QUESTION-06

$$Na_3PO_4 + Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + NaNO_3$$

$$Na_3PO_4 : \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \quad Ba(NO_3)_2 : \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix}, \quad Ba_3(PO_4)_2 : \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix}, \quad NaNO_3 : \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} \begin{array}{l} \rightarrow Na \\ \rightarrow P \\ \rightarrow O \\ \rightarrow Ba \\ \rightarrow N \end{array}$$

∴ The coefficients in the eq. are:

$$x_1 . Na_3PO_4 + x_2 . Ba(NO_3)_2 \rightarrow x_3 . Ba_3(PO_4)_2 + x_4 . (NaNO_3)$$

$$\therefore x_1 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & -1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \quad \text{Replace } R_1 \text{ and } R_2 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

$$R_2 + (-3)R_1 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \quad R_3 + (-4)R_1 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

$$\text{Replace } R_2 \,\&\, R_4 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 0 & 8 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \quad R_3 + (-6)R_2 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 18 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

$$R_5+(-2)R_2 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 18 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 0 & 6 & -1 & 0 \end{bmatrix} \quad R_3/18 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 0 & 6 & -1 & 0 \end{bmatrix}$$

$$R_4+(-6)R_3 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \end{bmatrix} \quad R_5+(-6)R_3 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2+(3)R_3 \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1+(2)R_3 \sim \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution is:

$x_1 = (1/3)x_4$, $x_2 = (1/2)x_4$, $x_3 = (1/6)x_4$ with x_4 free

Taking $x_4 = 6$

$x_1 = 2$

$x_2 = 3$

$x_3 = 1$

The balanced equation is:

$$2Na_3PO_4 + 3Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + 6NaNO_3$$

QUESTION-07

$NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2$

$$NaHCO_3: \begin{bmatrix}1\\1\\1\\3\end{bmatrix}, H_3C_6H_5O_7: \begin{bmatrix}0\\8\\6\\7\end{bmatrix}, Na_3C_6H_5O_7: \begin{bmatrix}3\\5\\6\\7\end{bmatrix}, H_2O: \begin{bmatrix}0\\2\\0\\1\end{bmatrix}, CO_2: \begin{bmatrix}0\\0\\1\\2\end{bmatrix} \begin{array}{l}\rightarrow Na\\ \rightarrow H\\ \rightarrow C\\ \rightarrow O\end{array}$$

The coefficients in the eq are:

$x_1 . NaHCO_3 + x_2 H_3C_6H_5O_7 \rightarrow x_3 . Na_3C_6H_5O_7 + x_4 . H_2O + x_5 . CO_2$

$$x_1 \begin{bmatrix}1\\1\\1\\3\end{bmatrix} + x_2 \begin{bmatrix}0\\8\\6\\7\end{bmatrix} = x_3 \begin{bmatrix}3\\5\\6\\7\end{bmatrix} + x_4 \begin{bmatrix}0\\2\\0\\1\end{bmatrix} + x_5 \begin{bmatrix}0\\0\\1\\2\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 1 & 8 & -5 & -2 & 0 & 0\\ 1 & 6 & -6 & 0 & -1 & 0\\ 3 & 7 & -7 & -1 & -2 & 0\end{bmatrix} \underset{\sim}{R_2+(-1)R_1} \begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 0 & 8 & -2 & -2 & 0 & 0\\ 1 & 6 & -6 & 0 & -1 & 0\\ 3 & 7 & -7 & -1 & -2 & 0\end{bmatrix}$$

$$\underset{\sim}{R_3+(-1)R_1} \begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 0 & 8 & -2 & -2 & 0 & 0\\ 0 & 6 & -3 & 0 & -1 & 0\\ 3 & 7 & -7 & -1 & -2 & 0\end{bmatrix} \underset{\sim}{R_4+(-3)R_1} \begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 0 & 8 & -2 & -2 & 0 & 0\\ 0 & 6 & -3 & 0 & -1 & 0\\ 0 & 7 & 2 & -1 & -2 & 0\end{bmatrix}$$

