QUADRATIC EQUATION & EXPRESSION

YEAR LONG REVISION EXERCISE
Not To Be Discussed in Class

SECTION - 1 : SINGLE CHOICE CORRECT QUESTIONS

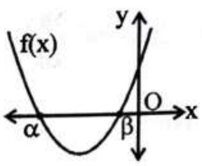
1. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is **[JEE 2002 (screening),3]**
(A) $(-\infty, -2) \cup (2, \infty)$
(B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(C) $(-\infty, -1) \cup (1, \infty)$
(D) $(\sqrt{2}, \infty)$

2. If the quadratic polynomial $P(x) = (p-3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of p can be
(A) $\frac{3}{2}$ (B) 4 (C) 6 (D) 7

3. If the roots of the quadratic equation $ax^2 + bx + c = 0$ are $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then $\left(\frac{a}{a+b+c}\right)^2$ equals
(A) k^2 (B) $(k + 1)^2$ (C) $(k + 2)^2$ (D) $k^2 (k + 1)^2$

4. If min. $(2x^2 - ax + 2) >$ max. $(b - 1 + 2x - x^2)$ then roots of the equation $2x^2 + ax + (2 - b) = 0$, are
(A) positive and distinct
(B) negative and distinct
(C) opposite in sign
(D) imaginary

5. The following figure shows the graph of $f(x) = ax^2 - bx + c$. Then which one of the following is correct?



(A) $\frac{b}{c} > 0$
(B) a and c are of opposite sign
(C) a and b are of same sign
(D) None

6. If p and q are the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha = 1$ $(\alpha \in R)$, then the minimum value of $(p^2 + q^2)$ is equal to
(A) 2 (B) 3 (C) 5 (D) 6

7. The product of all values of x which make the following statement true $(\log_3 x)(\log_5 9) - \log_x 25 + \log_3 2 = \log_3 54$, is
(A) $\sqrt{5}$ (B) 5 (C) $5\sqrt{5}$ (D) 25

8. If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of the equation $x^2 + px + q = 0$ are $(\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$, then- **[AIEEE-2002]**
(A) $p = 1$ and $q = 56$ (B) $p = 1$ and $q = -56$ (C) $p = -1$ and $q = 56$ (D) $p = -1$ and $q = -56$

9. If α and β be the roots of the equation $(x - a)(x - b) = c$ and $c \neq 0$, then roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are - **[AIEEE-2002]**
(A) a and c (B) b and c (C) a and b (D) a + b and b + c

10. If $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (where $\alpha \neq \beta$) is- **[AIEEE-2002]**
(A) $\frac{19}{3}$ (B) $\frac{25}{3}$ (C) $-\frac{19}{3}$ (D) None of these

11. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$, is- **[AIEEE-2003]**
(A) 4 (B) 1 (C) 3 (D) 2

12. The real number x when added to its inverse gives the minimum value of the sum at x equal to-
(A) 1 (B) –1 (C) –2 (D) 2 **[AIEEE-2003]**

13. Let two numbers have airthmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation- **[AIEEE-2004]**
(A) $x^2 + 18x - 16 = 0$ (B) $x^2 - 18x + 16 = 0$ (C) $x^2 + 18x + 16 = 0$ (D) $x^2 - 18x - 16 = 0$

14. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are- **[AIEEE-2004]**
(A) 0, – 1 (B) – 1, 1 (C) 0, 1 (D) – 1, 2

15. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is- **[AIEEE-2004]**
(A) 3 (B) 12 (C) $\frac{49}{4}$ (D) 4

16. If value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is- **[AIEEE-2005]**
(A) 2 (B) 3 (C) 0 (D) 1

17. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals-
(A) 1 (B) 2 (C) 3 (D) –2 **[AIEEE-2005]**

18. If x is real, then maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is- **[AIEEE-2006]**
(A) 1 (B) $\frac{17}{7}$ (C) $\frac{1}{4}$ (D) 41

19. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **[AIEEE-2007]**
(A) $(-3, \infty)$ (B) $(3, \infty)$ (C) $(-\infty, -3)$ (D) $(-3, -2) \cup (2, 3)$

20. The value of a for which one roots of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is **[AIEEE-2003]**
(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{2}{3}$

21. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the square of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in **[AIEEE-2003]**
(A) geometric progression (B) harmonic progression
(C) arithmetic-geometric progression (D) arithmetic progression

22. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval- **[AIEEE-2005]**
(A) [4, 5] (B) $(-\infty, 4)$ (C) $(6, \infty)$ (D) (5, 6)

23. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than –2 but less than 4, lie in the interval- **[AIEEE-2006]**
(A) $-1 < m < 3$ (B) $1 < m < 4$ (C) $-2 < m < 0$ (D) $m > 3$

24. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively then the value of $2 + q - p$ is- **[AIEEE-2006]**

(A) 0 (B) 1 (C) 2 (D) 3

25. Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$.. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then **[JEE 2006, 3]**

(A) $\lambda < \frac{4}{3}$ (B) $\lambda > \frac{5}{3}$ (C) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

26. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of 'r' is **[JEE 2007]**

(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$ (C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

SECTION - 2 : MULTIPLE CHOICE CORRECT QUESTIONS

27. For $x \in R$, the expression $\frac{x^2+34x-71}{x^2+2x-7}$ can not lie between,

(A) (5, 7) (B) (12, 19) (C) (1, 4) (D) (8, 9)

28. In which of the following inequalities, the set of all real values of x is same as the set of all real values of k for which the equation $kx^2 - 4x + k = 0$ has real roots and satisfying $1 - k \le 0$?

(A) $0 \le \log_2 x \le 1$ (B) $x^2 - 3x + 2 \le 0$

(C) $\sin(\pi x) \le 0$ in $[0, 2]$ (D) $|x - 1| \le 1$

29. If the vertex of the parabola $y = 3x^2 - 12x + 9$ is (a, b), then the parabola whose vertex is (b, a), is(are)

(A) $y = x^2 + 6x + 11$ (B) $y = x^2 - 7x + 3$

(C) $y = -2x^2 - 12x - 16$ (D) $y = -2x^2 + 16x - 13$

30. Let x and y be 2 real numbers which satisfy the equations $(\tan^2 x - \sec^2 y) = \frac{5a}{6} - 3$ and $(-\sec^2 x + \tan^2 y) = a^2$, then the value of a can be equal to

(A) $\frac{2}{3}$ (B) $\frac{-2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{-3}{2}$

31. Let a, b and c be real numbers. Which of the following statement(s) about the equation $(x-a)(x-b) = c$ is/are incorrect?

(A) If $c > 0$, then roots are always real. (B) If $c > 0$, then roots are always non-real.

(C) If $c < 0$, then roots are always real. (D) If $c < 0$, then roots are always non-real.

32. If quadratic equation $x^2 + 2(a + 2b)x + (2a + b - 1) = 0$ has unequal real roots for all $b \in R$ then the possible values of a can be equal to

(A) 5 (B) – 1 (C) – 10 (D) 3

33 If all values of x which satisfies the inequality $\log_{1/3}(x^2 + 2px + p^2 + 1) \ge 0$ also satisfy the inequality $kx^2 + kx - k^2 \le 0$ for all real values of k, then all possible values of p lies in the interval

(A) [– 1, 1] (B) [0, 1] (C) [0, 2] (D) [– 2, 0]

SECTION - 3 : MATRIX - MATCH QUESTIONS

34. The expression $y = ax^2 + bx + c$ (a, b, c $\in$ R and $a \neq 0$) represents a parabola which cuts the x-axis at the points which are roots of the equation $ax^2 + bx + c = 0$. Column-II contains values which correspond to the nature of roots mentioned in column-I.

	Column-I		Column-II
(A)	For $a = 1, c = 4$, if both roots are greater than 2 then b can be equal to	(P)	4
(B)	For $a = -1, b = 5$, if roots lie on either side of -1 then c can be equal to	(Q)	8
(C)	For $b = 6, c = 1$, if one root is less than -1 and the other root greater than $\frac{-1}{2}$ then a can be equal to	(R)	10
		(S)	no real value

35. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$

Match the expressions / statements in **Column I** with expressions / statements in **Column II**.

	Column I		Column II
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	(p)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, the $f(x)$ satisfies	(q)	$f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	(r)	$f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	(s)	$f(x) < 1$

[JEE 2007]

SECTION - 4 : NUMERICAL ANSWER BASED QUESTIONS

36. Find the values of 'a' for which one of the roots of the quadratic equation, $x^2 + (2a + 1)x + (a^2 + 2) = 0$ is twice the other root . Find also the roots of this equation for these values of 'a'.

37. If $y = \dfrac{x^2 + 2x - 3}{x^2 + 2x - 8}$ then find the interval in which y can lie for every $x \in R$ wherever defined.

38. If α and β be the roots of the equation $x^2 + 3x + 1 = 0$ then find the value of $\left(\dfrac{\alpha}{1+\beta}\right)^2 + \left(\dfrac{\beta}{\alpha+1}\right)^2$.

