FOM NOTES

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BASICS OF MATHEMATICS

1 Number Systems

Real nos.

- Rational (Q)
  numbers of form
  ① $\frac{p}{q}$ $(q \neq 0)$ $p, q \in \mathbb{Z}$
  or
  ② Terminating decimals
  or
  ③ Non Terminating but recurring.
  - Non Integral decimals
  - Integers ($\mathbb{Z}$ or I)
    - Even
    - odd $(2k \pm 1)$ $k \in \mathbb{Z}$
    - Whole numbers (W)
      - 0
      - Natural Nos. (N)
        - 1
        - prime
        - composite
    - negative Integers
- Irrational (Q')

* $\frac{0}{0}$ or $\frac{\text{Any real number}}{0}$ are not defined.

* $R = Q \cup Q^c$

Express $1.\overline{34}$ in $\frac{p}{q}$

short cut : $\dfrac{\text{All} - \text{nonBar}}{\underbrace{99..9}\,\underbrace{00..0}}$ → No. of digits without bar after decimal

no. of digits with bar after decimal

Ans $\dfrac{134-1}{99} = \dfrac{133}{99}$

Que: $1.3\overline{42} = \dfrac{1342-13}{990} = \dfrac{1329}{990}$

or $x = 1.3\overline{42}$

$10x = 13.\overline{42}$

$1000x = 1342.\overline{42}$

$1000x - 10x = 1342 - 13$

$x = \dfrac{1329}{990}$

Without division how to say that fraction/Q will be a terminating decimal or non terminating.

If denominator is of the form $2^a 5^b$ {a, b are whole numbers} Then $\frac{p}{q}$ is a terminating decimal.

Que: $\dfrac{7}{675}$ → Non Terminating recurring

($\because$ $675 = 5^2 \times 3^3$)

Que: $\dfrac{1}{625}$ → $\dfrac{1}{5^4 \times 2^0}$ Terminating.

Irrational: example: $\pi, e, \sqrt{2}, \sqrt{3}, \sqrt[3]{9}, \ldots$

* $x^{1/n}$ is irrational if x is not of the form α^n ($\alpha \in$ Rational)

E.g. square root of a non perfect square is irrational.

Rationalization

E.g. Rationalize denominator of $\dfrac{7}{2+\sqrt{3}}$

$2-\sqrt{3}$ is rationalizing factor ($\because (2+\sqrt{3})(2-\sqrt{3}) = 1$)

$\dfrac{7}{2+\sqrt{3}} \dfrac{(2-\sqrt{3})}{(2-\sqrt{3})} = \dfrac{7(2-\sqrt{3})}{4-3} = \dfrac{7(2-\sqrt{3})}{1}$

* denominator is rationalized

Now $1 \in Q$ is in the denominator rather than $2+\sqrt{3}$.

Que: Rationalizing factor of $2^{1/3}+4$?

Ans $(2^{1/3})^2 + (4)^2 - (2^{1/3})(4)$

$\because (2^{1/3}+4)\left((2^{1/3})^2 + (4)^2 - (2^{1/3})(4)\right)$

$= (2^{1/3})^3 + (4)^3$

$= 2 + 4^3 \in Q$

Prime Numbers

All natural numbers having only 2 factors (1 and itself)

* 2 is only even prime
* 12^{th} prime is 37, 21^{st} prime is 73 } Knowledge Just for fun
* 2017 → prime

It's an interesting prime number. If we insert 7 in between any 3 places will be prime. Again number will be prime.

E.g. 27017, 20717, 20177 are all prime.

* Any prime number > 3 is of the form 6K+1 or 6K−1 but converse may not be True.

E.g. $25 = 6(4)+1$ is not a prime number

* To check N is a prime:- divide all nos. between 2 to $[\sqrt{N}]$ (GIF/step function)

if divisible ⇒ N is not prime else it is a prime Number.

E.g. 2017 $\sqrt{2017} \simeq 44.\_$

$[\sqrt{2017}] = [44.\_] = 44$

2017 has no factor between 2 to 44

⇒ 2017 is a prime number.

