
  abc        abc
 +de        -de
  afg        hg
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Find the value of g if c = 3.
(1) 2 (2) 8 (3) 7 (4) 6

21. Find the last digit of 13^{96},
(1) 1 (2) 3 (3) 9 (4) 7

22. What is the unit's digit of the expression 1! + 2! + 3! + 4! +20!?
(1) 0 (2) 1 (3) 2 (4) 3

23. Find the units digit of $2^{458}+3^{343}+8^{2852}+6^{1234}$?
(1) 7 (2) 6 (3) 3 (4) 4

24. If one wants to measure all integral weights from 1 kg to 450 kgs using a common balance, what is the minimum number of weights needed? (1) 8
(2) 9 (3) 7 (4) 6

25. Find the total number of squares that can be observed in a 8 x 8 chess board.
(1) 64 (2) 204 (3) 86 (4) 210

ASSESSMENT PROBLEMS

1. If 7 saplings are planted in a row, there will not be place for 4 saplings. Instead, if 6 saplings are planted in a row 3 saplings will be left out. If the number of saplings is a greatest two-digit number, what is the number of saplings?
   (1) 84 (2) 81 (3) 83 (4) 82

2. If I pack either 6 garments or 8 garments in a carton, I will be left with 4 garments for one carton. But If I use a bigger size box and pack 10 garments per box then all boxes will be filled with garments. If the number of garments is a three-digit number what is the value?
   (1) 100 (2) 120 (3) 140 (4) 124

3. Three containers contain industrial alcohol with quantities 279 kgs, 372 kgs and 403 kgs. What biggest measure must be used to measure all the above quantities an exact number of times?
   (1) 9 (2) 12 (3) 13 (4) 31

4. The LCM of two numbers is 12 times their HCF. The sum of the LCM and HCF is 273. If one of the numbers is 63, what is the other number?
   (1) 36 (2) 48 (3) 84 (4) 63

5. What is the unit digit of 1!+3!+5!+ +47!+49!?
   (1) 8 (2) 7 (3) 6 (4) 5

6. Find the largest power of 8 in 160!
   (1) 20 (2) 22 (3) 52 (4) 44

7. What is the number of zeros at the end of 126!?
   (1) 26 (2) 12 (3) 13 (4) 30

8. What is the reminder if 3^{23} is divided by 10?
   (1) 3 (2) 9 (3) 7 (4) 1

9. What is the unit's digit for the expression $5^{123} + 6^{321} + 9^{456} + 8^{654}$
   (1) 5 (2) 6 (3) 4 (4) 1

10. Find the unit digit of $122^{122} \times 133^{133}$.
    (1) 2 (2) 4 (3) 6 (4) 8

11. A paper merchant had to weigh waste paper quantities from 1 kg to 100 kgs in integral weights using a common balance. What is the minimum number of weights required?
    (1) 7 (2) 6 (3) 5 (4) 4

12. What is the remainder when N = $(1! + 2! + 3! + \ldots\ldots + 1000!)^{40}$ is divided by 10?
(1) 1 (2) 3 (3) 7 (4) 8

13. There are two, 2-digit numbers ab & cd, ba is another 2-digit number prepared by reversing the digits of ab, if ab X cd = 493 and ba X cd = 2059, what is the value of sum of (ab + cd)?

(1) 43 (2) 45 (3) 47 (4) 46 (5) 49

14. Find the remainder of 15 X 17 X 19 when divided by 7.

(1) 5 (2) 3 (3) 1 (4) 0

15. Find the smallest and largest four digit numbers which are exactly divisible by 15, 21 and 12 when 6 is reduced from such numbers.

(1) 1266, 9654 (2) 1066, 9660 (3) 1266, 9666 (4) 1260, 9666

16. What is the smallest number that must be added to 1854 such that a remainder of 3 is left when the Number is divided by 8, 12 or 16?

(1) 21 (2) 18 (3) 15 (4) 39

17. Find the greatest number which when divides 3078 and 3906 leaves respective remainders of 3 and 6.

(1) 60 (2) 75 (3) 90 (4) 110

18. Find the greatest number which when divides 51,141 and 276 leaves the same remainder.
(1) 15 (2) 30 (3) 45 (4) 60

19. Find the largest two-digit number which leaves respective remainders of3 and 5 when it divides 2943 and 4835.
(1) 35 (2) 55 (3) 45 (4) 70

20. If six bells toll at intervals of 3, 4, 5, 6, 7 and seconds respectively, find when will they to together again given that these bells toll together at 11:15 a.m.
(1) 11:19 a.m. (2) 11:29 a.m. (3) 11:30 a.m. (4) 11:39 a.m.

21. Three numbers 97, 152 and 252 when divided by the same divisor leaves respective remainders of 1, 8 and 12. How many divisors satisfy this condition?
(1) 10 (2) 9 (3) 8 (4) 3

22. HCF of two numbers n1 and n2 is 5. Their product is 400. How many pairs of numbers (n1, n2) satisfy this condition?
(1) 3 (2) 2 (3) 1 (4) 5

23. The Difference between the HCF and LCM of two numbers is 24. How many pairs of numbers satisfy this condition?
(1) 4 (2) 8 (3) 6 (4) 2

24. Sum of two numbers and their LCM is 41. How many pairs of numbers satisfy this condition?
(1) 0 (2) 3 (3) 2 (4) 1

25. Ashok has to distribute 2002 pencils and 1430 pens to students. He wants to distribute an equal number of pens and pencils to each student. What is the maximum number of students to whom he can distribute the pens and pencils?
(1) 186 (2) 286 (3) 143 (4) 248

MODULE - 3

RATIO & PROPORTION

Ratio:

Comparison of two numbers or quantities having the same units is known as a Ratio.

The ratio of 'x' to 'y 'is written as x: y provided $y \neq 0$,

where 'x' is known as the antecedent in ratio and

'y' is the consequent in ratio.

A ratio is said to be in its simplest form if the HCF of the antecedent and the subsequent is 1.

Equivalent Ratios:

To get the equivalent fraction of a certain fraction, multiply or divide the numerator and denominator of the given fraction by the same number. In the same way, we get the equivalent ratio of a certain ratio.

Comparing ratios:

To compare two ratios, we express them as fractions with common denominators and then compare their numerators.

If a: b is a ratio, then:

- Duplicate ratio of (a: b) is $(a^2: b^2)$.
- Sub-duplicate ratio of (a: b) is $(a^{1/2}: b^{1/2})$.
- Triplicate ratio of (a: b) is $(a^3: b^3)$.
- Sub-triplicate ratio of (a: b) is $(a^{1/3}: b^{1/3})$.

Proportion

Proportion is represented by the symbol '= ' or ' :: '

If the ratio a: b is equal to the ratio c : d, then a, b, c, d are said to be in proportion.

Using symbols we write as a: b = c : d or a : b :: c : d

When 4 terms in proportion, then the product of the two extremes (i.e. the first and the fourth value) should be equal to the product of two middle values (i.e. the second and the third value)

FOURTH PROPORTIONAL:

If a: b = c : d, then d is called the fourth proportional to a, b, c. **d=** $\frac{bc}{a}$

THIRD PROPORTIONAL:

If a: b = b : c, then c is called the 3rd proportional a ,b c = $\frac{b^2}{a}$

2nd PROPORTIONAL or MEAN PROPORTIONAL:

Mean proportional between a and b is $\sqrt{ab}$.

DIRECT VARIATION:

Two quantities "x" and "y" are said to be in direct variation if an increase in one quantity results in increase in the other quantity and decrease in one results in decrease in the other quantity. If two quantities vary always in the same ratio, then they are in direct variation.

Examples:

Distance and Time are in Direct Variation, because more the distance travelled, the time taken will be more (if speed remains the same).

INVERSE VARIATION:

If two quantities 'x' and 'y' are such that an increase or decrease in 'x' leads to a corresponding decrease or increase in 'y' in the same ratio, then we can say, they vary indirectly or the variation is inverse. In notation, inverse variation is written as

$y \propto 1/x$ $y = p/x$

where p is constant of proportionality, $xy = p$. $x_1y_1 = x_2y_2$.

SOLVED EXAMPLES:

Example 1: Write any 4 equivalent ratios for 4: 3.

Sol: Given Ratio = 4: 3. The ratio in fractional form = 4/3, we can get equivalent ratios by "4" and "3" multiplied by 2, 3, 4, 5 and get the equivalent fractions of 4/3 are 8/6, 12/9, 16/12, 20/15,

∴ The equivalent ratios of 4: 3 are 8 : 6, 12 : 9, 16 : 12, 20 : 15

Example 2: Distribute Rs. 320 in the ratio 1: 3.

Sol: 1: 3 means the first quantity is 1 part and the second quantity in 3 parts. The total number of parts = 1 + 3 = 4. As 4 parts = Rs. 320

∴ 1 part = 320/4 = 80 ∴ 3 parts = 3 × 80 = Rs. 240

Example 3: What is the duplicate ratio of 2: 3?

Sol: Duplicate ratio of 2: 3 = 2^2: 3^2 = 4: 9.

Example 4: Triplicate ratio of two numbers is 27: 64. Find their duplicate ratio.

Sol: Triplicate ratio of two numbers is 27: 64,

so numbers should be $27^{1/3}$: $64^{1/3}$

numbers are in the ratio 3: 4.

duplicate ratio of 3: 4 = 3^2: 4^2 = 9: 16.

Example 5: The ratio of two numbers is 25: 36. Find their sub duplicate ratio.

Sol: Sub duplicate ratio of 25: 36 = $25^{1/2}$: $36^{1/2}$ = 5: 6.

Example 6: Prove that 16: 12 and 4: 3 are in proportion.

Sol: The product of the means = 12 × 4 = 48.

The product of the extremes = 16 × 3 = 48

As Product of Means = Product of Extremes

∴ 16: 12 & 4 : 3 are in proportion.

Example 7: Find the missing number in 3: 4 = 12: _____

Sol: Let the missing number is "a".

WKT, Product of means = Product of extremes.

Therefore 3 × a = 4 × 12; By dividing both sides by 3,

we get the missing term = (4 × 12)/3 = 16

Example 8: Taking 4 and 16 are means, write any two proportions.

Sol: Given 4 and 16 are means. So,: 4 = 16: _____

The product of Means is 4 × 16 = 64.

Hence the product of Extremes must also be 64

64 can be written as 4 × 16 or 2× 32 etc.

Two proportions are 2: 4: 16: 32 and16: 4: 16: 4.

Example 9: Find the fourth proportional of the numbers 12, 48, 16.

Sol: Let fourth proportional is x.

The product of extremes = The product of the means (Refer the concepts)

$$12/48 = 16/x$$

$$x = 64.$$

Example 10: Sam takes 2 hours to cover 40 km. Find the distance he will travel in 8 hours.

Sol: Let distance covered = y.

Distance = Speed x Time (Speed is Constant)

When time increases the distance also increases.

Therefore, they are in direct variation,

$$2: 8 = 40: y$$

$$y = (40 \times 8)/2 = 160 \text{ km.}$$

Sam will travel 160 km in 8 hours.

Example 11: Suppose that y varies inversely as x and that y = 12 when x = 6.

a) Form an equation connecting x and y.

b) Calculate the value of y when x = 18.

Sol: x and y are in inverse proportion. So $x_1y_1 = x_2y_2$

$$6 \times 12 = 18 \times y$$

So, y = 4

PRACTICE PROBLEMS:

1. When a sum of money was equally distributed among 49 children, each child received Rs. 20. If the same amount is equally distributed among children, such that each child gets Rs. 3.5, find the number of children.

   a) 280 b) 250 c) 300 d) 450

2. The cost of a diamond varies as the square of its weight. A diamond weighing 20 decigrams costs Rs. 4,800. Find the cost of a diamond of the same kind weighing 8 decigrams.

   a) 768 b) 750 c) 650 d) 675

3. The ratio of two numbers is 9: 5. If 9 is added to the greater number and 5 is subtracted from the smaller number, the greater number becomes thrice the smaller one. Find the numbers.

   a) 50,20 b) 40, 20 c) 36, 20 d) 55, 20

4. Find the ratio of the diagonal of a square of side 30 cm, to its side.

   a) $\sqrt{2}:\overline{1}$ b) 1: $\sqrt{3}$ c) 1: $\sqrt{5}$ d) 1: $\sqrt{10}$

5. The ratio of the first and second-class fares between the two stations is 6: 4 and the number of passengers travelling by first and second-class is 1: 30. If Rs. 2100 is collected as fare, what is the amount collected from first class passengers?

   a) 200 b) 100 c) 300 d) 500

6. The ratio of A's salary to B's was 4: 5. A's salary is increased by 10% and B's by 20%, what is the ratio of their salaries now?

   a) 11:15 b) 12:25 c) 15:16 d) 22:23

7. 200 g of 25% sulphuric acid solution was added to 300 g of 40% sulphuric acid solution. Find the concentration of the acid in the mixture.

   a) 34% b) 43% c) 50% d) 45%

8. In one alloy there is 60% gold in its total mass, while in another alloy it is 35%. 12 kg of the first alloy was melted together with 8 kg of the second one to form a third alloy. Find the percentage of gold in the new alloy.

   a) 34% b) 43% c) 50% d) 45%

9. Divide Rs. 390 among 3 persons A, B and C such that 3 times A's share, 2 times B's share and 4 times C's share are all equal. The shares of A, B and C are respectively

a) 120, 180, 90 b) 110, 80, 90

c) 110, 130, 60 d) 100, 170, 90

10. The ratio of the length and the breadth of a rectangle is 3: 5 and its area is 1.35 cm^2. Find the length of the rectangle.

a) 0.9cm b) 1cm c) 0.5cm d) 0.7cm

11. A, B and C enter into a partnership in the ratio $\frac{7}{4}:\frac{4}{3}:\frac{6}{5}$. After 4 months, A increases his share 50%. If the total profit at the end of one year be Rs. 21,600, then B's share in the profit is:

A. Rs. 2100 b) Rs. 2400 c) Rs. 3600 d) Rs. 4000

12. The ratio of two numbers is 25: 36. Find their sub duplicate ratio.

a) 5:6 b) 6:5 c) 13:11 d) 11:13

13. Find the missing number in 3: 4 = 12: _____

a) 200 b) 45 c) 16 d) 60

14. Find the fourth proportional of the numbers 12, 48, 16.

a) 64 b) 55 c) 65 d) 68

15. 300 coins consists of 1 rupee, 50 paise and 25 paise coins, their values being in the ratio of 10: 4: 3. Find the number of coins of each type.

a) 120, 180, 90 b) 100, 80, 120

c) 110, 130, 60 d) 100, 170, 90

16. The ratio of the present ages of Priya and her mother is 3: 7. The mother's age at the time of Priya's birth was 48 years. Find the mother's present age.

a) 84 b) 55 c) 65 d) 68

17. There are 145 students in the first three standards. The ratio of number of students in the first and the second standards is 2: 3, while that of students in standards second and third is 4: 3. Find the number of students in 2nd standard.

a) 84 b) 45 c) 60 d) 68

18. A piece of string 70 cm in length was cut into pieces, the ratio of whose lengths was 3: 7. Find the length of longest piece.

a) 49 b) 50 c) 55 d) 68

19. The ratio of the measures ∠A and ∠B of a triangle ABC is 3: 2. The ratio of the measures of ∠B and ∠C is 4: 5. Find the measure of largest angle of the triangle ABC.

a) 72-degree b) 82-degree c) 92-degree d) 68 degree

20. If the salaries of A, B, C are in the ratio of 4:5:6. If the increments of 20%,25% and 30% are allowed respectively in their salaries, then what will be the new ratio of their salaries?

a) 25:36:65 b) 85:36:65 c) 15:36:19 d) 96:125:156

21. The ratio of A's salary to B's was 4: 5. A's salary is increased by 10% and B's by 20%, what is the ratio of their salaries now?

a) 11:15 b) 15:16 c) 13:11 d) 9:16

22. In one alloy there is 60% gold in its total mass, while in another alloy it is 35%. 12 kg of the first alloy was melted together with 8 kg of the second one to form a third alloy. Find the percentage of gold in the new alloy.

a) 50% b) 68% c) 45% d) 44%

23. 60 kg of an alloy A is mixed with 100 kg of alloy B. If alloy A has aluminum and tin in ratio of 3:2 and alloy B has tin and zinc in ratio 1:4, then the amount of tin in new alloy is

(a) 44 kg (b) 36 kg (c) 48 kg (d) 48 kg

24. A and B together have Rs. 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have?

A. Rs. 460 B. Rs. 484

C. Rs. 550 D. Rs. 664

ASSESSMENT PROBLEMS

1. In the income statement of Asha and Ravenna, the ratio of their income in the year 2017 was 5 : 4. The ratio of Asha's income in the year 2018 to that in 2017 is 3 : 5 and the ratio of Ravenna's income in the year 2018 to that in 2017 is 3 : 2. If Rs. 10242 is the sum of the income of Asha and Ravenna in the year 2018, then find the income of Ravenna in the year 2017?

   a) Rs. 1024 b) Rs. 1138 c) Rs. 2776 d) Rs. 4552

2. . A, B and C enter into partnership. A invests 3 times as much as B invests and invests two-thirds of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B?

   a) 1200 b) 1500 c) 1700 d) 2000

3. Four milkman rented a pasture. A grazed 24 cows for 3 months, B 10 cows for 5 months; C 35 cows for 4 months and D 21 cows for 3 months.If A's share of rent is Rs. 720, find the total rent of the field.

   a) 3250 b) 3000 c) 2000 d) 3500

4. A bag contains certain number of coins of different denominations. The ratio of the number of Rs. 1 coins to Rs. 2 coins is 5: 7, respectively and the ratio of number of Rs. 2 coins to Rs. 5 coins is 7 : 6 respectively. Find the total value of the Rs. 5 coins, if the total value of the Rs. 1 coins in the bag is Rs. 15.

   a) Rs. 180 b) Rs. 90 c) Rs. 45 d) Rs. 115

5. A father distributed some chocolates among his four children and kept some with him. The eldest three children got chocolates in the ratio 3: 11: 7. The total number of chocolates with father and youngest child is three times the total chocolates with the three eldest children. The ratio of chocolates with father and that with all the children is 3: 4. Find the total number of chocolates if the youngest child has 81 chocolates with him?

   a) 273 b) 252 c) 278 d) 303

6. Anmol had 10 paise, 25 paise and 50 paise coins in the ratio of 10: 8: 9 respectively. After giving Rs. 20 his mother he has Rs. 40. How many 50 paise coins did he have?

a) 72 b) 60 c) 54 d) 35

7. Three numbers A, B and C are in the ratio of 12: 15: 25. If sum of these numbers is 312, ratio between the difference of B and A and the difference of C and B is –

a) 3: 7 b) 10: 3 c) 3: 10 d) 5: 7

8. The ratio of Nicotine to Heroine in four types of drugs of equal quantity is 2: 3, 3: 7, 4: 11 and 11 :9 respectively. The four drugs are mixed together. What is the ratio of Nicotine to Heroine after mixing?

a) 213: 91 b) 418: 189 c) 91: 149 d)149: 81

9. Out of three positive numbers, the ratio of the first and the second numbers is 3: 4 that of the second and the third numbers is 5: 6 if the product of the second and the third numbers is 4320. What is the sum of three numbers?

a)177 b) 165 c) 185 d) 160

10. The cost price of 2 shirts and 3 jeans is Rs. 2200 and the cost price of 2 jeans and 4 shirts is Rs. 2400. Find the ratio between the cost price of the jeans and the shirt.

a) 8: 9 b) 10: 7 c) 6: 5 d) 11: 10

11. The ratio of zinc and copper in a brass pieces are 13: 7. How much zinc will be there in 100 kg of such a piece?

a) 20 kg b) 35kg c) 55kg d) 65kg

12. The ratio of number of men and women in a factory of 720 workers is 7: 5. How many more women should be joined to make the ratio 1: 1?

a) 80 b) 100 c) 120 d) 150

13. What is the ratio whose terms differ by 40 and the measure of which is 2/7?

a) 12: 56 b) 16: 56 c) 23: 58 d) None

14. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B. Find the ratio of A: B is

a) 4: 3 b) 3: 4 c) 1: 1 d) 2: 3

15. In a certain school, the ratio of boys to girls is 7: 5. If there are 2400 students in the school, then how many girls are there?

a) 500 b) 700 c) 800 d) 1000

16. If a: b = 2: 3 and b: c = 5: 7, find the value of a: b: c?

A) 5: 7: 13 B) 10: 15: 17 C) 10: 15: 21 D) 7: 15: 13

17. What is the fourth proportional to 3, 7, and 15?

A) 15 B) 25 C) 30 D) 35

18. What is the mean proportion between 8 and 32?

A) 2 B) 4 C) 16 D) 32

19. What is the sub-duplicate ratio of 16: 25?

A) 2: 3 B) 4: 5 C) 3: 7 D) None

20. Divide Rs.1024 in the ratio 9: 7?

A) 576, 448 B) 235, 365 C) 123, 231 D) None

21. Ali and Ram invest in a business in the ratio 10: 9. If 5% of the total profit goes to charity and Ali's share is Rs 500, the total profit (in Rs) is

A) Rs 1000 B) Rs 1500 C) Rs 1750 D) Rs 1250

22. If 8 times P's capital is equal to 6 times Q's capital and twice of Q's capital is equal to thrice of R's capital, then the ratio of their capitals is :

A) 3 : 4 : 5 B) 9 : 12 : 8 C) 12 : 15 : 20 D) 20 : 15 : 12

23. If A, B and C are three partners in a business. If four the investment of A is equal to thrice the capital of B and the capital of B is three times the capital of C, out of a total profit of Rs 2500, the share of C is :

A) Rs 400 B) Rs 1200 C) Rs 800 D) Rs 900

24. A, B, C together invests Rs. 45000 for a business. If A subscribes Rs. 4000 more than B and B invests Rs. 4000 more than C, then out of a total profit of Rs. 9000, B receives:

A) Rs 1500 B) Rs 2000 C) Rs 3000 D) Rs 4400

25. A, B and C start a business. If A invests two times as much as B invests and B invests two-fifth of what C invests, then the ratio of capitals of A, B, C is

A) 4 : 2 : 5 B) 8 : 10 : 15 C) 5 : 3 : 2 D) 6 : 2 : 3

MODULE - 4

PERCENTAGE

"Percent" means "per 100"

One percent (**1%**) means 1 per 100.

50 percent (50%) means 50 per 100

Percentage can always be expressed as decimal or fraction or Ratio

PERCENTAGE EQUIVALENT OF FRACTIONS:

Any fraction can be expressed as percentage by multiplying the fraction by 100 and conversely any

percentage can be expressed as fraction by dividing the percentage by 100.

