EXERCISE-1 (SPECIAL DPP)

SPECIAL DPP-1

Q.1 $\dfrac{1}{\log_{\sqrt{bc}} abc} + \dfrac{1}{\log_{\sqrt{ca}} abc} + \dfrac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(A) 1/2 (B) 1 (C) 2 (D) 4

Q.2 If $5x^{\log_2 3} + 3^{\log_2 x} = 162$ then logarithm of x to the base 4 has the value equal to

(A) 2 (B) 1 (C) – 1 (D) 3/2

Q.3 If $\log(x+y) = \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y$, then

(A) $x+y=0$ (B) $xy=1$ (C) $x^2+xy+y^2=0$ (D) $x-y=0$

Q.4 If $\log_2(\log_3(\log_4 x)) = 0$, $\log_4(\log_3(\log_2 y)) = 0$ and $\log_3(\log_4(\log_2 z)) = 0$, then the correct option is

(A) $x>y>z$ (B) $x>z>y$ (C) $z>x>y$ (D) $z>y>z$

Q.5. The value of $\log_2\left(\dfrac{1}{7^{\log_7 0.125}}\right)$, is

(A) 1 (B) 2 (C) 3 (D) 4

Q.6 Let x satisfies the equation $\log_3(\log_9 x) = \log_9(\log_3 x)$ then the product of the digits in x is

(A) 9 (B) 18 (C) 36 (D) 8

Q.7 If $\log_3(\log_2 a) + \log_{\frac{1}{3}}\left(\log_{\frac{1}{2}} b\right) = 1$, then the value of ab^3 is

(A) 9 (B) 3 (C) 1 (D) $\frac{1}{3}$

Q.8 The value of $\log_{\frac{4}{3}}\left(\dfrac{56+\sqrt{56+\sqrt{56+\sqrt{56+\ldots\ldots\infty}}}}{\sqrt{64\sqrt{64\sqrt{64\ldots\ldots\infty}}}}\right)$ is equal to

(A) 0 (B) 2 (C) 3 (D) 4

Q.9 The value of $\log_{\frac{1}{6}} 2 \cdot \log_5 36 \cdot \log_{17} 125 \cdot \log_{\frac{1}{\sqrt{2}}} 17$, is equal to

(A) – 3 (B) – 6 (C) 6 (D) 12

Q.10

	Column-I		Column-II
(A)	$\log_2 x = -\log_{\frac{1}{2}} 7$, then the value of x is	(P)	$\frac{1}{49}$
(B)	$\log_3 y = \frac{-1}{\log_3 2}$, then the value of y is	(Q)	$\frac{1}{36}$
(C)	$\log_{\frac{1}{4}} z = \log_2 6$, then the value of z is	(R)	$\frac{1}{27}$
(D)	$\log_{\frac{1}{9}} w = \log_3 7$, then the value of w is	(S)	7
		(T)	$\frac{1}{6}$

SPECIAL DPP-2

Q.1 Product of all the solution of equation $x^{\log_{10} x} = \left(100 + 2^{\sqrt{\log_2 3}} - 3^{\sqrt{\log_3 2}}\right)x$ is

(A) $\frac{1}{10}$ (B) 1 (C) 10 (D) 100

Q.2 If $\log_7 2 = m$, then the value of $\log_{49} 28$ is

(A) $2(1+2m)$ (B) $\frac{1+2m}{2}$ (C) $\frac{2}{1+2m}$ (D) $1+m$

Q.3 If $P = \log_5(\log_5 3)$ and $3^{C+5^{-P}} = 405$ then C is equal to

(A) 3 (B) 4 (C) 81 (D) 5

Q.4 If $\frac{a+\log_4 3}{a+\log_2 3} = \frac{a+\log_8 3}{a+\log_4 3} = b$, then b is equal to

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{3}{2}$

Q.5 Let x, y and z be positive real numbers such that $x^{\log_2 7} = 8$, $y^{\log_3 5} = 81$ and $z^{\log_5 216} = \sqrt[3]{5}$.