39. Let M be the minimum value of $f(\theta) = (3\cos^2\theta + \sin^2\theta)(\sec^2\theta + 3\,\text{cosec}^2\theta)$, for permissible real values of θ and P denotes the product of all real solutions of the equation $\dfrac{(x-1)(50-10x)}{x^2 - 5x} = x^2 - 8x + 7$. Find (P M).

40. If the range of values of a for which the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ lie between the roots of the equation $x^2 - 2(a + 1)x + a(a - 1) = 0$ is (p, q), find the value of $\left(q + \dfrac{1}{p^2}\right)$.

41. Let x_1 and x_2 be the real roots of the equation $x^2 - kx + (k^2 + 7k + 15) = 0$. What is the maximum value of $(x_1^2 + x_2^2)$?

42. If $1-\log_x 2+\log_{x^2} 9-\log_{x^3} 64<0$, then range of x is (a, b). Find the minimum value of (a + 9b).

43. If α, β are roots of the equation $2x^2+6x+b=0$ where $b<0$, then find the least integral value of $\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)$.

44. Suppose that a, b, c, d are rationals which satisfy $a+b+c+d=10$, $(a+b)(c+d)=16$, $(a+c)(b+d)=21$ and $(a+d)(b+c)=24$, then find the value of $(a^2+b^2+c^2+d^2)$.

45. If sum of maximum and minimum value of $y=\log_2(x^4+x^2+1)-\log_2(x^4+x^3+2x^2+x+1)$ can be expressed in form $((\log_2 m)-n)$, where m and 2 are coprime then compute $(m+n)$.

46. If all the solutions of the inequality $x^2-6ax+5a^2\le 0$ are also the solutions of inequality $x^2-14x+40\le 0$ then find the number of possible integral values of a.

47. If roots of the equation $x^2-10cx-11d=0$ are a, b and those of $x^2-10ax-11b=0$ are c, d, then find the value of $a+b+c+d$. (a, b, c and d are distinct numbers) **[JEE 2006, 6]**

SECTION - 5 : SUBJECTIVE QUESTIONS

48. If α, β are the roots of the equation $x^2-2x+3=0$ obtain the equation whose roots are $\alpha^3-3\alpha^2+5\alpha-2$, $\beta^3-\beta^2+\beta+5$.

49. If one root of the equation $ax^2+bx+c=0$ be the square of the other, prove that $b^3+a^2c+ac^2=3abc$.

50. Show that if p, q , r & s are real numbers & $pr=2(q+s)$, then at least one of the equations $x^2+px+q=0$, $x^2+rx+s=0$ has real roots.

51. Find the range of values of a, such that $f(x)=\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}$ is always negative.

52. Find the values of 'a' for which $-3<\frac{x^2+ax-2}{x^2+x+1}<2$ is valid for all real x .

53. If the quadratic equations $x^2+bx+ca=0$ & $x^2+cx+ab=0$ (where $a\neq 0$) have a common root, prove that the equation containing their other roots is $x^2+ax+bc=0$.

54. The equation $x^2-ax+b=0$ & $x^3-px^2+qx=0$, where $b\neq 0, q\neq 0$, have one common root & the second equation has two equal roots. Prove that $2(q+b)=ap$.

55. Find all values of a for which the inequality $(a+4)x^2-2ax+2a-6<0$ is satisfied for all $x\in R$.

56. Find all values of a for which both roots of the equation $x^2-6ax+2-2a+9a^2=0$ are greater than 3.

57. Find all the values of the parameter 'a' for which both roots of the quadratic equation $x^2-ax+2=0$ belong to the interval (0, 3).

58. Find the values of K so that the quadratic equation $x^2+2(K-1)x+K+5=0$ has atleast one positive root.

59. If $a<b<c<d$ then prove that the roots of the equation; $(x-a)(x-c)+2(x-b)(x-d)=0$ are real & distinct.

60. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then show that $(ac^n)^{1/(n+1)} + (a^nc)^{1/(n+1)} + b = 0$.

61. Let $P(x) = x^2 + bx + c$, where b and c are integer. If P(x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of P(1).

62. Find the true set of values of p for which the equation : $p.2^{\cos^2 x} + p.2^{-\cos^2 x} - 2 = 0$ has real roots.

63. If the coefficients of the quadratic equation $ax^2 + bx + c = 0$ are odd integers then prove that the roots of the equation cannot be rational number.

64. If the three equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root, find a and b and the roots of the equations.

65. If the quadratic equation $ax^2 + bx + c = 0$ has real roots, of opposite sign in the interval (–2, 2) then prove that $1+\frac{c}{4a}-\left|\frac{b}{2a}\right|>0$.