For example

$$50\,\% = \frac{50}{100} = \frac{1}{2} \qquad 25\,\% = \frac{25}{100} = \frac{1}{4} \qquad 20\,\% = \frac{20}{100} = \frac{1}{5}$$

This fraction will help us to do calculation faster,

Example: 12.5 % (4888.48) =?

$$12.5\,\% = \frac{12.5}{100} = \frac{1}{8}$$

So , $\frac{1}{8}(4888.48) = 611.06$

Fraction values And Equivalent Percentage Values

Fraction Values	Percentage	Fraction Values	Percentage
$\frac{1}{2}$	50%	$\frac{1}{8}$	12.5%
$\frac{1}{3}$	33.33%	$\frac{1}{9}$	11.11% or 11 $\frac{1}{9}$%
$\frac{1}{4}$	25%	$\frac{1}{10}$	10%
$\frac{1}{5}$	20%	$\frac{1}{11}$	9.09% or 9 $\frac{1}{11}$%
$\frac{1}{6}$	16.66% or 16 $\frac{2}{3}$%	$\frac{1}{12}$	8.33%
$\frac{1}{7}$	14.287% or 14 $\frac{2}{7}$%		

Marks scored by A is 50 and marks scored by B is 40. Then

Question	Formula or expression	Answers
(i) A is what % more than B?	$\frac{A-B}{B} * 100$	25%
(ii) B is what % less than A?	$\frac{B-A}{A} * 100$	20%
(iii) A is what % of B?	$\frac{A}{B} * 100$	125%
(iv) B is what % of A?	$\frac{B}{A} * 100$	80%

Ex 2: The ratio of A's salary to B's salary is 6 : 5.

(a) By what % does A earn more than B?

(b) By what % does B earn less than A?

Solution:

(a) $\frac{A-B}{B} * 100 = \frac{6-5}{5} * 100 = \frac{1}{5} * 100 = 20\%$

(b) $\frac{A-B}{A} * 100 = \frac{6-5}{6} * 100 = \frac{1}{6} * 100 = 16.66\%$

Ex 5: A is taller than B by 25%, by what percentage B is shorter than A?

Solution:

Method 1 :

Let us Assume B = 100 then A = 125

$= \frac{125-100}{100} * 100 = 25\%$

Answer = 25%

Method 2 :

25 % = $\frac{1}{4}$ Then A= 5 & B= 4

$= \frac{5-4}{4} * 100 = 25\%$

Ex 6: Income of A is more than B. A spends 30% of income on food and B spends 25% of his income on food. Who spends more on food?

Sol: Income of A is more than B and he spends higher % of income. (as 30% > 20%) on food.

Hence A spends more on food than B.

Ex 7: In the above example if income of A is less than B then who spends more on food?

Sol: As a percentage, 30% > 25%. But here we are taking. higher % (ie. 30%) of lower base

(i.e A) may be more or less than 25% of B. Hence, answer is cannot be determined.

PERCENTAGE CHANGE:

$$\% \text{ Increase} = \frac{Final\ Value - Initial\ Value}{Initial\ Value} * 100$$

$$\% \text{ decrease} = \frac{Initial\ Value - Final\ Value}{Initial\ Value} * 100$$

For example, present population of city A is 45000 and last year it was 35000. Then we can find % increase.

Actual Increase = 45000 – 35000 = 10000

Initial populations = 35000

$$\% \text{ Increase} = \frac{45000 - 35000}{35000} * 100$$

$$= 28.56\%.$$

Ex 8:

(a) Production of wheat in 1996 was 60,000 tons and in 1997, it was 75,000 tons. By what percentage did the production increase?

Sol:

$$\% \text{ change} = \frac{75000 - 60000}{60000} * 100 = 25\%$$

(b) In the year 1999, the production of pulses increases by 25% over that of the preceding year and reached 400 thousand tons. What was the quantity of pulses produced in 1998?

Sol:

If production in 1998 = 100%.

then production in 1999 = 125%. out of given 125%. We want 100%.

Hence, prod. in 1998 = $\frac{100}{125} * 400 = 320$

(c) The quantity of jowar produced in the year 1997 was 21,000 tons. During the next year the production increased by 33.33 %. What was theproduction in 1998?

Sol:

33.33 % is 1/3

Hence 4/3 (21000) = 28,000 tons

Ex 9 : If the price of an item is increased by 25%, by what % should it be brought down to bring back to the original level?

Sol:

Assume, Original Price = 100 then New Price = 125

From 125 we should have brought back to 100,

$$\% \text{ change} = \frac{125 - 100}{125} * 100 = 20\%$$

Ex 10: If the price of an item is decreased by 25% by what % should it be raised to bring it back to original level?

Sol:

Assume , Original Price = 100 then New Price = 75

From 75 we should brought back to 100,

$$\% \text{ change} = \frac{100 - 75}{75} * 100 = 33.33\%$$

Ex 11: If the price of tea is decreased by 20% then by what % should the consumption be increased in order to maintain constant expenditure?

Sol:

PRICE * QUANTITY = EXPENDITURE

Assume the initial Price is 5Rs then new price is 4Rs,

Assume the quantity is 4Kg as the expenditure are same then the new quantity is 5

	PRICE	**QUANTITY**	**EXPENDITURE**
INITIAL	Rs 5	4Kg	Rs 20
NEW	Rs 4	5Kg	Rs 20

Hence, $\% \text{ change} = \frac{5 - 4}{4} * 100 = 25\%$

Ex 12: The production of wheat formed 20% of the total food grain production in 1996. In the next year, the share of wheat in the total food grain production went up by 5 percentage points. Find the % change in production of what from1996 to 1997 if the total food grain production increased is by 20% from 1996 to 1997

Sol:

	1996	**1997**
Total Food grain production	100	120
Wheat production	20	30

Hence, % change = $\frac{30-20}{20} * 100 = 50\%$

SUCCESSIVE PERCENTAGE FORMULA.

The value is increased by a% and b% then the

$$\textbf{Overall percentage change} = a + b + \frac{ab}{100}\%$$

Ex 13: If A's income is 16% more than B's income and B's income is 25% more than C's income, by what percentage is A's income more than C's income?

$$\textbf{Overall percentage change} = (16 + 25 + \frac{16*25}{100})\% = 45\%$$

PRACTICE PROBLEMS

1. If 24 carat gold is considered as 100% purity what is purity percentage of 22 carat gold?

   (1) 91 2/3% (2) 83.33% (3) 166% (4) 42%

2. A charity in Chennai raised Rs 1,20,000. This amount was 125% of the amount raised in the last year. What was the amount raised last year?

   (1) 1,50,000 (2) 1,05,000 (3) 90,000 (4) 96,000

3. Which of the following represents a percentage increase of 20%?

   (1) 20 to 30 (2) 30 to 40 (3) 50 to 60 (4) 90 to 100

4. 23% of a + 23% of b = 207. What is the average of a and b?

   (1) 207 (2) 20700/23 (3) 450 (4) CBD

5. If all the 6's are replaced by 9's, then by what percentage [approximately] will the algebraic sum of all the numbers from 1 to 100 [both inclusive] vary?

   (1) 15% (2) 1.6% (3) 6.5% (4) 18%

6. From a barrel containing 500 ml of alcohol, 3 cups of alcohol are poured into another barrel containing 500 ml of water. After mixing, 3 cups of the mixture are poured into the barrel of alcohol. Between the percentage of water in the barrel of alcohol and the percentage of the alcohol in the barrel of water which one is more?

   (1) Percentage of water in the barrel of alcohol is more

   (2) Percentage of alcohol in the barrel of water is more

   (3) It will be same

   (4) None of these

7. Price of cooking gas is increased by 10%. By what percentage should the consumption be reduced in order to maintain the same expenditure level?

   (1) 9.09% (2) 10% (3) 11.11% (4) 110%

8. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B. Find the ratio A : B.
   (1) 4:5 (2) 2:3 (3) 3:4 (4) 4:3

9. A basket contains a certain number of apples. Ameen picks 1 less than 50% of the apples. Then Nano picks 1 less than 50% of the remaining apples. Then Srikant picks 1 less than 50% of the remaining apples. Finally only3 apples are left in the basket. What is the number of apples in the basket initially? How many apples did Nano pick?

   (1) 14 , 4 (2) 10, 2 (3) 15, 5 (4) 24, 10

10. In a list of weights of candidates recorded in computer, the weight of A is marked as 50 kg instead of 40 kg. Find the percentage of correction required.

(1) 11.6% (2) 16% (3) 20% (4) 25%

11. Spice jet reduces their airfares by 20%. What should be the increase in number of tickets sold such that the revenue remains constant?

(1) 20% (2) 25% (3) 45% (4) 50%

12. Dr .Badrinath spends 80% of his income. If his income and expenditure increase by 30% and 15% respectively what will the percentage increase in his savings?

(1) 30% (2) 90% (3) 12.33% (4) 83.15%

13. In an examination, 52% of the candidates failed in English, 42% in Mathematics and 17% in both. What is the number of students who passed in both the subjects if the total strength of the class is 100?

(1) 14 (2) 20 (3) 19 (4) 23

14. If 240 liters of oil is poured into a tank, it is still 20% empty. How much more oil is to be poured to fill the tank?

(1) 60 liters (2) 192 liters (3) 288 liters (4) 160 liters

15. A batsman scored 120 runs which included 11 fours and 6 sixes. What percentage of his total score did he make by running between the wickets?

(1) 40% (2) 41% (3) 33.33% (4) 41.5%

16. 60% of batsmen are right-handed and 40% of right-handed batsmen score over 500 runs per year. If 30% of all batsmen score over 500 runs per year, what percentage of left-handed batsmen score under 500 runs per year?

(1)60% (2) 68% (3) 70% (4) 72%

17. AB is a two-digit number, both A and B are distinct. When AB is increased by 33.33%, AB becomes BA. How many such numbers are possible?

(1) 1 (2) 2 (3) 12 (4) 0

18. If the length of a rectangle is increased by 10% and the breadth by 20% what is the percentage increase in area.

(1)20% (2) 25% (3) 30% (4) 32%

19. If the length of the rectangle increases by 10% and the breadth by 20%, Find the minimum percentage of the new perimeter more than its old from the given options?

(1) 2.7% (2) 15% (3) 30% (4) 32%

20. In an exam Gappu secured 20% of the total marks and he failed by 20 marks. However, his cousin Pappu passed the exam securing 40% of total marks, which was 10, more than the passing mark. What was the passing mark?

(1)35 (2)40 (3)45 (4)50

21. A class of 100 students, there are 60 girls and 40 boys. If 60% of the girls and 80% of the boys passed the exam what was the % of students who passed?

1. 60% 2. 68% 3. 70% 4. 72%

22. The length, breadth and height of a room are in the ratio 3:2:1. If the breadth and height are halved while the length is doubled, then the total area of the four walls of the room will be?

1. remain the same
2. decrease by 13.64%
3. decrease by 18.75%
4. decrease by 30%

23. Till 2010, Dada won 38% of the matches in which he captained India. In 2011, he played 40 more matches as a captain and won 22 of them. Now Dada has an overall captaincy record of more than 40% wins. What is the largest number of matches he could have captained till the end of 2011?

1. 339 2. 299 3. 321 4. None of these

24. A student has taken six tests in a subject with each test carrying equal marks. He has scored marks in the ratio of 7:8:9: 6:6:6. If he scored 70% of total marks, then in how many tests did he score over 80% marks?

1. 1 2. 2 3. 3 4. 4

25. A school has gathered 80% of the required donation from 60% of the donors who gave an average of Rs. 500 per head. How much, on average, should the remaining donors contribute so that the school receives exactly the money it requires?

1. Rs. 150 2. Rs. 170 3. Rs. 180 4. Rs. 187.5

ASSESSMENT PROBLEMS

1. A speaks the truth 70% of the time and B speaks the truth 80% of the time. Find the probability in terms of percentage that both are contradicting each other?

   (a) 96% (b) 12% (c) 9% (d) 38%

2. A alone can complete 25% of a work in a day. B alone can complete 8.5% of work in a day. If they work together what percentage of work would be finished in two days?

   (a) 33.33% (b) 25% (c) 66.67% (d) 20%

3. B is 50% faster than A. A starts at 9 a.m and B starts at 10 am. A travels at a speed of 50 kmph. If A and B are 300 kms apart and they travel in opposite direction what is the exact time when they meet each other?

   (a) 11 am (b) 12.55 pm (c) 10.45 am (d) None

4. A person has Rs100. If he wins he gains 10% and if he loses the game, he loses 10%. He wins twice and loses twice. How much money did he have at the end of four games?

   (a) 95 (b) 100 (c) 99 (d) None

5. If the numerator of a fraction is increased by 100% and the denominator is increased by 50% the resultant fraction is 8/15. What is the original fraction?

   (a) 3/2 (b) 5/4 (c) 2/5 (d) 5/11

6. The total strength of a class is 80. If 20% more boys and 30% more girls join the class, then the class strength becomes 99. What is the number of boys and girls in the class initially?

   (a) 30, 50 (b) 40, 40 (c) 12, 68 (d) 50, 30

7. A shop sells floor tiles at Rs 48 per square meter. A building contractor employs a machine that polishes the tiles that damages 10% of the total number of tiles which cannot be used any more. Calculate the amount that needs to be paid by contractor to tile shop owner, if the hall is square in shape and has a perimeter of 400 meters?

   (a) ` 4,32,000 (b) ` 5,28,000 (c) ` 69,12,000 (d) ` 6,91,200

8. Prakash scores 30% in an examination and fails the exam by 20 marks whereas Sumit scores 45% and passes the exam by 10 marks. What is the maximum mark one can score in this exam?

   (a) 225 (b) 180 (c) 200 (d) 250

9. A reduction of 10% in price of sugar enables a housewife to buy 5 kg more for ` 300/-. What is the price of sugar per kg after the reduction?
   (a) 20 (b) 40 (c) 6 (d) None

10. In a company 40% of the employees are men and 75% of the men earn more than Rs 25,000 per year. If 45% of the company's employees earn more than Rs 25,000 per year, what fraction of the women employed bythe company earn at most ` 25,000 per year?

    (a) 3/4 (b) 7/8 (c) 2/33 (d) None

11. In an examination, 100 questions are divided into three groups A, B and C such that each group contains at least one question. Each question in group A carries 1 mark, each question in group B carries 2 marks and each question in group C carries 3 marks. It is known that the questions in group A together carry at least 60 % of the total marks. If a person attends all the questions what are the marks scored by him?

    (a) 68 (b) 92 (c) 96 (d) CBD

12. Total amount with A, B, C and D is Rs 240. A has 50% of B, C and D together. B has 33.33% of A, C and D together and C has 25% of A, B and D together. What is the amount with C?
    (a) 80 (b) 155 (c) 48 (d) 30

13. Arvind is standing in a queue. The number of persons in front of him is 40% of number of people standing behind him. What is the possible number of people in the queue?

    (a) 101 (b) 41 (c) 8 (d) Cannot be determined

14. 75% of a number when added to 75 is equal to the number. What is the number?

    (a)200 (b) 75 (c) 250 (d) 300

15. The salary of A is 10% higher than B's salary. By what percentage is B's salary less than A's salary?

    (a) 10% (b) 9.09% (c) 9% (d) 9.18%%

16. There are 3 vessels A, B, C of different capacities. Initially vessel A is filled with water. All the water from A is transferred to to vessels B and C. Volume of vessel C = 1/3rd of A. If 300 ml of water transferred to B had been transferred to C then B would have 50% more water than C. What is the capacity of C?
    (a) 900 ml (b) 3,000 ml (c) 4,500 ml (d) 1,500 ml

17. The population of a town is 5000. If the males increase by 6% and females by 14% the population will be 5500. Find the number of females in the town.
    (a) 5000 (b) 2 0 0 0 (c) 3500 (d) 2500.

18. A dishonest milkman professes to sell his milk at cost price but he mixesit with water and there by gains 25%. The percentage of water in the mixture is?
    (a) 25% (b) 20% (c) 15% (d) 60%

19. There are 2 equal numbers "x" and "y". x is increased by 16 % and y is decreased by 16 %. By what percentage is the value of x more than the value of y?
    (a) 33 % (b) 30% (c) 40% (d) 70%

20. Guru goes to shop and buys a sofa costing Rs.13080. The rate of sales tax is 9%. She asks the shopkeeper to reduce the price of sofa so that she pays Rs.13080 inclusive of tax. Find the percentage reduction needed to satisfy her requirement?
    (a) 8.33% (b) 8.26% (c) 9% (d) 8.5%

21. India scored 48 runs in the first 10 over of a 50 over match. If that was 16% of the target, then how many more runs does India need to win the match?
    A. 220 B. 252 C. 270 D. 300

22. Rakesh earns a sum of Rs 50,000 every month. He spends 20% of rent, 15% on food and groceries, 25% on children's education. What is the money left with him for other expenses?
    A. 10,000 B. 20,000 C. 30,000 D.40,000

23. The price of Milk has been increased by 20% and it now costs Rs 24. What was the original cost of Milk?
    A. Rs 16 B. Rs 18 C. Rs 20 D. Rs 24

24. If the length of a rectangle is increased by 10% and the breadth by 20% what is the percentage increase in area.
    A. 20% B. 25% C. 30% D. 32%

25. 11. A family consumes 20 kg of sugar every month. If the price of sugar increases by 10%, what should be approximate reduction in consumption such that expenditure doesn't increase?
    A. 1 kg B. 1.8Kg C. 2.4Kg D. None of the above

MODULE - 5

PROFIT AND LOSS

Cost Price:

The price at which an item is bought is cost price.

Selling Price:

The price at which an item is sold is selling price.

Profit = selling price – cost price

Loss = cost price – selling price

$$\text{Profit\%} = \frac{PROFIT}{COSTPRICE}\ X\ 100$$

$$\text{Loss\%} = \frac{LOSS}{COSTPRICE}\ X\ 100$$

The price at which an item is marked is marked price.

Discount = Marked price – Selling price

$$\text{Discount\%} = \frac{DISCOUNT}{MARKED}\ X\ 100$$

If a seller offers successive discounts of d1 and d2, the effective discount is d1 + d2 – (d1 x d2)/100

Ex 1:

(a) The C.P & S. P of an article are Rs.400 & Rs.600 respectively. What is profit %?

Sol : Profit% = $\frac{600-400}{400} X\ 100 = 50\%$

(b) If the price above given are interchanged what is % of loss.

Sol : Loss% = $\frac{400-600}{600} X\ 100 = 33.33\%$

Ex 2: An article brought at Rs. 700 was sold at a profit of 20% what is the S.P?

Sol : S.P = 120%(C.P) = 120%(700)= Rs 840

Ex 3: An article is sold for Rs. 400 at profit of 25%. What is C. P?

Sol : C.P = $\frac{100}{125}\ (S.P) = Rs320$

Ex 4: 'A, sells an article to 'B' at 25% profit B sells same to 'C' at 20% profit.

(a) By what % is the C.P of 'C' more than 'A'?

Sol :

A's C.P	A's S.P or B's C.P	B's S.P or C's C.P
100Rs	125%(100) =125	120%(125) = 150
100Rs		150

% Increase = $\frac{C-A}{A} * 100 = \frac{150-100}{100} * 100 = 50\%$

(b) If the C. P of 'A' is 600. How much did it cost to 'C'.

With the reference of the previous Solution, C.P of A is assumed as 100 Hers it is mentioned as 600 then C.P Of C can be written as 150*6 = 900 Rs

$\frac{C.P\ of\ A}{C.P\ of\ C} = \frac{100}{150} = \frac{600}{?}$

C.P of C is 900

(c) If 'C' brought it for Rs. 1200 at what price did 'A' buy it.

$$\frac{C.P\ of\ A}{C.P\ of\ C} = \frac{100}{150} = \frac{?}{1200}$$

C.P of A is 800

Ex 5: The C.P. of two articles is same. If one item is sold at 10% P and other at 10% L, what is % P or %L on the whole?

Solution :

	C.P	S.P	Profit or Loss
A	100	110	10 Profit
B	100	90	10 Loss
Total	200	200	0

No Profit or Loss

Ex 6: Two articles sold at same price. On one article there is a loss of 10% and on the other profit of 10% what is P% or L% in the whole.

	C.P	S.P	Profit or Loss
A	(Assume) 10	11	
B	10	9	
Total			

Given that, Selling price is same in both the cases, make it S.P same

	C.P	S.P	Profit or Loss
A	90	99	9 Profit
B	110	99	-11 Loss
Total	200	198	-2

$$\text{Loss\%} = \frac{198-200}{200} X\ 100 = 1\%$$

Short Cut Method $\quad \text{Loss\%} = \frac{x^2}{100}\ \% = 1\%$

PRACTICE PROBLEMS

1. Cost Price = ` 40, Selling Price = ` 50 and ratio between SP/CP = 50/40 = 1.25. What do you infer from the given data.

   (1) If SP/CP is greater than 1, then there must be a profit

   (2) To calculate the profit %, we can also find the ratio of SP/CP as one more method

   (3) Whenever SP is greater than CP, there

   (4) All of the above

2. A buy a shirt for ` 100 and sells it to B for ` 120. B now sells it to C for ` 144. Had A sold it directly to C at the price C paid for it, then his profit % would have been _____

   (1) 10% (2) 20% (3) 44% (4) 46%

3. The selling price of 15 apples is equal to the cost price of 20 apples. What is the profit or loss percent?

   (1) 22% (2) 33.33% (3) 25% (4) 18%

4. An advertisement states the following - "Buy 2 shirts and get 1 free". Find the discount %?

   (1) 50% (2) 66.67% (3) 75% (4) 33.33%

5. If CP = ` 50, SP = ` 100 and Marked Price = ` 120 then Profit % and Discount % are (in that order)

   (1) 50%, 20% (2) 100%, 20% (3) 37%, 50% (4) 100%, 16.67%

6. A college student buys a second-hand bike for ` 22,500. He spends ` 1,500 on an overhaul and other repairs. He then sells the bike for ` 30,000 to his friend. His profit % is _____

   (1) 20% (2) 25% (3) 30% (4) 40%

7. If the selling price of an article is 7/6 times its cost price, the profit percentage is _____

   (1) 33.33% (2) 25% (3) 16.67% (4) 20%

8. Ravi buys mangoes at the rate of Rs 21 for 3 kg and sells them at Rs 50 for 5 kg. How many kgs should he sell to earn a profit of ` 51?

(1) 17 kg (2) 5 kg (approx.) (3) 34 kg (4) None

9. If books bought at prices ranging from Rs 200 to Rs 300 are sold at prices ranging from Rs 300 to Rs 400, what is the greatest possible profit % that can be made by selling 10 books?

(1) 10% (2) 20% (3) 300 % (4) 100%

10. John bought a car for Rs 5,00,000. He sold at a 30% profit. From the money raised from this sale, he bought another car, which unfortunately he had to sell for a 30% loss. What was his overall profit% or loss%?

(1) Loss 9% (2) Profit 9% (3) Loss 5% (4) Profit 10%

11. By giving a 20% discount on the marked price of an item the profit is reduced by 60. What is the marked price of the item?