* 2 and 3 are the only consecutive prime nos. or prime number with difference 1 $3-2=1$

Que: a, b, c are prime and $b = c^2 - a^2$ find $a+b+c$?

$b = c^2 - a^2 = (c-a)(c+a) = 1 \times b$ ($\because b \in$ prime)

$a, b, c \in$ prime

⇒ $a, b, c \in N$

⇒ $c-a < c+a$

⇒ $c-a = 1$ $c+a = b$

a and c are primes ⇒ $c = 3$ $a = 2$ (difference of 2 primes $=1$)

$\Rightarrow b = c + a = 3 + 2 = \underline{\underline{5}}$

* **Twin Primes:** * If $a, b \in$ Primes and $a - b = \pm 2$ or $|a-b| = 2$

Then a and b are known as Twin Primes. E.g. 3 and 5, 7 and 5

* **co-primes or relatively prime**

If a and b are Two Numbers and they have no common factor (or if $HCF(a,b) = 1$)

E.g. 3 and 17 are coprimes

3 and 16 are coprimes

3 and 39 are not coprimes

* Two prime numbers are always co-prime.

HCF or G.C.D (Highest Common Divisor) (Greatest common Divisor)

The product of least powers of common primes involved in prime factorization.

Eg
$24 = 2^3 \times 3$
$126 = 2 \times 3^2 \times 7$
$39 = 3 \times 13$

common prime $= 3$
least power $= 1$
$HCF = \underline{\underline{3^1}}$

LCM (least common multiple)

The product of Highest powers of all the primes involved.

In $\underline{24, 126, 39}$

Primes involved $\rightarrow 2, 3, 7, 13$

Highest powers $\rightarrow 3, 2, 1, 1$

$\Rightarrow LCM = 2^3 \times 3^2 \times 7^1 \times 13^1$

LCM of Fractions $= \dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

HCF of Fractions $= \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

e.g. $LCM \left\{ \frac{1}{2}, \frac{2}{4} \right\} = \dfrac{LCM\{1,2\}}{HCF\{2,4\}} = \dfrac{2}{2} = 1$

$HCF \left\{ \frac{1}{2}, \frac{2}{4}, \frac{6}{5} \right\} = \dfrac{HCF\{1,2,6\}}{LCM\{2,4,5\}} = \dfrac{1}{20}$

Que: $LCM\{2, \pi\} = ?$

Does not exist / Not defined.

* A rational and an irrational can never have a common multiple.

Few facts:-

① rat $\pm$ rat $\rightarrow$ rat

② Irr $\pm$ Irr $\rightarrow$ rational or Irrational

E.g. $\underbrace{2+\sqrt{3}}_{Q^c} - \underbrace{\sqrt{3}}_{Q^c} = \underbrace{2}_{Q}$

③ rational $\times$ Irrational gives you rational only in one case i.e. rational with which you have multiplied is 0.

Algebraic Formulae: (Identities ∴ They are True $\forall x, y \in R$)

① $(x+y)^2 = x^2 + y^2 + 2xy$
$= (x-y)^2 + 4xy$

② $(x-y)^2 = x^2 + y^2 - 2xy$
$= (x+y)^2 - 4xy$

③ $(x^2 - y^2) = (x-y)(x+y)$

④ $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$
$= a^2 + b^2 + c^2 + 2abc\left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b}\right)$

⑤ $(a-b+c)^2 = a^2 + (-b)^2 + c^2 + 2(a)(-b) + 2(-b)(c) + 2ca$

replace b with -b in ④

$= a^2 + b^2 + c^2 - 2ab - 2bc + 2ca.$

⑥ $a^2 + b^2 + c^2 - ab - bc - ca$
$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

(∵ multiply & divide by 2)

⑦ $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

$= \frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$\Rightarrow$ If $a^3+b^3+c^3-3abc=0$

Then $(a+b+c)=0$

or $(a-b)^2+(b-c)^2+(c-a)^2 = 0.$

$\Rightarrow$ $a+b+c=0$ or $a=b=c$

or if $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

or if $a=b=c \Rightarrow a^3+b^3+c^3 = 3abc$

or if $a^3+b^3+c^3=0 \Rightarrow a+b+c=0$ or $a=b=c$.