66. Find the maximum possible value of $8\cdot 27^{\log_6 x} + 27\cdot 8^{\log_6 x} - x^3$, where $x > 0$.

67. For $a \le 0$, determine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0$.

68. The equation $x^n + px^2 + qx + r = 0$, where $n \ge 5$ & $r \ne 0$ has roots $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots\ldots \alpha_n$.

Denoting $\sum_{i=1}^{n} \alpha_i^k$ by S_k.

(a) Calculate S_2 & deduce that the roots cannot all be real.

(b) Prove that $S_n + pS_2 + qS_1 + nr = 0$ & hence find the value of S_n.

69. Find the number of integral values of a so that the inequation $x^2 - 2(a + 1)x + 3(a - 3)(a + 1) < 0$ is satisfied by atleast one $x \in R^+$.

70. Let a, b, c be real numbers with $a \ne 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. **[JEE 2001, Mains, 5 out of 100]**

71. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of 'a' for which equation has unequal real roots for all values of 'b'. **[JEE 2003, Mains-4 out of 60]**

72. Find the range of values of *t* for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. **[JEE 2005(Mains), 2]**

73. A quadratic polynomial $f(x) = x^2 + ax + b$ is formed with one of its zeros being $\frac{4+3\sqrt{3}}{2+\sqrt{3}}$ where *a* and *b* are integers. Also $g(x) = x^4 + 2x^3 - 10x^2 + 4x - 10$ is a biquadratic polynomial such that $g\left(\frac{4+3\sqrt{3}}{2+\sqrt{3}}\right) = c\sqrt{3} + d$ where *c* and *d* are also integers. Find the values of a, b, c and d.

SECTION - 6 : ASSERTION–REASON QUESTIONS

Assertion & Reason

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I

(C) Statement-I is true, Statement-II is false

(D) Statement-I is false, Statement-II is true

74. **Statement-I** – If $a + b + c > 0$ and $a < 0 < b < c$, then the roots of the equation $a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$ are of both negative.

Because

Statement-II – If both roots are negative, then sum of roots < 0 and product of roots > 0

(A) A (B) B (C) C (D) D

75. **Statement-I** – Let $(a_1, a_2, a_3, a_4, a_5)$ denote a re-arrangement of $(1, -4, 6, 7, -10)$. Then the equation $a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0$ has at least two real roots.

Because

Statement-II – If $ax^2 + bx + c = 0$ and $a + b + c = 0$, (i.e. in a polynomial the sum of coefficients is zero) then $x = 1$ is root of $ax^2 + bx + c = 0$.

(A) A (B) B (C) C (D) D

76. **Statement-I** – If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$.

Because

Statement-II – If a, b, c are odd integer then the roots of the equation $4\,abc\,x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

(A) A (B) B (C) C (D) D

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ANSWERS

YEAR LONG REVISION EXERCISE

SECTION - 1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	D	D	D	C	C	D	C	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	B	A	C	D	A	D	D	D
Que.	21	22	23	24	25	26				
Ans.	B	B	A	D	A	D				

SECTION - 2

Que.	27	28	29	30	31	32	33
Ans.	AD	AB	AC	AD	BCD	BC	ABC

SECTION - 3

Que.	34	35
Ans.	(A)-s; (B)-qr; (C)-p	(A)-prs; (B)-qs; (C)-qs; (D)-prs

SECTION - 4

36. $a = 4$; roots are -3 and -6 **37.** $y \in \left(-\infty, \frac{4}{9}\right] \cup (1, \infty)$ **38.** 18 **39.** 24

40. 17 **41.** 18 **42.** 25 **43.** 10 **44.** 39 **45.** 5 **46.** 0

47. 1210

SECTION - 5

48. $x^2 - 3x + 2 = 0$ is the required equation. **51.** $a \in \left(-\infty, -\frac{1}{2}\right)$

52. $-2 < a < 1$ **55.** $a \in (-\infty, -6)$ **56.** $\left(\frac{11}{9}, \infty\right)$ **57.** $\left[2\sqrt{2}, \frac{11}{3}\right)$

58. $k \le -1$ **61.** 4 **62.** $p \in \left[\frac{4}{5}, 1\right]$

64. $a = -7$ and $b = -8$, (3, 4), (3, 5) and (3, 12) are the root. **66.** 216.

67. $x = a(1 - \sqrt{2})$, $x = a(\sqrt{6} - 1)$ **68.** **(a)** $S_2 = 0$, **(b)** $S_n = -nr$

69. 5 **70.** $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$

71. $a > 1$ **72.** $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$

73. $a = 2, b = -11, c = 4, d = -1$

SECTION - 6

74. (D) **75.** (A) **76.** (B)

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