(1) 80 (2) 150 (3) 300 (4) 200

12. The Military canteen gives a 15% discount to senior citizens and an 8% discount to students. Robert, an sixth-grade student, got Rs 320 off on his purchase and his grandfather – a senior citizen got 300 off on his purchase. What is the marked price of each item?

(1) Rs 3,000, Rs 2,000 (2) Rs 5,000, Rs 6,000
(3) Rs 12,000, Rs 14,000 (4) Rs 2,000, Rs 4,000

13. A grocer buys two kinds of rice at Rs 1.8/kg and Rs 1.2/kg. Both the items are mixed in the ratio of 1:2. The mixture so obtained was sold at Rs 1.75 /kg. Find the profit percentage?

(1) 25% (2) 13% (3) 33.33% (4) 23%

14. A dishonest shopkeeper sells his goods at cost price but uses a weight of 800 gm instead of one kilogram weight. What is his profit percentage?

(1) 25% (2) 40% (3) 20% (4) None

15. A shopkeeper offers three successive discounts of 10%, 20% and 30% to a customer. If the marked price of the item is ` 10,000, what is the price the customer has to pay the shopkeeper?

(1) Rs 4000 (2) Rs 5040 (3) Rs 9940 (4) Rs 8465

16. An intelligent vegetable vendor gets 20% more vegetables than what he pays for at the time of making his purchase and delivers to the customer 20% less than what he charges for. If his normal profit percentage is 10% what is his overall profit percentage?

(1) 68% (2) 65% (3) 72% (4) 50%

17. The selling price of 20 apples is equal to the cost price of 15 apples. What is the profit or loss percentage?

(1) 22% (2) 33.33% (3) 25% (4) 18%

18. By selling an article at a discount of 40%, a trader makes a profit of 20%, By what percentage does he markup the cost price?

(1) 119% (2) 200% (3) 100% (4) 40.20%

19. A person has Rs 100. If he wins he gains 10% and if he loses the game, he loses 10%. He wins twice and loses twice. How much money did he have at the end of four games?

(1) ` 95 (2) ` 100 (3) ` 99 (4) None of these

20. By selling 40 cricket bats, Sreesanth makes a profit equal to the selling price of 10 cricket bats. What was the profit % made?

(1) 16.66% (2) 20% (3) 25% (4) 33%

21. Mr Sidhu sells 25 cricket bats and makes a loss equal to selling price of 5 Cricket bats? What was the loss %

(1) 16.66% (2) 20% (3) 25% (4) 33%

22. Rahman sold a book for Rs 133 and made a profit of 10% on the transaction. What is his cost price?

(1) Rs 120 (2) Rs 125 (3) Rs 130 (4) Rs 115

23. By selling twelve marbles for a rupee, a shopkeeper loses 20%. In order to gain, 20% in the transaction, he should sell the marbles at the rate of how many marbles for a rupee

(1) 8 (2) 9 (3) 10 (4) 11

24. By selling a bicycle at Rs 900, Abdul made a loss of 25%. If he had sold it for Rs 1440, what would be his profit %?

(1) 16.66% (2) 20% (3) 25% (4) 33%

25. A shopkeeper allows a discount of 10% on the goods. For cash payments he further allows a discount of 20%. How much single discount will be equivalent to this offer?

(1) 16.66% (2) 20% (3) 25% (4) 28%

ASSESSMENT PROBLEMS:

1. At what percentage above the cost price must an article be marked so as to gain 35% after allowing the customer a discount of 10%?

   (1) 50% (2) 45% (3) 20% (4) 30%

2. An item should be sold for Rs. 50 more to get a profit of 10% instead of a loss of 15%. Find the cost price of the item?

   (1) Rs 170 (2) Rs 200 (3) Rs 220 (4) Rs 250

3. Three successive discounts of 20% each is equivalent to a single discount of what %?

   (1) 84% (2) 60% (3) 48.88% (4) 47%

4. A trader professes to sell his goods at cost price but uses 800gms weight for every kg. What is his profit percentage?

   (1) 25% (2) 33.33% (3) 37.5% (4) 30%

5. A trader loss 25%, if an article is sold at Rs. 243.find the selling price of the article to get a profit of 25%.

   (1) 300 (2) 675 (3) 405 (4) 240

6. A sells to B an iron box at 10% profit, B sells it to C, for 20% profit.If C pays Rs.528 for the product what is A cost price?

   (1) 300 (2) 675 (3) 405 (4) 240

7. If goods are purchased for Rs.900, one third of them sold at a profit at 10%, at what profit percent should the remaining be sold to obtain a profit percent of 15%?

   (1) 50% (2) 33.33% (3) 37.5% (4) 30%

8. A trader marks an article 30% above its cost price. If he allows a discount of 30%, then find his profit/loss percent.

   (1) 8% (2) 12.5% (3) 37.5% (4) 30%

9. The listed price of an article is Rs.1000. Successive discounts of 10% and 15% are offered on it. What is going to be the new price?

(1) 300 (2) 675 (3) 765 (4) 240

10. The successive discount of 25% and 20% are equal to a single discount of(1)

40% (2) 12.5% (3) 37.5% (4) 30%

11. At what percentage above the cost price must an article be marked so as to gain 35% after allowing the customer a discount of 10%?
1. 50% 2. 40% 3. 25% 4. 30%

12. Harbhajan Singh purchases 400 cricket balls for Rs 6400. If he sells 1/4th of them at a profit of 20% and 3/8th of them at a loss of 20%, then what should be profit on the remaining such that an overall profit of 5% is made?
1. 16.66% 2. 20% 3. 25% 4. 30%

13. A reduction of 20% in the price of sugar enables a purchaser to obtain 2.5 Kg more for Rs 160. Find the original price per kg of sugar
1. 16 2. 20 3. 24 4. None of these

14. An item is bought for Rs. 350 and sold to a middleman at a 20% profit who sells it to the final customer at a 20% profit. Find the profit made by the middleman.
1. Rs. 70 2. Rs. 78 3. Rs. 84 4. Rs. 90

15. A fruit trader sells 60% of his mangoes at a 10% loss and the rest at a 20% profit. Find the net profit percentage in the transaction.
1. 1% 2. 2% 3. 5% 4. 7%

16. An item should be sold for Rs. 50 more to get a profit of 10% instead of a loss of 15%. Find the cost price of the item?
1. Rs. 170 2. Rs. 200 3. Rs. 220 4. Rs. 250

17. Selling an item at 90% of the marked price will ensure a profit of 50%. What will be the profit percentage if the item is sold at marked price?
1. 55% 2. 66.66% 3. 75% 4. None of these

18. A vendor finds out that 10% of his bulbs are damaged. By what percent should he increase the selling price of the remaining bulbs to make up for the loss caused on account of the damaged bulbs?
1. 10% 2. 11.11% 3. 15% 4. 20%

19. If the cost of an item is decreased by 10% and its selling price is increased by 10% the profit percentage becomes 10%. Find the initial profit or loss percent.
1. No profit No loss 2. 5% profit 3. 5% loss 4. 10% loss

20. A dishonest vendor claims to sell his items at cost price, but instead of using a 1 kg weight he uses faulty 950 gm weight. Find his actual profit or loss percentage?
1. 5% 2. 5.2% 3. 5.26% 4. 5.3%

21. Selling an item at 90% of the marked price will ensure a profit of 50%. What will be the profit percentage if the item is sold at marked price?

1. 55% 2. 66.66% 3. 75% 4. None of these

22. A vendor finds out that 10% of his bulbs are damaged. By what percent should he increase the selling price of the remaining bulbs to make up for the loss caused on account of the damaged bulbs?

1. 10% 2. 11.11% 3. 15% 4. 20%

23. If the cost of an item is decreased by 10% and its selling price is increased by 10% the profit percentage becomes 10%. Find the initial profit or loss percent.

1. No profit No loss 2. 5% profit 3. 5% loss 4. 10% loss

24. A dishonest vendor claims to sell his items at cost price, but instead of using a 1 kg weight he uses faulty 950 gm weight. Find his actual profit or loss percentage?

1. 5% 2. 5.2% 3. 5.26% 4. 5.3%

25. A dealer increases the cost of his goods by 20% once in a while and decreases them by 20% later. The selling price was Rs. 1,843.20. What is the initial cost of the item?

1. Rs. 2500 2. Rs. 2200 3. Rs. 2000 4. Rs. 1920

MODULE-6

AVERAGES, MIXTURES AND ALLIGATIONS

- **Average** $= \frac{\text{Sum of observations}}{\text{Number of observations}}$

Properties of average are:

When the difference between all the items is same (and the number of terms is odd), then the average is equal to the middle term.

- If x is added to all the items, then the average increases by x.
- If x is subtracted from all the items, then the average decreases by x.
- If every item is multiplied by x, then the average also gets multiplied by x.
- If every item is divided by x, then the average also gets divided by x.

Important Points (Shortcuts):

- When a person replaces another person then:
  - If the average is increased, then
    Age of new person= Age of person who left + (Increase in average * total number of persons)
  - If the average is decreased, then
    Age of new person= Age of person who left - (Decrease in average * total number of persons)
- When a person joins the group:
  - In case of an increase in average,
    Age of new member= Previous average+ (Increase in average * Number of members including new member)
  - In case of decrease in average,
    Age of new member= Previous average- (Decrease in average * Number of members including new member)
- In the Arithmetic Progression there are two cases:
  - When the number of terms is odd - the average will be the middle term.
  - When number of terms is even - the average will be the average of two middle terms.

SOLVED EXAMPLES:

Problem 1:
The average salary of 20 officers in a company is 300. The average salary of non-officers in the same company is 120. Find the number of the non-officers if the average of the whole organization is 150.

Solution:
There are two methods to solve this. One is the conventional method. Let the number of Non-officers be N. Now the equation can be made, knowing that the sum of the salaries of all the officers and non-officers would give the total salary of the organization. The equation will be 20 × 300 + 120 × N = 150 (20 + N). Solving this, N = 100.

Problem 2:
The average weight of a class of 24 students is 36 years. When the weight of the teacher is also included, the average weight increases by 1kg. What is the weight of the teacher?

Solution:
We know that, in case of increase in average,
Age of new member= Previous average+ (Increase in average X Number of members including new member)
Therefore,
Age of new member = 36 + (1 × 25)
Age of new member = 36 + 25 = 61

Problem 3:
Find the average of first 30 natural numbers.

Solution:
Sum of first n natural numbers = n(n+1)/2
Therefore, Average=(30 (30+1))/(2*30)
= (30*31)/(2*31) = 15.5

Problem 4:
A merchant has a sale of Rs. 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs. 6500?

Solution:
Total Sale of 5 months = 6435+6927+6855+7230+6562 = 34009
Required Sale = 6500 * 6 = 39000
Sale required in 6^{th} month = 39000 - 34009 = Rs 4991

Problem 5:
The average age of the teacher and six students is 12 which is reduced by 5 if the age of the teacher is excluded. What is the age of the teacher?

Solution:

Total age of six students and teacher = 12 * 7 = 84
New average when teacher is excluded = 7
Therefore, total age of six students = 6 * 7 = 42
Therefore, age of teacher = 84 - 42 = 42

Problem 6:
19 persons went to a hotel for a combined dinner party. 13 of them spent Rs 79 each on their dinner and the rest spent Rs 4 more than the average expenditure of all the 19. What was the total approximate money spent by them?
Solution:
Assume that "×" is the average expenditure of 19 persons.
Then, 19× = 13 * 79 + 6 (×+4)
13× = 1051
× ≅81 (approximately, as in the question approximate money is asked)
Therefore, total expenditure = 81 * 19= Rs 1539

Problem 7:
The average marks of 13 students 40. The average marks of first 7 students are 42 and that of the last seven are 35. Find the marks of the 7th student.
Solution:
Total marks of 13 students = 13 * 40 = 520 Marks of first 7 students = 7 * 42 = 294
Marks of last 7 students = 7 * 35 = 245
Marks of 7th student = (294+245) - 520 = 19

Problem 8:
Aman has a certain average for 9 innings in the tenth innings; he scores 100 runs, thereby increasing his average by 8 runs. His new average is
Solution:
Let his total average b "×"
His total runs in 9 innings will be 9×
New total; 9x + 100 = 10 (×+8)
× = 20
Therefore, new average = 20 + 8 = 28

A mixture is created when two or more substances are mixed in a certain ratio

Types of mixtures

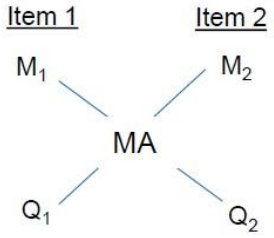
1. Simple mixture

A simple mixture is formed by the mixture of two or more different substances.

Ex. Water and Wine mixture

2. Compound mixture

Compound mixture is formed by the mixture of two or more simple mixtures.



If M_1 and M_2 are the values, Q_1 and Q_2 are the quantities of item 1 and item 2 respectively and M_A is the weighted average of the two items, then

$$\frac{Q_1}{Q_2} = \frac{M_2 - MA}{M_A - M_1}$$

Weighted average M_A can be calculated by, $M_A = \frac{Q_1M_1 + Q_2M_2}{Q_1 + Q_2}$

The alligation rule can be applied when cheaper substance is mixed with expensive substance

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{Price of dearer} - \text{Mean price}}{\text{Mean price} - \text{Price of cheaper}}$$

If two mixtures M_1 and M_2,having substances S_1 and S_2 in the ratio a:b and p:q respectively are mixed, then in the final mixture,

$$\frac{\text{Quantity of } S_1}{\text{Quantity of } S_2} = \frac{M_1\left[\frac{a}{a+b}\right] + M_2\left[\frac{p}{p+q}\right]}{M_1\left[\frac{b}{a+b}\right] + M_2\left[\frac{q}{p+q}\right]}$$

If there is a container with 'a' liters of liquid A and if 'b' liters are withdrawn and equal amount is replaced by another liquid B and if the operation is repeated for 'n' times

After nth operation,

- Liquid A in the container = $\left[\frac{a-b}{a}\right]^n \times$ Initial quantity of A in the container
- $\frac{\text{Liquid A after nth operation}}{\text{Liquid B after nth operation}} = \frac{\left[\frac{a-b}{a}\right]^n}{1-\left[\frac{a-b}{a}\right]^n}$

SOLVED EXAMPLES:

1. A Container contains 192 litres of Milk. A seller draws out x% of Milk and replaced it with the same quantity of water. He repeated the same process for 3 times. And thus, Milk content in the mixture is only 81 litres. Then how much percent he withdraws every time?

Explanation: $81 = 192(1-x/100)^3$

$x = 25$

2. A Jar contains 30 litres mixture of Milk and Water in the ratio of x:y respectively. When 10 litre of the mixture is taken out and replaced it water, then the ratio becomes 2:3. Then what is the initial quantity of Milk in the Jar?

Explanation: $x+y = 30$

$(x-10*x/x+y)/ (y-10*y/(x+y) + 10) = 2/3$

$2x-4/3y = 20$

$x = 18$

3. 'X' litres of the mixture contains Milk and Water in the ratio 4:3. If 13 litres of Water is added then the ratio becomes 1:1. Then what is the final quantity of water in the mixture?

Explanation: $4x/3x+13 = 1$

$x = 13$

Water = $3x+13 = 39+13 = 52$

4. A Jar contains 200 litres of Milk a thief stole 'X' litres of Milk and replaced it with water. Next, he stole 40 litres of Milk and replaced it with water. Again, he stole 50 litres of Milk and replaced with water. If the quantity of water in the final mixture is 92 litres. Then what is the value of X?

Explanation: Milk = 200-92 =108

$108 = 200*(200-x)/200*160/200*150/200$

$x = 20$ Litre

5. A Jar contains a mixture of Milk and Water 18 and 12 Litres respectively. When 'x' litre of the mixture is taken out and replaced with the same quantity of Water, then the ratio of Milk and Water becomes 2:3. Then what is the quantity of Water in final Mixture?

Explanation: $(18-x*18/30)/(12- x*12/30+x) = 2/3$

$x = \underline{10}$

Water = $12+3/5*10 = 18$

PRACTICE PROBLEMS:

1. The average of 4 terms is 20 and the 1st term is 1/3 of the remainingterms. What will be the first number?
   (a) 30 (b) 2 0 (c) 6 0 (d) 80

2. The average of 7 consecutive numbers is n. If the next two numbers are included, the average will be:
   (a) increased by 2 (b) no change (c) increased by 1 (d)increased by 2

3. For 9 innings, Boman has an average of 75 runs. In the tenth inning, he scores 100 runs, thus increasing his average. His new average is:
   (a) Rs. 75 (b) R s . 100 (c) Rs. 72 (d) Rs. 77.5

4. In a family of 8, the men eat on average 72kg of food and women eat on an average 50 kg of food. The men and women are equal in number. A hungry woman named Neetu joined the family for dinner and the average consumption became 67. How much did Neetu eat (in kgs)?
   (a) Rs. 115 (b) Rs. 80 (c) Rs. 90 (d) Rs. 85

5. In a hotel, the tariff for every odd date is Rs.1000 and for even dates is Rs. 2000. If the man paid total of 30000 in all. For how many days did he stay in the hotel given that the first day is 5th date of the month?
   (a) 50 (b) 20 (c) 40 (d) 60

6. The average age of x, y and z is 45. x is as much more than the average and y is less than the average. Find the value of z.
   (a) 45 years (b) 35 years (c) 60 years (d) 15 years

7. The average for some number of terms (say 'n') is zero. How many maximum numbers of terms can be negative?
   (a) 0 (b) N (c) n+1 (d) n-1

8. The average salary of 30 officers in a firm is Rs.120 and the average salary of people is Rs. 40. Find the total number of people if the average salary of the firm is Rs. 50.
   (a) Rs. 180 (b) Rs. 420 (c) Rs. 240 (d) Rs. 210

9. In a class, the average marks of 40 students was calculated to be 52.15. It was later discovered that the marks of a student were taken to be 49, instead of 85. Find the real average of the class.
   (a) 53.05 (b) 53.15 (c) 52.85 (d) 52.95

10. In 40 overs game, in first 20 overs of a game of cricket, the run rate was only 5. What should be the run rate for the remaining overs so that the total score reaches 300?
(a) 15 (b) 10 (c) 28 (d) 20

11. Vikram covered 180 km distance in 10 hours. The first part of his journey he covered by Car, then he hired a Rickshaw. The speed of the car and rickshaw is 25 kmph and 15 kmph respectively. Find the ratio of the distances covered by the car and the rickshaw.
(a) 7:5 (b) 6:5 (c) 5:1 (d) 5:7

12. A mixture of wheat is sold at Rs.3 per Kg. This mixture is formed by mixing the Wheat of Rs.2.10 per kg and Rs.2.52 per kg. What is the ratio of price of cheaper to the costlier quality in the mixture if the profit of 25% is being earned?
(a) 5:3 (b) 6:5 (c) 2:5 (d) 4:3

13. From the 50 litres of a chemical solution, 5 litres of chemical solution is taken out and after it, 5 litres of water is added to the rest amount of chemical solution. Again 5 litres of chemical solution and water is drawn out and it was replaced by 5 litres of water. If this process is continued similarly for the third time, Find the amount of chemical solution left after the third replacement.
(a) 36.45 L (b) 38.45 L (c) 36.54 L (d) 35.5 L

14. From a container of milk, which contains 200 litres of milk, the seller replaces each time with water when he sells 40 litres of milk (or mixture). Every time he sells out only 40 litres of milk (or mixture). After replacing the milk with water 4th time, find the total amount of water in the mixture.
(a) 118.08L (b) 112.5 L (c) 119.05 L (d) 117.08 L

15. A jar was full with Milk. A person used to draw out 20% of the Milk from the jar and replaced it with water. He repeated the same process 4 times and thus there was only 512 gm of milk left in the jar, the rest part of the jar was filled with the water. Find the initial amount of milk in the jar.
(a) 1250 gms (b) 1225 gms (c) 1280 gms (d) 1295 gms

16. From a container of Milk, a thief has stolen 15 litres of milk and replaced it with same quantity of water. He again repeated the same process. Thus, in three attempts, the ratio of Milk and water became 343:169. Find the initial amount of Milk in the container?
(a) 130 L (b) 150 L (c) 100 L (d) 120 L

17. The ratio of Solution "A" and Solution "B" in the container is 3:2 when 10 litres of the mixture is taken out and is replaced by the Solution "B", the ratio become 2:3. Find the total quantity of the mixture in the container.
(a) 30 L (b) 60 L (c) 50 L (d) 35 L

18. From a container, 6 litres Solution "A" was drawn out and was replaced by water. Again 6 litres of the mixture was drawn out and was replaced by the water. Thus, the quantity of Solution "A" and water in the container after these two operations is 9:16. Find the quantity of the mixture.
(a) 13 L (b) 12 L (c) 10 L (d) 15 L

19. The diluted Milk contains only 8 litres of Milk and the rest is water. A new mixture whose concentration is 30%, is to be formed by replacing Milk. How many litres of the mixture shall be replaced with pure Milk if there was initially 32 litres of water in the mixture?
(a) 3 L (b) 4 L (c) 6 L (d) 5 L

20. In a school, the average weight of boys in a class is 30 kg and the average weight of girls in the same class is 20kg. If the average weight of the whole class is 23.25 kg, what could be the possible strength of boys and girls respectively in the same class?
(a) 13 & 27 (b) 15 & 29 (c) 12 &22 (d) 18 & 32

21. The average of the two-digit numbers, which remain the same when the digits interchange their positions, is:
(a) 50 (b) 44 (c) 66 (d) 55

22. The average of 20 numbers is zero. Of them at the most, how many may be greater than zero?
(a) 0 (b) 1 (c) 19 (d) 10

23. The mean of 50 observations was 36. It was found later that an observation 48 was wrongly taken as 23. The corrected new mean is?
(a) 35.2 (b) 36.1 (c) 36.5 (d) 39.1

24. How many kgs of sugar costing Rs 9/kg must be mixed with 27 kgs of sugar costing Rs 7/kg so that there may be a gain of 10% by selling the mixture at Rs 9.24
(a) 56 (b) 63 (c) 75 (d) 30

25. In what ratio water must be mixed with milk costing Rs 12/ltr to obtain a new mixture worth of Rs 8/ltr ?
(a) 1:2 (b) 4:5 (c) 3:7 (d) None

ASSESMENT EXERCISE:

1. Yogi writes 9 exams and scores an average of 65. If he scores an average of 63 in the first 5 subjects & 66 in the last 5 subjects, how much did he score in the 5th subject?
   (a) 60 (b) 62 (c) 58 (d) 65

2. The average age of a group of 5 friends is 33. If Rani is replaced by Raju, the average age of the group becomes 31.8.What is the age difference between Raju and Rani?
   (a) 1.2 yrs (b) 4 yrs (c) 6 yrs (d) None

3. If the average of 6, 11, 19 and 'd' lies between 19 and 'd' (19 < d), which of the following is true?
   (a) d < 40 (b) d = 40 (c) d > 40 (d) None