$$\underset{\sim}{R_2 * \frac{1}{8}} \begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 0 & 1 & -1/4 & -1/4 & 0 & 0\\ 0 & 6 & -3 & 0 & -1 & 0\\ 0 & 7 & 2 & -1 & -2 & 0\end{bmatrix} \underset{\sim}{R_3+(-6)R_2} \begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 0 & 1 & -1/4 & -1/4 & 0 & 0\\ 0 & 0 & 3/2 & 3/2 & -1 & 0\\ 0 & 7 & 2 & -1 & -2 & 0\end{bmatrix}$$

$$\underset{\sim}{R_4+(-7)R_2} \begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 0 & 1 & -1/4 & -1/4 & 0 & 0\\ 0 & 0 & -3/2 & 3/2 & -1 & 0\\ 0 & 0 & 15/4 & 3/4 & -2 & 0\end{bmatrix} \underset{\sim}{R_3 * (-2/3)} \begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 0 & 1 & -1/4 & -1/4 & 0 & 0\\ 0 & 0 & 1 & -1 & 2/3 & 0\\ 0 & 0 & 15/4 & 3/4 & -2 & 0\end{bmatrix}$$

$$R_4 + \left(-\tfrac{15}{4}\right)R_3 \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 9/2 & -9/2 & 0 \end{bmatrix} \quad R_4 \times \left(\tfrac{2}{9}\right) \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$R_3 + R_4 \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \quad R_2 + \left(\tfrac{1}{4}\right)R_4 \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$R_2 + \left(\tfrac{1}{4}\right)R_3 \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \quad R_1 + (3)R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

The general solution is

$x_1 = x_5$, $x_2 = \left(\frac{1}{3}\right)x_5$, $x_3 = \left(\frac{1}{3}\right)x_5$, $x_4 = x_5$

where x_5 is free

Taking $x_5 = 3$.

$x_1 = 3$

$x_2 = 1$

$x_3 = 1$

$x_4 = 3$

The balanced equation is

$$3NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$$

QUESTION-08

$$KMnO_4 + MnSO_4 + H_2O \longrightarrow MnO_2 + K_2SO_4 + H_2SO_4$$

$$KMnO_4 : \begin{bmatrix}1\\1\\4\\0\\0\end{bmatrix}, MnSO_4 : \begin{bmatrix}0\\1\\4\\1\\0\end{bmatrix}, H_2O : \begin{bmatrix}0\\0\\1\\0\\2\end{bmatrix}, MnO_2 : \begin{bmatrix}0\\1\\2\\0\\0\end{bmatrix}, K_2SO_4 : \begin{bmatrix}2\\0\\4\\1\\0\end{bmatrix}, H_2SO_4 : \begin{bmatrix}0\\0\\4\\1\\2\end{bmatrix} \begin{matrix}\to K\\ \to Mn\\ \to O\\ \to S\\ \to H\end{matrix}$$

The coefficients in the eq. are:

$$x_1 . KMnO_4 + x_2 . MnSO_4 + x_3 H_2O \longrightarrow x_4 MnO_2 + x_5 K_2SO_4 + x_6 H_2SO_4$$

$$x_1 . \begin{bmatrix}1\\1\\4\\0\\0\end{bmatrix} + x_2 \begin{bmatrix}0\\1\\4\\1\\0\end{bmatrix} + x_3 \begin{bmatrix}0\\0\\1\\0\\2\end{bmatrix} = x_4 \begin{bmatrix}0\\1\\2\\0\\0\end{bmatrix} + x_5 \begin{bmatrix}2\\0\\4\\1\\0\end{bmatrix} + x_6 \begin{bmatrix}0\\0\\4\\1\\2\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0&-2&0&0\\1&1&0&-1&0&0&0\\4&4&1&-2&-4&-4&0\\0&1&0&0&-1&-1&0\\0&0&2&0&0&-2&0\end{bmatrix} \underset{\sim}{R_2+(-1)R_1} \begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\4&4&1&-2&-4&-4&0\\0&1&0&0&-1&-1&0\\0&0&2&0&0&-2&0\end{bmatrix}$$

