

## Numerical 2023

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### Question.1

If the sum of the series

$$\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2 \cdot 3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{2^2 \cdot 3} + \frac{1}{2 \cdot 3^2} - \frac{1}{3^3}\right) +$$

$$\left(\frac{1}{2^4} - \frac{1}{2^3 \cdot 3} + \frac{1}{2^2 \cdot 3^2} - \frac{1}{2 \cdot 3^3} + \frac{1}{3^4}\right) + \dots$$

is  $\frac{\alpha}{\beta}$ , where  $\alpha$  and  $\beta$  are co-prime, then  $\alpha + 3\beta$  is equal to \_\_\_\_\_.

JEE Main 2023 (Online) 15th April Morning Shift

### Question:2

The sum to 20 terms of the series  $2 \cdot 2^2 - 3^2 + 2 \cdot 4^2 - 5^2 + 2 \cdot 6^2 - \dots$  is equal to \_\_\_\_\_.

JEE Main 2023 (Online) 13th April Morning Shift

### Question:3

For  $k \in \mathbb{N}$ , if the sum of the series  $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots$  is 10, then the value of  $k$  is \_\_\_\_\_.

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### Question:4

Let  $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \dots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$ . Then the value of  $(16S - (25)^{-54})$  is equal to \_\_\_\_\_.

JEE Main 2023 (Online) 11th April Morning Shift

### Question:5

Suppose  $a_1, a_2, 2, a_3, a_4$  be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is  $\frac{49}{2}$ , then  $a_4$  is equal to \_\_\_\_\_.

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**Question:6** The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are not divisible by 3, is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 10th April Morning Shift**

**Question:7**

Let  $0 < z < y < x$  be three real numbers such that  $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$  are in an arithmetic progression and  $x, \sqrt{2}y, z$  are in a geometric progression. If  $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$ , then  $3(x + y + z)^2$  is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 8th April Evening Shift**

**Question:8**

The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, ..., 399,

2, 5, 8, 11, ..., 359 and

2, 7, 12, 17, ..., 197,

is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 1st February Evening Shift**

**Question:9**

Let  $a_1 = 8, a_2, a_3, \dots, a_n$  be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is \_\_\_\_\_.

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**Question:10**

The sum  $1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + 5 \cdot 9^2 - \dots + 15 \cdot 29^2$  is \_\_\_\_\_.

**JEE Main 2023 (Online) 31st January Evening Shift**

**Question:11**

Let  $a_1, a_2, \dots, a_n$  be in A.P. If  $a_5 = 2a_7$  and  $a_{11} = 18$ , then

$12 \left( \frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \dots + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}} \right)$  is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 31st January Morning Shift**

**Question:12**

The 8<sup>th</sup> common term of the series

$$S_1 = 3 + 7 + 11 + 15 + 19 + \dots$$

$$S_2 = 1 + 6 + 11 + 16 + 21 + \dots$$

is :

**JEE Main 2023 (Online) 30th January Evening Shift****Question:13**

Let  $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$ , where  $a, b, c \in \mathbb{Z}$  and  $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ . Then  $a^2 - b + c$  is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 30th January Morning Shift****Question:14**

Let  $a_1 = b_1 = 1$  and  $a_n = a_{n-1} + (n - 1)$ ,  $b_n = b_{n-1} + a_{n-1}$ ,  $\forall n \geq 2$ . If  $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$  and  $T = \sum_{n=1}^8 \frac{n}{2^{n-1}}$ , then  $2^7(2S - T)$  is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 29th January Evening Shift****Question:15**

Let  $\{a_k\}$  and  $\{b_k\}$ ,  $k \in N$ , be two G.P.s with common ratios  $r_1$  and  $r_2$  respectively such that  $a_1 = b_1 = 4$  and  $r_1 < r_2$ . Let  $c_k = a_k + b_k$ ,  $k \in N$ . If  $c_2 = 5$  and  $c_3 = \frac{13}{4}$  then  $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$  is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 29th January Evening Shift****Question:16**

Let  $a_1, a_2, a_3, \dots$  be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then  $a_1a_9 + a_2a_4a_9 + a_5 + a_7$  is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 29th January Morning Shift**

**Question:17**

For the two positive numbers  $a, b$ , if  $a, b$  and  $\frac{1}{18}$  are in a geometric progression, while  $\frac{1}{a}, 10$  and  $\frac{1}{b}$  are in an arithmetic progression, then  $16a + 12b$  is equal to \_\_\_\_\_.