The value of $x^{(\log_2 7)^2} + y^{(\log_3 5)^2} + z^{(\log_5 216)^2}$, is

(A) 526 (B) 750 (C) 874 (D) 974

Q.6 If $x = 500$, $y = 100$ and $z = 5050$, then the value of $\left(\log_{xyz} x^z\right)\left(1+\log_x yz\right)$ is equal to

(A) 500 (B) 100 (C) 5050 (D) 10

Q.7 Suppose n be an integer greater than 1, let $a_n = \dfrac{1}{\log_n 2002}$. Suppose $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then $(b - c)$ equals

(A) $\dfrac{1}{1001}$ (B) $\dfrac{1}{1002}$ (C) -1 (D) -2

Q.8 If $L = \sum_{r=7}^{2400} \log_7\left(\dfrac{r+1}{r}\right)$, $M = \prod_{r=2}^{1023} \log_r(r+1)$ and $N = \sum_{r=2}^{2011}\left(\dfrac{1}{\log_r p}\right)$

where $p = (1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \ldots\ldots\ldots\ldots \cdot 2011)$, then

(A) $L + M = 13$ (B) $M^2 + N^2 = 101$ (C) $L - M + N = 6$ (D) $LMN = 30$

Q.9 Which of the following vanishes?

(A) $\log\tan 1^\circ . \log\tan 2^\circ . \log\tan 3^\circ \ldots\ldots. \log\tan 89^\circ$

(B) $\log\sin 1^\circ . \log\sin 2^\circ . \log\sin 3^\circ \ldots\ldots. \log\sin 90^\circ$

(C) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

(D) $\log\tan 1^\circ + \log\tan 2^\circ + \log\tan 3^\circ + \ldots\ldots + \log\tan 89^\circ$.

Q.10

	Column-I		Column-II
(A)	Given $x > 1$ and $\log_x\left(x^{x^2}\right) + \log_x\left(x^{-5x}\right) = \log_x\left(\dfrac{1}{x^6}\right)$. The sum of all values of x that satisfying the equation, is	(P)	2
(B)	Let $0 < x < \pi$, $3^{\tan x} = 27^{\sin x}$, then the value of $\sec x$, is	(Q)	3
(C)	Let $a = x - 2$ and $b = x - 4$. The value of x satisfying the equation $\dfrac{\log_a(x-3)\log_b(x+10)}{\log_b(x-3)} = 2$, is	(R)	4
		(S)	5
(D)	The real values of x so that all terms are real and satisfy the equation $\sqrt{2x} = \sqrt{x+7} - 1$, is	(T)	6

Q.11 If $\prod_{r=3}^{26} \log_r(r+1) = 3^x$, then find the value of x.

SPECIAL DPP-3

Q.1 If $10^{\log_a(\log_b(\log_c x))} = 1$ and $10^{(\log_b(\log_c(\log_a x)))} = 1$ then, a is equal to

(A) $\dfrac{a}{b}$ (B) $c^{a/b}$ (C) ab (D) $c^{b/c}$

Q.2 If $x \in R$, then number of real solution of the equation $2^x + 2^{-x} = \log_5 24$ is

(A) 0 (B) 1 (C) 2 (D) more than 2

Q.3 If $x \geq y > 1$ then the maximum value of $\log_x\left(\dfrac{x}{y}\right) + \log_y\left(\dfrac{y}{x}\right)$ is equal to

(A) -2 (B) 0 (C) 2 (D) 4

Q.4 If $\log_5(3^x - 4^y) = 3$ and $3^{\frac{x}{2}} - 2^y = 5$, then $\frac{x}{y}$ is equal to

(A) $\frac{2(\log_2 5)-2}{1+\log_2 5}$ (B) $\frac{(\log_3 5)+2}{1+\log_2 5}$ (C) $\frac{2(\log_3 5)+2}{1+\log_2 5}$ (D) $\frac{2(\log_3 5)+1}{1+\log_2 5}$

Q.5 Let $x = 4^{\log_2 \sqrt{9^{k-1}+7}}$ and $y = \frac{1}{32^{\log_2 \sqrt[5]{3^{k-1}+1}}}$ and $xy = 4$, then the sum of the cubes of the real value(s) of k is

(A) 1 (B) 5 (C) 8 (D) 9

Q.6 Number of real solution(s) of the equation $9^{\log_3(\ln x)} = \ln x - (\ln^2 x) + 1$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

Q.7 The value of the expression $\frac{1}{\log_4(18)} + \frac{1}{2\log_6(3)+\log_6(2)} + \frac{5}{\log_3(18)}$, is

(A) odd (B) an irrational

(C) even composite (D) twin prime with 5

Q.8 Which of the following real numbers is(are) non-positive?

