DETERMINANTS

All Mathematical truths are relative and conditional. — C.P . STEINMETZ

0.1 Introduction

In the previous chapter, we have studied about matrices and algebra of matrices. We have also learnt that a system of algebraic equations can be expressed in the form of matrices. This means, a system of linear equations like

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

can be represented as $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$. Now, this system of equations has a unique solution or not, is determined by the number $a_1b_2 - a_2b_1$. (Recall that if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or, $a_1b_2 - a_2b_1 + 1 \neq 0$ then the system of linear equations has a unique solution).The number $a_1b_2 - a_2b_1$ which determines uniqueness of solution is associated with the matrix $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ and is called the determinant of A or det A. Determinants have wide applications in Engineering, Science, Economics, Social Science, etc. In this chapter, we shall study determinants up to order three only with real entries. Also, we will study various properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle, adjoint and inverse of a square matrix, consistency and inconsistency of system of linear equations and solution of linear equations in two or three variables using inverse of a matrix.

0.2 Determinants

To every square matrix $A = [a_{ij}]$ of order n, we can associate a number (real or complex) called determinant of the square matrix A, where $a_{ij} = (i.j)^{th}$ element of A. This may be thought of as a function which associates each square matrix with a unique number (real or complex). If M is the set of square matrices, K is the set of numbers (real or complex) and $f : M \to K$ is defined by $f(A) = k$, where $A \epsilon M$ and $k \epsilon K$, then $f(A)$ is called the determinant of A. It is also denoted by $|A|$ or det$A$ or $\triangle$. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = det(A)$

Remarks

i. For matrix A, $|A|$ is read as determinant of A and not modulus of A

ii. Only square matrices have determinants.