$$\underset{\sim}{R_3+(-4)R_1} \begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\0&4&1&-2&4&-4&0\\0&1&0&0&-1&-1&0\\0&0&2&0&0&-2&0\end{bmatrix} \underset{\sim}{R_3+(-4)R_2} \begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\0&0&1&2&-4&-4&0\\0&1&0&0&-1&-1&0\\0&0&2&0&0&-2&0\end{bmatrix}$$

$$\underset{\sim}{R_4+(-1)R_2} \begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\0&0&1&2&-4&-4&0\\0&0&0&1&-3&-1&0\\0&0&2&0&0&-2&0\end{bmatrix} \underset{\sim}{R_5+(-2)R_3} \begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\0&0&1&2&-4&-4&0\\0&0&0&1&-3&-1&0\\0&0&0&-4&8&6&0\end{bmatrix}$$

$$\underset{\sim}{R_5+(4)R_4} \begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\0&0&1&2&-4&-4&0\\0&0&0&1&-3&-1&0\\0&0&0&0&-4&2&0\end{bmatrix} R_5 * \left(-\frac{1}{4}\right) \begin{bmatrix}1&0&0&0&-2&0&0\\0&1&0&-1&2&0&0\\0&0&1&2&-4&-4&0\\0&0&0&1&-3&-1&0\\0&0&0&0&1&-1/2&0\end{bmatrix}$$

$$R_4+(3)R_5 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix} \quad R_3+(4)R_5 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & -6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix}$$

$$R_2+(-2)R_5 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix} \quad R_1+(2)R_5 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix}$$

$$R_3+(-2)R_4 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix} \quad R_2+R_4 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix}$$

The general solution is :-

$x_1 = x_6$, $x_2 = \left(\frac{3}{2}\right)x_6$, $x_3 = x_6$, $x_4 = \left(\frac{5}{2}\right)x_6$, $x_5 = \left(\frac{1}{2}\right)x_6$

where x_6 is free

Taking $x_6 = 2$

$x_1 = 2$

$x_2 = 3$

$x_3 = 2$

$x_4 = 5$

$x_5 = 1$

The balanced equation is :

$$2KMnO_4 + 3MnSO_4 + 2H_2O \rightarrow 5MnO_2 + K_2SO_4 + 2H_2SO_4$$

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QUESTION-09

$$PbN_6 + CrMn_2O_8 \rightarrow Pb_3O_4 + Cr_2O_3 + MnO_2 + NO$$

$$PbN_6:\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix},\ CrMn_2O_8:\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix},\ Pb_3O_4:\begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix},\ Cr_2O_3:\begin{bmatrix}0\\0\\2\\0\\3\end{bmatrix},\ MnO_2:\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix},\ NO:\begin{array}{c l}\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix} & \begin{array}{l}\rightarrow Pb\\ \rightarrow N\\ \rightarrow Cr\\ \rightarrow Mn\\ \rightarrow O\end{array}\end{array}$$

The coefficients of eq. are:

$$x_1\, PbN_6 + x_2\, CrMn_2O_8 \rightarrow x_3\, Pb_3O_4 + x_4\, Cr_2O_3 + x_5\, MnO_2 + x_6\, NO$$

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix} = x_3\begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix} + x_4\begin{bmatrix}0\\0\\2\\0\\3\end{bmatrix} + x_5\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix} + x_6\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}1&0&3&0&0&0&0\\6&0&0&0&0&1&0\\0&1&0&2&0&0&0\\0&2&0&0&1&0&0\\0&8&4&3&2&1&0\end{bmatrix} \underset{\sim}{\text{Replace } R_2 \text{ and } R_3} \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\6&0&0&0&0&-1&0\\0&2&0&0&-1&0&0\\0&8&-4&-3&-2&-1&0\end{bmatrix}$$

$$\underset{\sim}{R_3+(-6)R_1}\begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&18&0&0&-1&0\\0&2&0&0&-1&0&0\\0&8&-4&-3&-2&-1&0\end{bmatrix} \underset{\sim}{x_3 * (1/18)} \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&1&0&0&-1/18&0\\0&2&0&0&-1&0&0\\0&8&-4&-3&-2&-1&0\end{bmatrix}$$

