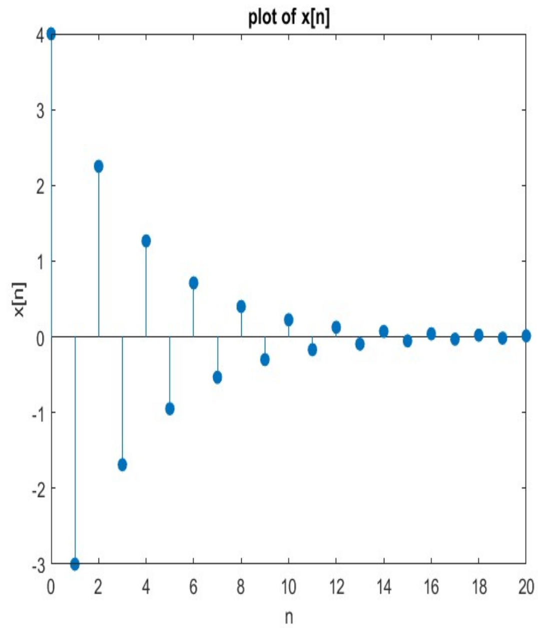
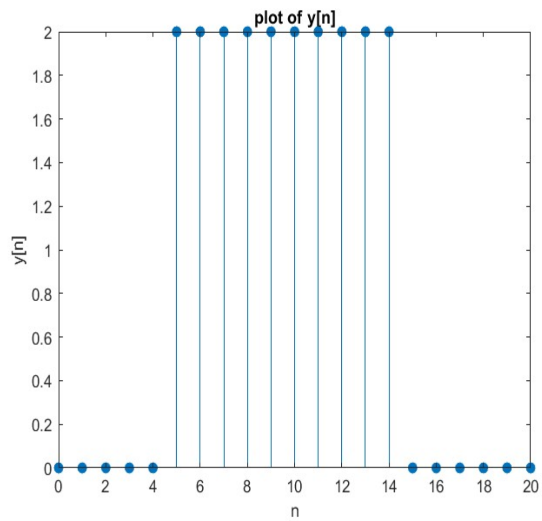
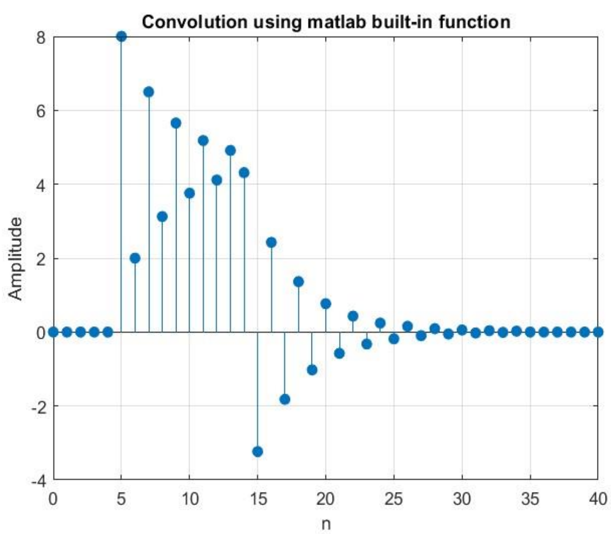
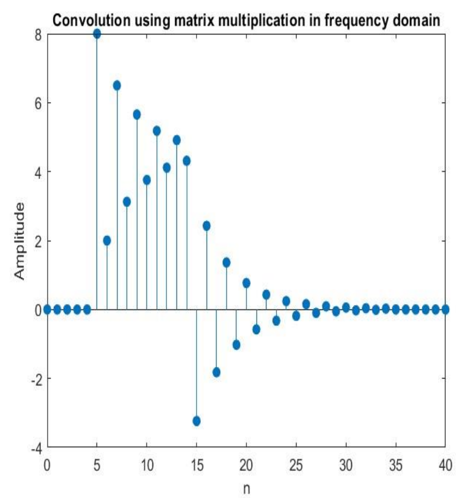
Exercise 1:





I am plotting the signals x[n] and y[n] in the above taking n from 0 to 20. So, both of them have an array of size 21.