4. In a class of 45 students, the average age of the first 10 students is 15 years and that of the next 15 students is 18 years and that of the remaining students is 19.5 years. Hence, the overall average age is
   (a) 25 (b) 24 (c) 20 (d)18

5. A man employs 20 men, 15 women, and 'x' children. He pays daily wages of Rs.10 per man, Rs. 8 per woman, and Rs. 4 per child. His daily average wages bill works out to Rs. 8.50 per person. What is the value of 'x'?
   (a) 5 (b) 10 (c) 15 (d) Indeterminate

6. A person's salary is always 6/5 times his previous month salary. His average salary for 4 months was Rs.1342.Find first month's salary (in Rs)?
   (a) 800 (b) 1000 (c) 1200 (d) 900

7. A milkman claims to sell milk at the cost price but actually mixes water and milk in the ratio of 1: 4. By selling this mixture, his revenue is Rs. 600 every day. The amount of milk in the mixture sold remains the same every day. One festival day, his revenue is Rs. 560 by selling the mixture at its normal fixed price of Rs. 10 per litre. What is the ratio of water to the milk in the milk mixture on that day?
   (a) 1: 5 (b) 1:2 (c) 1:7 (d) 1:6

8. At the time of the birth of twins, the average age of a couple was 24 years. When the twins attained the mother's age, the average age of this family of four was 35 years. What was the father's age (in years) when the twins attained their mother's age (at their birth)?
   (a) 54 (b) 62 (c) 48 (d) 59

9. Given three numbers, the difference of the two greater numbers is added to the smallest number to obtain a new number. The average of the new number and the two greater numbers is 10 more than the average of the given three numbers. What number is added to the smallest number?
   (a) 10 (b) 20 (c) 25 (d) 30

10. A set of consecutive positive integers beginning with 1 is written on the blackboard. A student came and erased one number. The average of the remaining numbers is . What was the number erased?
(a) 11 (b) 19 (c) 9 (d) None

11. Two solutions of Hydrochloric acid, A and B are 65% and 90% concentration respectively are mixed in the ratio 2: 3. What is the concentration of Hydrochloric acid in the resultant mixture?
(a) 60% (b) 75% (c) 80% (d) 90%

12. There are two varieties of rice, R1 and R2. R1 costs Rs. 32/kg and R2 costs Rs. 37.50/kg. 15 kg of R1 is mixed with 18 kg of R2 to get a new variety of rice R3. What should be the selling price of R3 in order to make 10% profit? (in Rs.)
(a) 38 (b) 35 (c) 38.50 (d) 39

13. In 30 litres of salt solution 5% is salt. Another mixture has 2% salt. How many litres of the second mixture should be added to the first mixture to get a mixture that has 3% salt?
(a) 50 L (b) 60 L (c) 55 L (d) 58 L

14. In two alloys iron and copper are in the ratio 5: 3 and 3: 2 respectively. 16 kg of first alloy and 15 kg of second alloy are mixed with a certain quantity of pure copper and the resulting alloy consists of iron and copper in equal proportions. Then the quantity of pure copper added is
(a) 5 kg (b) 6 kg (c) 7 kg (d) None

15. A dishonest grocer professes to sell pure butter at cost price, but he mixes it with adulterated fat and thereby gains 25%. Find the percentage of adulterated fat in the mixture assuming that adulterated fat is freely available
(a) 20% (b) 25% (c) 33.33% (d) 40%

16. A merchant purchased two qualities of pulses at the rate of Rs 200 per quintal and Rs 260 per quintal. In 52 quintals of the second quality, how much pulse of the first quality should be mixed so that by selling the resulting mixture at Rs 300 per quintal, he gains a profit of 25%?
(a) 100 quintals (b) 104 quintals (c) 26 quintals (d) None

17. There are two mixtures of honey and water, the quantity of honey in them being 25% and 75% of the mixture. If 2 gallons of the first are mixed with three gallons of the second, what will be the ratio of honey to water in the new mixutre?
(a) 11 : 2 (b) 11 : 9 (c) 9 : 11 (d) 2 : 11

18. In what ratio must a person mix three kinds of wheat costing him Rs 1.20, Rs1.44/kg and Rs1.74/kg, so that the mixture may be worth Rs 1.41/ kg?

(a) 1 : 2 : 3 (b) 4 : 5 : 7 (c) 12 : 7 : 7 (d) 13 : 7 : 9

19. Two solutions of 90% and 97% purity are mixed resulting in 21 litres of mixture of 94% purity. How much is the quantity of the first solution in the resulting mixture?
(a) 15 litres (b) 12 litres (c) 9 litres (d) 6 litres

20. In three vessels each of 10 litres capacity, mixture of milk and water is filled. The ratios of milk and water are 2 : 1, 3 : 1 and 3 : 2 in the three respective vessels. If all the three vessels are emptied into a single large vessel, find the proportion of milk and and water in the mixture.
(a) 181 : 49 (b) 101 : 49 (c) 121 : 59 (d) 131 : 69

21. There is a mixture of milk and water in which ratio is 11:18. If 15 litres of milk is added to it, and 30 litres of water removed then, find the ratio of milk and water respectively if initial mixture was 435 litres?
(a) 4:3 (b) 4:5 (c)5:6 (d)None of these

22. A man consumes mixture of two varieties of milks total of 30 liters daily. First type of milk has 20% water concentration and second type of milk contains 30% of water. If daily consumption of water is 7.8 liters, find the quantity of second type of milk?
(a) 12 liters (b) 15 liters (c) 18 liters (d) 20 liters

23. There are two vessels A and B and the capacity of both vessels are equal. Vessel A contains the mixture of milk and water in the ratio of 5:4 and vessel B contains the mixture of milk and water in the ratio of 3:2. If the mixtures of both vessels are mixed, then what is the ratio of the milk and water in the final solution
(a) 26:19 (b)24:17 (c) 27:20 (d) 28:13

24. Vessel A contains the mixture of milk and water in the ratio of 5:4 and vessel B contains the mixture of milk and water in the ratio of 3:2. If the mixture from vessel A and B is mixed, then the quantity of milk and water in the final solution is 32 liters and 24 liters respectively. Find the initial quantity of vessel A?
(a) 90 liters (b) 72 liters (c) 45 liters (d) 36 liters

25. A vessel contains mixture of milk and water in which milk is 375% of water. If 228 litres of mixture is removed and replaced by 28 litres water only then % of milk becomes 250% of water. Find the initial quantity of milk in the vessel.
(a) 290 litres (b) 195 litres (c) 390 litres (d) 420 litres

MODULE 7

TIME AND WORK

CONCEPTS:

Man - Work - Hour Formula:

- M men can do a piece of work in T hours, then Total effort or work = MT
- If 'A' can do a piece of work in 'D' days, then A's 1 day's work = $\frac{1}{D}$

  Part of work done by 'A' for t days = $\frac{t}{D}$
- If A's 1 day's work =$\frac{1}{D}$, then 'A' can finish the work in 'D' days.
- $\frac{MDH}{W}$ = **Constant**

  M = Number of men
  D = Number of days
  H = Number of hours per day
  W = Amount of work
- If $M1$ men can do $W1$ work in $D1$ days working $H1$ hours per day and $M2$ men can do $W2$ work in $D2$ days working $H2$ hours per day, then
  - $\frac{M1D1H1}{W1} = \frac{M2D2H2}{W2}$
- If A is x times as good a workman as B, then:
  - Ratio of work done by A and B = x:1
  - Ratio of times taken by A and B to finish a work = 1:x
  - That is, A will take $(\frac{1}{X})^{th}$ of the time taken by B to do the same work.

Shortcuts:

- A and B can do a piece of work in 'a' days and 'b' days respectively, then working together:

  They will complete the work in $\frac{ab}{a+b}$ days

  In one day, they will finish $\frac{a+b}{ab}th$ part of work.
- If 'A' can do a piece of work in 'a' days, 'B' can do in 'b' days and 'C' can do in 'c' days then, A, B and C together can finish the same work in $\frac{abc}{ab+bc+ca}$ days
- If A can do a work in x days and A and B together can do the same work in y days then,

  Number of days required to complete the work if B works alone = $\frac{xy}{x-y}$ *days*
- If A and B together can do a piece of work in x days, B and C together can do it in y days and C and A together can do it in z days, then number of days required to do the same work:If A, B, and C working together = $\frac{2xyz}{xy+yz+zx}$

- If A and B can together complete a job in x days.
  If A alone does the work and takes a days more than A and B working together. If B alone does the work and takes b days more than A and B working together, then $\boldsymbol{x = \sqrt{ab}}$ **days**
- If m_1 men or b_1 boys can complete a work in D days, then m_2 men and b_2 boys can complete the same work in $\frac{Dm1b1}{m2b1+m1b2}$ days.
- If the number of men to do a job is changed in the ratio a:b, then the time required to do the work will be changed in the inverse ratio b:a
- ***Note:*** *If people work for same number of days, ratio in which the total money earned has to be shared is the ratio of work done per day by each one of them.*
  A, B, C can do a piece of work in x, y, z days respectively. The ratio in which the amount earned should be shared is $\frac{1}{x} : \frac{1}{y} : \frac{1}{z} = yz:zx:xy$

Solved Examples:

Problem 1:
If Art and Rita can do a job in 8 hours (working together at their respective constant rates) and Art can do the job alone in 12 hours. In how many hours can Rita do the job alone?
Solution:
Let Rita does the work in R days. Using basic work formula, the equation would be
1/12 + 1/R = 1/8
⇒ 8R + 96 = 12R
⇒ 96 = 4R
⇒ 24 = R Working alone, Rita can do the job in 24 hours.

Problem 2:
A can do a piece of work in 60 days, which B can do in 40 days. Both started the work but A left 10 days before the completion of the work. The work was finished in how many days?
Solution
A left the job 10 days before the completion. So, B worked alone for the last 10 days. First, we will calculate B's 10 days work, which he did alone.
In 10 days B will do 10 × 1/40 = 1/4th of the work.
Remaining work 1 - ¼ = ¾ (Which A and B have done together). A and B can do 1/60 + 1/40 work in 1 day.
Their one-day's work is 1/60 + 1/40 = (2 + 3)/120 = 5/120 = 1/24. They can finish the work in 24 days.
They would have done three-fourth of the work in 24 × 3/4 = 18 days.
⇒ Total days = 18 + 10 = 28.

Problem 3:
A can do a piece of work in 24 days and B in 20 days but with the help of C they finished the work in 8 days. C alone can do the work in how many days?
Solution:

Using work formula here (1/A) + (1/B) + (1/C) = (1/8)
(1/C) = (1/8) - (1/A) - (1/B) ⇒ (1/C) = (1/8) -(1/24) = (1/20) ⇒ (1/C) = (1/30)
C can do this work in 30 days.

Problem 4:
If machine X can produce 1,000 bolts in 8 hours and machine Y can produce 1,000 bolts in 24 hours. In how many hours can machines X and Y, working together at these constant rates, produce 1,000 bolts?
Solution:
Using formula for work: 1/8 + 1/24 = 1/h ⇒ 4/24 = 1/6. Working together, machines X and Y can produce 1,000 bolts in 6 hours.

Problem 5:
A and B can do a piece of work in 36 days, B and C in 48 days, A and C can do this work in 72 days. In what time can they do it all working together?
Solution:
A and B's one day's work = 1/36. B and C's one day's work = 1/48.
C and A's one day's work = 1/72.
If we add all this it will give us the work of 2A, 2B and 2C in 1 day i.e.
(1/36) + (1/48) + (1/72) + (1/16)
That also implies that A, B and C's one day's work will be half of this i.e.
(1/2) x (1/16) = (1/32)
From here it can be found that they will complete the work in 32 days.

Problem 6:
A can do as much work in 6 days as C in 10 days. B can do as much work in 6 days as C can do in 4 days. What time would B require to do a work if A takes 48 days to finish it?
Soluution:
A : C :: 6 : 10 or (A/C) = (3/5) and B : C :: 6 : 4 or (B/C) = (3/2),

(B/A) = (B/C) x (C/A) = (3/2) x (5/3) = (5/2)
Therefore, B = (5/2) x A ⇒ (5/2) x 48 = 120 days.

Problem 7:
A can do a piece of work in 48 days and B in 72 days but with the help of C they finished the work in 24 days. Out of the total payment of Rs. 3000, how much should be given to C?
Solution:
The payment made to anybody is in the proportion of the work done and not in the ratio of days spent. Using work and time formula in 24 days working alone A & B would have done 24/48 = 1/2 and 24/72 = 1/3 of the work. That means they together did 1/2 + 1/3 = 5/6th of the work. Remaining 1/6th of the work must be done by C, the only person present. Now as he did 1/6th of the work, he should be paid 1/6th of the money i.e. 3000 × 1/6 = Rs. 500.

PRACTICE PROBLEMS:

1. A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then find the fraction of the work that is left.
   (a) 9/15 (b)15/9 (c) 8/15 (d) 17/19

2. A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work if he is assisted by B and C on every third day?
   (a) 10 (b) 12 (c) 15 (d) 20

3. A alone can do a piece of work in 6 days and B alone 8 days. A and B undertook to do it forRs.3200. With the help of C, they completed the work in 3 days. How much is to be paid to C?
   (a) 280 (b) 385 (c) 400 (d) 53

4. A can do a piece of work in 4hours; B and C together can do it in 3 hours, while A and C together can do it in 2 hours. In how long will B alone take to do it?
   (a) 12 hrs (b) 14 hrs (c) 22 hrs (d)13.5 hrs

5. A can do a certain work in the same time in which B and C together can do it. If A and B together could do it in 10 days and C alone in 50 days, then in how many days B alone could do it?
   (a) 25 (b) 18 (c) 12 (d) 29

6. Sakshi can do a piece of work in 20 days. Tanya is 25% more efficient than Sakshi. Then find the number of days taken by Tanya to do the same piece of work?
   (a) 14 days (b) 24 days (c) 19 days (d)16 days

7. A and B can complete a work in 15 days and 10 days respectively. They started doing the work together but after 2 days B had to leave and A alone completed the remaining work. In how many days whole work was completed?
   (a) 5 days (b) 12 days (c) 7 days (d) 14 days

8. A works twice as fast as B. If B can complete a work in 12 days independently, in how many days A and B can together finish the work?
   (a) 5 days (b) 3 days (c) 4 days (d) 7 days

9. P is thrice as efficient as Q and is therefore able to finish a piece of work in 60 days less than Q. Find the time in which P and Q can complete the work individually.
   (a) 60, 90 days (b) 90, 60 days (c) 30, 90 days (d) 45, 60 days

10. A tub can be filled in 20 minutes but there is a leakage in it which can empty the full tub in 60minutes. In how many minutes it can be filled?
(a) 25 min (b) 28 min (c) 32 min (d) 30 min

11. A can do a piece of work in 14 days while B can do it in 21 days. In how many days, working together they will complete the whole work?
(a) 10.2 days (b) 8.4 days (c) 6 days (d) 10.4 days

12. A is thrice as efficient as B. Working together they complete the work in 3 days. If B takes 8 days more than A, what is the number of days taken by A to finish the whole work, alone?
(a) 4 days (b) 6 days (c) 8 days (d) 10 days

13. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?
(a) 6 days (b) 12 days (c) 13 days (d) 5 days

14. A and B undertake to do a piece of work for Rs.600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.
(a) 100,230,80 (b) 300,225, 75 (c) 200,150,75 (d) 250,150,90

15. A is twice as good a workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?
(a) 22 days (b) 25 days (c) 27 days (d) 29 days

16. A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the job?
(a) 50 days (b) 60 days (c) 90 days (d) 120 days

17. Time taken by A to finish a piece of work is twice the time taken B and thrice the time taken by C. If all three of them work together, it takes them 2 days to complete the entire work. How much work was done by B alone?
(a)2/3 (b)3/5 (c)1/2 (d) 1/3

18. Sonal and Preeti started working on a project and they can complete the project in 30 days. Sonal worked for 16 days and Preeti completed the remaining work in 44 days. How many days would Preeti have taken to complete the entire project all by herself?

(a) 60 days (b) 80 days (c) 110 days (d) 120 days

19. Dev completed the school project in 20 days. How many days will Arun take to complete the same work if he is 25% more efficient than Dev?
(a) 12 days (b) 16 days (c) 15 days (d) 20 days

20. A house can be painted by 7 men and 2 women in 8 days, if they work for 12 hours a day. Working at the same rate, 6 men and 20 women can do the same job in 4 days. In how many hours can 8 men and 8 women paint the house?
(a) 12 hours (b) 16 hours (c) 32 hours (d) 64 hours

21. If the ratio of the work done by (x + 1) men in (x − 2) days to the work done by (x + 2) men in (x − 4) days is 9: 8, then the value of x is
(a) 12 (b) 14 (c) 10 (d) 16

22. Raju can do a piece of work in 10 days, Vicky in 12 days and Tinku in 15 days. They all start the work together, but Raju leaves after 2 days and Vicky leaves 3 days before the work is completed. In how many days is the work completed?
(a) 5 days (b) 6 days (c) 7 days (d) 8 days

23. P and Q can do a work in 12 and 16 days respectively. If they work on alternate days, beginning with Q, then in how many days will the work be completed?
(a) $13\frac{1}{4}$ (b) $13\frac{2}{3}$ (c) $13\frac{3}{4}$ (d) $13\frac{1}{3}$

24. Three taps A, B and C together can fill a tank in 6 hours. All the taps are opened simultaneously. After 2 hours, C was closed and the tank was filled in 8 more hours. Find the time (in hours) in which C can fill the tank
a) 12 hours (b) 10 hours (c) 14 hours (d) 18 hours

25. P and Q are filling pipes. If the bottom $\frac{3}{4}$ th of the tank is filled by P and the rest is filled by Q, the tank will be filled in 45 minutes. If the bottom $\frac{3}{4}$ th of the tank is filled by Q and the rest is filled by P, the tank will be filled in 39 minutes. Find the time taken by the two pipes working together to fill the tank.
(a) $18\frac{4}{7}$ minutes (b) $24\frac{6}{7}$ minutes (c) $20\frac{4}{7}$ minutes (d) $16\frac{2}{7}$ minutes

ASSESMENT PROBLEMS:

1. Two men are as efficient as 3 women who are as efficient as 4 machines. The number of men, women and machines that worked together and completed a job is in the ratio 3: 4 : 5. They are paid a total of Rs 4900 for the job. Find the total share of women
   (a) 1200 (b) 1800 (c) 1500 (d) none of these

2. A, B and C working alone, can complete a work in 8, 12 and 24 days respectively. All the three of them started the work and then A left after 2 days. B left after another 3 days and C then completed the remaining work. If the total wages earned by the three of them are Rs 6000, find the share of A.
   (a) 1,000 (b) 2,500 (c) 2,000 (d) 1,500

3. A house can be painted by 7 men and 2 women in 8 days, if they work for 12 hours a day. Working at the same rate, 6 men and 20 women can do the same job in 4 days. In how many hours can 8 men and 8 women paint the house?
   (a) 12 hours (b) 16 hours (c) 32 hours (d) 64 hours

4. In a garrison with 600 men, there are provisions which are sufficient for 24 days. After 18 days, 300 additional men join. In addition, the average consumption per head also changes. As a result, the provisions last for only three more days. What is the percentage increase in the average consumption per day for the last three days?
   (a) 25% (b) 50% (c) $33\frac{1}{3}\%$ (d) $66\frac{2}{3}\%$

5. 12 men and 16 boys can do a piece of work in 5 days, 13 men and 24 boys can do it in 4 days. Then the ratio of daily work done by a man to that ofa boy is
   (a) 2: 1 (b) 3 : 1 (c) 3 : 2 (d) 5 : 4

6. A tyre has two punctures. The first puncture along would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat?
   (a) $1\frac{1}{4}$ minutes (b) $3\frac{1}{2}$ minutes (c) $3\frac{3}{5}$ minutes (d) $4\frac{1}{4}$ minutes

7. Howard and John take 12 days and 16 days respectively to complete a job. Gerard is at least as fast as John but at most as fast as Howard. Gerard and John work on alternate days and completed the job in x days. Which of the following can be the value of x?
   (a) 12 (b) 13 (c) 14 (d) 17

8. A garrison had provisions for 1500 men for 30 days. After some days, 300 more men joined the garrison. The provisions lasted for a total of 26 days from the beginning. After how many days did the new men join?
(a) 24 (b) 6 (c) 4 (d) 26

9. A is 80% more ef cient than B who is 60% more efficient than C. A takes 40 days less than B to complete a work. A starts the work and works for 25 days and then B takes over. B then works for the next 30 days and then C takes over. In how much more time can C complete the remaining work?
(a) 20 days (b) 24 days (c) 32 days (d) 40 days

10. A, B and C are three taps connected to a tank. Time taken by C to fill the tank is $\frac{9}{2}$ times the time taken by A and B to fill it. The time taken by A to fill it is $\frac{5}{3}$ times the time taken by B and C to fill it. A, B and C take $\frac{20}{11}$ hours to fill it. Find the time taken by B to fill it (in hours).
(a) 3 (b) 5 (c) 6 (d) 10

11. Some workers have been divided into two groups—A and B—depending on their rate of doing work. Three workers from A and six from B take 20 days to complete a job. Eight from A and 4 from B take 10 days to complete it. Find the time taken by one worker from each group to complete it (in days).
(a) 90 (b) 108 (c) 72 (d) 54

12. Pipes A and B can fill a tank in 20 minutes and 30 minutes respectively and C can empty it in 15 minutes. A is opened for a minute and thenclosed. B is then opened for a minute and then closed. C is then openedfor a minute and then closed. This process is repeated until the tank isfilled. Find the time taken to fill the tank (in minutes).
(a) 169 (b) 170 (c) 167 (d) 166

13. A pump can be operated both for filling a tank and for emptying it. The capacity of tank is 2400 m^3. The emptying capacity of the pump is 10 m^3 per minute higher than its filling capacity. Consequently, the pump needs 8 minutes less to empty the tank to fill it. Find the filling capacity of pump.
(a) 50 m^3/min (b) 60 m^3/min (c) 58 m^3/min (d) None of these

14. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank ?
(a) 20 hrs (b) 25 hrs (c) 35 hrs (d) None of these

15. Two pipes A and B can fill a tank in 15 hours and 20 hours respectively while a third pipe C can empty the full tank in 25 hours. All the three

pipes are opened in the begining. After 10 hours, C is closed. In how much time, will the tank be full?

(a) 12 hrs (b) 13 hrs (c) 16 hrs (d) 18 hrs

16. A contractor decided to complete a work in 60 days and employed 60 men at the beginning and 30 men additionally after 20 days and got the work completed as per schedule. If he had not employed the additional men, how many extradays would he have needed to complete the work?

(a) 60 days (b) 40 days (c) 20 days (d) 55 days

17. To complete a job, P takes half as long as Q and R together take. Q takes 8 times as long as P and R together take. All the three together can complete the job in $\frac{20}{3}$ days. Find the time taken by each of P, Q and R to complete it.

