Sets

NCERT

– by Deepneet

Sets And Their Representations

collections

In everyday life → shoe collection
→ A pack of cards
→ A cricket Team.

In mathematics → odd natural nos.
→ Equilateral Triangles in a plane.
→ Prime factors of 5 → ?

Ans $\{5\}$ ✓

$\{1, 5\}$ ✗

→ 1 is not a prime
→ 1 is neither prime nor composite

* All the above collections are well-defined

* well defined means there is no confusion about what to include in or exclude from a collection.

e.g. in a collection of odd natural nos.

2 → Exclude → is not odd
5 → include
$\frac{3}{5}$ → Exclude (not a Natural no.)

If I give you any random element you will be able to see if it is odd Natural number or not. without any confusion

⇒ this set is well-defined.

* A set of Intelligent students in your class.
Not a well-defined set.

* should I add chauhan in the set
→ some says yes he's intelligent
→ some says no

confusion will be there regarding whom to add whom not to

⇒ set of intelligent students is not a well-defined set.

* collection of famous/renowned mathematicians in this world
→ not well-defined.
who is famous, How to decide, confusion is there.

Set: A well-defined collection of objects or

A well defined collection of distinct objects.

generally we do not repeat elements in a set.

e.g. $\{1, 2, 1, 3\}$ is written as $\{1, 2, 3\}$

* 1 will be written one time.

no. of elements in $\{1, 2, 1, 3\} = 3$, $\neq 4$

elements are 1, 2 and 3.

* order/position of the elements in a set does not matter.

⇒ $\{1, 2, 3\} = \{1, 3, 2\} = \{3, 2, 1\}$
all are same sets.

Representation of a set

* Sets are represented by capital letters A to Z and their elements/objects/members by small letters.

* Sets are always written in curly braces. $\{\ \}$.

* some of the most commonly used sets in mathematics are:

N → set of all Natural nos.
Z → set of all Integers
Q → The set of all Rational Numbers
R → set of all real numbers
Z^+ → set of positive Integers
Q^+ → The set of Positive Rational numbers.
R^+ → The set of positive Real numbers.

Symbol $\in$ → "belongs to"

greek symbol "epsilon" $\in$ denotes the phrase "belongs to"

$\notin$ denotes "doesn't belongs to".

There are two methods to represent any set:

① Roaster Form / Tabular Form

② set Builder Form.

① In Roaster Form all the elements are listed, seperated by commas, and are enclosed within curly braces.

E.g. set of all days of a week starting from S.

Then, A = {sunday, saturday}   (I am using capital 'A' to represent my set)

E.g. set of letters or English alphabets used in word MISSISSIPPI

Then $\boxed{S = \{M, I, S, P\}}$ → set is in Roaster Form

* M, I, S and P will come only once in a set.

Hence, M, I, S and P are the alphabets which are present in the word MISSISSIPPI

set of Natural nos. in Roster Form.

$N = \{1, 2, 3, \dots\}$

$3 \leq x < 5$ in roster form can't be written

$\{3, ?$ now which element to write next?

② Set-Builder Form

Format $S = \{x : \text{rule for } x\}$

E.g. $S = \{x : 3 \leq x < 5, x \in R\}$

read as S is a set of all those x such at x is a real number greater than equal to 3 and less than 5.

for Example;

$A = \{2, 3, 4, 5\}$ is a set of natural nos. between 1 and 6

I can say that

$3 \in A$ and $1 \notin A$

(because 3 belongs to set A or 3 is present in set A or 3 is an element of set A but 1 is not.)

set of Natural nos. in set builder form

$N = \{x : x = n, n \in N\}$

write $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\}$ in set builder form

$\{x : x = \frac{n}{n+1}, n \in N \text{ and } n \leq 6\}$

Cardinal Number / Order / Cardinality of a set

denoted by n(A), O(A) or |A| which means number of elements of set A.