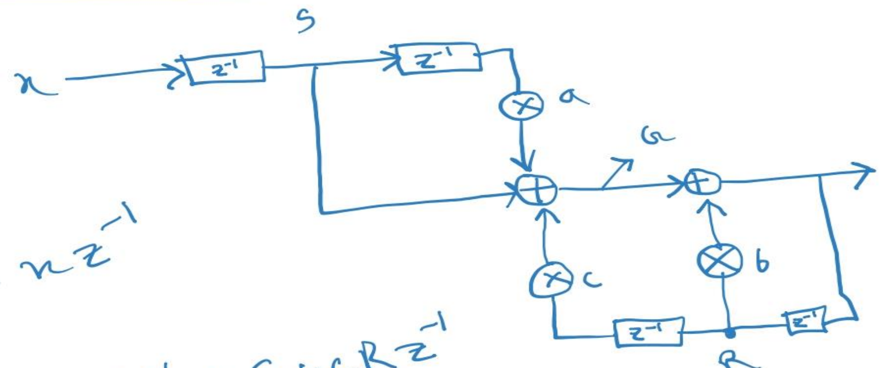




In the above figures, I am displaying convolution done in two different ways. The first one shows convolution from matlab built-in function conv. In the second figure, I am doing convolution by multiplying the matrices of x and y in frequency domain. I first took the FFT of both x and y, then I have created appropriate matrix for multiplication in the frequency domain. Then I use IFFT to again bring it back to time domain and plot it using stem function. The convolution has a size of 41, which verifies the formula size of x + size of y -1 = 21+21-1=41. Observing the graph, it is seen that the convolution signal starts becoming non-zero from n = 5 and returns to being zero again at n = 35. So, the extension is ext{z} = [5,35] and the duration is 35-5 = 30. It also verifies the theory that the duration of convoluted signal equals the duration of both the original signals. The duration of x is 20 and the duration of y is 10, so total is 30. Thus, the extension and duration of convoluted signal is verified this way as well.

Exercise 2:

exercise-2



$s = xz^{-1}$

$G = asz^{-1} + s + cRz^{-1}$

$R = yz^{-1}$

$G + bR = y$

$\Rightarrow \; y = asz^{-1} + s + cRz^{-1} + bR$

$\Rightarrow \; y = axz^{-2} + xz^{-1} + cyz^{-2} + byz^{-1}$

$\Rightarrow \; y\left(1 - cz^{-2} - bz^{-1}\right) = x\left(az^{-2} + z^{-1}\right)$

$$\Rightarrow \; H(z) = \frac{y}{x} = \frac{az^{-2} + z^{-1}}{1 - cz^{-2} - bz^{-1}}$$

$$= \frac{z + 2}{z^2 - bz - c}$$

Given,

$a = 2$

$b = 3/5$

$c = 1/9$

$$H(z) = \frac{z+2}{z^2 - bz - c}$$

$$= \frac{z+2}{(z+0.15)(z-0.75)}$$

$$= \frac{\alpha_1}{(z+0.15)} + \frac{\alpha_2}{(z-0.75)}$$

$$\alpha_1 = \lim_{z \to -0.15} \frac{z+2}{(z-0.75)} = \frac{-0.15+2}{-0.15-0.75}$$

$$= \frac{1.85}{-0.9}$$

$$\alpha_2 = \lim_{z \to 0.75} \frac{z+2}{(z+0.15)} = \frac{2.75}{0.9}$$

So, we can rewrite as,

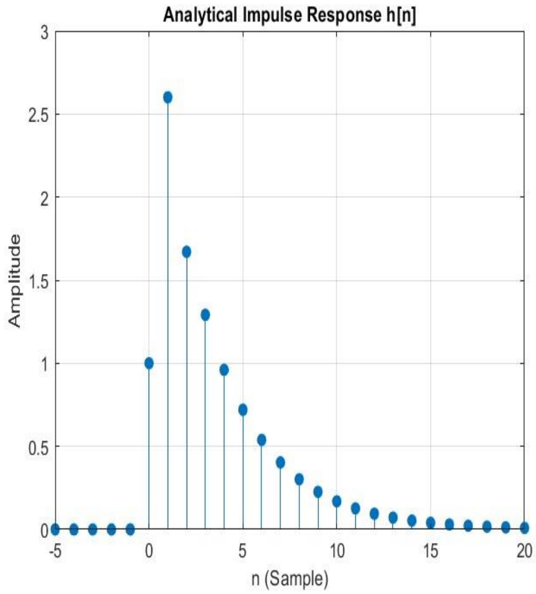
$$\frac{3}{(z-.75)} - \frac{2}{(z+0.15)}$$