(a) 10,60,30 (b) 20,40,60 (c) 15,30,45 (d) 15,90,60

18. A group of 30 women takes 36 days to complete a piece of work for which they are paid a total of Rs 60,000. The rate at which a man works as well as his daily wage is double that of a woman. How many men must join 15 women to complete the work in 24 days? How much more money is earned by the men than by the women?

(a) 10 men, 15,000 (b) 15 men, 20,000
(c) 15 men, 30,000 (d) 10 men, 30,000

19. A tank of dimensions 18 m × 10 m × 8 m is filled to its capacity. A drain pipe can empty the tank in 50 hours. An inlet pipe supplies water at the rate of 6 litres/sec. If both the pipes are opened at the same time, how much time would they take to empty the tank?

(a) 100 hours (b) 150 hours (c) 300 hours (d) 200 hours

20. A frog, which is at the bottom of a 50 m deep well, is trying to come out of it. In every jump it covers 1.25 m but slips 0.75 m. In how many jumps would it come out of the well?

(a) 98 (b) 99 (c) 100 (d) 101

21. A mason employed a certain number of workers to finish constructing a wall in a certain scheduled time. After some time, he realized that the work would get delayed by a fourth of the scheduled time. So, he immediately increased the number of workers by a third and thus managed to finish constructing the wall on time. Sometime after the workforce was increased, all the newly added workers left and the remaining workers reduced their rate of work by half. Finally, the work got completed with a delay of 50% of the scheduled time. What fraction of the total work was still incomplete by the end of the scheduled time?

(a) 22.5% (b) 25% (c) 20% (d) 16.6%

22. Imran and Irfan are two tailors. Imran takes three hrs to stitch 10 shirts and four hrs to stitch 12 pants. Irfan can stitch 12 pants in three hrs and 10 shirts in four hrs. They get an order for the delivery of 200 shirts and 200 pants. What is the quickest time in which they can deliver the order?
(a) 59 hrs 6 mins (b) 66 hrs 6 mins
(c) 63 hrs 12 mins (d) 55 hrs 43 mins

23. 45. Mohan can complete a work in 25 days. He worked for 5 days and left the work; and then Bhim completed the remaining work in 30 days. Had Bhim started the work and left it after 15 days, how much more time would Mohan have taken to complete the remaining work?
(a) 7.5 days (b) 11.5 days (c) 12 days (d) 15 days

24. X takes 16 days to complete 4/5th of a work, Y takes 12 days to complete 3/7th of the same work and Z takes 20 days to complete 5/9th of the same work. If they work together for 5 days and then X and Z leaves the work, then find the number of days to complete the whole work?
(a) $17\frac{1}{9}$ days (b) $16\frac{2}{3}$ days (c) $15\frac{3}{5}$ days (d) $12\frac{1}{9}$ days

25. Efficiency of P is 60% more than Q and Q takes 40 days to complete a piece of work. Both started the work and work for 6 days and then they decided to work alternatively starting with Q. Find the total time taken by both of them to complete the whole work?
(a) 15 days (b) 18 days (c) 25 days (d) 21 days

MODULE-8
TIME, SPEED & DISTANCE-1

- The speed of a body is defined as the distance covered by it in unit time.
- Speed = Distance / Time

Important Points:

- To covert km/h to m/s use: **m/s= km/h x $\frac{5}{18}$**
- To covert m/s to km/h use: **km/h= m/s x $\frac{18}{5}$**
- If the distance covered is constant, then the speed is inversely proportional to time.
  - S α $\frac{1}{T}$ => **$S_1T_1 = S_2T_2$**
- If the time is constant, then the distance covered is directly proportional to the speed.
  - D α S => $\frac{D1}{S1} = \frac{D2}{S2}$
- If the speed is constant, then the distance covered is directly proportional to the time.
  - D α T => $\frac{D1}{T1} = \frac{D2}{T2}$

Average Speed:

If a body travels d1, d2, d3..... dn distances, with speeds s1, s2, s3.....sn in time t1, t2, t3.....tn respectively then the average speed of the body through the total distance is given by:

Average speed = (Total Distance)/(Total Time)

Note: Average Speed = $\frac{2xy}{x+y}$ where x km/hr is a speed for certain distance and y km/hr is a speed at for same distance covered.

Relative Speed:

Case 1: When one object is moving and the other is stationary the relative speed between them is the speed of the moving object. For ex. when a train crosses a stationary person on a platform the relative speed of the train and the person will be the speed of the train.

Case 2: When two objects are moving in opposite directions the relative speed between them is the sum of their speeds. For ex. the relative speed between two trains moving towards each other is the sum of their individual speeds.
Relative Speed=S_1+S_2

Case 3: When two bodies are moving in the same direction the relative speed between them is the difference of their speeds. For ex. the relative speed between two trains moving in the same direction on parallel tracks is the difference between their individual speeds.

Relative Speed=S_1-S_2

Circular tracks:

- When two persons are running around a circular track in the same direction, the difference in the distances covered by the faster and slower person between two meeting points is equal to the perimeter of the circular track and the relative speed is equal to difference in speeds of two persons.
- When two persons are running around a circular track in opposite directions, the relative speed is equal to the sum of their speeds and from one meeting point to another, the sum of the distances travelled by them is equal to the perimeter of circular track.
- Two persons starting a race on a circular track at the same time and from the same point, will meet for the first time when the faster person gains one complete round over the slower person. The time taken for this = **length of track / relative speed**
- Two persons starting a race on a circular track at the same time and from the same point, will meet for the first time at the starting point after a time which is the LCM of time taken by each one of them to complete one lap of the track.

Three persons starting a race on a circular track and from the same starting point, will meet for the first time after the start at a time which is equal to the LCM of the time taken by the fastest to gain a complete round over each of the other two

Solved Examples:

Problem 1:
A man covers a distance of 600m in 2min 30sec. What will be the speed in km/hr?
Solution:
Speed =Distance / Time
Distance covered = 600m, Time taken = 2min 30sec = 150sec
Therefore,
Speed= 600 / 150 = 4 m/sec
4m/sec = (4*18/5) km/hr = 14.4 km/ hr.

Problem 2:
A boy travelling from his home to school at 25 km/hr and came back at 4 km/hr. If whole journey took 5 hours 48 min. Find the distance of home and school.
Solution:
In this question, distance for both speed is constant.
Average speed = (2xy/ x+y) km/hr, where x and y are speeds
Average speed = (2*25*4)/ 25+4 =200/29 km/hr
Time = 5hours 48min= 29/5 hours
Now, Distance travelled = Average speed * Time
Distance Travelled = (200/29)*(29/5) = 40 km
Therefore distance of school from home = 40/2 = 20km.

Problem 3:
If a person goes around an equilateral triangle shaped field at speed of 10, 20 and 40 kmph on the first, second point, then find his average speed during the journey.

Solution:
Let the measure of each side of triangle is D km. The person travelled the distance from A to B with 10 kmph, B to C with 20 kmph and C to A with 40 kmph.
If TAB = Time taken by the person to travel from A to B,
TBC = Time taken by the person to travel from B to C and
TCA = Time taken by the person to travel from C to A.
Then total time = TAB + TBC + TCA
D/10 + D/20 + D/40 = D((8+4+2)/80) = 7D/40
Total distance travelled = D + D + D = 3D
Hence, average speed =3D/(7D/40) = 120/7 = 171/7 kmph.

Problem 4:
A boy goes to school with the speed of 3 km an hour and returns with a speed of 2 km/hr. If he takes 5 hrs in all, find the distance in km between the village and the school.

Solution:
Here s1 = 3, s2 = 2 and T = 5.
The distance between the village and the school= $T\left(\frac{S_1S_2}{S_1+S_2}\right) = 5\left(\frac{3\times2}{3+2}\right)$

= 6 km.

Problem 5:
A car during its journey travels 40 min at a speed of 30 km/hr, another 50 min at a speed of 60 km/hr and 1 hr at a speed of 30 km/hr. Find the average speed of the car.

Solution:
Here $T_1 = 40/60$, $T_2 = 50/60$, $T_3 = 1$, $S_1 = 30$, $S_2 = 60$, $S_3 = 30$

$\therefore$ Average speed of the car $= \frac{S_1T_1 + S_2T_2 + S_3T_3}{T_1 + T_2 + T_3}$

$$= \frac{30\times \frac{40}{60} + 60 \times\frac{50}{60} + 30 \times 1}{\frac{40}{60} + \frac{50}{60} + 1} = 40 \text{ km}$$

Problem 6:
In a 600 m race, *P* gives *Q* a start of 200 m. Ratio of the speeds of *P* and *Q* is 5 : 4. Who wins the race? By what distance does the winner beat the loser?

Solution:
Q has to run 400 m to finish the race. In the time *Q* runs 400 m, *P* can run 500 m. When *Q* finished the race, *P* would have another 100 m to run.
Q wins the race and he beats *P* by 100 m.

Problem 7:
Ramu is 50% faster than Somu. In a race, Ramu gave Somu a head start of 200 m. Both finished the race simultaneously. Find the length of the race.
Solution:
Let the length of the race be X m .

$$\frac{X}{X-200} = \frac{150}{100}$$

$$X = 600.$$

PRACTICE PROBLEMS:

1. Walking 7/6 of his usual speed, Ram reaches school 4 minutes early. Find the original time taken by him to reach the school.
   (a) 15 min (b) 28 min (c) 17 min (d) 25 min

2. Ram covers a distance in 42 minutes. He covers 2/3 of it at 4 km/hr and the remaining at 5 km/h. Find the approximate total distance.
   (a) 3 Kms (b) 4 Kms (c) 6 Kms (d) 8 Kms

3. A man travels on his motorcycle from Kolkata to Delhi at a speed of 77 km/hr and comes back at 33 km/hr. Find the average speed of the journey.
   (a) 40.5 Km/hr (b) 46.5 Km/hr (c) 38 Km/hr (d) 40 Km/hr

4. A car covers 420 km with a constant speed. If its speed were 10 km/hr more it would have taken 1 hour less to cover the distance. What is the speed of the car?
   (a) 60 Km/hr (b) 65 Km/hr (c) 75 Km/hr (d) 80 Km/hr

5. A and B start from P and Q towards each other. If A walks at speed of 4 km/hr and B walks at 8 km/hr find when will they meet given that the distance between P and Q is 144 kms?
   (a) 12 hrs (b) 14 hrs (c) 18 hrs (d) 10 hrs

6. Two trains P & Q start from station A and B respectively towards each other. The trains take 4 hour and 9 hours respectively to reach B and A after crossing each other. Find the ratio of speed of P & Q.
   (a) 1:2 (b) 2:3 (c) 3:2 (d) 4:5

7. Karan and Arjun sit in the bus X & Y respectively. The buses start from the same point and move in opposite direction at respective speed of 30 kmph and 24 kmph. How far is Karan from Arjun after 4 hours of journey?
   (a) 216 kms (b) 218 kms (c) 220 kms (d) 240 kms

8. If a man rides at 30 kmph, he will reach office 1 hour late. If he rides at 40 kmph, he will reach office 1 hour early. Find his usual travel distance.
   (a) 200 kms (b) 350 kms (c) 240 kms (d) 280 kms

9. In a race of 1000 m, A beats B by 200 m or 20 seconds. What is the speed of B?
   (a) 10 m/s (b) 7.5 m/s (c) 5.8 m/s (d) 7 m/s

10. Two trains travel towards each other on the same track at speeds of 60 km/hr and 80 km/hr, separated initially by a distance of 280 km. A bird flying from the faster train to the slower train and back at a speed of 150

km/hr continues doing so till the trains collide. What is the total distance flown by the bird till the trains collide?
(a) 200 km (b) 250 km (c) 300 km (d) 350 km

11. By walking at 3/4th of his usual speed, a man reaches office 20 minutes later than usual. What is his usual time?
(a) 1 hr (b) 0.4 hr (c) 0.6 hr (d) 1.2 hr

12. A police jeep parked by the roadside notices a car to pass by it with a speed of 60 km/hr. Realizing that the vehicle is travelling at a speed over the maximum prescribed limit it starts the chase exactly after 15 minutes with a speed of 72 km/hr. In how much time will the jeep manage to catch up with the speeding vehicle? (time is being calculated from the moment the police vehicle begins the chase)
(a) 28 min (b) 32 min (c) 60 min (d) 75 min

13. The speed of three cars in the ratio 2:3:4. Then find the ratio between the times taken by these cars to travel the same distance.
(a) 5:4:1 (b) 7:3:5 (c) 6:2:5 (d) none of these

14. In a circular race of 1200m, A and B start from the same point and at the same time with speeds of 27 kmph and 45 kmph. Find when will they meet again for the first time on the track when they are running in the same direction.
(a) 240 s (b) 180 s (c) 72 s (d) 305 s

15. P, Q run around a circular track 1200 m long with speed of 10, 12, m/s. If they start at the same point and at the same time in the same direction, when will they meet again at the starting point?
(a) 10 min (b) 15 min (c) 12 min (d) 8 min

16. Arun started his bike journey at a speed of "s" kmph. After 2 hours, he reduced his speed by 10 kmph and travelled for 1 hour. He again increased his speed by 10% and travelled for 2 hours. Find the value of "s" if the total distance he covered is 98 km.
(a) 20 kmph (b) 25 kmph (c) 30 kmph (d) 35 kmph

17. There was a race of 2250 meters between P and Q on a circular track of 1080 meters. After 6 minutes of starting the race they meet for the first time during the race. Find the time taken by Q to complete the race, if he runs at one-third of the speed of P?
(a) 25 mins (b) 15 mins (c) 18 mins (d) 16 mins

18. A bus driver drives the bus at 24 kmph from Nagercoil to Madurai. Another driver drives at 26 kmph from Madurai to Nagercoil, which are 250 km

apart. After what time, will they cross each other, if they start at the same time?
(a) 2 hours (b) 3 hours (c) 4 hours (d) 5 hours

19. A man travels a certain distance at an average speed of 60 km/hr without stoppage and with stoppage he covers the same distance at an averagespeed of 40 km/hr. How many minutes per hour does he stop?
(a) 20 minutes (b) 35 minutes (c) 30 minutes (d) 40 minutes

20. Time taken by a car running at 30 km/h to cover a certain distance is 2 hours less than time taken by car to cover 10 km more than previous distance with speed of 24 km/h. find the time taken by car to cover 35 km more than initial distance with speed of 12.5 km/h?
(a) 20 hours (b) 22 hours (c) 18 hours (d) 15 hours

21. Two cities X and Y are x km apart, person A starts from X at 65 kmph and after 2 hours' person B starts from city Y at 85 kmph, after 3 more hours both of them met. In what time person A completes his journey, if speed of A is 7 km/hr less than that of the person who start his journey from city X?
(a) 12 hours (b) 10 hours (c) 8 hours (d) 9 hours

22. A boy travelled to his school which is at the distance of 20 km in 3 hours. He rode a bicycle at 6 kmph for some distance. Then he passed the remaining distance at 7 kmph by bus. What is the distance travelled by bus?
(a) 7 km (b) 14 km (c) 21 km (d) 28 km

23. The ratio of speed of car A and car B is 4:3. Speed of car C is 10 m/s more than speed of car A and car D covered 420 km distance in 6 hours which is 1 hr more than that of C, then find the speed of car B.
(a) 50 km/hr (b) 48 km/hr (c) 36 km/hr (d) 60 km/hr

24. A person travelling from P to Q at an average speed of 68 km/h and reached at point Q in 3.5 hours. If person covers same distance with stoppage to its average speed reduced by 20.4 km/h as compared to without stoppage speed, then find the number of stoppages if each stoppage is 36 minutes.
(a) 12 (b) 6 (c) 5 (d) 4

25. A person is travelling from Ajmer to Manali by driving Bike at speed of 120 km/h without any halt. Has he taken 6 halts of 12 minutes each, then the total time taken to cover distance from Ajmer to Manali would have been 5 hours. Find the total distance from Ajmer to Manali in Km?
(a) 450 km (b) 456 km (c) 432 km (d) 336 km

ASSESMENT PROBLEMS:

1. In a 600 m race, *A* gives *B* a start of 200 m and beats him by 120 m. Find the ratio of the speeds of *A* and *B*
   (a) 15: 4 (b) 15: 7 (c) 10: 3 (d) 12: 5

2. Three cyclists with respective speeds of 5 m/sec, 10 m/sec and 20 m/sec are cycling around a circular track of length 100 meters. If they are cycling in the same direction, after how many seconds will all the three meet forthe first time?
   (a) 60 (b) 30 (c) 40 (d) 20

3. Three cyclists start cycling simultaneously from the same point on a circular track 900 m long in the same direction with speeds of 10 m/sec, 20 m/sec and 15 m/sec, respectively. How long will they take (in seconds) before they meet for the first time?
   (a) 90 (b) 180 (c) 360 (d) 45

4. The speeds of three runners A, B and C are in the ratio 1: 2: 3. If A takes 2 hours more than C to cover a certain distance, and the time taken by B to cover the same distance.
   (a) 2 hours (b) 2.5 hour (c) 3 hours (d) 1.5 hours

5. Lucky is a thief. He steals a TV from the house of Mr. Batra and escapes on his Lambretta at 6:00 a.m. at a speed of 40 kmph. At 8:00 a.m., Mr. Batra realizes that there had been a burglary and immediately starts chasing Lucky on his Luna at a speed of 60 kmph. At what time will Mr. Batra catch up with Lucky?
   (a) 12 pm (b) 1 pm (c) 2 pm (d) 1.20 pm

6. Two riders on the horseback with a gun and a bullet proof shield were moving towards each other at a constant speed of 20 km/h and 5 km/h respectively. When they were 100 km apart, they started firing bullets at each other at the speed of 10 km/h. When a bullet of rider 1 hits the shield of rider 2, rider 2 fires a bullet and the process continues vice versa. Neglecting the time lag at the instant when the bullet hits the shield and the rider fires the shot, find the total distance covered by all the bullets shot by both the riders.
   (a) 50 km (b) 4 0 km (c) 2 5 km (d) None of these

7. A passenger train departs from Ahmedabad at 6 pm for Bombay. At 9 p.m. an express train, whose average speed exceeds that of the passenger train by 15 km/h, leaves Bombay for Ahmedabad. Two trains meet each other mid-route. At what time do they meet, given that the distance between the cities is 1080 km?
   (a) 4 pm (b) 2 pm (c) 12 midnight (d) 6 am

8. A car covers a distance of 715 km at a constant speed. If the speed of the car had been 10 km/h more, then it would have taken 2 h less to cover the same distance. What is the original speed of the car?
(a) 55 km/h (b) 50 km/h (c) 45 km/h (d) 65 km/h

9. A train leaves station X at 5 a.m. and reaches station Y at 9 a.m. Another train leaves station Y at 7a.m. and reaches station X at 10: 30 a.m. At what time do the two trains cross each other?
(a) 7 : 36 am (b) 7 : 56 am (c) 8 : 36 am (d) 8 : 56 am

10. A train covered a certain distance at a uniform speed. If the train had been 6 km/h faster, then it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, then the train would have taken 6 hours more than the scheduled time. The length of the journey is
(a) 700 km (b) 740 km (c) 720 km (d) 760 km

11. Shyam's house, his office and his gym are all equidistant from each other. The distance between any 2 of them is 4 km. Shyam starts walking from his gym in a direction parallel to the road connecting his office and his house and stops when he reaches a point directly east of his office. He then reverses direction and walks till he reaches a point directly south of his office. The total distance walked by Shyam is
(a) 6 km (b) 9 km (c) 16 km (d) 12 km

12. A dog after travelling 50 km meets a swami who counsels him to go slower. He then proceeds at 3/4 of his former speed and arrives at his destination 35 minutes late. Had the meeting occurred 24 km further the dog would have reached its destination 25 minutes late. The speed of the dog is
(a) 48 km/h (b) 36 km/h (c) 54 km/h (d) 58 km/h

13. Ramesh and Somesh are competing in a 100 m race. Initially, Ramesh runs at twice the speed of Somesh for the first fifty m. After the 50 m mark, Ramesh runs at 1/4th his initial speed while Somesh continues to run at his original speed. If Somesh catches up with Ramesh at a distance of 'N' m from the finish line, then N is equal to
(a) 35 (b) 10 (c) 45 (d)None of these

14. *A*, *B*, and *C* are three participants in a kilometer race. If *A* can give *B* a start of 40 metres and *B* can give *C* a start of 25 metres, how many metres of a start can *A* give to *C*?
(a) 60 m (b) 64 m (c) 62 m (d) 66 m

15. A monkey ascends a greased pole 12 metres high. He ascends 2 metres in first minute and slips down 1 metre in the alternate minute. In which minute, he reaches the top ?

(a) 21 st (b) 22 nd (c) 23rd (d) 24 th

16. A tiger is 50 of its own leaps behind a deer. The tiger takes 5 leaps per minute to the deer's 4. If the tiger and the deer cover 8 m and 5 m per leap respectively, what distance will the tiger have to run before it catches the deer?
(a) 600 m (b) 700 m (c) 800 m (d) 1000 m

17. A candle of 6 cm long burns at the rate of 5 cm in 5 h and another candle of 8 cm long burns at the rate of 6 cm in 4h. What is the time required by each candle to remain of equal lengths after burning for some hours, when they start to burn simultaneously with uniform rate of burning?
(a) 1 h (b) 1.5 h (c) 2 h (d) None of these

18. Two persons start from the opposite ends of a 90 km straight track and run to and fro between the two ends. The speed of first person is 30 m/s and the speed of other is 125/6 m/s. They continue their motion for 10 hours. How many times they pass each other?
(a) 10 (b) 9 (c) 12 (d) None of these

19. A beats B by 100 m in a race of 1200 m and B beats C by 200 m in a race of 1600 m. Approximately by how many metres can A beat C in a race of 9600 m?
(a) 1600 m (b) 1800 m (c) 1900 m (d) 2400 m

20. A gives both B and C a start of 60 m in a 1500 m race. However, while B finishes with him, C is 15 m behind them when A and B cross the finishing line. How much start can B give C for the 1500 m race course?
(a) $7\frac{6}{23}$ m (b) $15\frac{5}{8}$ m (c) $7\frac{11}{16}$ m (d) $5\frac{5}{24}$ m

21. An aircraft was to take off from a certain airport at 8 a.m. but it was delayed by 30 mins. To make up for the lost time, it was to increase its speed by 250 km/hr from the normal speed to reach its destination 1500 km away, on time. What was the normal speed of the aircraft?
(a) 650 km/hr (b) 750 km/hr (c) 850 km/hr (d) 1000 km/hr

22. Raja was on a long-distance trip. He travelled by air 2/5 of the distance which was 1200 km. Then he hired a car and travelled 1/3 of the whole trip. Thereafter, he completed the rest of the journey by train. Calculate the distance that Raja travelled by train.
(a) 480 km (b) 800 km (c) 1600 km (d) 1800 km

23. A train T1 starts from Ahmedabad to Mumbai at 7 a.m. and reaches at 12 noon. A second train T2 starts at 7 a.m. from Mumbai reaches Ahmedabad at 1 p.m. When did the two trains cross each other?