E.g. $A = \{\{3, 4\}, 5\}$

$n(A) = 2$

$\{3, 4\} \in A$

$3 \notin A$

$\phi \notin A$

Types of sets Based on cardinality

① Empty set / Null set / Void Set :

set with zero elements or
set with $n(A) = 0$.
denoted by ϕ or $\{\}$

E.g. * set of odd natural nos. divisible by 2.
* set of students of school studying in class XI^{th} and XII^{th} simultaneously.

This is null set as, schools will not give admission in class XI^{th} and XII^{th} simultaneously.

* $\{x : x$ is a natural no, $x < 5$ and $x > 7\}$

How can any number be less than 5 and also greater than 7 at the same time.

E.g. $4 < 5$ True
but $4 \not> 7$

* $\{y : y$ is a point common to an two parallel lines$\}$

||el lines never meet
$\Rightarrow$ no common point exists

② Singleton Set

A set with $n(A) = 1$.
* A set with only one element.

E.g. $\{x : x^2 - 1 = 0,\ x \in N\} = \{1\}$

$x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$
but $1 \in N$ and $-1 \notin N$
$\Rightarrow x$ can be $+1$ only

③ Finite set

A set with finite number of elements.

E.g. set of all human beings on this Earth.

* population is countable
infinite humans $\Rightarrow$ no free space even to move
Then everywhere humans should be there.

2022 population: 795.09 crores

E.g. set of all months in a year.

E.g. $\{1, 2, 3, 4\}$

④ Infinite Set

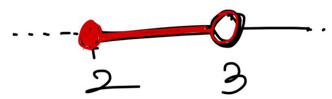
A set with infinite number of elements.

E.g. set of Natural nos.

E.g. $\{x : 2 \leq x < 3,\ x \in R\}$
$= [2, 3)$

Interval: A set in which all the numbers between first and last number are present.

E.g. $[2, 3) \rightarrow$ It has 2 and all numbers between 2 and 3.



$[a,b]$ or $(a,b]$ or (a,b) or $[a,b)$ all have same length of interval $= \underline{b-a}$

E.g. length of interval $(2,5)$
$= 5-2 = \underline{3}$

length of interval $(6,8]$
$= 8-6 = \underline{2}$

doesnot matter if ends are included in interval or not.

Subsets

set A is a subset of B if all elements of A are also in B.

denoted by $A \subset B$.

E.g. $\{3,5\} \subset \{5,8,3\}$

because 3 and 5 are in right set too

$\{3,5\} \not\subset \{3, \{3,5\}, 8\}$

because $5 \in \{3,5\}$

but $5 \notin \{3, \{3,5\}, 8\}$

mathematical definition of a subset

$A \subset B$ if $a \in A \Rightarrow a \in B$

$A \subset B \Rightarrow$ A is a subset of B

and B is called superset of A

Equivalent sets:

sets with equal number of elements are called equivalent sets.

E.g. $\{3,\pi\}$ and $\{\{3,\pi\}, 2\}$ are equivalent sets

i.e. Both has 2 elements.

E.g. $\{3,\pi\}$ and $\{3,\pi\}$ are equivalent sets.

Equal sets: sets when elements and no. of elements are same,

or if $A \subset B$ and $B \subset A$
$\Rightarrow A = B$.

E.g. $\{1,2,3\} = \{3,2,1\}$

no. of elements are 3 in both and are same i.e. 1, 2 and 3.

Note: $\{1,2,3\} \subset \{3,2,1\}$
and $\{3,2,1\} \subset \{1,2,3\}$

* NOTE All equal sets are equivalent sets.

because, Equal sets have same no. of elements.