(A) $\log_{0.\bar{3}}\left(\frac{\sqrt{5}+2}{\sqrt{5}-2}\right)$ (B) $\log_7\left(\sqrt{83}-9\right)$

(C) $\log_{2-\sqrt{3}}\left(\sqrt{2}+1\right)$ (D) $\log_2\sqrt{9\cdot\sqrt[3]{27^{\frac{-5}{3}}\cdot 243^{\frac{-7}{5}}}}$

Q.9 Given that $\frac{1}{\log_7 2} + \frac{1}{\log_9 4} = x$, then which of the following will divide $(4)^x$?

(A) 3 (B) 7 (C) 9 (D) 21

Q.10

	Column-I		Column-II
(A)	The value of expression $3^{\sqrt{\log_{27}8}} - 2^{\sqrt{\log_{32}243}} - 5^{\sqrt{\log_{625}81}} + 3^{\sqrt{\log_9 25}} + \sqrt{2}^{\log_2 9} + 3^{\log_4 25} - 5^{\log_4 9}$, is less than	(P)	2
(B)	The value of x satisfying the equation $2^{\log_2 e^{\ln 5^{\log_5 7^{\log_7 10^{\log_{10}(8x-3)}}}}} = 13$, is	(Q)	3
		(R)	4
(C)	The number $N = \left(\frac{1}{\log_2 \pi} + \frac{1}{\log_6 \pi}\right)$ is less than	(S)	5
(D)	Let $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)\left(1+9^{\log_3 8}\right)^{\log_{65} 5}$ then $(l+m)$ is divisible by	(T)	6

Q.11 If $\log_2(\log_8 x) = \log_8(\log_2 x)$, find the value of $(\log_2 x)^2$.

SPECIAL DPP-4

Q.1 If $\log_{30}(3)=\alpha$ and $\log_{30}(5)=\beta$, then $\log_{30}(8)$ is equal to
(A) $3(1+\alpha-\beta)$ (B) $3(1+\alpha+\beta)$ (C) $3(\alpha+\beta)$ (D) $3(1-\alpha-\beta)$

Q.2 The equation $(\log_{10}x+2)^3+(\log_{10}x-1)^3=(2\log_{10}x+1)^3$ has
(A) no natural solution
(B) two rational solutions
(C) no prime solution.
(D) one irrational solution.

Q.3

	Column-I		Column-II
(A)	If $\log_b 3=4$ and $\log_{b^2} 27=\frac{3a}{2}$, then the value of (a^2-b^4) is equal to	(P)	2
		(Q)	3
(B)	If number of digits in 12^{11} is 'd', and number of cyphers after decimal before a significant figure starts in $(0.2)^9$ is 'c', then $(d-c)$ is equal to	(R)	6
(C)	If $N=\text{antilog}_3\left(\log_6\left(\text{antilog}_{\sqrt{5}}(\log_5 1296)\right)\right)$, then the characteristic of $\log N$ to the base 2, is equal to	(S)	13

Q.4

	Column-I		Column-II
(A)	If $\left(\log_2\left(\log_{\frac{1}{2}}(\log_2 a)\right)\right)^2+\left(\log_3\left(\log_{\frac{1}{3}}(\log_3 b)\right)\right)^2=0$, then (a^2+b^3) is greater than	(P)	1
(B)	If $11^{\log_{10}x}=242-x^{\log_{10}11}$ then x is coprime with	(Q)	2
(C)	If $p=\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}$, then the value of (p^3+3p+1) is less than	(R)	3
(D)	If $\log_{\frac{\sqrt{x}}{2}}\left(\log_9\left(\sqrt{3}+\sqrt{12}\right)\right)=2$, then the value of x is twin prime with	(S)	4
		(T)	5

Q.5 Given that $\log 2=0.301$, find the number of digits before decimal in the solution to the equation $\log_5(\log_4(\log_3(\log_2 x)))=0$.

Q.6 Let $N=\log_3\left(\dfrac{\log_3 3^{3^{3^3}}}{\log_{3^3} 3^{3^3}}\right)$, then find the sum of digits in N.

Q.7 Find the sum of all integral values of x satisfying $(\log_5 x)^2+\log_{5x}\left(\dfrac{5}{x}\right)=1$.