Que: Find the minimum value of

$a^2+29+4b^2-6a+8b$

$a^2+25+4b^2-6a+8b+4$

$a^2+25-6a+4(b^2+2b+1) = 0$

$a^2-6a+9+16+4(b^2+2b+1)=0$

$(a-3)^2+16+4(b+1)^2=0$

$\Rightarrow$ minimum value is $\underline{16}$

at $a=3$ $b=-1$

Euclid's Division Lemma

If we have two positive integers a and b, then there exist unique integers q and r which satisfies the condition

$a = bq + r$ where $0 \le r < b$.

E.g. For 3 and 2 There exists 2 unique integers such that $3 = 2q + r$

$0 \le r < 2$

$2\overline{)3}^{\,1}$, $\frac{2}{1}$ } → we got $q=1$ $r=1$

If you have 3 values x, y, z

Then $\sum x$ means $x+y+z$

$\sum(x-y)$ means $(x-y)+(y-z)+(z-x)$

Que: if x, y, z are distinct real nos. Then value of $\left(\frac{1}{x-y}\right)^2+\left(\frac{1}{y-z}\right)^2+\left(\frac{1}{z-x}\right)^2$

let $\frac{1}{x-y}=a$ $\frac{1}{y-z}=b$ $\frac{1}{z-x}=c$

we know that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

$\Rightarrow (a+b+c)^2 = a^2+b^2+c^2+2abc(0)$

$\because$ $2ab+2bc+2ca = 2abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$\Rightarrow ab+bc+ca=0$.

$\Rightarrow \boxed{(a+b+c)^2} = a^2+b^2+c^2$

Answer

options are

① $\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2$

② $\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2 - 2\sum\underbrace{\frac{1}{(x-y)(y-z)}}_{=0}$

③ $\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2 + 2\sum\underbrace{\frac{1}{(x-y)(y-z)}}_{=0}$

④ $\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2 + \sum\underbrace{\frac{1}{(x-y)(y-z)}}_{=0}$

we know $\sum\frac{1}{(x-y)(y-z)} = \sum ab$

$= ab+bc+ac = 0.$

Hence all options are true.

Few more Identities

⑧ $(a+b)^3 = a^3+b^3+3ab^2+3a^2b$

$= a^3+b^3+3ab(a+b)$

⑨ $(a-b)^3 = a^3-b^3-3a^2b+3ab^2$

$= a^3-b^3-3ab(a-b)$

⑩ $a^3+b^3 = (a+b)^3-3ab(a+b)$

$= (a+b)(a^2+b^2-ab)$

⑪ $a^3-b^3 = (a-b)^3+3ab(a-b)$

$= (a-b)(a^2+b^2+ab)$

12. $a^4+a^2+1 = (a^2+a+1)(a^2-a+1)$

Add a^2 subtract a^2 from a^4+a^2+1

Que based on Identity 7

Que: Find the value of $\dfrac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$

since $(a-b)+(b-c)+(c-a) = 0$

$\Rightarrow (a-b)^3+(b-c)^3+(c-a)^3 = 3(a-b)(b-c)(c-a)$

Ans 3

Que: If $a^4+a^2+1 \in$ prime ; $a \in \mathbb{Z}$

Find number of values of a.

we know $a^4+a^2+1 = a^4+2a^2+1-a^2$

$= (a^2+1)^2-a^2$

$= (a^2+1+a)(a^2+1-a)$

$a^4+a^2+1 \in$ prime

$\Rightarrow a^2+1+a=1$ & $a^2+1-a = a^4+a^2+1$

or

$a^2+1+a = a^4+a^2+1$ & $a^2+1-a=1$

solve

$a^2+1+a=1$

$\Rightarrow \boxed{a=0 \text{ or } -1}$

put $a=0$

$\Rightarrow 1=1$ True

$a=-1$

$\Rightarrow 1+1+1 = 1+1+1$ True

solve

$a^2+1-a=1$

$\Rightarrow \boxed{a=0 \text{ or } 1}$

put values

$0+1+0 = 0+0+1$ True

$a=1$

$1+1+1 = 1+1+1$ True

$\Rightarrow$ possible values of a are $0, \boxed{1, -1}$

but a can not be 0.