* ϕ is a subset of all sets

* Every set is a subset of itself

Que: Write subsets of $\{1,2,3\}$

Ans: ϕ, $\{1\}$, $\{2\}$, $\{3\}$,
$\{1,2\}$, $\{1,3\}$, $\{2,3\}$
$\{1,2,3\}$

Total 8 subsets $= 2^3$

Que: write subsets of $\{1,2,3,4\}$

Ans: ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$,
$\{1,2\}$, $\{1,3\}$, $\{1,4\}$, $\{2,3\}$,
$\{2,4\}$, $\{3,4\}$,
$\{1,2,3\}$, $\{1,2,4\}$, $\{1,3,4\}$
$\{2,3,4\}$,
$\{1,2,3,4\}$

Total 16 subsets $= 2^4$

Que: write all subsets of $\{2, \{3,5\}, 4\}$

Ans ϕ, $\{2\}$, $\{\{3,5\}\}$, $\{4\}$,
$\{2, \{3,5\}\}$, $\{2,4\}$, $\{\{3,5\}, 4\}$,
$\{2, \{3,5\}, 4\}$

Total $8 = 2^3$

* no. of subsets of set $A = 2^{n(A)}$

Que: no. of subsets of A if:

(i) $A = \{a, 7, 9, 2, \pi, -1\}$

Ans $n(A) = 6$

no. of subsets $= 2^6$

(ii) $n(A) = 50$

Ans no. of subsets $= 2^{n(A)} = 2^{50}$

Proper Subsets : All subsets excluding itself

E.g. proper subsets of $\{1,2\}$ are

ϕ ✓, $\{1\}$ ✓, $\{2\}$ ✓, ~~$\{1,2\}$~~

Total $3 = 4-1 = 2^2 - 1$

$\Rightarrow$ no. of proper subsets of A $= 2^{n(A)} - 1$

Que: set A has m elements and set B has n elements. If the total no. of subsets of A is 112 more than the subsets of B, then value of $m.n = ?$

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given $n(A) = m$, $n(B) = n$

$2^m = 112 + 2^n$ → Total no. of subsets of B

$\Rightarrow 2^m - 2^n = 112$ → From this $m > n$.

Take 2^n common.

$2^n(2^{m-n} - 1) = 112$

$2^n(2^{m-n} - 1) = 2^4 \times 7$

2^n → even; $2^{m-n}-1$ → even $- 1$ = odd

$$\begin{array}{r|l} 2 & 112 \\ 2 & 56 \\ 2 & 28 \\ 2 & 14 \\ & 7 \end{array}$$

$\Rightarrow 2^n = 2^4$, $2^{m-n} - 1 = 7$

$\Rightarrow n = 4$

$\Rightarrow 2^{m-4} - 1 = 7$

$2^{m-4} = 8 = 2^3$

$\Rightarrow m - 4 = 3$

$m = 7$

Ans $m.n = 7 \times 4 = 28$ **Ans**

True/False

1. ϕ is subset of every set ✓
2. Every set is a subset of itself ✓
3. If $n(A) = \alpha$, then no. of proper subsets of $A = 2^{n(A)} - 1$ ✓
4. $\phi = \{0\}$ ✗
5. $\phi = \{\}$ ✓
6. $\{0\}$ is a null set ✗ It is a singleton set with one element '0'.
7. $\{\phi\} = \{\{\}\}$ is a null set ✗ It is a singleton set with one element 'ϕ'.
8. ϕ, $\{0\}$, $\{\phi\}$ are different sets ✓
9. no. of subsets of ϕ is 0. ✗

Ans is 1 $\quad n(\phi) = 0$

subsets of ϕ are : ϕ $\quad$ no. of subsets $= 2^{n(A)} = 2^0 = 1$

Que: fill the blanks with correct symbol $\quad A = \{1, 2, 3, 4\}$

(i) $2 \in A$

(ii) $\{2\} \subseteq A$

(iii) $\phi \subseteq A$

(iv) $7 \notin A$

(v) $\{5,4\} \not\subset A$ or $\not\subseteq$

True/False

(vi) $\phi \notin A$ True

(vii) $\{\phi\} \subset A$ True

(viii) $\{\{2\}\} \subset A$ false because $\{2\} \notin A$

(ix) $3 \in A$ True

(x) A is subset of itself True

Power Set

denoted by $P(A)$ = Power set of A

= set of all subsets of A.