SPECIAL DPP-5

Q.1 If A is the number of integers whose logarithms to the base 10 have characteristic 11 and B the number of integers, the logarithm of whose reciprocals to the base 10 have characteristic -4, then the value of $(\log_{10}A - \log_{10}B)$ is equal to

(A) -7 (B) 7 (C) 8 (D) 9

Q.2 The expression $\dfrac{\left(\log_{\frac{a}{b}} p\right)^2+\left(\log_{\frac{b}{c}} p\right)^2+\left(\log_{\frac{c}{a}} p\right)^2}{\left(\log_{\frac{a}{b}} p+\log_{\frac{b}{c}} p+\log_{\frac{c}{a}} p\right)^2}$, wherever defined, simplifies to

(A) 1 (B) 2 (C) 3 (D) 4

Q.3 Number of ordered pair(s) of (x, y) satisfying the system of equations,

$\log_2 xy = 5$ and $\log_{1/2}\dfrac{x}{y} = 1$ is :

(A) one (B) two (C) three (D) four

Paragraph for question nos. 4 to 6

$\log_M N = \alpha + \beta$, where α is an integer & $\beta \in [0, 1)$

Q.4 If M & α are prime & $\alpha + M = 7$ then the greatest integral value of N is

(A) 64 (B) 63 (C) 125 (D) 124

Q.5 If M & α are twin prime & $\alpha + M = 8$ then the greatest integral value of N is

(A) 624 (B) 625 (C) 728 (D) 729

Q.6 If M & α are relative prime & $\alpha + M = 7$ then minimum integral value of N is

(A) 25 (B) 32 (C) 6 (D) 81

Q.7

	Column-I		Column-II
(A)	The expression $x = \log_2 \log_9 \sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$ simplifies to	(P)	an integer
(B)	The number $N = 2^{(\log_2 3\cdot\log_3 4\cdot\log_4 5.........\cdot\log_{99}100)}$ simplifies to	(Q)	a prime
(C)	The expression $\dfrac{1}{\log_5 3}+\dfrac{1}{\log_6 3}-\dfrac{1}{\log_{10} 3}$ simplifies to	(R)	a natural
(D)	The number $N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$ simplifies to	(S)	a composite

Q.8 If sum of the integral values of x satisfying the equation $|x-1|^{\log^2 x-\log x^2}=|x-1|^3$ is N, then find characteristic of logarithm of N to the base 5.

Q.9 If x satisfies the equation $\log_{125} x^3 - 3\sqrt{\log_{25} x^2}=4$, then find the number of digits in x. [Use: $\log 2 = 0.3010$]

Q.10 Find the sum of all possible values of x satisfying the equation

$$\sqrt{x^2-4x+4}=(\log_2 9)\left(\log_3 \sqrt{5}\right)(\log_{25} 256)$$

EXERCISE-2

Q.1 Let A denotes the value of $\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right)+\log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$

when $a = 43$ and $b = 57$

and B denotes the value of the expression $\left(2^{\log_6 18}\right)\cdot\left(3^{\log_6 3}\right)$.

Find the value of $(A \cdot B)$.

Q.2(a) If $x = \log_3 4$ and $y = \log_5 3$, find the value of $\log_3 10$ and $\log_3\left(\frac{6}{5}\right)$ in terms of x and y.

(b) If $k^{\log_2 5} = 16$, find the value of $k^{(\log_2 5)^2}$.

Q.3 Prove that: (a) $\frac{\log_2 24}{\log_{96} 2}-\frac{\log_2 192}{\log_{12} 2}=3$; (b) $\frac{\log_3 135}{\log_{15} 3}-\frac{\log_3 5}{\log_{405} 3}=3$

Q.4 Given that $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2}(8)=\frac{2}{s^3+1}$. Write $\log_2\frac{a^2b^5}{c^4}$ as a function of 's' $(a, b, c > 0,\ c \neq 1)$.

Q.5 Simplify the following:

(a) $4^{5\log_{4\sqrt{2}}\left(3-\sqrt{6}\right)-6\log_8\left(\sqrt{3}-\sqrt{2}\right)}$

(b) $\frac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\cdot\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}}-(125)^{\log_{25} 6}\right)$

(c) $5^{\log_{1/5}\left(\frac{1}{2}\right)}+\log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}}+\log_{1/2}\frac{1}{10+2\sqrt{21}}$.