$$\underset{\sim}{R_4+(-2)R_2}\begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&1&0&0&-1/18&0\\0&0&0&4&-1&0&0\\0&8&-4&-3&-2&-1&0\end{bmatrix} \underset{\sim}{R_5+(-8)R_2} \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&1&0&0&-1/18&0\\0&0&0&4&-1&0&0\\0&0&-4&13&-2&-1&0\end{bmatrix}$$

$$\underset{\sim}{R_5+(4)R_3}\begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&1&0&0&-1/18&0\\0&0&0&4&-1&0&0\\0&0&0&13&-2&-11/9&0\end{bmatrix} \underset{\sim}{R_4 * (1/4)} \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&1&0&0&-1/18&0\\0&0&0&1&-1/4&0&0\\0&0&0&13&-2&-11/9&0\end{bmatrix}$$

$$R_5 + (-13)R_4 \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5/4 & -11/9 & 0 \end{bmatrix} \quad R_5 \times (4/5) \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

$$R_4 + (\tfrac{1}{4})R_5 \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix} \quad R_2 + (2)R_4 \sim \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

$$R_1 + (3)R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

The general solution is:
$x_1 = \left(\frac{1}{6}\right)x_6$, $x_2 = \left(\frac{22}{45}\right)x_6$, $x_3 = \left(\frac{1}{18}\right)x_6$, $x_4 = \left(\frac{11}{45}\right)x_6$, $x_5 = \left(\frac{44}{45}\right)x_6$
with x_6 free

Taking $x_6 = 90$

$x_1 = 15$
$x_2 = 44$
$x_3 = 5$
$x_4 = 22$
$x_5 = 88$

The balanced eq. is

$$15\,PbN_6 + 44\,CrMn_2O_8 \rightarrow 5\,Pb_3O_4 + 22\,Cr_2O_3 + 88\,MnO_2 + 90\,NO$$

QUESTION - 10

$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \rightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O$

$MnS: \begin{bmatrix}1\\1\\0\\0\\0\\0\end{bmatrix}, As_2Cr_{10}O_{35}: \begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix}, H_2SO_4: \begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix}, HMnO_4: \begin{bmatrix}1\\0\\0\\0\\4\\1\end{bmatrix}, AsH_3: \begin{bmatrix}0\\0\\1\\0\\0\\3\end{bmatrix}, CrS_3O_{12} \begin{bmatrix}0\\3\\0\\1\\12\\0\end{bmatrix}, H_2O \begin{bmatrix}0\\0\\0\\0\\1\\2\end{bmatrix}$

The coefficients of eq. are:

$x_1 . MnS + x_2 . As_2Cr_{10}O_{35} + x_3 . H_2SO_4 \rightarrow x_4 HMnO_4 + x_5 AsH_3 + x_6 . CrS_3O_{12} + x_7 H_2O$

$$x_1 . \begin{bmatrix}1\\1\\0\\0\\0\\0\end{bmatrix} + x_2 \begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix} + x_3 \begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix} = x_4 \begin{bmatrix}1\\0\\0\\0\\4\\1\end{bmatrix} + x_5 \begin{bmatrix}0\\0\\1\\0\\0\\3\end{bmatrix} + x_6 \begin{bmatrix}0\\3\\0\\1\\12\\0\end{bmatrix} + x_7 \begin{bmatrix}0\\0\\0\\0\\1\\2\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&-1&0&0&0&0\\1&0&1&0&0&-3&0&0\\0&2&0&0&-1&0&0&0\\0&10&0&0&0&-1&0&0\\0&35&4&-4&0&-12&-1&0\\0&0&2&-1&-3&0&-2&0\end{bmatrix} \underset{\sim}{R_2+(-1)R_1} \begin{bmatrix}1&0&0&-1&0&0&0&0\\0&0&1&1&0&-3&0&0\\0&2&0&0&-1&0&0&0\\0&10&0&0&0&-1&0&0\\0&35&4&-4&0&-12&-1&0\\0&0&2&-1&-3&0&-2&0\end{bmatrix}$$