$\Rightarrow n(P(A)) = 2^{n(A)}$

E.g. $A = \{1, \phi, 3\}$ write $P(A)$.

$P(A) = \{\phi, \{1\}, \{\phi\}, \{3\}, \{1,\phi\}, \{1,3\}, \{\phi,3\}, \{1,\phi,3\}\}$

no. of Elements $= 8 = 2^3 = 2^{n(A)}$

Operation Between Two Sets

① **Union** : combined parts of two sets.

denoted by $(A \cup B)$ or $(A+B)$ or $(A$ or $B)$ or atleast one of A or B.

$\Rightarrow$ $A \cup B$ will have elements which is in atleast one of A or B.

$\Rightarrow$ All Elements of A and all Elements of B will be in $A \cup B$.

E.g. $A = \{1, 2\}$ $B = \{3, 4\}$

$A \cup B = \{1, 2, 3, 4\}$

1 is in A
2 is in A
3 is in B
4 is in B

} all are in $A \cup B$.

E.g. $A = \{1, 2\}$ $B = \{2, 3\}$

$A \cup B = \{1, 2, 3\}$

1 is in A
2 is in both A and B
3 is in B
4 is in neither A nor B

} 1, 2 and 3 will be in $A \cup B$.

② **Intersection**

common parts | Elements of two sets.

denoted by: $A \cap B$, AB or A and B.

E.g. $A = \{1, 2\}$ $B = \{3, 4\}$

$A \cap B = \phi$

E.g. $A = \{1, 2\}$ $B = \{2, 3\}$

$A \cap B = \{2\}$

* If $A \cap B = \phi$ Then A and B are called Disjoint sets.

③ Difference of Two sets

$\boxed{A - B = A - (A \cap B)}$ → only A.

$B - A = B - (A \cap B)$

E.g. $A = \{1, 2, 3\}$

$B = \{2, 5\}$

$A - B = \{1, \cancel{2}, 3\}$

$= \{1, 3\}$

$B - A = \{5\}$ → out of B remove Elements that are in A

Eg $A = \{5, 6\}$, $B = \{5, \phi\}$

$A - B = \{6\}$

Eg $A = \{7, 8\}$

$B = \{2\}$

$A - B = \{7, 8\}$

Universal set : A kind of universe for all sets.

↓

Superset of all sets.

mostly universal set will be given in the question.

* universal set is denoted by U.

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(4) **Complement of a set** = Everything except the set.

$\boxed{A' = U - A}$ denoted by A' or A^c or $\bar{A}$

for set U is everything

Few Properties of sets

(1) commutative laws for Intersection and Union

→ $A \cup B = B \cup A$

→ $A \cap B = B \cap A$

(2) Assossiative laws

→ $A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$

→ $A \cap (B \cap C) = (A \cap B) \cap C = A \cap B \cap C$

(3) Idempotent laws

→ $A \cup A = A$

→ $A \cap A = A$

(4) Law of Identity Element

$A \cup \phi = A$ → ϕ is identity for union

$A \cap U = A$ → U is identity for Intersection.

* Eg. 1 is identity for multiplication

⇒ if I multiply 1 with any no. I will get back the same no.

Eg $1 \times x = x = x \times 1$

* 0 is identity for addition.

⇒ if I add 0 with any no. I will get back the same no.

Eg $x + 0 = x = 0 + x$

similarly Any set $A \cup \phi = A = \phi \cup A$ → ϕ is identity for union.

Set $A \cap U = A = U \cap A$

(5) $A \cup U = U$

$A \cap \phi = \phi$

(6) $(A')' = A$ → Double complement law

$A' = U - A$

$(A')' = U - A'$

$= U - (U - A)$

$= \cancel{U} - \cancel{U} + A = \underline{A}$

(7) $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

} De-morgan's law

* we will prove this later by Venn Diagram.

(8) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

} Distributive laws

Que: Given $U = \{2, 7, 9, 11, \pi, 3.14\}$

$A = \{7, \pi\}$

Find A'.