**JEE Main 2023 (Online) 25th January Evening Shift****Question:18**

If  $\frac{1^3+2^3+3^3+\dots \text{ up to } n \text{ terms}}{1 \cdot 3+2 \cdot 5+3 \cdot 7+\dots \text{ up to } n \text{ terms}} = \frac{9}{5}$ , then the value of  $n$  is

**JEE Main 2023 (Online) 24th January Evening Shift****Question:19**

The 4<sup>th</sup> term of GP is 500 and its common ratio is  $\frac{1}{m}$ ,  $m \in \mathbb{N}$ . Let  $S_n$  denote the sum of the first  $n$  terms of this GP. If  $S_6 > S_5 + 1$  and  $S_7 < S_6 + \frac{1}{2}$ , then the number of possible values of  $m$  is \_\_\_\_\_

**JEE Main 2023 (Online) 24th January Morning Shift****Numerical Answers Key**

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1. Ans. (7)      19. Ans. (12)

2. Ans. (1310)

3. Ans. (2)

4. Ans. (2175)

5. Ans. (16)

6. Ans. (9525)

7. Ans. (150)

8. Ans. (321)

9. Ans. (754)

10. Ans. (6952)

11. Ans. (8)

12. Ans. (151)

13. Ans. (126)

14. Ans. (461)

15. Ans. (9)

16. Ans. (60)

17. Ans. (3)

18. Ans. (5)

## MCQ 2023

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### Question:1

Let  $A_1$  and  $A_2$  be two arithmetic means and  $G_1, G_2, G_3$  be three geometric

means of two distinct positive numbers. Then  $G_1^4 + G_2^4 + G_3^4 + G_1^2G_3^2$  is equal to :

JEE Main 2023 (Online) 15th April Morning Shift

**A**  $(A_1 + A_2)^2 G_1 G_3$

**B**  $(A_1 + A_2) G_1^2 G_3^2$

**C**  $2(A_1 + A_2) G_1^2 G_3^2$

**D**  $2(A_1 + A_2) G_1 G_3$

### Question:2

Let  $a_1, a_2, a_3, \dots$  be a G.P. of increasing positive numbers. Let the sum of its 6<sup>th</sup> and 8<sup>th</sup> terms be 2 and the product of its 3<sup>rd</sup> and 5<sup>th</sup> terms be  $\frac{1}{9}$ . Then  $6(a_2 + a_4)(a_4 + a_6)$  is equal to

JEE Main 2023 (Online) 13th April Evening Shift

**A**  $2\sqrt{2}$

**B** 2

**C**  $3\sqrt{3}$

**D** 3

### Question:3

Let  $s_1, s_2, s_3, \dots, s_{10}$  respectively be the sum to 12 terms of 10 A.P.s whose first terms are 1, 2, 3, ..., 10 and the common differences are 1, 3, 5, ..., 19 respectively. Then  $\sum_{i=1}^{10} s_i$  is equal to :

### JEE Main 2023 (Online) 13th April Morning Shift

A 7360

B 7220

C 7260

D 7380

### Question:4

Let  $\{a_n\}$  be a sequence such that  $a_1 + a_2 + \dots + a_n = \frac{n^2+3n}{(n+1)(n+2)}$ . If  $28 \sum_{k=1}^{10} \frac{1}{a_k} = p_1 p_2 p_3 \dots p_m$ , where  $p_1, p_2, \dots, p_m$  are the first  $m$  prime numbers, then  $m$  is equal to

### JEE Main 2023 (Online) 12th April Morning Shift

A 5

B 7

C 6

D 8

**Question:5**

Let  $a, b, c$  and  $d$  be positive real numbers such that  $a + b + c + d = 11$ . If the maximum value of  $a^5b^3c^2d$  is  $3750\beta$ , then the value of  $\beta$  is

**JEE Main 2023 (Online) 11th April Evening Shift**

A 110

B 108

C 90

D 55

**Question:6**

Let  $x_1, x_2, \dots, x_{100}$  be in an arithmetic progression, with  $x_1 = 2$  and their mean equal to 200 . If  $y_i = i(x_i - i)$ ,  $1 \leq i \leq 100$ , then the mean of  $y_1, y_2, \dots, y_{100}$  is

**JEE Main 2023 (Online) 11th April Morning Shift**

A 10051.50

B 10049.50

C 10100

D 10101.50

**Question:7**

If  $S_n = 4 + 11 + 21 + 34 + 50 + \dots$  to  $n$  terms, then  $\frac{1}{60}(S_{29} - S_9)$  is equal to

**JEE Main 2023 (Online) 10th April Evening Shift**

**A** 227

**B** 226

**C** 220

**D** 223

**Question:8**

Let the first term  $\alpha$  and the common ratio  $r$  of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033, then the sum of these three terms is equal to