(d) $49^{\left(1-\log_7 2\right)}+5^{-\log_5 4}$

Q.6 Find the square of the sum of the roots of the equation

$$\log_3 x\cdot\log_4 x\cdot\log_5 x=\log_3 x\cdot\log_4 x+\log_4 x\cdot\log_5 x+\log_5 x\cdot\log_3 x.$$

Q.7 Let a and b be real numbers greater than 1 for which there exists a positive real number c, different from 1, such that

$$2(\log_a c + \log_b c) = 9\log_{ab} c.$$ Find the largest possible value of $\log_a b$.

Q.8 If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$.

Find the value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$.

Q.9 (a) $\log(\log x) + \log(\log x^3 - 2) = 0$; where base of log is 10 everywhere.
(b) $\log_x 2 \cdot \log_{2x} 2 = \log_{4x} 2$
(c) $5^{\log x} + 5\, x^{\log 5} = 3\ (a > 0)$; where base of log is a.
(d) $x^{\log x + 4} = 32$, where base of logarithm is 2.

Q.10 Solve the system of equations:

$$\log_a x \ \log_a (xyz) = 48$$
$$\log_a y \ \log_a (xyz) = 12,\ a > 0,\ a \neq 1.$$
$$\log_a z \ \log_a (xyz) = 84$$

Q.11 $\log_{x+1}(x^2 + x - 6)^2 = 4$

Q.12 $\log_5 120 + (x-3) - 2 \cdot \log_5(1 - 5^{x-3}) = -\log_5(0.2 - 5^{x-4})$

Q.13 $\log 4 + \left(1 + \frac{1}{2x}\right)\log 3 = \log\left(\sqrt[x]{3} + 27\right)$.

Q.14 If 'x' and 'y' are real numbers such that, $2\log(2y - 3x) = \log x + \log y$, find $\frac{x}{y}$.

Q.15 Find the sum of all solutions of the equation $3^{(\log_9 x)^2 - \frac{9}{2}\log_9 x + 5} = 3\sqrt{3}$.

Q.16 Positive numbers x, y and z satisfy $xyz = 10^{81}$ and $(\log_{10}x)(\log_{10}yz) + (\log_{10}y)(\log_{10}z) = 468$.
Find the value of $(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2$.

Q.17
(a) Given: $\log_{10}34.56 = 1.5386$, find $\log_{10}3.456$; $\log_{10}0.3456$ & $\log_{10}0.003456$.
(b) Find the number of positive integers which have the characteristic 3, when the base of the logarithm is 7.
(c) If $\log_{10}2 = 0.3010$ & $\log_{10}3 = 0.4771$, find the value of $\log_{10}(2.25)$.
(d) If $N = \text{antilog}_3\left(\log_6\left(\text{antilog}_{\sqrt{5}}(\log_5 1296)\right)\right)$, then find the characteristic of $\log N$ to the base 2.
(e) Let L be the number of digits in 3^{40} and M be the number of zeroes in 3^{-40} after decimal before a significant digit, then find $(L - M)$.

Q.18 If $\log_{3x} 45 = \log_{4x} 40\sqrt{3}$ then find the characteristic of $\log x^3$ to the base 7.

Q.19 If (x_1, y_1) and (x_2, y_2) are the solution of the system of equation

$$\log_{225}(x) + \log_{64}(y) = 4,$$
$$\log_x(225) - \log_y(64) = 1,$$

then find the value of $\log_{30}(x_1y_1x_2y_2)$.

Q.20 Solve for x : $\log^2(4-x) + \log(4-x) \cdot \log\left(x+\frac{1}{2}\right) - 2\log^2\left(x+\frac{1}{2}\right) = 0.$

EXERCISE-3

(JEE-ADVANCED Previous Year's Questions)

Q.1 Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}.$$

Then x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6 [JEE 2011, 3]

Q.2 The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots\cdots}}}\right)$ is [JEE 2012, 4]

Q.3 If $3^x = 4^{x-1}$, then x =

(A) $\frac{2\log_3 2}{2\log_3 2-1}$ (B) $\frac{2}{2-\log_2 3}$ (C) $\frac{1}{1-\log_4 3}$ (D) $\frac{2\log_2 3}{2\log_2 3-1}$

[JEE ADV. 2013, 3 (–1)]

EXERCISE-4

(Potential Problems Based on CBSE)

Q.1 Simplify:

(i) $\log\left(\frac{1}{x}+\frac{1}{y}\right) - \log(x+y) + \log x + \log y.$ (ii) $\frac{\log x^3z - \log zy^3}{\log x - \log y}.$