(a) 10.13 a.m. (b) 10.00 a.m. (c) 9.43 a.m. (d) 9.35 a.m.

24. If a child walks at the rate of 5 m/min from his home, he is 6 mins late for school; if he walks at the rate of 7 m/min, he reaches half an hour earlier. How far is his school from his home?
(a) 450 mins (b) 540 mins (c) 630 mins (d) 360 mins

25. A circular playground has an area of 616 sq. m. What time will it take for a runner to run around the circular ground at the speed of 22 km/hr?
(a) 4 hrs (b) 3 hrs (c) 2 hrs (d) None of these

MODULE-9
TIME, SPEED & DISTANCE-2
(Trains & Boats)

- The speed of a body is defined as the distance covered by it in unit time.
- Speed = Distance / Time

Boats & Streams:

- Let the speed of a boat in still water be u km/hr and the speed of the stream be v km/hr, then
- Speed of a boat upstream (Su)= speed of boat in still water - speed of the stream= (u - v)
- Speed of a boat downstream (Sd) = speed of boat in still water + speed of stream= (u+v)
- Speed of boat in still water = (downstream speed + upstream speed) / 2 = $\frac{Sd+Su}{2}$
- Speed of stream = (downstream speed – upstream speed) / 2 = $\frac{Sd-Su}{2}$

Problems on Trains:

- Speed of the Train = Total distance covered by the train / Time taken

- If the length of two trains is given, say a and b, and the trains are moving in **opposite directions** with speeds of x and y respectively, then the

  - **Time taken by trains to cross each other = {(a+b) / (x+y)}**

- If the length of two trains is given, say a and b, and they are moving in the **same direction**, with speeds x and y respectively, then the

  - **Time taken to cross each other = {(a+b) / (x-y)}**

- When the **starting time of two trains is the same from x and y towards each other** and after crossing each other, they took t1 and t2 time in reaching y and x respectively, then the
  - **Ratio between the speed of two trains = $\sqrt{t2}$: $\sqrt{t1}$**

- If two trains leave x and y stations at time t1 and t2 respectively and travel with speed L and M respectively, then distanced from x,

  - where two trains meet is = (t2 – t1) × {(product of speed) / (difference in speed)}

- The average speed of a train without any stoppage is x, and with the stoppage, it covers the same distance at an average speed of y, then
  - **Rest Time per hour = (Difference in average speed) / (Speed without stoppage)**

- If two trains of equal lengths and different speeds take t1 and t2 time to cross a pole, then the t**ime taken by them to cross each other if the train is moving in opposite direction = (2×t1×t2) / (t2+t1)**

- If two trains of equal lengths and different speeds take t1 and t2 time to cross a pole, then the **time taken by them to cross each other if the train is moving in the same direction = (2×t1×t2) / (t2-t1)**

Solved Examples:

Problem 1:

A train travelling at 60 kmph crosses a man in 6 seconds. What is the length of the train?

Solution:
Speed in m/sec = 60 *(5/18) = 50/3 m/sec
Time taken to cross the man = 6 secs
Therefore, Distance = (50/3) * 6 = 100 meters (i.e. the length of the train)

Problem 2:
A train travelling at 60 kmph crosses another train travelling in the same direction at 50 kmph in 30 seconds. What is the combined length of both the trains?
Solution:
Speed of train A = 60 kmph = 60* (5/18) = 50/3 m/sec
Speed of train B = = 50 kmph = 50 *(5/18) = 125/9 m/sec
The relative speed =(50/3)-(125/9)=25/9 m/s (we have subtracted the two values because both the trains are going in the same direction)
Time taken by train A to cross train B = 30 secs
Distance = Speed * Time
Distance =25/9 * 30 = 250/3 meters (i.e. the combined length of both trains)

Problem 3:
Time taken by two trains running in opposite directions to cross a man standing on the platform in 28 seconds and 18 seconds respectively. It took 26 seconds for the trains to cross each other. What is the ratio of their speeds?
Solution:
Let the speed one train be x and the speed of the second train be y
Length of the first train = Speed × Time = 28x
Length of second train = Speed × Time = 18y
So, {(28x+18y) / (x+y)} = 26
⇒ 28x+18y = 26x+26y
⇒ 2x = 8y ⇒ Therefore, x: y = 4:1

Problem 4:
The speed of the boat in still water is 20 km/hr and the speed of the stream is 5 km/hr. Find the downstream speed and the upstream speed.

Solution:
Here we have b = 20 km/hr and w = 5 km/hr
So the downstream speed = b + w = 20 + 5 = 25km/hr
The upstream speed = b – w = 20 - 5 = 15 km/hr

Problem 5:
Speed of the boat in still water is 15 km/hr and the speed of the current is 7 km/hr. Find the total time taken by a man rowing to a place at a distance of 88 km and back.
Solution:
The downstream speed is 15 + 7 = 22 km/hr and the upstream speed = 15 - 7 = 8 km/hr.
The total time taken = (88/22) + (88/8) = 4+11 = 15hours.

Problem 6:
A boat running downstream covers a distance of 24 km in 3 hours while for covering the same distance upstream, it takes 6 hours. What is the speed of the current?
Solution:
The downstream speed = 24/3 = 8 km/hr
The upstream speed = 24/6 = 4 km/hr
The speed of the current = (8-4)/2 = 4/2 = 2 km/hr

Problem 7:
A man can row 8 km/hr in still water. If the speed of the current is 2 km/hr and it takes 4 hours to a man to row a place and come back, then how far is the place?
Solution:
The downstream speed = 8 + 2 = 10 km/hr and the upstream speed = 8 - 2 = 6 km/hr
Let the distance is 'x' km. We have
(x/10) + (x+6) = 4
⇒ x((3+5) / 30) = 4
⇒ x = (4*30) / 8 = 15 km.

PRACTICE PROBLEMS:

1. A train 180 m long is running at a speed of 54 km/hr. Time taken by the train to cross a bridge 270 m long is
   (a) 20 s (b) 25 s (c) 15 s (d) 30 s

2. If a train 60 m long crosses a lamp post in 6 seconds, then the time taken (in seconds) by it to cross a railway platform 200m long is
   (a) 26 s (b) 18 s (c) 12 s (d) 21 s

3. A man rows upstream 24 km and downstream 36 km taking 6 hours each time. The speed of the current is
   (a) 1 km/hr (b) 2.5 km/hr (c) 2 km/hr (d) 1.8 km/hr

4. A boat goes 60 km upstream in 10 hrs and a distance of 70 km downstream in 7 hrs. The speed of the boat in standing water is
   (a) 6 km/hr (b) 5 km/hr (c) 8 km/hr (d) 6.5 km/hr

5. Ram can row 20 kmph in still water. If the river is flowing at 4 kmph, it takes him 60 mins to row to a place and back. How far is the place?
   (a) 19.5 kms (b) 19.2 kms (c) 19.8 kms (d) 19.1 kms

6. How many seconds will a train 60 m in length, travelling at the rate of 42 km/hr, take to pass another train 84 m long, proceeding in the same direction at the rate of 30 km/hr?
   (a) 20 s (b) 30 s (c) 40 (d) None of these

7. Two trains running at the rate of 45 km/hr and 36 km/hr an hour respectively, on parallel tracks in opposite directions, are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rate as before, a person sitting in the faster train observes that he passes the other in 30 seconds. Find the length of trains.
   (a)70m, 100m (b) 65m, 95m (c)75m, 105m (d) 80m , 110m

8. Two stations P and Q are 110 km apart on a straight line. One train starts from P at 7 am and travels towards Q at 20 km/hr speed. Another train starts from Q at 8 am and travels towards P at a speed of 25 km/hr. At what time will they meet?
   (a) 9 am (b) 10 am (c) 10.30 am (d) 11 am

9. A train 75 meters long overtook a person who was walking at the rate of 6 km/hr and passed him in $7\frac{1}{2}$ seconds. Subsequently, it overtook a second

person, and passed him in $6\frac{3}{4}$ seconds. At what rate was the second person travelling?

(a) 1 km/hr (b) 2 km/hr (c) 3 km/hr (d) 4 km/hr

10. A train covers a distance between two stations A and B in 45 minutes. If the speed is reduced by 5 km/h, it will cover the same distance in 48 minutes. What is the distance between the two stations A and B(in km)? Also find the speed of the train.

(a) 80 km, 60 km/hr (b) 90 km , 70 km/hr
(c) 60 km, 80 km/hr (d) 70 km, 70 km/hr

11. The speed of a boat in still water is 6 km/h and the speed of the stream is 1.5 km/h. A man rows to a place at a distance of 22.5 km and comes back to the starting point. Find the total time taken by him.

(a) 4 hours (b) 6 hours (c) 8 hours (d) 10 hours

12. A person can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of $2\frac{1}{4}$ km/hr, find how far he can row in an hour in still water.

(a) $10\frac{1}{4}$ km/hr (b) $11\frac{1}{4}$ km/hr (c) $12\frac{1}{4}$ km/hr (d) $13\frac{1}{4}$ km/hr

13. A boat travels upstream from B to A and downstream from A to B in 3hours. If the speed of the boat in still water is 9 km/hr and the speed ofthe current is 3 km/hr, find the distance between A and B in kilometres.

(a) 10 km (b) 12 km (c) 14 km (d) 16 km

14. The current of a stream runs at 1 km/hr. A motor boat goes 35 km upstream and back again to the starting point in 12 hours. The speed of the motor boat in still water in km/hr is

(a) 4 (b) 6 (c) 8 (d) 12

15. A boat covers 24 km upstream and 36 km downstream in 6 hours, while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hours, then the velocity of the current is

(a) 1 km/hr (b) 1.5 km/hr (c) 2 km/hr (d) 2.5 km/hr

16. If a man's rate with the current is 12 km/h and the rate of current is 1.5 km/h, then the man's rate against the current in km/h is

(a) 10 (b) 9 (c) 8 (d) 6

17. A man can row at 5km/hr in still water and the velocity of the current is 1 km/hr. it takes him 1 hour to row to a place and back. How far is the place?

(a) 1.2 km (b) 1.8 km (c) 2.4 km (d) 3 km

18. The speed of a boat in still water is 6 km/hr. It takes the boat twice as long to row up as to row down the river. Find the rate of the stream.
(a) 1 km/hr (b) 2km/hr (c) 3 km/hr (d) 4 km/hr

19. A man can row 30 km upstream and 44 km downstream in 10 hours, Also,he can row 40km upstream and 55 km downstream in 13 hours. Find the rate of the current and the speed of the man in still water.
(a) 1 km/hr (b) 2 km/hr (c) 3 km/hr (d) 4 km/hr

20. A person can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of 3 km/hr, find the speed of the person in still water.
(a) 12 km/hr (b) 15 km/hr (c) 18 km/hr (d) 20 km/hr

21. Rajdhani Express train travelling at a uniform speed clears a platform 200 metres long in 10 seconds and passes a telegraph post in 6 seconds. The speed of train is
(a) 150 km/hr (b) 180 km/hr (c) 200 km/hr (d) 175 km/hr

22. A train 100 metres long meets a man going in opposite direction at 5 km/hr and passes him in 71 Seconds. The speed of the train is
(a) 40 km/hr (b) 45 km/hr (c) 36 km/hr (d) 52 km/hr

23. A passenger in train 'P' travelling at 1 km/hr min uses his stop watch and finds that another train 'Q' travelling in the opposite direction, completely passed his windows in 3 seconds. If the length of the train 'Q' is 87.50 m. find its speed (km/hr).
(a) 36 (b) 54 (c) 48 (d) 45

24. Local trains leave from a station at an interval of 15 minutes at a speed of 16 km/hr. A man moving from opposite side meets the trains at an interval of 12 minutes. The speed of the man is
(a) 5 km/hr (b) 4 km/hr (c) 5.5 km/hr (d) 3 km/hr

25. Two trains are travelling in opposite directions at uniform speeds of 50 km/hr and 6 km/hr timely. They take 6 seconds to cross each other. If the two trains had travelled in the same, then a passenger sitting in the faster train would have overtaken the slower train in 27 sec length of the faster train is more than the slower train by:
(a) 108.33 m (b) 75 m (c) 150 m (d) 33.33 m

ASSESMENT PROBLEMS:

1. A boat rows 16 km up the stream and 30 km downstream taking 5 h each time. The velocity of the current
   (a) 1.1 km/hr (b) 1.2 km/hr (c) 1.4 km/hr (d) 1.5 km/hr

2. Vijay can row a certain distance downstream in 6 h and return the same distance in 9 h. If the stream flows at the rate of 3 km/h, find the speed of Vijay in still water.
   (a) 12 km/hr (b) 13 km/hr (c) 14 km/hr (d) 15 km/hr

3. Subbu can row 6 km/hr in still water. When the river is running at 1.2 km/hr, it takes him 1 hour to row to a place and back. How far in the place?
   (a) 2.88 km (b) 2.00 km (c) 3.12 km (d) 2.76 km

4. A dog is passed by a train in 8 seconds. Find the length of the train if its speed is 36 kmph.
   (a) 70 m (b) 80 m (c) 85 m (d) 90 m

5. A lazy man can row upstream at 16 km/h and downstream at 22 km/h. Find the man's rate in still water (in kmph).
   (a) 19 (b) 14 (c) 17 (d) 18

6. A man can row 30 km upstream and 44 km downstream in 10 hours. It is also known that he can row 40 km upstream and 55 km downstream in 13 hours. Find the speed of the man in still water.
   (a) 4 km/hr (b) 6 km/hr (c) 8 km/hr (d) 12 km/hr

7. In a stream that is running at 2 km/hr, a man goes 10 km upstream and comes back to the starting point in 55 minutes. Find the speed of the man in still water.
   (a) 20 km/h (b) 22 km/h (c) 24 km/h (d) 28 km/h

8. A man goes down stream at x km/hr and upstream at y km/hr. The speed of the boat in still water is
   (a) 0.5 $(x + y)$ (b) 0.5 $(x - y)$ (c) $x + y$ (d) $x - y$

9. A motorboat whose speed in still water is 15 kmph goes 30 km downstream and comes back in a total 4 hours 30 min. Determine the speed of the stream.
   (a) 2 kmph (b) 3 kmph (c) 4 kmph (d) 5 kmph

10. A boat went down the river for a distance of 20 km. It then turned back and returned to its starting point, having travelled a total of 7 hours. On

its return trip, at a distance of 12 km from the starting point, it encountered a log, which had passed the starting point at the moment at which the boat had started downstream. The downstream speed of the boat is

(a) 7 kmph (b) 13 kmph (c) 16 kmph (d) 10 kmph

11. Two boats go downstream from point X to point Y. The faster boat coversthe distance from X to Y 1.5 times as fast as the slower boat. It is knownthat for every hour the slower boat lags behind the faster boat by 8 km. However, if they go upstream, then the faster boat covers the distance fromY to X in half the time as the slower boat. Find the speed of the faster boatin still water.

(a) 12 kmph (b) 20 kmph (c) 24 kmph (d) 25 kmph

12. A jogger running at 9 kmph alongside a railway track is 240 metres ahead of the engine of a 120 metres long train running at 45 kmph in the same direction. In how much time will the train pass the jogger?

(a) 3.6 sec (b) 18sec (c) 36 sec (d) 72 sec

13. Two train each 100m long moving in opposite directions cross each other in 8 seconds. If one is moving twice faster than the other, then the speedof the faster train is?

(a) 30kmph (b) 45kmph (c) 60kmph (d) 75kmph

14. A goods train leaves a station at a certain time and at a fixed speed. After 8 hours, an express train leaves the same station and moves in the same direction at a uniform speed of 120 kmph, this train catches up the goods train in 7 hours. Find the speed of the goods train.

(a) 50 (b) 48 (c) 56 (d) 60

15. Two trains are running on parallel lines in the same direction at speeds of 60 km/hr and 35 km/hr respectively. The faster train crosses a man in the slower train in 54 second. If the length of the slower train is 4/5th of the faster train, find the length of the slower train.

(a) 250 m (b) 375 m (c) 450 m (d) None of these

16. A train of length 500 m crosses a platform of length 50% more than the length of the train in 50 seconds. Find the time taken by this train to cross another train of same length running with double the speed of first train in the opposite direction.

(a) $13\frac{1}{3}$ s (b) $13\frac{1}{2}$ s (c) 13 s (d) $15\frac{1}{2}$

17. The ratio of the length of two trains X and Y is 4: 7 and the ratio of the time taken by both trains to cross a man standing on a platform is 2: 3. If the speed of the train X is 36 km/hr find the speed of the train Y in m/s
(a) $\frac{35}{3}$ m/s (b) 30 m/s (c) $\frac{20}{3}$ m/s (d) 15 m/s

18. If telegraph poles on rail road are 50m apart, then how many will be passed by train in 4 hrs travelling at 45kmph?
(a) 3604 (b) 3 6 0 1 (c) 3602 (d) 3603

19. Two trains for B leave A at 6:30am and 7:40am at 30 kmph and 40 kmph respectively. How many kms from A will the trains meet?
(a) 140 (b) 150 (c) 35 (d) 100

20. Two trains from prince town leave prince town at 9:00am and 9:30 am and travel 36 kmph and 54 kmph respectively. At what distance from prince town two trains are together?
(a) 36kms (b) 54kms (c) 72kms (d)None of these

21. The ratio of the speed of the boat in downstream to upstream is 7: 4. A boy takes 4 hours to cover the total distance of 88km upstream. What is the speed of the boat in still water?
(a) 30.25kmph (b) 35 kmph (c) 20.15kmph (d) 15kmph

22. A boat, going downstream in a river covered a distance of 40 km at an average speed of 50 km/hr. While returning to the same place, or upstream journey at an average speed of 30 km/hr to cover the same distance. Find the average speed of the boat during the whole journey?
(a) 42 km (b) 37.5 km (c) 32.75 km (d) 28.50 km

23. If the speed of current is 25% less than the speed of boat in still water and the boat covers 315 km downstream in 9 hours, what is the time taken by the same boat covers 100 km upstream?
(a) 10 hours (b) 12.5 hours (c) 20 hours (d) 25 hour

24. Train A crosses a car running opposite direction at the speed of 12 kmph in 18 seconds and the length of train A is 300 m. If train A increased the speed by 25% and crosses train B running opposite direction at the speed of 60 kmph in 21.6 seconds, then find the length of train B?
(a) 450 m (b)360 m (c) 400 m (d) 420 m

25. A train can cross a platform of 720m in 48 seconds, and a bridge which is 3/2 of length of platform in 66 seconds. Find the speed of train in km/h.
(a) 60km/hr (b) 20km/h (c) 36km/hr (d) 72km/hr

MODULE-10
SIMPLE INTEREST AND COMPOUND INTEREST

Simple Interest:

The principal amount at the beginning of any year is same as the original principal. Thus, if a borrower borrows a principal amount (P) `10,000 at the rate of 10% p.a. for n (=1, 2, 3 etc.) years, the Simple Interest will be calculated as follows:

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Principal	10000	10000	10000	10000	10000	10000
Interest for the year	1000	1000	1000	1000	1000	1000
Cumulative interest till the year (I)	1000	2000	3000	4000	5000	6000
Net amount payable (P+I)	11000	12000	13000	14000	15000	16000

Simple Interest on Principal Amount P borrowed for t years @ r% p.a., will fetch an interest of Prt/100, as described in the table above. Thus, net payable amount (A) at the end of t years is given by:

$A = P^* (1+\frac{r}{100})^t$

Compound Interest:

The principal amount at the beginning of any year is same as the net payable amount till the end of previous year. Thus, if a borrower borrows a Principal amount (P) of `10,000 at the rate of 10% p.a. for n (=1, 2, 3 etc.) years, the Compound Interest will be calculated as follows:

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Principal	10000	11000	12100	13310	14641	16105
Interest for the year	1000	1100	1210	1331	1464	1611
Cumulative interest till the year (I)	1000	2100	3310	4641	6105	7716
Net amount payable (P+I)	11000	12100	13310	14641	16105	17716

Compound Interest for Principal amount 'P' borrowed @ r% p.a., under compound interest, at the end of 1st year, will become P*(1+r/100), which in

turn will be treated as the principal at the beginning of 2nd year, as discussed earlier. Thus, net amount payable at the end of 2nd year will become $P*(1+r/100)^2$; Thus, at the end of 't' years, net amount payable (A) is be given by:

$A = P* (1+\frac{r}{100})^t$

Compounding more than once per annum:

If compounding is done k times in a year, then till the end of 't' year total number of times compounding will be applied is (k x t) and every time effectiverate of interest will be (r / k) % p.a. Thus, in that case net payable amount (A) will be:

$A= P*(1+\frac{r/k}{100})^{kt}$

Half Yearly: $A= P*(1+\frac{r/2}{100})^{2t}$

Quarterly: $A= P*(1+\frac{r/4}{100})^{4t}$

Solved Examples:

1. Sohan takes a loan of Rs 1000 from the Central bank for a period of one year. The given rate of interest is 10% per annum. Find the interest and the amount Sohan has to pay at the end of one year.

Solution: Let's write down the given information,

Here, the loan amount = Principal = Rs 1000

Time for which it is borrowed = T = 1 year

Rate of interest = R = 10%

Thus, the Simple Interest for a year, SI = (P × R ×T) / 100

= (1000 × 10 × 1) / 100

= Rs 100

Amount = Principal + Interest

The amount that Sohan has to pay to the bank at the end of one year = Principal + Interest = 1000 + 100 = Rs1100.

2. Ram borrowed a sum of Rs 5000 for 2 years at the rate of 3% per annum. Find the interest accumulated on the sum of at the end of 2 years and calculate the total amount.

Solution: Let's write down the given information,

P = Rs 5000

R = 3%

T = 2 years

SI = (P × R ×T) / 100 = (5000 ×3 × 2) / 100 = Rs 300

Now, let's calculate the amount of money at the end of two years,

Amount = Principal + Interest

So, The amount that Ram has to pay to the bank at the end of two years = 5000 + 300 = Rs 5300.

3. Mahi pays Rs 5000 as an amount on the sum of Rs 2000 which he had borrowed for 3 years. What is the rate of interest?

Solution: Let's write down the given information,

Amount at the end of three years = Rs 5000

Principal= Rs 2000

SI = Amount – Principal = 5000 – 2000 = Rs 3000

Time = 3 years

Rate =?

R = (Simple Interest × 100) / (Principal× Time)

R = (3000 × 100 /5000 × 3) =0.2%

Thus, R = 0.2%

4. The count of a certain breed of bacteria was found to increase at the rate of 5% per hour. What will be the growth of bacteria at the end of 3 hours if the count was initially 6000.

Solution:

Since the population of bacteria increases at the rate of 5% per hour,

We know the formula for calculating the amount,

Amount= Principal $(1 + R/100)^n$

Thus, the population at the end of 3 hours = $6000(1 + 3/100)^3$

$= 6000(1 + 0.03)^3$

$= 6000(1.03)^3$

= Rs 6556.36.