**JEE Main 2023 (Online) 10th April Morning Shift**

**A** 241

**B** 231

**C** 220

**D** 210

**Question:9**

Let  $a_n$  be the  $n^{\text{th}}$  term of the series  $5 + 8 + 14 + 23 + 35 + 50 + \dots$  and  $S_n = \sum_{k=1}^n a_k$ . Then  $S_{30} - a_{40}$  is equal to

**JEE Main 2023 (Online) 8th April Evening Shift**

**A** 11280

**B** 11290

**C** 11310

**D** 11260

**Question:10**

Let  $S_K = \frac{1+2+\dots+K}{K}$  and  $\sum_{j=1}^n S_j^2 = \frac{n}{A} (Bn^2 + Cn + D)$ , where  $A, B, C, D \in \mathbb{N}$  and  $A$  has least value. Then

**JEE Main 2023 (Online) 8th April Morning Shift**

**A**  $A + B + C + D$  is divisible by 5

**B**  $A + C + D$  is not divisible by  $B$

**C**  $A + B = 5(D - C)$

**D**  $A + B$  is divisible by  $D$

**Question:11**

If  $\gcd(m, n) = 1$  and  $1^2 - 2^2 + 3^2 - 4^2 + \dots + (2021)^2 - (2022)^2 + (2023)^2 = 1012 m^2 n$  then  $m^2 - n^2$  is equal to

**JEE Main 2023 (Online) 6th April Evening Shift**

**A** 220

**B** 200

**C** 240

**D** 180

**Question:12**

The sum of the first 20 terms of the series  $5 + 11 + 19 + 29 + 41 + \dots$  is

**JEE Main 2023 (Online) 6th April Morning Shift**

**A** 3420

**B** 3450

**C** 3250

**D** 3520

**Question:13**

The sum  $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$  is equal to :

JEE Main 2023 (Online) 1st February Evening Shift

A  $\frac{11e}{2} + \frac{7}{2e}$

B  $\frac{13e}{4} + \frac{5}{4e} - 4$

C  $\frac{11e}{2} + \frac{7}{2e} - 4$

D  $\frac{13e}{4} + \frac{5}{4e}$

Question:14 The sum of 10 terms of the series

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots \text{ is}$$

JEE Main 2023 (Online) 1st February Morning Shift

A  $\frac{58}{111}$

B  $\frac{56}{111}$

C  $\frac{55}{111}$

D  $\frac{59}{111}$

Question:15

Let  $a_1, a_2, a_3, \dots$  be an A.P. If  $a_7 = 3$ , the product  $a_1 a_4$  is minimum and the sum of its first  $n$  terms is zero, then  $n! - 4a_{n(n+2)}$  is equal to :

JEE Main 2023 (Online) 31st January Evening Shift

A 24

B  $\frac{381}{4}$

C 9

D  $\frac{33}{4}$

**Question:16** If the sum and product of four positive consecutive terms of a G.P, are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

JEE Main 2023 (Online) 31st January Morning Shift

A 7

B 14

C 3

D  $\frac{9}{2}$

**Question:17**

Let  $a, b, c > 1$ ,  $a^3, b^3$  and  $c^3$  be in A.P, and  $\log_a b, \log_c a$  and  $\log_b c$  be in G.P. If the sum of first 20 terms of an A.P., whose first term is  $\frac{a+4b+c}{3}$  and the common difference is  $\frac{a-8b+c}{10}$  is  $-444$ , then  $abc$  is equal to :

JEE Main 2023 (Online) 30th January Evening Shift

A 343

B 216

C  $\frac{343}{8}$

D  $\frac{125}{8}$

### Question:18

If  $a_n = \frac{-2}{4n^2 - 16n + 15}$ , then  $a_1 + a_2 + \dots + a_{25}$  is equal to :

### JEE Main 2023 (Online) 30th January Morning Shift

A  $\frac{51}{144}$

B  $\frac{49}{138}$

C  $\frac{50}{141}$

D  $\frac{52}{147}$

### Question:19

For three positive integers p, q, r,  $x^{pq^2} = y^{qr} = z^{p^2r}$  and  $r = pq + 1$  such that  $3, 3 \log_y x, 3 \log_z y, 7 \log_x z$  are in A.P. with common difference  $\frac{1}{2}$ . Then r-p-q is equal to

### JEE Main 2023 (Online) 24th January Morning Shift

**A** 12

**B** -6

**C** 6

**D** 2

## MCQ Answers Key

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1. Ans. (C)
2. Ans. (D)
3. Ans. (C)
4. Ans. (C)
5. Ans. (C)
6. Ans. (B)
7. Ans. (D)
8. Ans. (B)
9. Ans. (B)
10. Ans. (D)
11. Ans. (C)
12. Ans. (D)
13. Ans. (B)
14. Ans. (C)
15. Ans. (A)
16. Ans. (A)
17. Ans. (B)
18. Ans. (C)
19. Ans. (D)