if $a=0 \Rightarrow a^4+a^2+1 = 1 \notin$ prime

Ans a can be 1 or -1

Que: If $x = 2+\sqrt{3}$

Find $x+\frac{1}{x} = 4$

$x^2+\frac{1}{x^2} = 14$

$x^4+\frac{1}{x^4}$ H.W

$x^5+\frac{1}{x^5}$ H.W

$x = 2+\sqrt{3} \Rightarrow \frac{1}{x} = \frac{1}{2+\sqrt{3}} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

$= 2-\sqrt{3}$

$\Rightarrow x+\frac{1}{x} = 2+\sqrt{3}+2-\sqrt{3}$

$= 4$

$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$(4)^2 = x^2+\frac{1}{x^2}+2$

$x^2+\frac{1}{x^2} = 16-2 = 14$

for $x^5+\frac{1}{x^5}$

do $\left(x^3+\frac{1}{x^3}\right)\left(x^2+\frac{1}{x^2}\right)$

$= x^5+\frac{1}{x^5}+\left(x+\frac{1}{x}\right)$

and $x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$= (4)^3-3(4)$

$= (4)(16-3) = 4\times 13$

Que: $x = 1+\sqrt{2}$ Then $x^4 - x^3 - 2x^2 - 3x + 1 = ?$

$x^2 = 3 + 2\sqrt{2}$

$x^3 = (3+2\sqrt{2})(1+\sqrt{2})$

$= 3 + 5\sqrt{2} + 4$

$= 7 + 5\sqrt{2}$

$x^4 - x^3 - 2x^2 - 3x + 1$

$x^3(x-1) - 2(3+2\sqrt{2}) - 3(1+\sqrt{2}) + 1$

$(7+5\sqrt{2})(\sqrt{2}) - 6 - 4\sqrt{2}$

$- 3 - 3\sqrt{2} + 1$

$\cancel{7\sqrt{2}} + 10 - 8 - \cancel{7\sqrt{2}}$

$= \underline{\underline{2}}$ **Ans**

Polynomials: Algebraic Expression in which the power of variable is whole number.

* No variable term should be in denominator.

* **degree:** Highest power of variable in a polynomial. (for poly. with one variable involved.)

→ $f(x) = 3x + 4$ → Polynomial with 2 Terms.

→ $g(x) = 3 = 3x^0$ → Polynomial with 1 term.

→ $h(x) = \dfrac{1}{(x-2)}$ → $(x-2)^{-1}$ degree is not defined

(can apply binomial powers x will go on to ∞)

⇒ degree is not defined.

$\boxed{r(x) = 0}$ zero polynomial with degree non defined

$\because 0 = 0x^0 = 0x^1 = 0x^2 = \ldots$

so highest power is not defined

Polynomials

- Based on degree
  - deg 1 ⇒ linear polynomial
  - deg 2 ⇒ Quadratic
  - deg 3 ⇒ cubic
  - deg 4 ⇒ Biquadratic Polynomial
- Based on number of Terms
  - 1 Term ⇒ monomial
  - 2 Terms ⇒ binomial
  - 3 Terms ⇒ Trinomial
  - ⋮

* Polynomial does not mean Alg. Expression with many Terms $\because f(x) = 1$ is also a polynomial.

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* degree $= 0$

⇒ constant polynomial

E.g. $3x^0 = 3$.

Euclid's Division Algorithm

Dividend = Divisor × Quotient + remainder

degree of remainder < degree of Divisor.

Ans = Quotient

Divisor ⟌ Dividend

remainder

Que: divide $x^4 - 3x^3 + 4x^2 + 2$ by $x^2 - 6x + 7$.