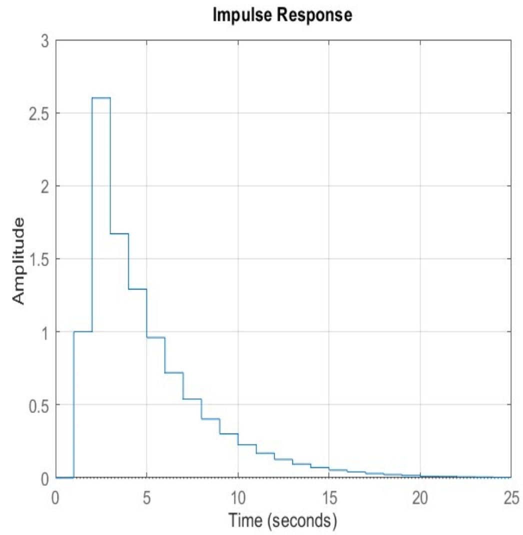
Applying the formula of inverse z-transform

$$= 3(0.75)^n u[n] - 2(-0.15)^n u[n]$$

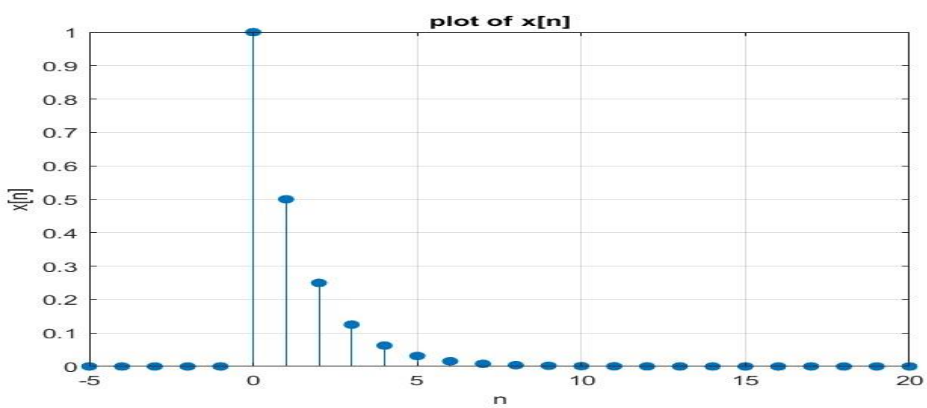
$$= 3(0.75)^n u[n] - 2(-0.15)^n u[n]$$

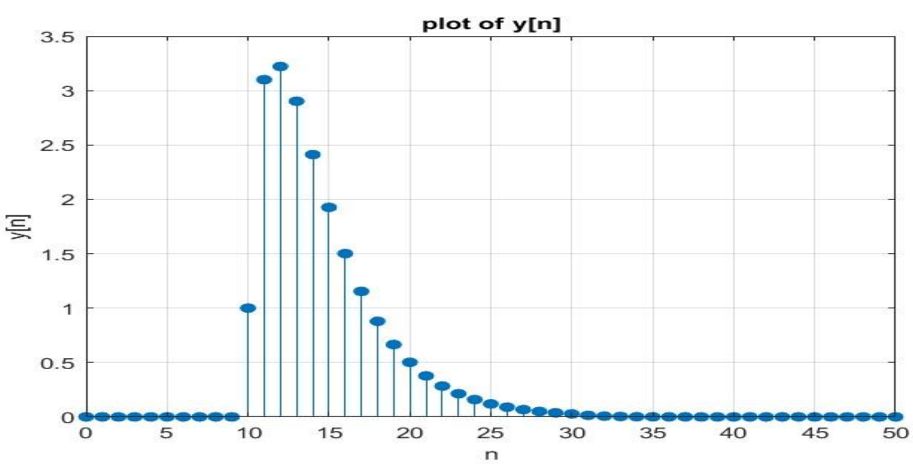
After getting the transfer function in time domain from the above calculation, I have plotted it to get the impulse response using analytical calculations and taking n from -5 to 20. Then I found the impulse response again with MATLAB built-in function and transfer function in Z domain. I compare the plot of impulse response in both cases and find they are almost same. The figures are given below. First one is analytical, second one using built-in function.





The difference equation is y[n] = x[n-1] + a*x[n-1] + b*y[n-1] + c*y[n-2]. Poles of the transfer are 0.7485 and -0.1485 and only one zero at -2. Region of convergence is maximum absolute value of pole which is 0.7485 and this indicates the system is inside the unit circle, so it is also BIBO stable. Then taking n = -5 to 20, x[n] is determined and plotted. Taking same n, y[n] = h[n] * x[n] is found and plotted. Then we compare the values and it is almost similar. The graphs are given below.