5. Calculate the Amount and Compound Interest on Rs. 2000 for 2 years at 10% per year?

Solution:

We need to calculate the value of Amount using this formula:

$$A = P (1 + R/100) T.$$

Putting the values in this formula, given

P = Rs. 2000,
R = 10% and
T = 2 years

$$A= 2000(1 + 10/100)^2 = 2420$$

So, the Amount = Rs. 2420.

Hence, Compound Interest = Rs. 2420 – Rs. 2000 = Rs. 420.

6. Find the compound interest on Rs. 12,800 for 2 years at per annum.

Solution:

Here, P = Rs. 12,800, R = 25/2% p.a., T = 2 years

Therefore, $A = RS.\ P (1 + R/100)^n = RS.12800[1 + 25/ (2 X 100)]^2$

$= RS.12800 (1 + 25/200)^2 = RS.12800 (1 + 1/8)^2$

$= RS.12800 [(8 + 1)/8]^2$

Hence, the Amount = Rs. 16200

Now, Compound interest = A – P = Rs. 16200 – Rs. 12800 = Rs. 3400

7. At what rate percent per annum will a sum of Rs. 10,000 amount to Rs. 14,641 in 4 years compounded annually?

Solution:

Let the required rate be R% per annum
A = 14641, P = Rs. 10000
We know that $A = P (1 + R/100)^n$

$$14641 = 10000 (1 + R/100)^4$$
$$14641/10000 = (1 + R/100)4$$
$$11/10 = 1 + R/100$$
$$R = 10\% \text{ p.a.}$$

8. Calculate the compound interest on Rs. 12000 for 1 years at 10% per annum when compounded half-yearly?

Solution:

Principal P = Rs. 12000, R = 20% per annum and n = 1 years.

therefore, Amount after 2 years = $P\ (1 + R/200)^{2n}$

$= RS.12000\ (1 + 10/200)^{2x1}$

$= RS.12000\ (1 + 1/20)^2$

$= RS.12000\ (21/20)^2$

$= RS.12000\ X\ 21/20\ X\ 21/20$

$= RS.12000\ X\ 441/400 = RS.13230$

therefore, Compound interest = Rs. 13230 – Rs. 12000 = Rs. 1230

9. Shyam deposited in a bank Rs. 7500 for 6 months at the rate of 8% p.a. interest compounded quarterly. Find the amount he received after 6 months.

Solution:

P = Rs. 7500, R = 8% per annum and n = 6 months = 6/12 = ½ year.

therefore, Amount after 6 months = $P\ (1 + R/400)^{4n}$

$= RS.7500\ X\ (1 + 8/400)^{4x1/2}$

$= RS.7500\ X\ (1 + 1/50)^2$

$= RS.7500\ X\ (51/50)^2$

$= RS.7803$

10. In what time will Rs. 2,560,000 amounts to Rs. 2,825,761 at 5% per annum, interest being compounded half-yearly?

Solution:

Principal P = Rs. 2,560,000, Amount A = Rs. 2,825,761, rate R = 5% per annum

Since, the interest is compounded half-yearly

therefore, $A = P\ (1 + R/200)^{2n}$, where n is the no. of years

$2,825,761 = 2,560,000(1 + 5/200)^{2n}$

$2,825,761/2,560,000 = (41/40)^{2n}$

$(41/40)^4 = (41/40)^{2n}$

$2n = 4$

$n = 4/2$ years = 2 years

PRACTICE PROBLEMS:

1. If Rs. 4 becomes Rs. 10 in 50 years at simple interest, the rate % p.a. is
   a) 5 % b) 2 (1/2) % c) 3 1/3 % d)3%

2. If Rs. 460 amounts to Rs. 640 in 6 years, what will it amount to in 2 years at the same rate %?

   a) Rs.520 b) Rs. 585 c) Rs. 700 d) Rs. 640

3. A certain sum of money amounts to 7/4 of itself in 3 years. The rate percent p.a. is
   a) 25 % b) 20 % c) 15% d) 10 %

4. The simple interest on a sum of money is 25% of the principal and the numberof years is equal to the rate % p.a. The rate % p.a. is
   a) 5 % b) 2 1/3% c) 10 % d) 9 %

5. Anoop borrowed Rs. 800 at 6 % p.a. & Rs. 1200 at 7 % p.a. for the same duration. He had to pay Rs. 1584 in all as interest. Find the time period.
   a) 10 years b) 11 years c) 12 years d) 13 years

6. A man invested 1/3 of his capital at 7%; ¼ at 8% and the remainder at 10%. If his annual income is Rs.561, What is his capital?
   a) Rs. 6600 b) Rs. 6585 c) Rs. 6700 d) Rs. 6640

7. The rate at which a sum becomes 4 times of itself in 15 years at S.I will be?
   a) 25 % b) 20 % c) 15% d)18%

8. A person invests money in three different schemes for 6 years, 10 years and 12 years at 10%, 12% and 15% Simple Interest respectively. At the completion of each scheme, he gets the same interest. What is the ratio of his investment?
   a) 6: 3: 2 b) 6:2:3 c) 1:2:3 d) none of these

9. A watch is sold for Rs.440 cash or for Rs.200 cash down payment together with Rs.244 to be paid after one month. Find the rate of interest (S.I) charged in the installment scheme.
   a) 10% b) 15% c) 20% d) 25%

10. Kishore purchases a track suit for Rs.2400 cash or for Rs.1000 cash down payments and two monthly instalments of Rs.800 each. Find the rate of interest (SI).
    a) 75% b) 120% c) 50% d) None of these

11. The difference between the CI and SI on a certain amount at 10% per annum for 2 years, compounded annually is Rs.372. Find the principal.
    a) Rs.32200 b) Rs.35000 c) Rs.37200 d) None

12. Find the minimum number of complete years required so that the sum of money put out at 20 % compound interest will be more than double.
a) 1 Year b) 2 Year c) 3 Year d) 4 Year

13. A sum of money at compound interest amounts to four times of itself in 2 years. In how many years will it be 16 times of itself?
a) 4 Year b) 27 Year c) 9 Year d) 6 Year

14. In what time will a man receive Rs. 51 as compound interest on Rs. 625 at 4% p.a. compounded annually?
a) 4.5 Year b) 2.5 Year c) 2 Year d) 3 Year

15. I invested a sum of money at compound interest. It amounted to Rs. 4840 in 2 years and to Rs. 5324 in 3 years. Find the rate of interest.
a) 8 % b) 12 % c) 10 % d) 15 %

16. A certain sum of money invested @ 40 % p.a. compounded half-yearly amounts to Rs. 5760 in 1 year. What is the sum (in Rs.)?
a) 2100 b) 4000 c) 1900 d) 2000

17. A sum of money is invested at compound interest compounded annually. The interests in two successive years were Rs. 150 and Rs. 168. Then the rate percent was
a) 1.5 % b) 3 % c) 6 % d) 12 %

18. If a certain sum of money invested at a certain rate of compound interest doubles in 5 years. In how many years will it become 4 times?
a) 7 years b) 10 years c) 11 years d) 12 years

19. If a certain sum of money invested at a certain rate of compound interest doubles in 6 years. In how many years will it become 8 times?
a) 16 b) 18 c) 20 d) 24

20. Find the difference between Compound Interest and Simple Interest on Rs. 4000 for 1 year at 10% p.a., if the interest is compounded half-yearly.
a) Rs.40 b) Rs.35 c) Rs.25 d) Rs.10

21. What would be the simple interest obtained on an amount of Rs 64,728 at the rate of 10 p.c.p.a after 7 years?
A) Rs 45309.6 B) Rs 39214.2 C) Rs 48077.5 D) Rs 40223.1

22. In how much time would the simple interest on a principal amount be 0.125 times the principal amount at 10% per annum?
A) 1.25 years B) 3.5 years C) 2 years D) 1 year

23. A sum of money at a certain rate per annum of simple interest doubles in the 5 years and at a different rate become three times in 12 years. The lower rate of interest per annum is:
A) 15% B) 20% C) $6\frac{2}{3}\%$ D) $16\frac{2}{3}\%$

24. In how many years will a sum of money double itself at 25/4 % simple interest per annum?
    A) 24 years B) 20 years C) 16 years D) 12 years

25. If a sum of money amounts to 12900 and 14250 at the end of 4th year and 5th year respectively a certain rate of simple interest, then the rate of interest is
    A) 10% B) 12% C) 18% D) 20%

ASSESSMENT PROBLEMS

1) A certain amount earns simple interest of Rs. 1750 after 7 years. Had the interest been 2% more, how much more interest would it have earned?

   A) ₹ 35 B) 350 C) 245 D) CBD

2) The Interest received at 15% per annum simple interest after 3 yrs is Rs. 630. What was the principal (in Rs)?
   A) 1200 B) 1750 C) 1400 D) 2000

3) A sum of money becomes 9 times in 20 years. Find the 10 times of rate of interest.
   A) 350% B) 45% C) 400% D) 250%

4) A sum becomes 6-fold at 5% per annum. At what rate, the sum becomes 12-fold?
   A) 10% B) 12% C) 9% D) 11%

5) The rates of simple interest in two banks x and y are in the ratio of 10: 8. Rajini wants to deposit her total savings in two banks in such a way that she receives equal half-yearly interest from both. She should deposit the savings in banks x and y in the ratio of
   A) 4: 5 B) 3: 5 C) 5: 4 D) 2: 1

6) Find the simple interest on Rs 3000 at 25/4% per annum for the period from 4th Feb, 2005 to 18th April, 2005.
   A) Rs 45.70 B) Rs 34.65 C) Rs 38.50 D) Rs 37.50

7) A sum at the simple interest at 27/2% % per annum amounts to Rs 2502.50 after 4 years. find the sum?
   A) Rs. 1345 B) Rs. 1625 C) Rs. 2502 D) Rs. 1825

8) A sum of Rs, 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, it would amount to how much?
   A) Rs.652 B) Rs.752 C) Rs.992 D) Rs. 562

9) Geeta borrowed some money at the rate of 6% p.a for the first two years, at the rate of 9% p.a for the next three years, and at the rate of 14% p.a for the period beyond five years. If she pays a total interest of Rs.11400 at the end of nine years, how much did she borrow?
   A) `Rs 10,000 B) Rs 11,000 C) Rs 12,000 D) Rs 14,000

10) A certain sum of money amounts to Rs. 1008 in 2 years and to Rs. 1164 in 7/2 years. Find the rate of interest.

A) 10% B) 11% C) 12% D) 13%

11) The rate at which a sum becomes four times of itself in 15 years at S.I, will be:
A) 12% B) 15% C) 20% D) 25%

12) A sum of Rs.1550 was lent partly at 5% and partly at 8% p.a. simple interest. The total interest received after 3 years was Rs. 300. The ratio of the money lent at 5% to that lent at 8% is:
A) 5: 8 B) 8: 5 C) 16: 15 D) 31: 6

13) A certain sum of money amounts to 5/4 of itself in 5 years. What is the simple rate of interest per annum?
A) 6% B) 5% C) 4% D) 8%

14) Out of certain sum, 1/3rd is interested at 3%, 1/6th at 6% and rest at 8%. If the simple interest for 2 years from all these investments amounts to Rs. 600. Find the original sum.
A) 4000 B) 15000 C) 5000 D) 4975

15) Namrata deposited Rs. 8,000 which amounted 9200 after 3 years at S.I. had the interest been 2% more, she would get how much?
A) 9480 B) 9580 C) 9660 D) 9680

16) S.I for a sum of 1550 for 2 years is rupees 20 more than the S.I for 1450 for the same duration. Find the rate of interest.
A) 5% B) 10% C) 15% D) 24%

17) In a certain time, a sum becomes 3 times at the rate of 5% per annum. At what rate of interest the same sum becomes 6 times in same duration?
A) 7% B) 9% C) 12.5% D) 17%

18) At what rate per cent per annum calculated in simple interest will a sum of money double in 10 years?
A) 10% B) 12% C) 12.5% D) 13.5%

19) A sum of money lent on simple interest triples itself in 15 years and 6 months. In how many years will it be doubled?
A) 6 yr 3 months B) 7 yr 9 months
C) 8 yr 3 months D) 9 yr 6 months

20) A sum of money becomes 3 times in 5 years. in how many years will be thesame sum become 6 times at the same rate of simple interest?
A) 15-year B) 12.5 year
C) 10-year D) 7.5 year

21) A sum becomes 3 times its original value in 6 years 8 months, when kept at simple interest. If the rate of interest is halved and compounded annually, then in which year does the sum become double?
(A) 6th year (B) 5th year (C) 3rd year (D) 4th year

22) In 4 years Rs. 6,000 amounts to Rs. 8,000. In what time at the same rate will Rs. 525 amount to Rs. 700?
(A) 2 years (B) 3 years (C) 4 years (D) 5 years

23) Find the proportion in which Rs 6,000 can be divided into two parts so that simple interest on the first part for 2years at 6% p.a. may be equal to the simple interest on the second part for 3 years at 8% p.a.
(A) Rs. 4,000; Rs. 2,000 (B) Rs. 5,000; Rs. 1,000
(C) Rs. 3,000; Rs. 3,000 (D) Rs. 2,000; Rs. 1,000

24) Rs. 200 tribled in 7 years when compounded annually. How many more years will it take to get another Rs. 4,800 compound interest?
(A) 7 years (B) 14 years (C) 21 years (D) None of these

25) The cost of a good is Rs. 700 at present. At 10% inflation per annum, what will be the cost of the same article 2 years henceforth?
(A) Rs. 800 (B) Rs. 840 (C) Rs. 847 (D) Rs. 870

MODULE-11

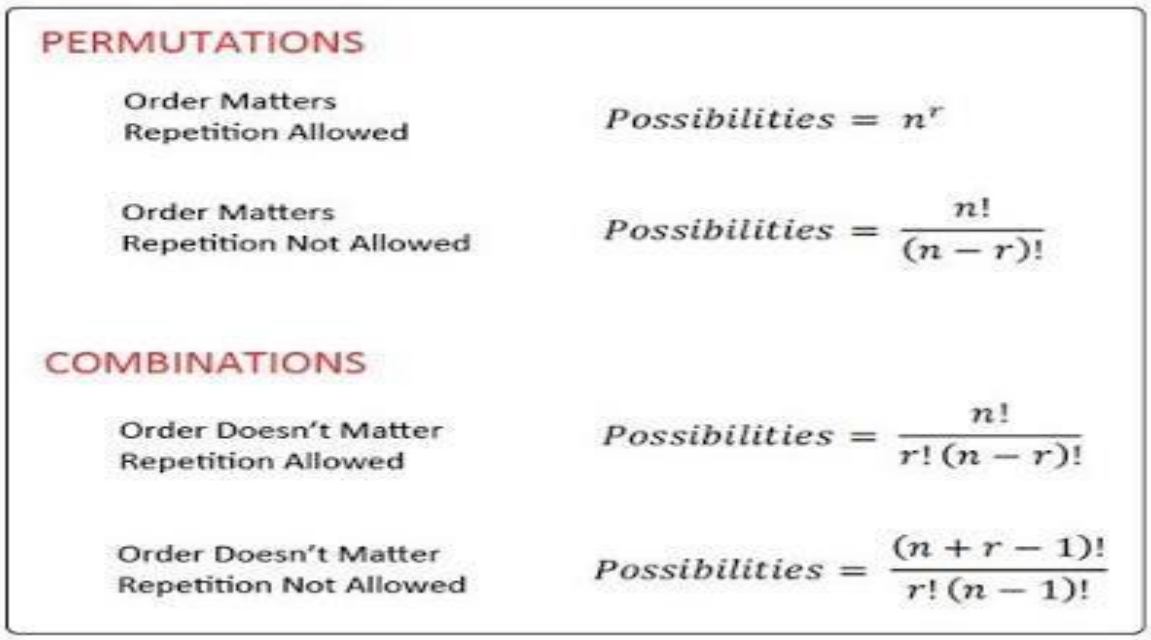
PERMUTATION AND COMBINATION

Multiplication Rule:

If any event can occur in m ways and after it happens in any one of these ways, a second event can occur in n ways, then both the events together can occur in 'm' × 'n' ways.

If one event with n outcomes occurs r times with repetition allowed, then the number of ordered arrangements is 'n^r'

Distinctly ordered sets are called arrangements or permutations. The number of permutations of n objects taken r at a time is given by: nP_r



If we have n elements of which 'x' are alike of one kind, 'y' area like of another kind, 'z' are alike of another kind then the number of ordered selections or permutations is given by:

n! / (x! y! z!)

Circular Arrangements:

The number of arrangements = (n – 1)! in a circle

- If 'n' objects are arranged in a circular way and if the clockwise and anti-clockwise arrangement is different, then the formula is (n-1)! ways.
- When there is no difference between clockwise and anticlockwise arrangements. In those cases, the total possible arrangements are half of the original ways of arrangements,

  i.e. (n-1)! /2

Principle of Grouping:

'n' identical items in **'r'** distinct groups	No restrictions: $^{n+r-1}C_{r-1}$
	No group empty: $^{n-1}C_{r-1}$
'n' distinct objects in **'r'** distinct groups	No restrictions: r^n
	Arrangement in a group is important:

Solved examples:

Example 1: How many four-digit numbers can be formed using the digits 0, 3, 4, 5, 6, 7 if

i. Repetition of digits is not allowed?

ii. Repetition of digits is allowed?

Sol: (i) In a four-digit number, 0 cannot appear in the thousand's place. So, the thousand's place can be filled in 5 ways (viz. 3, 4, 5, 6, and 7). Since, the repetition of digits is not allowed and 0 can be used at hundred's place, so the hundred's place can be filled in 5 ways. Now, any one of the remaining four digits can be used to fill up ten's place. So, ten's place can be filled in 4 ways. One's place can be filled with the remaining three digits in 3 ways. Hence, the required number of ways = 5 × 5 × 4 × 3 = 300.

(ii) For a four-digit number, we have to fill up four places and 0 cannot appear in the thousand's place. So, thousand's place can be filled in 5 ways. Since, repetition of digits is allowed, so each of the three remaining places viz

hundred's, ten's and one's place can be filled in 6 ways. Hence, the required number of ways = 5 × 6 × 6 × 6 = 1,080.

Example 2: It is required to seat 5 Indians and 4 Americans in a row so that all Americans occupy the even places. How many such arrangements are possible?

Sol: In all, 9 persons are to be seated in a row and in the row of 9 positions; there are exactly four even places viz. second, fourth, sixth and eighth. It is given that these four even places are to be occupied by 4 Americans. This can be done in ${}^4P_4$ ways. The remaining five positions can be filled by the 5 Indians in ${}^5P_5$ ways. So, by the fundamental principle of counting, the number of seating arrangements as required is ${}^4P_4 \times {}^5P_5 = 4! \times 5! = 24 \times 120 = 2{,}880$.

Example 3: Find the sum of all the numbers that can be formed with the digits 2, 3, 7, 8 taken all at a time.

Sol: The total number of numbers formed with the digits 2, 3, 7 and 8 taken all at a time = Number of arrangements of 4 digits taken = ${}^4P_4 = 4! = 24$. To find the sum of these 24 numbers we will find the sum of digits at units, tens, hundreds and thousand's place in all these numbers. Consider the digits in the unit's place in all these numbers. Each of the digits 2, 3, 7 and 8 occur in 3! = 6 times in the unit's place.

So, the total ways for the digits in the unit's place in all these numbers = (2 + 3 + 7 + 8) x 3! = 120. Similarly, the sum of the digits in the ten's, hundreds and thousand's places in all these numbers = (2 + 3 + 7 + 8) × 3! = 120 each. Hence the sum of all the numbers = $(10^0 + 10^1 + 10^2 + 10^3) \times 120 = 133320$.

Example 4: How many words can be formed from the letters of the word 'HALFTIME' so that the vowels never come together?

Sol: The total number of words formed by using all the eight letters of the word 'HALFTIME' is ${}^8P_8 = 8! = 40{,}320$. Now we will find the words in which the vowels are together. There are three vowels A, I and E. Let's take them as one unit. So we have 5 letters and one unit of vowels. These 6 can be arranged in 6! ways. Also, the 3 vowels can be arranged in 3! ways. So the total number of words in which the vowels are together = 6! × 3! = 720 × 6 = 4320. So, the total number

of words in which vowels are never together = Total number of words – Number of words in which vowels are together = 40,320 – 4,320 = 36,000.

Example 5: How many four digit numbers divisible by 4 can be made with the digits 1, 2, 7, 4, 9 if the repetition of digits is not allowed?

Sol: A number is divisible by 4 if the number formed by the last two digits is divisible by 4.

So, there are four two-digit numbers divisible by 4, which can be made with the help of these digits. These are 12, 24, 72 and 92.

Now corresponding each such way, the remaining three digits at thousand's and hundred's places can be arranged in ${}^3P_2$ ways.

Hence the required number of numbers = ${}^3P_2 \times 4 = 3! \times 4 = 24$.

Example 6: Find the number of ways in which 12 different flowers can be arranged to form a garland.

Sol: 12 different flowers can be arranged in circular form in (12 – 1)! = 11! ways. Since there is no distinction between the clockwise and anticlockwise arrangements, so, the required number of arrangements = (11! /2)

Example 7: Out of 5 boys and 2 girls, a committee of 3 is to be formed. In how many ways can it be done if at least one girl is to be included?

Sol: The committee can be constituted in the following ways:

i. By selecting 2 boys and 1 girl.

ii. By selecting 1 boy and 2 girls

Combination Calculator: 2 boys out of 5 boys and 1 girl out of 2 girls can be chosen in ${}^5C_2 \times {}^2C_1$ ways and 1 boy out of 5 boys and 2 girls out of 2 girls can be chosen in ${}^5C_1 \times {}^2C_2$ ways. ∴The total number of ways of forming the committee = ${}^5C_2 \times {}^2C_1 + {}^5C_1 \times {}^2C_2 = 20 + 5 = 25$.

Example 8: How many different 11 letter words can be formed with the letters ***ppppeeeeuuk***?

Sol: There are 11 letters in the given word of which 4 are p's, 4 are e's and 2 are u's. The total number of words is the arrangement of 11 things, of which 4 are alike of one kind, 4 are alike of second kind and 2 are of third kind i.e. (11! /4!4!2!). Hence, the total number of words = (11! /4!4!2!) = 34,650.

Example 9: A gentleman has 5 friends to invite. In how many ways can he send invitation cards to them if he has four servants to carry the cards?

Sol: Here each card can be carried by any of the four servants. ∴ Required number = $4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$

Example 10: In an examination, a candidate is required to pass all five different subjects. The number of ways he can fail is:

Sol: The candidate will fail if he fails either in 1 or 2 or 3 or 4 or 5 subjects, ∴ Required number of ways ${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 31$

PRACTICE PROBLEMS:

1. Ramesh Kumar Joshi is taken to a toy-shop. "ToysRUs" containing 15 distinct toy cars, 17 distinct toy dolls and 6 distinct toy guns. Find the number of ways in which Ramesh can choose a toy.