Quotient: $x^2 + 3x + 15$

$$
\begin{array}{r|l}
x^2-6x+7 & x^4 - 3x^3 + 4x^2 + 2 \\
 & \underline{x^4 - 6x^3 + 7x^2} \\
 & \quad 3x^3 - 3x^2 + 2 \\
 & \quad \underline{3x^3 - 18x^2 + 21x} \\
 & \quad\quad 15x^2 - 21x + 2 \\
 & \quad\quad \underline{15x^2 - 90x + 105} \\
 & \quad\quad\quad 69x - 103
\end{array}
$$

$\frac{x^4}{x^2} = x^2$ ⇒ multiply x^2 with (x^2-6x+7) and write & subtract

$\frac{3x^3}{x^2} = 3x$

$3x(x^2 - 6x + 7) = 3x^3 - 18x^2 + 21x$

$69x - 103$ → remainder

⇒ $x^4 - 3x^3 + 4x^2 + 2 = (x^2 - 6x + 7)(x^2 + 3x + 15) + 69x - 103$.

Que: $f(x) = ax^7 + bx^3 + cx - 4$

$f(5) = 4$

$f(-5) = ?$

$f(5) = 4$

⇒ $a(5)^7 + b(5)^3 + c(5) - 4 = 4$

⇒ $a(5)^7 + b(5)^3 + c(5) = 8$

$f(-5) = a(-5)^7 + b(-5)^3 + c(-5) - 4$

$= -a \times 5^5 - b \times 5^3 - 5c - 4$

$= -(5^5 a + 5^3 b + 5c) - 4$

$= -(8) - 4 = \underline{\underline{-12}}$

Theorems based on Polynomials:

Remainder Theorem: If $f(x)$ is divided by $(x-a)$ then $f(a)$ is remainder

$\Rightarrow f(x) \rightarrow$ dividend, $(x-a) \rightarrow$ divisor.

Hence, we can write

$f(x) = (x-a) \times \text{Quotient} + \text{Remainder}.$

put $x=a$

$\Rightarrow f(a) = (a-a) \times \text{Quotient} + \text{Remainder}$

$\Rightarrow \boxed{f(a) = \text{Remainder}}$

Factor Theorem: If $f(a)=0$ Then $(x-a)$ is a factor of $f(x)$ & vice versa.

e.g. $f(x) = x^2 - 2x - 3 = (x-3)(x+1)$

$f(3) = 3^2 - 2(3) - 3 = 9-6-3 = 0$

$\Rightarrow (x-3)$ is one of the factor

Que: Remainder when $f(x)$ is divided by $(x-3)$ is 2 and when $f(x)$ is divided by $(x-1)$ rem. is 4. what is the remainder when $f(x)$ is divided by $(x-1)(x-3)$?

① $\Rightarrow f(x) = (x-3)\,\text{Quo} + 2$

② $\rightarrow f(x) = (x-1)\,Q' + 4$

deg. of rem < deg of divisor
$\Rightarrow$ deg rem < deg of $(x-3)$
$\Rightarrow$ deg (rem) < 1
$\Rightarrow$ deg (rem) = 0,
= constant poly.

$f(x) = (x-1)(x-3)\,Q'' + \text{remainder}$

deg (rem) < deg $((x-1)(x-2))$

deg (rem) < 2

Take remainder = $ax+b$.

③ $\rightarrow f(x) = (x-1)(x-3)\,Q'' + ax+b.$

Now, from Equation ① $f(3)=2$

from ② $f(1) = 4$

put values in ③

$\Rightarrow f(3) = 0 + a(3)+b$

$\Rightarrow \boxed{2 = a(3)+b}$ ✓

from ② $f(1)=4$ put $x=1$ in ③

$f(1) = 0 + a(1)+b$

$\boxed{4 = a+b}$ ✓

after solving both $a = -1$ $b = 5$

Ans $\boxed{-x+5}$ is the required remainder

Exponents and Powers

a^b: $b \rightarrow$ power, $\rightarrow$ Exponent; $a \rightarrow$ Base

* If power is non Integral rational then Exponent is called surd. (fractional powers involved)