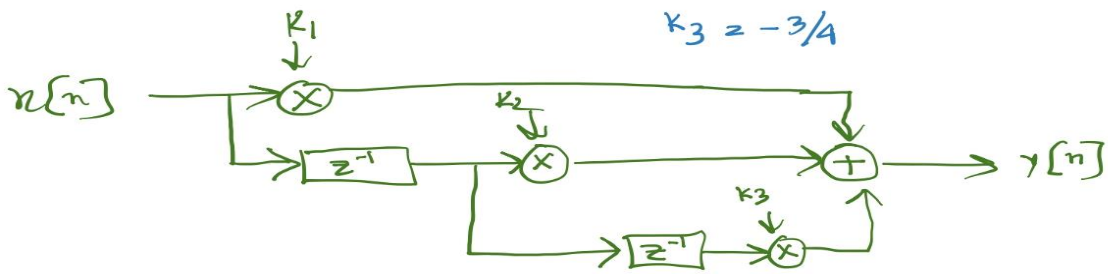
Exercise 3: In the following, I have constructed a block diagram and determined transfer function in Z domain. I have also written the transfer function in the format for finding impulse response.

ex-3: $y[n] = K_1 x[n] + K_2 x[n-1] + K_3 x[n-2]$

$K_1 = 1/4$

$K_2 = 3/4$

$K_3 = -3/4$



$$Y[z] = K_1 X[z] + K_2 z^{-1} X[z] + K_3 z^{-2} X[z]$$
$$= X[z]\left[K_1 + K_2 z^{-1} + K_3 z^{-2}\right]$$

$$H(z) = \frac{Y[z]}{X[z]} = K_1 + K_2 z^{-1} + K_3 z^{-2}$$

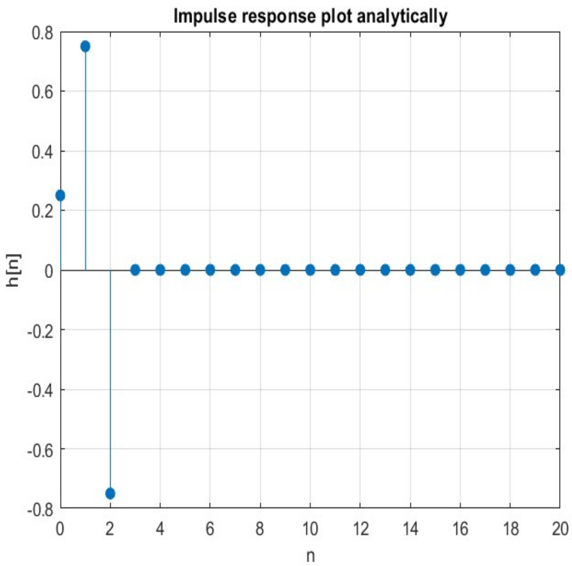
Transfer function, $= \dfrac{K_3 + K_2 z + K_1 z^2}{z^2}$

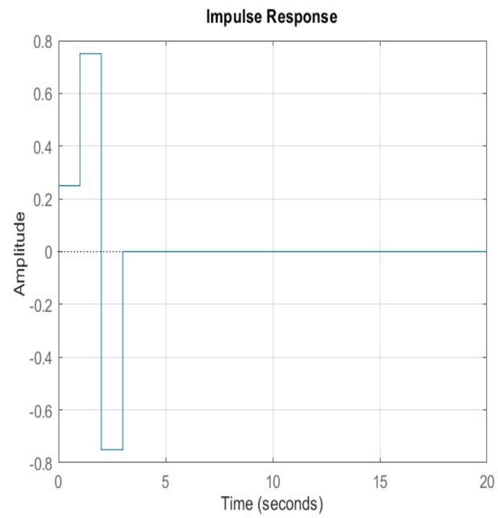
$$H = \frac{0.25z^2 + 0.75z - 0.75}{z^2}$$

impulse response,

$$y = K_1 \delta(n) + K_2 \delta(n-1) + K_3 \delta(n-2)$$

The zeros of the transfer function are -3.7913, 0.7913 and the poles are 0,0. The region of convergence ROC = 0. The impulse response is plotted both analytically and using MATLAB function. Both graphs are given below for comparison. The first one is from analytical calculation and second one from MATLAB function.





After that x[n] is computed and plotted using appropriate n. Y[n] is also computed both analytically and using matlab functions and then verified. The graph of them are given below.