   (a) 36 (b) 34 (c) 38 (d) 40

2. A man has 3 jackets, 10 shirts and 5 pairs of slacks. If an outfit consists of a jacket, shirt and a pair of slacks, how many different outfits can the man make?

   (a) 150 (b) 130 (c) 125 (d) 140

3. There are five routes for going from a place A to another place B and six routes for going from the place B to a third place C. Find the number of different ways through which a person can go from A to C via B.

   (a) 40 (b) 20 (c) 30 (d) 35

4. There are 7 special buses running from Indore to Bhopal and 9 boats playing down the stream from Bhopal to Bairagrah and 4 boats intended for the return journey from Bairagarh to Bhopal. Find in how many different ways can a gentleman have a trip to Bairagarh from Indore via Bhopal and back to Bhopal again?

   (a) 252 (b) 250 (c) 248 (d) 200

5. Find the number of different ways in which four persons can be accommodated in three different chairs.

   (a) 12 (b) 18 (c) 24 (d) 20

6. A shop has 8 doors and 12 windows.

   a. In how many ways can a thief enter the shop, if he may enter through a door or a window?

   (a) 16 (b) 18 (c) 17 (d) 20

   b. In how many ways can the thief rob the shop by entering through a window and exiting through a door?

   (a) 90 (b) 92 (c) 96 (d) 94

7. A department in an organisation contains 17 male employees and 15 female employees.

   a. In how many ways, can an employee be chosen from the department to represent the department in the annual meeting?

   (a) 32 (b) 24 (c) 16 (d) 20

   b. In how many ways, can a male and a female employee be chosen from the department to represent the department in the employee's union meeting?

(a) 255 (b) 150 (c) 240 (d) 155

8. How many words (with or without meaning) of three distinct letters of the English alphabets are there?

(a) 15600 (b) 13800 (c) 12500 (d) 14600

9. Find the total number of ways in which 4 persons can take their place in a cab having 6 seats.

(a) 300 (b) 240 (c) 360 (d) 180

10. In how many ways can 6 people stand in a queue?

(a) 700 (b) 720 (c) 600 (d) 780

11. How many 4-digit numbers can be made by using digits 1 to 7 (repetition is not allowed), if the digit 4 will always be there in the number?

(a) 400 (b) 480 (c) 360 (d) 240

12. How many different 3-letter words can be made by 5 vowels, if vowel 'A' will never be included?

(a) 12 (b) 48 (c) 24 (d) 36

13. How many 3-digit numbers can be made by using digits 1 to 7, if repetition is allowed?

(a) 343 (b) 300 (c) 289 (d) 321

14. Find the number of permutations of the letters of the word ASSASSINATION.

(a) 13! / (4! 3! 2! 3!) (b) 11! / (3! 3! 2! 4)

(c) 11! / (4! 2! 3!) (d) 13! / (4! 3! 2! 2!)

15. Find the number of permutations of the letters of the words 'DADDY DID A DEADLY DEED'.

(a) 19! / (9! 3! 2! 3!) (b) 17! / (9! 2! 3! 3!)

(c) 19! / (9! 2! 2! 3!) (d) 17! / (9! 3! 2! 2!)

16. How many different words can be formed with the letters of word 'ORDINATE'?

(a) 500 (b) 576 (c) 550 (d) 600

17. In how many ways can 7 people sit down at a round table?

(a) 700 (b) 680 (c) 720 (d) 740

18. In how many ways can a garland of 10 different flowers be made?

(a) 9! /2! (b) 10! (c) 8! / 2 (d) 10! / 2!

19. In how many ways a hockey team of eleven can be selected from 16 players?

(a) 4362 (b) 4368 (c) 3460 (d) 3648

20. In class of 25 students, find the total number of ways to select two representatives, if a particular person will be never selected.

(a) 256 (b) 276 (c) 225 (d) 290

21. In how many ways we can make two groups of 8 and 3 students out of total 11 students.

(a) 150 (b) 165 (c) 155 (d) 160

22. In a city no persons have identical set of teeth and there is no person without a tooth. Also, no person has more than 32 teeth. If we disregard the shaped and size of tooth and consider only the positioning of the teeth, then find the maximum population of the city. (Assume no two persons have similar configuration regarding positioning of teeth).

(a) 2^{32}-1 (b) 2 * 32 (c) 232 (d) 63

23. Find the total number of combinations of 5 alphabets A, B, A, B, B taking some or all at time.

(a) 10 (b) 12 (c) 11 (d) 9

24. Out of 6 consonants and 5 vowels, how many words of 3 consonants and 2 vowels can be formed?

(a) 22000 (b) 24000 (c) 28000 (d) 20000

25. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. Determine the number of ways in which the seating arrangement can be made.

(a) 11C5 * 9! * 9! (b) 10C5 * 9! * 8!

(c) 10C5 * 9! * 9! (d) 11C5 * 9! * 8!

ASSESSMENT PROBLEMS:

Directions (1-2): Kindly study the following informationcarefully and answer the question that follows:

Five students are to be arranged on five chairs for a photograph. Three of these are girls and the rest are boys.

1. Find out the total number of ways in which three girls are together.
   A.36 B. 84 C. 100 D. 120

2. Five students are to be arranged on five chairs for a photograph. Three of these are girls and the rest are boys. Find out the number of ways in which all three girls do not occupy consecutive seats.
   A. 120 B. 36 C. 84 D.136

3. Directions (3-4): Kindly study the following informationcarefully and answer the question that follows:
   Using all the letters of the word LINEAR.
   How many words start with a vowel but end with a consonant?
   A. 224 B. 316 C. 212 D. 216

4. Using all the letters of the word LINEAR.
   How many different words can be formed that start and end with vowel?
   A. 126 B. 108 C. 144 D. 216

5. A six letter word is to be formed by using at least two vowels in it. How many such words can be formed (not necessarily meaningful) if all the letters in word are different?
   A. 53349120 B. 53439120 C. 53431920 D. 54339120

6. In a language, there are six different words. A sentence can be formed by at least 2 words. If order of words is changed in a sentence, we get a different sentence. How many different sentences can be formed in this language?
   A. 1870 B. 1970 C. 1950 D. 2190

7. How many 3 - letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?
   A. 720 B. 420 C. 5040 D. 120

8. There are 2 shirts, 3 jeans, 3 socks and 2 skirts. In how many ways a shopkeeper can arrange these things so that all the socks come together and all the skirts come together?
   A. 60,720 B.60,480 C. 60,540 D. 60,690

9. In how many different ways can the letters of the word WINDOW be arranged in such a way that the vowels never come together?
   A. 350 B. 250 C. 720 D. 240

10. The number of ways in which 8 different books can be arranged on a shelf so that 3 particular books shall not be together:
A. 11! – 3! B. 361000 C. 8! X 3! – 5! D. 36000

11. 16 persons shake hands with one another in a party. How many shake hands took place?
A. 124 B. 120 C. 165 D. 150

12. How many 3 - letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?
A. 720 B. 420 C. 5040 D. 120

13. In how many different ways can the letters of word 'POPULAR' be arranged?
A. 1650 B. 1780 C. 6800 D. None of these

14. A team of 7 children is to be selected out of 7 girls and 5 boys such that it contains at least 5 girls. In how many different ways can the selection be made?
A. 105 B. 246 C. 100 D. 128

15. In how many different ways can the letters of word RABBIT be arranged?
A. 360 B. 240 C. 300 D. 275

16. In how many ways can the letters of the word 'PARAGLIDING' be arranged such that all the vowels occur together?
A. 88322 ways B. 120960 ways C. 740 ways D. 144868 ways

17. Five people out of whom only two can drive are to be seated in a five seater car with two seats in front and three in the rear. The people who know driving don't sit together. Only someone who knows driving can sit on the driver's seat. Find the number of ways the five people can be seated.
A. 40 B. 60 C. 48 D. 36

18. A boy is playing a Snake & Ladder game; he is on 91 and has to get to 100 to complete the game. There is a snake on 93 and 96. In how many wayshe can complete the game, if he doesn't want to roll the dice more than three times.
A. 20 B.15 C.16 D.18

19. 8 members are to be selected from a group of 9 males and 7 females. In how many ways will the members with at most 3 females and at least 4 males be selected?
A. 6472 ways B. 6286 ways C. 6435 ways D. 6225 ways

20. A chess board has rows and columns marked A to H and 1-8. A man has a knight and a rook which he has to place on the board such that the two

pieces are not in same row or column, what is total number of ways he can place the two pieces?
A. 3072 B. 3136 C. 6272 D. 6144

21. In a class there are 10 boys and 8 girls. The teacher wants to select a boy and a girl to represent the class in a function. In how many ways can the teacher make this select ion?
a. 70 b. 80 c. 90 d. 60

22. There are three candidates for musical, four candidates for a Mathemat ical and five for a Science scholarship. In how many ways can these scholarships be awarded?
a. 12 b. 60 c. 23 d. 17

23. A room has 8 doors. In how many ways can a man enter the room through one door and come out through a different door?
a. 15 b. 56 c. 64 d. 16

24. Find the total number of ways of answering 5 objective questions, each questions having 4 choices.
a. 120 b. 3125 c. 1024 d. None of these

25. How many four-digit numbers, each divisible by 4 can be formed using the digits 5, 6, 7, 8, 9, repetition of digits being allowed in any number?
a. 100 b. 100 c. 75 d. 150

MODULE-12

PROBABILITY

Event and outcome

An **Outcome** is a result of a random experiment. For example, when we roll a dice getting six is an outcome.

An **Event** is a set of outcomes. For example, when we roll dice the probability of getting a number less than five is an event.
Note: An Event can have a single outcome.

Probability,

P(E) = Number of Outcomes Favorable to E / Number of all possibleoutcomes of the experiment

Here we assume that the outcomes of the experiment are **equally likely**.

Sum of Probabilities

The **sum** of the probabilities of all the **elementary events** of an experiment is **one**.
Example: take the coin-tossing experiment. P(Heads) + P (Tails)

= (1/2) + (1/2) =1

Impossible event

An event that has **no chance of occurring** is called an **Impossible event**, i.e.

P(E) = 0.

E.g.: Probability of getting a 7 on a roll of a die is 0. As 7 can never be an outcome of this trial.

Sure event

An event that has a **100% probability** of occurrence is called a **sure event**. The probability of occurrence of a **sure event** is **one**.

E.g. What is the probability that a number obtained after throwing a die is less than 7?
So, P(E) = P(Getting a number less than 7) = 6/6= 1

Solved Problems

Example 1: A coin is thrown 3 times. What is the probability that at least one head is obtained?

Sol: Sample space = [HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]

Total number of ways = 2 × 2 × 2 = 8. Fav. Cases = 7

P (A) = 7/8

OR

P (of getting at least one head) = 1 – P (no head) ⇒ 1 – (1/8) = 7/8

Example 2: Find the probability of getting a numbered card when a card is drawn from the pack of 52 cards.

Sol: Total Cards = 52. Numbered Cards = (2, 3, 4, 5, 6, 7, 8, 9, 10) 9 from each suit 4 × 9 = 36

P (E) = 36/52 = 9/13

Example 3: There are 5 green 7 red balls. Two balls are selected one by one without replacement. Find the probability that first is green and second is red.

Sol: P (G) × P (R) = (5/12) x (7/11) = 35/132

Example 4: What is the probability of getting a sum of 7 when two dice are thrown?

Sol: Probability math - Total number of ways = 6 × 6 = 36 ways.

Favourable cases = (1, 6) (6, 1) (2, 5) (5, 2) (3, 4) (4, 3) --- 6 ways.

P (A) = 6/36

= 1/6

Example 5: 1 card is drawn at random from the pack of 52 cards.

(i) Find the Probability that it is an honor card.

(ii) It is a face card.

Sol:

(i) honor cards = (A, J, Q, K) 4 cards from each suits = 4 × 4 = 16

P (honor card) = 16/52 = 4/13

(ii) face cards = (J, Q,K) 3 cards from each suit = 3 × 4 = 12

Cards.P (face Card) = 12/52 = 3/13

Example 6: Two cards are drawn from the pack of 52 cards. Find the probability that both are diamonds or both are kings.

Sol: Total no. of ways = $^{52}C_2$

Case I: Both are diamonds = $^{13}C_2$

Case II: Both are kings = $^{4}C_2$

P (both are diamonds or both are kings) = ($^{13}C_2$ + $^{4}C_2$) / $^{52}C_2$

Example 7: Three dice are rolled together. What is the probability as getting at least one '4'?

Sol:

Total number of ways = 6 × 6 × 6 = 216.

Probability of getting number '4' at least one time

= 1 – (Probability of getting no number 4)

= 1 – (5/6) x (5/6) x (5/6) = 91/216

Example 8: A problem is given to three persons P, Q, R whose respective chances of solving it are 2/7, 4/7, 4/9 respectively. What is the probability that the problem is solved?

Sol:

Probability of the problem getting solved = 1 – (Probability of none of them solving the problem)

$$P(P)=\frac{2}{7}\Rightarrow P(\overline{P})=1-\frac{2}{7}=\frac{5}{7},\ P(Q)=\frac{4}{7}\Rightarrow P(\overline{Q})=1-\frac{4}{7}=\frac{3}{7},\ P(R)=\frac{4}{9}\Rightarrow P(\overline{R})=1-\frac{4}{9}=\frac{5}{9}$$

PRACTICE PROBLEMS:

1. What is the probability of getting a multiple of 3 while rolling a dice?

(1) 3/6 (2) 1/3 (3) 2/3 (4) 1/6

2. Two dice are thrown simultaneously. The probability of getting a sum of 10 or 11 is

(1) 7/12 (2) 5/36 (3) ¼ (4) 1/6

Directions for the questions 3 – 5: A box contains 5 red and 4 blue balls. Two balls are drawn at random from the box. Find the probability that both of them are red if

3. The balls are drawn together

(1) 25/72 (2) 5/72 (3) 5/36 (4) 5/18

4. The balls are drawn one after the other with replacement

(1) 25/72 (2) 5/18 (3) 25/81 (4) 10/81

5. The balls are drawn one after the other without replacement

(1) 5/18 (2) 25/72 (3) 25/81 (4) 5/72

Directions for the questions 6 – 7: The probabilities that A and B will tell the truth are 2/3 and 4/5 respectively. What is the probability

6. They agree with each other

(1) 8/15 (2) 1/15 (3) 3/5 (4) 2/5

7. They contradict each other while giving a testimony in the court(1)

2/15 (2) 4/15 (3) 8/15 (4) 2/5

8. A company wants to make a core committee to take major decisions. The company has 5 directors 6 managers and 4 executives. Find the probability that the core committee will contain 2 directors, 2 managers and one executive?

(1) 200/3003 (2) 600/1001 (3) 2000/3003 (4) 200/1001

Directions for the questions 9 – 10: From a pack of 52 playing card. One card is drawn at random. What is the probability

9. Face card

(1) 12/13 (2) 3/13 (3) 3/52 (4) 1/13

10. Honour card

(1) 4/13 (2) 3/13 (3) 2/13 (4) 1/13

11. There are two children in a family, find the probability that there in at least one girl in the family

(1) 1/2 (2) 1/4 (3) 3/4 (4) 1/3

Directions for the questions 12 – 13: From a group of 5 men and 4 women, 4 persons are to be chosen. What is the probability

12. Equal number of men & women are selected?

(1) 1/4 (2) ½ (3) 1/21 (4) 10/21

13. More number of men are selected as compared to women?

(1) 5/126 (2) 121/126 (3) 20/63 (4) 5/14

14. From a pack of 52 playing cards 2 cards are drawn at random. What is the probability that either both are red or both are aces?

(1) 331/1326 (2) 55/221 (3) 55/1326 (4) 28/52c2

15. A die is rolled and an odd number appears at the top. What is the probability that it is greater than 1?

(1) 1/3 (2) 5/6 (3) 1/2 (4) 2/3

16. Two Natural numbers are selected at random from first 9 natural numbers. If the sum of two numbers in even, the probability that both the numbers are odd will be

(1) 1/2 (2) 3/8 (3) 5/8 (4) None

17. The probability of selecting two numbers their sum being odd out of first a positive integer is

(1) 5/9 (2) ½ (3) 5/8 (4) 5/36

18. A couple has two children. Find the probability that both one boys, if it is known that one of the children is a boy

(1) 1/9 (2) 1/3 (3) 2/3 (4) 1/4

19. From question 18, find the probability that both are boys, if it is known that the older child is a boy.

(1) 1/4 (2) 1 (3) 3/4 (4)1/2

20. A four-digit number is formed with the digits 1,2,4,5 without repetition. Find the chance of it being divisible by 5

(1) 3/4 (2) 9/16 (3) 1/2 (4) ¼

21. Three coins are tossed simultaneously, what is the probability of getting at least two tails?

(1) 0.375 (2) 0.735 (3) 0.5 (4)0.75

22. Four coins are tossed simultaneously, what is the probability of getting at least one head?

(1) 0.735 (2) b. 0.75 (3) 0.5 (4)0.875

23. Four coins are tossed simultaneously, what is the probability of getting exactly two tails?

(1) 1/8 (2) 3/8 (3) 5/8 (4)7/8

24. A fair dice is rolled twice, what is the probability of getting one even number and once odd number?

(1) 1/6 (2) 1/4 (3) 1/2 (4)1/3

25. A fair dice is rolled twice, what is the probability of getting the number 5 twice?

(1) 1/6 (2) 1/12 (3) 1/24 (4)1/36

ASSESSMENT PROBLEMS:

1. A shop sells 10 tube lights out of which 3 are defective. Salman buys four tube lights. Find the probability that at least two of the tube lights that he buys work.

   A) 29/30 B) 34/35 C) 14/15 D) 24/25

2. A shop sells 10 tube lights out of which 3 are defective. Salman buys four tube lights. Find the probability that all of Salman's tube lights work.

   A) 1/15 B) 1/25 C) 1/30 D) None of these

Directions (Set of 2 Questions): Kindly study the following information carefully and answer the question that follows:

3. From a box containing 8 yellow and 5 white pens, three are drawn one after the other. Find the probability of all three pens being yellow if the pens drawn are not replaced?

   A) 336/1716 B) 128/429 C) 113/1716 D) 336/2197

4. From a box containing 8 yellow and 5 white pens, three are drawn one after the other. Find the probability of all three pens being yellow if the pen drawn is replaced by another yellow-colored pen before the next pen is picked.

   A) 336/2197 B) 512/2197 C) 40/2197 D) 57/91

Directions (Set of 2 Questions): Kindly study the following information carefully and answer the question that follows:

5. A box contains 4 white, 6 green, 2 red and 5 yellow pens. If two pens are picked at random, what is the probability that both of them are green?

   A) 5/136 B) 1/136 C) 15/136 D) 8/15

6. A box contains 21 balls numbered 1 to 21. A ball is drawn and then another ball is drawn without replacement. What is the probability that both balls are even numbered?

   A) 2/7 B) 8/21 C) 3/14 D) 5/21

7. There are 3 green, 4 oranges and 5 white color bulbs in a bag. If a bulb is picked at random, what is the probability of having either a green or a white bulb?

   A) ¾ B) 2/3 C) 4/3 D) 2/5

8. A box contains slips with numbers from 1 to 50 written on them. A slip is drawn and replaced. Then another slip is drawn and after replacing anotherslip is drawn. What is the probability that an even number appears on the?

first draw, an odd number on the second draw and a number divisible by 3 on the third draw?

A) 1/25 B) 2/25 C) 8/25 D) 4/25

9. When 4 fair coins are tossed together what is the probability of getting at least 3 heads?

A) ¼ B) ¾ C) 5/16 D) 3/8

10. A committee of 3 members is to be made out of 6 men and 5 women. What is the probability that the committee has at least two women?

A) 10/33 B) 14/33 C) 14/15 D) 13/25

11. A box contains 21 balls numbered 1 to 21. A ball is drawn and then another ball is drawn without replacement. What is the probability that both balls are even numbered?

A) 2/7 B) 8/21 C) 3/14 D) 5/21

12. There are 3 green, 4 orange and 5 white color bulbs in a bag. If a bulb is picked at random, what is the probability of having either a green or a white bulb?

A) ¾ B) 2/3 C) 4/3 D) 2/5

13. A box contains slips with numbers from 1 to 50 written on them. A slip is drawn and replaced. Then another slip is drawn and after replacing another slip is drawn. What is the probability that an even number appears on thefirst draw, an odd number on the second draw and a number divisible by 3on the third draw?

A) 1/25 B) 2/25 C) 8/25 D) 4/25

14. When 4 fair coins are tossed together what is the probability of getting at least 3 heads?

A) ¼ B) ¾ C) 5/16 D) 3/8

15. A committee of 3 members is to be made out of 6 men and 5 women. What is the probability that the committee has at least two women?

A) 10/33 B) 14/33 C) 14/15 D) 13/25

16. The names of 5 students from section A, 6 students from section B and 7 students from section C were selected. The age of all the 18 students was different. Again, one name was selected from them and it was found that it was of section B. What was the probability that it was the youngest student of the section B?

A) 1/18 B) 1/15 C) 1/6 D) 1/12

17. A bag contains 35 balls of three different colors viz. red, orange and pink.The ratio of red balls to orange balls is 3: 2, respectively and probability of choosing a pink ball is 3/7. If two balls are picked from the bag, then what is the probability that one ball is orange and one ball is pink?

A) 24/119 B) 60/119 C) 96/595 D) 3/17

18. There are total 18 balls in a bag. Out of them 6 are red in color, 4 are green in color and 8 are blue in color. If Vishal picks three balls randomly from the bag, then what will be the probability that all the three balls are not ofthe same color?

A) 95/102 B) 19/23 C) 21/26 D) 46/51

19. Ram and Shyam are playing chess together. Ram knows the two rows in which he has to put all the pieces in but he doesn't know how to place them. What is the probability that he puts all the pieces in the right place?

A) 8! /16! B) 8! / (2 × 15!) C) 8! /15! D) (2 × 8!)/16!

20. A child paints the six faces of a cube with six different colors red, blue, pink, yellow, green and orange. What is the probability that red, pink and blue faces share a common corner?

A) 1/6 B) 1/20 C) 1/10 D) 1/5

21. Two balls are drawn from it simultaneously, what is the probability that the two balls are of different color?

a. 35/66 b. 35/132 c. 35/144 d. 32/121

22. A card is drawn from a well shuffled pack of cards, what is the probability of it being a red color card?

a. ¼ b. 1/2 c. 1/8 d. 1/6

23. A card is drawn from a well shuffled pack of cards, what is the probability of it a queen card?

a. 1/52 b. 1/13 c. 1/26 d. ¼

24. A card is drawn from a well shuffled pack of cards, what is the probability that the card is number card?

a. 7/13 b. 9/13 c. 5/13 d. 3/13

25. A card is drawn from a well shuffled pack of cards, what is the probability that the card is a face card?

a. 7/13 b. 9/13 c. 5/13 d. 3/13