Q.2 Show that :

(i) $\log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80} = 1$

(ii) $\log\frac{81}{8} - 2\log\frac{3}{2} + 3\log\frac{2}{3} + \log\frac{3}{4} = 0$

Q.3 Solve for x :

(i) $\log(x+1) + \log(x-1) = \log 99$ (ii) $\frac{\log 16}{\log 4} = \log x$

(iii) $\log(3x-2) + \log(3x+2) = 5\log 2$ (iv) $\log 5 + \log(5x+1) = \log(x+5) + 1$

Q.4 Simplify each of the following :

(i) $\log_8 \sqrt{6} + \log_8\left(\sqrt{\frac{2}{3}}\right) - \log_8(\log_3 9)$ (ii) $\log_2 [\log_2\{\log_3 (\log_3 27^3)\}]$

Q.5 (i) If $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$, show that $a^2 + b^2 = 6ab$

(ii) If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$, show that $\frac{a+b}{2} = \sqrt{ab}$ and $a^2 + b^2 = 2ab$.

(iii) If $a^2 + b^2 = 7ab$, prove that $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$.

Q.6 (i) If $a = \log_{24} 12$, $b = \log_{36} 24$ and $c = \log_{48} 36$, show that $1 + abc = 2bc$.

(ii) If $x = \log \frac{2}{3}$, $y = \log \frac{3}{5}$ and $z = \log \frac{5}{2}$, show that $x + y + z = 0$

(iii) If $y = x^{\frac{1}{m}}$, show that $m = \log_y x$.

Q.7 Prove that

(i) $\log_3 \log_2 \log_{\sqrt{3}} 81 = 1$

(ii) $\log_a x \times \log_b y = \log_b x \times \log_a y$

(iii) $\log_2 \log_2 \log_2 16 = 1$

(iv) $\log_a x = \log_b x \times \log_c b \times \ldots\ldots. \times \log_n m \times \log_a n$

Q.8 Simplify:

$$\frac{\log_9 11}{\log_5 13} \div \frac{\log_3 11}{\log_{\sqrt{5}} 13}$$

Q.9 (i) If $\log_4 10 = x$, $\log_2 20 = y$ and $\log_5 8 = z$.

Prove that $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$.

(ii) If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$.

Prove that $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$.

Q.10 (i) Prove that $\frac{1}{1+\log_b a + \log_b c} + \frac{1}{1+\log_c a + \log_c b} + \frac{1}{1+\log_a b + \log_a c} = 1$.

(ii) Show that $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots\ldots.. + \frac{1}{\log_{43} n} = \frac{1}{\log_{43!} n}$.

EXERCISE-5 (Rank Booster)

Paragraph for question nos. 1 to 3

Let A denotes the sum of the roots of the equation $\frac{1}{5-4\log_4 x}+\frac{4}{1+\log_4 x}=3$.

B denotes the value of the product of m and n, if $2^m=3$ and $3^n=4$.

C denotes the sum of the integral roots of the equation $\log_{3x}\left(\frac{3}{x}\right)+(\log_3 x)^2=1$.

Q.1 The value of A + B equals
(A) 10 (B) 6 (C) 8 (D) 4

Q.2 The value of B + C equals
(A) 6 (B) 2 (C) 4 (D) 8

Q.3 The value of A + C ÷ B equals
(A) 5 (B) 8 (C) 7 (D) 4

Q.4 Find the sum of all possible values of x satisfying simultaneous the equations

$$\log^2 x-3\log x=\log(x^2)-4 \text{ and } \log^2(100x)+\log^2(10x)=14+\log\left(\frac{1}{x}\right).$$

[Note : Assume base of logarithm is 10.]

Q.5 Let k be the unique positive value satisfying the equation

$$(4k)^{\log 2}-(9k)^{\log 3}=0, \text{ then find the value of } (72k).$$

Q.6 Given $\frac{\log_2\left(\frac{b^3}{8}\right)}{\log_3\left(\frac{27}{a^2}\right)}=1$ and $\log_3\left(\frac{9}{a}\right)=\log_2\left(\frac{b}{4}\right)$. If the largest single digit number which can divide the value of $\left(\frac{a}{b}\right)$ is m, then find the value of m.

Q.7 If $\log_3 M=a_1+b_1$ and $\log_5 M=a_2+b_2$ where $a_1, a_2 \in N$ and $b_1, b_2 \in [0, 1)$. If $a_1 a_2=6$, then find the number of integral values of M.