eg $2^{1/2}$, $2^{3/4}$

Laws of Exponents

① $x^{1/n} = \sqrt[n]{x}$

② $(a^m)^n = a^{mn}$ $(a \neq 0)$

eg $(2^3)^2 = 2^6$

$(2^3)^2 \neq 2^{3^2}$

$2^{3^2} = 2^9 = 512$

$(2^3)^2 = 2^6 = 64$

③ $a^m a^n = a^{m+n}$ eg $2^3 \times 2 = 2^4$

④ $\dfrac{a^m}{a^n} = a^{m-n}$ e.g. $\dfrac{2^3}{2} = 2^2$

⑤ $a^0 = 1$

if $m=n$

$\Rightarrow \dfrac{a^m}{a^m} = a^{m-m}$

$1 = a^0$

Ratio And Proportion

$a:b = \dfrac{a}{b}$

$a:b = c:d \Rightarrow \dfrac{a}{b} = \dfrac{c}{d} \Rightarrow a:b::c:d$

* if $\dfrac{a}{b} = \dfrac{c}{d}$ $\Rightarrow \dfrac{a}{c} = \dfrac{b}{d}$ (called Alternando)

* $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{b}{a} = \dfrac{d}{c}$ (called invertendo)

* $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a}{b}+1 = \dfrac{c}{d}+1 \Rightarrow \dfrac{a+b}{b} = \dfrac{c+d}{d}$ (called componendo)

* $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a}{b}-1 = \dfrac{c}{d}-1 \Rightarrow \dfrac{a-b}{b} = \dfrac{c-d}{d}$ (called dividendo)

* $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$ or $\dfrac{b+a}{b-a} = \dfrac{d+c}{d-c}$ (componendo And dividendo rule)

Que: Solve for x if

$$\frac{3x^4+x^2-2x-3}{3x^4-x^2+2x+3} = \frac{5x^4+2x^2-7x+3}{5x^4-2x^2+7x-3}$$

$$\Rightarrow \frac{(3x^4+x^2-2x-3)+(3x^4-x^2+2x+3)}{(3x^4+x^2-2x-3)-(3x^4-x^2+2x+3)} = \frac{(5x^4+2x^2-7x+3)+(5x^4-2x^2+7x-3)}{(5x^4+2x^2-7x+3)-(5x^4-2x^2+7x-3)}$$

$$\Rightarrow \frac{\cancel{2}(3x^4)}{\cancel{2}(x^2-2x-3)} = \frac{\cancel{2}(5x^4)}{\cancel{2}(2x^2-7x+3)}$$

$$\frac{3x^4}{x^2-2x-3} = \frac{5x^4}{2x^2-7x+3}$$

$$x^4(6x^2-21x+9) = (5x^2-10x-15)x^4$$

$$x^4(x^2-11x+24) = 0.$$

$$x = \frac{11 \pm \sqrt{121-96}}{2} \quad \text{or } x^4 = 0.$$

$$x = \frac{11 \pm \sqrt{25}}{2}$$

$$= \frac{11 \pm 5}{2} = \underline{8 \text{ or } 3}$$

Ans $0, 8, 3$

* never cancel variables

If you would have cancelled x^4 by x^4 $\Rightarrow$ you will not get $x=0$ as a solution

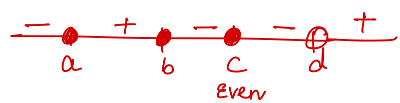
Wavy Curve Method

1. Make RHS or LHS zero
2. Factorize all polynomials to convert it in product of linears
3. Make coefficients of x positive
4. Plot roots of all factors on number line.
5. Exclude denominator roots. Include numerator roots only if Equality is possible
6. Start from extreme right mark signs as $+ - + - + \ldots$

start from + & then alternate signs

* if any factor has even power $\Rightarrow$ don't change the sign.

Que: $\dfrac{(x-a)(x-b)(x-c)^2}{(x-d)} \le 0$



Ans $(-\infty, a] \cup [b, c] \cup [c, d)$

$\Rightarrow (-\infty, a] \cup [b, d)$

* For modulus Questions.

Find roots of modulus

plot roots on number line

see sign of modulus for all cases and solve & check the range of x.

* Practice | do all questions discussed in class.