Ans :- $A' = U - A$

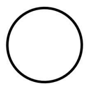
$= \{2, 7, 9, 11, 3.14\}$

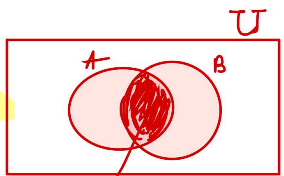
Venn DIAGRAM Representations

* It is one of the way to represent sets

* Elements are placed inside circles.

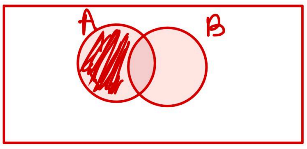
(1) U = Universal set ⇒ 

2 Any set ⇒ 

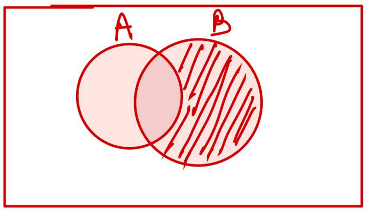
(3) $A \cap B$ = 

AB or $A \cap B$ or A and B

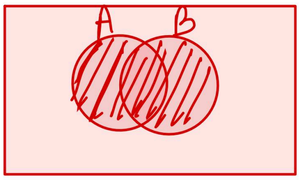
④ only A or Exactly A or $A-B$ or $A-(A\cap B)$



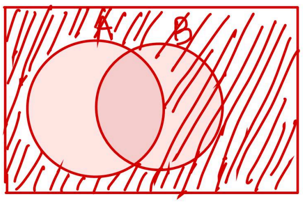
⑤ only B or Exactly B or $B-A$ or $B-(A\cap B)$



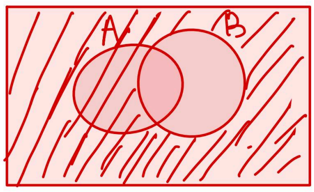
⑥ $A\cup B$ = A or B = atleast one of A or B



⑦ $A' = U - A$

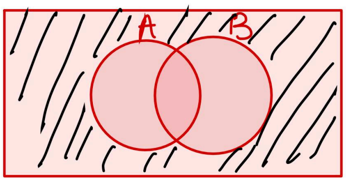


⑧ $B' = U - B$

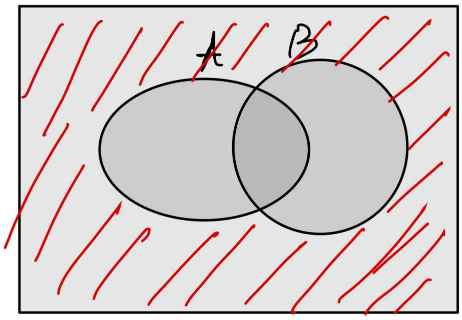


common shaded Area of Both

⑨ $(A\cup B)'$



$A'\cap B' =$



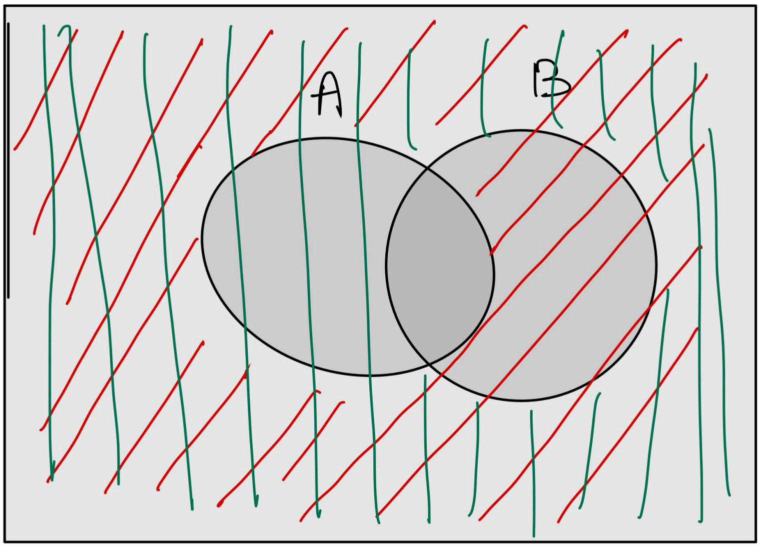
$= (A\cup B)'$

$\Rightarrow A'\cap B' = (A\cup B)'$

Hence, De-Morgan's law is proved.