Q.8 Solve : $\log_3\left(\sqrt{x}+\left|\sqrt{x}-1\right|\right)=\log_9\left(4\sqrt{x}-3+4\left|\sqrt{x}-1\right|\right)$

Q.9 Prove that : $2^{\left(\sqrt{\log_a \sqrt[4]{ab}+\log_b \sqrt[4]{ab}}-\sqrt{\log_a \sqrt[4]{\frac{b}{a}}+\log_b \sqrt[4]{\frac{a}{b}}}\right)\cdot\sqrt{\log_a b}}=\begin{cases}2 & \text{if } b\ge a>1\\ 2^{\log_a b} & \text{if } 1<b<a\end{cases}$

Q.10 Find the value of x satisfying the equation,

$$\sqrt{\left(\log_3\sqrt[3]{3x}+\log_x\sqrt[3]{3x}\right)\cdot\log_3 x^3}+\sqrt{\left(\log_3\sqrt[3]{x/3}+\log_x\sqrt[3]{3/x}\right)\cdot\log_3 x^3}=2.$$

ANSWER KEY

EXERCISE-1

SPECIAL DPP-1

Q.1 B Q.2 D Q.3 D Q.4 B Q.5 C Q.6 D Q.7 C
Q.8 A Q.9 D Q.10 (A) S : (B) R ; (C) Q ; (D) P

SPECIAL DPP-2

Q.1 C Q.2 B Q.3 B Q.4 C Q.5 D Q.6 C Q.7 C
Q.8 A, B, D Q.9 A, B, C, D Q.10 (A) S; (B) Q; (C) T; (D) P Q.11 0001

SPECIAL DPP-3

Q.1 D Q.2 A Q.3 B Q.4 C Q.5 D Q.6 B Q.7 A, D
Q.8 A, B, C, D Q.9 A, B, C, D Q.10 (A) R, S, T; (B) P; (C) Q, R, S, T; (D) P, R, S
Q.11 27

SPECIAL DPP-4

Q.1 D Q.2 B, C, D Q.3 (A) S, (B) R, (C) Q
Q.4 (A) P, Q, R, S ; (B) P, R; (C) S, T; (D) T Q.5 0025 Q.6 0007 Q.7 6

SPECIAL DPP-5

Q.1 C Q.2 A Q.3 B Q.4 D Q.5 C Q.6 C
Q.7 (A) P, (B) P, R, S, (C) P, R, (D) P, Q, R Q.8 0004 Q.9 0012 Q.10 0004

EXERCISE-2

Q.1 12 Q.2 (a) $\frac{xy+2}{2y}, \frac{xy+2y-2}{2y}$; (b) 625 Q.4 $2s + 10s^2 - 3(s^3 + 1)$

Q.5 (a) 9, (b) 1, (c) 6, (d) $\frac{25}{2}$ Q.6 3721 Q.7 2 Q.8 469

Q.9 (a) x=10 (b) $x = 2^{\sqrt{2}}$ or $2^{-\sqrt{2}}$ (c) $x = 2^{-\log a}$ where base of log is 5, (d) x = 2 or 1/32

Q.10 (a^4, a, a^7) or $\left(\frac{1}{a^4}, \frac{1}{a}, \frac{1}{a^7}\right)$ Q.11 x = 1 Q.12 x = 1 Q.13 $x \in \phi$ Q.14 $\frac{4}{9}$

Q.15 2196 Q.16 5625 Q.17 (a) 0.5386; $\bar{1}.5386$; $\bar{3}.5386$ (b) 2058 (c) 0.3522 (d) 3 ; (e) 1

Q.18 2 Q.19 12 Q.20 $\left\{0, \frac{7}{4}, \frac{3+\sqrt{24}}{2}\right\}$

EXERCISE-3

Q.1 C Q.2 4 Q.3 ABC

EXERCISE-4

Q.1 (i) 0; (ii) 3 Q.3 (i) x = 10 (ii) x = 100 (iii) x = 2 (iv) x = 3
Q.4 (i) 0; (ii) 0 Q.8 1

EXERCISE-5

Q.1 C Q.2 A Q.3 B Q.4 10 Q.5 0002 Q.6 9 Q.7 54
Q.8 $[0, 1] \cup \{4\}$; Q.10 $x \in [1/3, 3] - \{1\}$