Replace R_2 and R_3 and $R_2 * (\frac{1}{2})$

$$\begin{bmatrix}1&0&0&-1&0&0&0&0\\0&1&0&0&-1/2&0&0&0\\0&0&1&1&0&-3&0&0\\0&10&0&0&0&-1&0&0\\0&35&4&-4&0&-12&-1&0\\0&0&2&-1&-3&0&-2&0\end{bmatrix} \underset{\sim}{R_4+(-10)R_2} \begin{bmatrix}1&0&0&-1&0&0&0&0\\0&1&0&0&-1/2&0&0&0\\0&0&1&1&0&-3&0&0\\0&0&0&0&5&-1&0&0\\0&35&4&-4&0&-12&-1&0\\0&0&2&-1&-3&0&-2&0\end{bmatrix}$$

$$R_5+(-35)R_2 \sim \begin{bmatrix}1&0&0&-1&0&0&0&0\\0&1&0&0&-1/2&0&0&0\\0&0&1&1&0&-3&0&0\\0&0&0&0&5&-1&0&0\\0&0&4&-4&35/2&-12&-1&0\\0&0&2&-1&-3&0&-2&0\end{bmatrix}$$

Replace R_4 and R_6 and $R_4 * -1$ $\sim$

$$\begin{bmatrix}1&0&0&-1&0&0&0&0\\0&1&0&0&-1/2&0&0&0\\0&0&1&1&0&-3&0&0\\0&0&-2&1&3&0&2&0\\0&0&4&-4&35/2&-12&-1&0\\0&0&0&0&5&-1&0&0\end{bmatrix}$$

Replace R_5 and R_6 and $R_5/5$ $\sim$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & -2 & 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & 4 & -4 & \frac{35}{2} & -12 & -1 & 0 \end{bmatrix}$$

$R_4+(2)R_3$ and $R_6+(-4)R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & -6 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 & -8 & \frac{35}{2} & 0 & -1 & 0 \end{bmatrix}$$

$R_4 * \left(\frac{1}{3}\right)$ $\sim$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 & -8 & \frac{35}{2} & 0 & -1 & 0 \end{bmatrix}$$

$R_6+(8)R_4$ $\sim$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{51}{2} & -16 & \frac{13}{3} & 0 \end{bmatrix}$$

$R_6+\left(\frac{-51}{2}\right)R_5$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{109}{10} & \frac{13}{3} & 0 \end{bmatrix}$$

$R_6 * \left(-\frac{10}{109}\right)$ $\sim$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{130}{327} & 0 \end{bmatrix}$$

$R_5+\left(\frac{1}{5}\right)R_6$ $\sim$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -\frac{26}{327} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{130}{327} & 0 \end{bmatrix}$$

$R_4+(2)R_6$ and $R_3+(3)R_6$ $\sim$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -\frac{130}{109} & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & -\frac{14}{109} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -\frac{26}{327} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{130}{327} & 0 \end{bmatrix}$$

$R_4+(-1)R_5$ and $R_2+(\frac{1}{2})R_5$ $\sim$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\frac{13}{327} & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -\frac{130}{109} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{16}{327} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -\frac{26}{327} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{130}{327} & 0 \end{bmatrix}$$

R_1+R_4 and $R_3+(-1)R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -\frac{16}{327} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\frac{13}{327} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{374}{327} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{16}{327} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -\frac{26}{327} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{130}{327} & 0 \end{bmatrix}$$

The general solution is:

$$x_1 = \left(\frac{16}{327}\right)x_7$$

$$x_2 = \left(\frac{13}{327}\right)x_7$$

$$x_3 = \left(\frac{374}{327}\right)x_7$$

$$x_4 = \left(\frac{16}{327}\right)x_7$$

$$x_5 = \left(\frac{26}{327}\right)x_7$$

$$x_6 = \left(\frac{130}{327}\right)x'$$

where x_7 is free

Taking $x_7 = 327$

$x_1 = 16$, $x_2 = 13$, $x_3 = 374$, $x_4 = 16$, $x_5 = 26$, $x_6 = 130$

The balanced equation is:

$$16\,MnS + 13\,As_2Cr_{10}O_{35} + 374\,H_2SO_4 \rightarrow 16\,HMnO_4 + 26\,AsH_3 + 130\,CrS_3O_{12} + 327\,H_2O$$