Similarly;

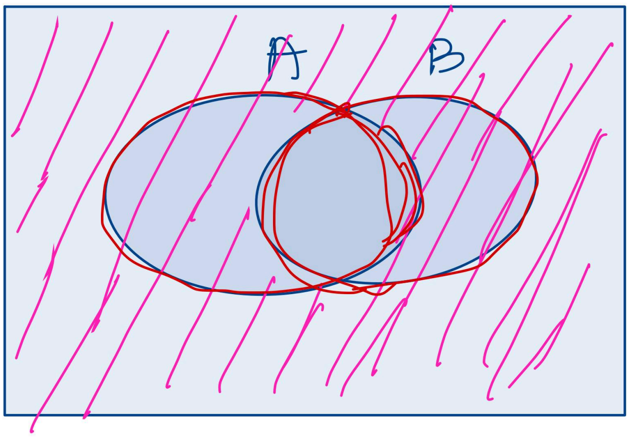
$(A\cap B)' = A'\cup B'$



red ⇒ A'
→ Everything outside A.
green ⇒ B'

now $A' \cup B'$ ⇒ every shaded region either green or red

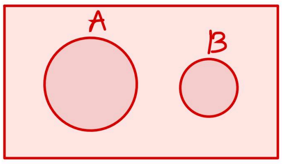
=



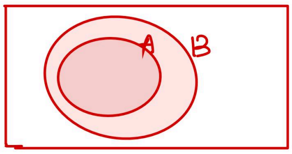
= Everything except $A \cap B$

⇒ $A' \cup B' = (A \cap B)'$ → De-morgan's law.

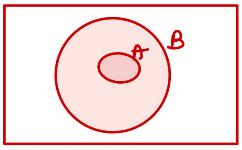
For disjoint sets $\Rightarrow A \cap B = \phi$



* $A \subset B$



$B \subset A$



Few formulas

① $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

② $n(A' \cap B') = n((A \cup B)')$ (De Morgan's law)

$= n(U) - n(A \cup B)$

③ $n(A \cap B') = n(A) - n(A \cap B)$

$A \cap B' = A - B = A - (A \cap B)$

Formulas for 3 sets A, B and C

④ $n(A \cup B \cup C) = S_1 - S_2 + S_3$

$= n(A) + n(B) + n(C)$
$- n(A \cap B) - n(A \cap C) - n(B \cap C)$
$+ n(A \cap B \cap C)$

Proof:- $n(A \cup B \cup C)$

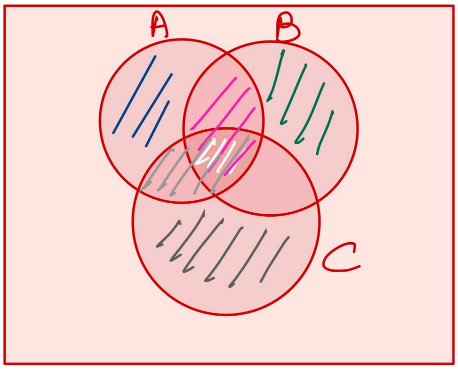
$= n(A \cup (B \cup C))$ Associative law

$= n(A) + n(B \cup C) - n(A \cap (B \cup C))$

$= n(A) + n(B) + n(C) - n(B \cap C)$
$- n((A \cap B) \cup (A \cap C))$ Distributive law

$= n(A) + n(B) + n(C) - n(B \cap C)$
$- [n(A \cap B) + n(A \cap C) - n((A \cap B) \cap (A \cap C))]$

