PART 3 - MATHEMATICS

SECTION–I : (Maximum Marks : 80)

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :
  Full Marks : +4 If only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases

1. The sum to infinity of the series

$1+\frac{2}{5}+\frac{6}{5^2}+\frac{10}{5^3}+\frac{14}{5^4}+.....$ is

(A) $\frac{4}{7}$ (B) $\frac{5}{4}$

(C) $\frac{7}{4}$ (D) $\frac{6}{5}$

2. If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3+\left(1\frac{1}{2}\right)^3+\left(2\frac{1}{4}\right)^3+3^3+\left(3\frac{3}{4}\right)^3+.......$ is equal to 225 k, then k is equal to:

(A) 9 (B) 27

(C) 108 (D) 54

3. $\frac{1}{\log_a x}+\frac{1}{\log_c x}=\frac{2}{\log_b x}$, then a, b, c are in

$(x>0,\ x\neq 1)$

(A) A.P.

(B) G.P.

(C) H.P.

(D) None of these

4. If $x_1,\ x_2,\ x_3,\ x_4,\ x_5,$ ... is a geometric progression of natural numbers and $x_1x_2x_3x_4=64,$ then complete set of solutions of x_4 is

(A) $\{1\}$

(B) $\{8\}$

(C) $\{1,8\}$

(D) Can not be determined

5. Let S_n denote the sum to n terms of an arithmetic progression whose first term is a. If the common difference is equal to $S_n-kS_{n-1}+S_{n-2}$, then k =

(A) 1 (B) 2

(C) 3 (D) None of these

6. Let positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are in

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

7. If $\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}(x\neq 0)$, then a, b, c, d are in

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

8. Two consecutive numbers from 1, 2, 3, ……. 50 are removed. A.M. of the remaining numbers is $\frac{105}{4}$. Then the numbers are

(A) 7, 8 (B) 8, 9

(C) 6, 7 (D) None of these

9. The first and last terms of an A.P. are a and l respectively. If s be the sum of all the terms of the A.P., then common difference is

(A) $\frac{l^2-a^2}{2s-(l+a)}$ (B) $\frac{l^2-a^2}{2s-(l-a)}$

(C) $\frac{l^2+a^2}{2s+(l+a)}$ (D) $\frac{l^2+a^2}{2s-(l+a)}$

10. a, b, c are distinct positive real numbers such that a > b > c. If 2 log (a – c), log ($a^2 - c^2$), log($a^2 + 2b^2 + c^2$) are in A.P., then

(A) a, b, c are in A.P.

(B) $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ are in A.P.

(C) a, b, c are in G.P.

(D) a, b, c are in H.P.

11. Which one is true? (a, b, c are positive distinct numbers)

(A) $(a + b + c)^3 \geq 27\ abc$

(B) $a^3 + b^3 > 2a^{3/2}\ b^{3/2}$

(C) $b^2 + c^2 \geq 2bc$

(D) All of these

12. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, ……., where n consecutive terms have the value n, the 1025th term is

(A) 2^9 (B) 2^{10}

(C) 2^{11} (D) 2^8

13. If HM : GM = 4 : 5 for two positive numbers, then the ratio of the numbers is

(A) 4 : 1 (B) 3 : 2

(C) 3 : 4 (D) None of these

14. The sum to n terms of an A.P. is $cn(n-1)$, then sum of the squares of these terms is

(A) $\frac{2}{3}c^2(n-1)(2n-1)n$

(B) $\frac{2}{3}c^2(n+1)(2n-1)n$

(C) $\frac{1}{6}c^2n(n+1)(2n+1)$

(D) None of these

15. If n arithmetic means a_1, a_2, ..., a_n are inserted between 50 and 200 and n harmonic means h_1, h_2, ..., h_n are inserted between the same two numbers, then a_2h_{n-1} is equal to

(A) 500 (B) $\frac{10000}{n}$

(C) 10000 (D) $\frac{250}{n}$

16. If the sum of the first 2n terms of the A.P. 2, 5, 8, ……, is equal to the sum of the first n terms of the A.P., 57, 59, 61, ……, then n must be equal to

(A) 10 (B) 12

(C) 11 (D) 13

17. If $\sum_{r=1}^{n} t_r = \frac{1}{12}n(n+1)(n+2)$, then the value of $\sum_{r=1}^{n}\frac{1}{t_r}$ is

(A) $\frac{2n}{n+1}$ (B) $\frac{n}{(n+1)}$

(C) $\frac{4n}{n+1}$ (D) $\frac{3n}{n+1}$

18. Let $\{a_n\}$ and $\{b_n\}$ are two sequences given by $a_n = (x)^{\frac{1}{2^n}} + (y)^{\frac{1}{2^n}}$ and $b_n = (x)^{\frac{1}{2^n}} - (y)^{\frac{1}{2^n}}$ for all $n \in N$. The value of $a_1a_2a_3..........a_n$ is equal to

(A) $x - y$ (B) $\frac{x+y}{b_n}$

(C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

19. If 1, 2, 3 ..... are first terms: 1, 3, 5 .... are common differences and S_1, S_2, S_3, ... are sums of n terms of given p AP's; then $S_1 + S_2 + S_3 + ... + S_p$ is equal to

(A) $\frac{np(np+1)}{2}$ (B) $\frac{n(np+1)}{2}$

(C) $\frac{np(p+1)}{2}$ (D) $\frac{np(np-1)}{2}$

20. The H.M. between two numbers is $\frac{16}{5}$, their A.M. is A and G.M. is G. If $2A + G^2 = 26$, then the numbers are

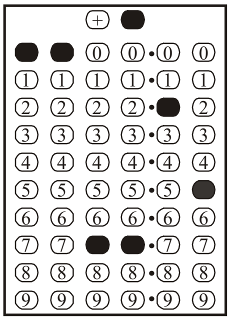
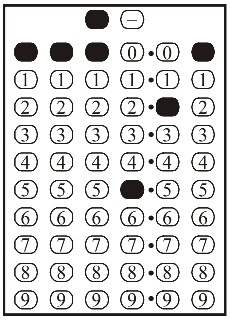
(A) 6, 8 (B) 4, 8

(C) 2, 8 (D) 1, 8

SECTION-II : (Maximum Marks: 20)

- This section contains **FIVE** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.

  For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.

- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If ONLY the correct numerical value is entered as answer.

  Zero Marks : 0 In all other cases.

1. $(241)\sum_{k=1}^{15}\left(\frac{k}{k^4+k^2+1}\right)$ is equal to:

2. The value of x + y + z is 15 if a, x, y, z, b are in AP while the value of $\left(\frac{1}{x}\right)+\left(\frac{1}{y}\right)+\left(\frac{1}{z}\right)$ is $\frac{5}{3}$ if a, x, y, z, b are in HP. Then a^2+b^2 is?

3. If a, b and c are positive real numbers such that the minimum value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ is λ, then $\frac{8\lambda}{3}$ is

4. If $\lambda = 1^2-\frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}+\frac{5^2}{5^4}-\frac{6^2}{5^5}+..........\infty$, then 54λ

5. If $\lambda=\sum_{k=1}^{\infty}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$, then 16λ is