$= n(A) + n(B) + n(C)$
$- n(A \cap B) - n(A \cap C)$
$- n(B \cap C)$
$+ n(A \cap B \cap C)$



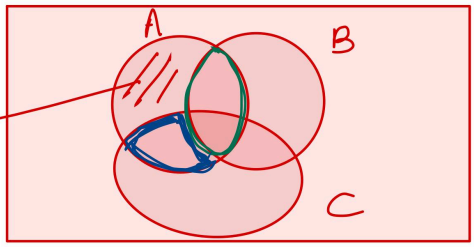
only A   only B

only C

$A \cap B \cap C$

$A \cap B$

$A \cap C$



only A

⑤ $n(\text{only } A)$
$= n(A \cap B' \cap C')$
$= n(A) - n(\text{Green}) - n(\text{Blue})$
$= n(A) - n(A \cap B)$
$- [n(A \cap C) - n(A \cap B \cap C)]$
$= n(A) - n(A \cap B) - n(A \cap C)$
$+ n(A \cap B \cap C)$

similarly $n(\text{only } B)$

$= n(A' \cap B \cap C')$

$= n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

* $n(\text{only } C) = n(A' \cap B' \cap C)$

$= n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$n(\text{Exactly one of the set})$

$= n(\text{only } A) + n(\text{only } B) + n(\text{only } C)$

$= n(A) + n(B) + n(C)$

$\quad - 2\big(n(A \cap B) + n(A \cap C) + n(B \cap C)\big)$

$\quad + 3\big(n(A \cap B \cap C)\big)$

$= S_1 - 2S_2 + 3S_3$

Que: if $P(A) = P(B)$, show that $A = B$.

Proof:- let $x \in A$ ✓

$\Rightarrow \{x\} \subset A$

$\Rightarrow \{x\} \in P(A)$

$\Rightarrow \{x\} \in P(B)$ $\because P(A) = P(B)$

$\Rightarrow \{x\} \subset B$

$\Rightarrow x \in B$ ✓

→ To show $A \subset B$ and $B \subset A$

→ To show $x \in A \Rightarrow x \in B$ and $y \in B \Rightarrow y \in A$

$\Rightarrow A \subset B$

similarly, $B \subset A$

$\Rightarrow A = B$.

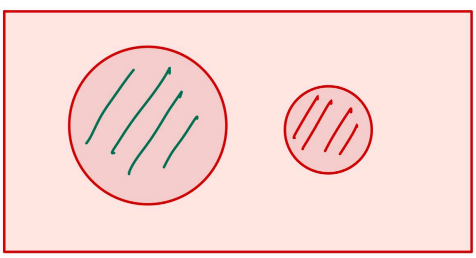
Homework

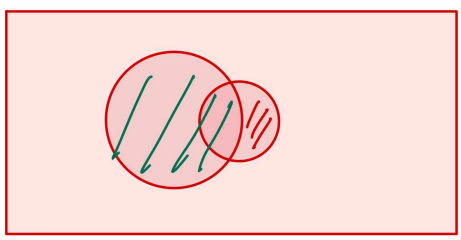
Important: Ex 1.6, misc. Examples, misc. Exercise.

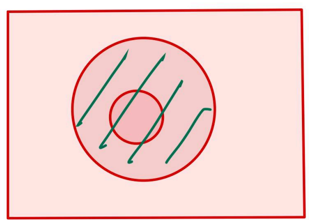
*

$$\max\{n(A), n(B)\} \le n(A \cup B) \le \min\{n(A) + n(B), n(U)\}$$

This inequality can decide max and min. value to $n(A \cap B)$ in questions.







$A \cup B$ is max.

$= n(A) + n(B)$

but if $n(A) + n(B) > n(U)$ which is not possible

$A \cup B$ is min.

= bigger set

$= \max\{n(A), n(B)\}$

Then $n(U)$ will be max. value of $n(A \cup B)$