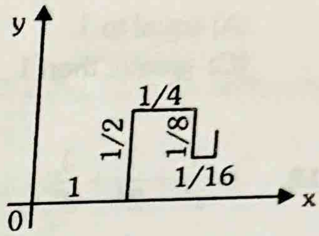
SEQUENCE AND SERIES

YEAR LONG REVISION EXERCISE
Not To Be Discussed in Class

SECTION - 1 : SINGLE CHOICE CORRECT QUESTIONS

1. A particle begins at the origin and moves successively in the following manner as shown, 1 unit to the right, $\frac{1}{2}$ unit up, $\frac{1}{4}$ unit to the right, $\frac{1}{8}$ unit down, $\frac{1}{16}$ unit to the right etc. The length of each move is half the length of the previous move and movement continues in the 'zigzag' manner indefinitely. The co-ordinates of the point to which the 'zigzag' converges is -



(A) $\left(\frac{4}{3},\frac{2}{3}\right)$ (B) $\left(\frac{4}{3},\frac{2}{5}\right)$ (C) $\left(\frac{3}{2},\frac{2}{3}\right)$ (D) $\left(2,\frac{2}{5}\right)$

2. If p, q, r in harmonic progression and p & r be different having same sign then the roots of the equation $px^2 + qx + r = 0$ are -
(A) real and equal (B) real and distinct (C) irrational (D) imaginary

3. If $\ln(a + c)$, $\ln(c - a)$, $\ln(a - 2b + c)$ are in A.P., then :
(A) a, b, c are in A.P. (B) a^2, b^2, c^2 are in A.P (C) a, b, c are in G.P. (D) a, b, c are in H.P.

4. The sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals. Then bc^2, ca^2 and ab^2 are in -
(A) AP (B) GP (C) HP (D) None of these

5. In quadratic equation $ax^2 + bx + c = 0$, if α, β are roots of equation, $\Delta = b^2 - 4ac$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P. then **[JEE 2005 (screening)]**
(A) $\Delta \neq 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

6. The sum $\sum_{k=1}^{\infty} \frac{2^{k+2}}{3^k}$ equal to
(A) 12 (B) 8 (C) 6 (D) 4

7. The arithmetic mean of the nine numbers in the given set {9, 99, 999, ... 999999999} is a 9 digit number N, all whose digits are distinct. The number N does not contain the digit
(A) 0 (B) 2 (C) 5 (D) 9

8. If $\frac{1+3+5+.....\text{upto } n \text{ terms}}{4+7+10+....\text{upto } n \text{ terms}} = \frac{20}{7\log_{10} x}$ and $n = \log_{10} x + \log_{10} x^{\frac{1}{2}} + \log_{10} x^{\frac{1}{4}} + \log_{10} x^{\frac{1}{8}} + ... + \infty$, then x is equal to
(A) 10^3 (B) 10^5 (C) 10^6 (D) 10^7

9. Consider the A.P. $a_1, a_2, ..., a_n, ...$
the G.P. $b_1, b_2, ..., b_n, ...$
such that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then
(A) $b_6 = 27$ (B) $b_7 = 27$ (C) $b_8 = 81$ (D) $b_9 = 18$

10. Let α, β, γ be the roots of the equation $x^3 + 3ax^2 + 3bx + c = 0$. If α, β, γ are in H.P. then β is equal to -

(A) $-\frac{c}{b}$ (B) $\frac{c}{b}$ (C) $-a$ (D) a

11. If a, b and c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ is -

(A) equal to 1 (B) less than 1
(C) greater than 1 (D) any real number

12. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots + \text{to } \infty = \frac{\pi^4}{90}$, then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots + \text{to } \infty$ is equals to -

(A) $\frac{\pi^4}{96}$ (B) $\frac{\pi^4}{45}$ (C) $\frac{89\pi^4}{90}$ (D) none of these

13. Let a_n be the n^{th} term of a G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ & $\sum_{n=1}^{100} a_{2n-1} = \beta$ such that $\alpha \neq \beta$. Then the common ratio of the G.P. is -

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ (D) $\sqrt{\frac{\beta}{\alpha}}$

14. Let $f(x) = x^2 + x^4 + x^6 + x^8 + \ldots\ldots \infty$ for all real x such that the sum converges. Number of real x for which the equation $f(x) - x = 0$ holds, is

(A) 0 (B) 1 (C) 2 (D) 3

15. If $x > 1, y > 1, z > 1$ are in G.P., then $\frac{1}{1+\ell n x}, \frac{1}{1+\ell n y}, \frac{1}{1+\ell n z}$ are in -

(A) A.P. (B) H.P. (C) G.P. (D) none of above

16. If there exists a geometric progression containing 27, 8 and 12 as three of its terms (not necessarily consecutive) then how many progressions are possible ?

(A) 1 (B) 2 (C) infinite (D) None of these

17. In a geometric sequence, the first term is a and the common ratio is r. If S_n denotes the sum to n, terms and $U_n = \sum_{n=1}^{n} S_n$ then $rS_n + (1 - r)U_n =$

(A) n a (B) n (C) n^2 a (D) None of these

18. Consider an A.P. $t_1, t_2, t_3, \ldots\ldots$ If 5^{th}, 9^{th} and 16^{th} terms of this A.P. form three consecutive terms of a G.P. with non zero common ratio q, then the value of q is

(A) $\frac{4}{7}$ (B) $\frac{2}{7}$ (C) $\frac{7}{4}$ (D) none

19. Suppose p is the first of $n(n > 1)$ AM's between two positive numbers a and b, then value of p is -

(A) $\frac{na+b}{n+1}$ (B) $\frac{na-b}{n+1}$ (C) $\frac{nb+a}{n+1}$ (D) $\frac{nb-a}{n+1}$

20. Let s_1, s_2, s_3 .... and t_1, t_2, t_3 .... are two arithmetic sequences such that $s_1 = t_1 \neq 0$; $s_2 = 2t_2$ and $\sum_{i=1}^{10} s_i = \sum_{i=1}^{15} t_i$. Then the value of $\frac{s_2 - s_1}{t_2 - t_1}$ is

(A) $\frac{8}{3}$ (B) $\frac{3}{2}$ (C) $\frac{19}{8}$ (D) 2

21. Suppose x, y, z is a geometric series with a common ratio of 'r' such that $x \neq y$. If x, 3y, 5z is an arithmetic sequence then the value of 'r' equals

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C) $\frac{3}{5}$ (D) $\frac{2}{3}$

22. Let $a_n = 16, 4, 1, ...$ be a geometric sequence. Define P_n as the product of the first n terms. The value of $\sum_{n=1}^{\infty} \sqrt[n]{P_n}$

(A) 8 (B) 16 (C) 32 (D) 64

23. Let a_n, $n \in N$ is an A.P. with common difference 'd' and all whose terms are non-zero. If n approaches infinity, then the sum $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + ... + \frac{1}{a_n a_{n+1}}$ will approach

(A) $\frac{1}{a_1 d}$ (B) $\frac{2}{a_1 d}$ (C) $\frac{1}{2a_1 d}$ (D) $a_1 d$

24. Let S_1, S_2, S_3 be the sums of the first n, 2n and 3n terms of an A.P. respectively. If $S_3 = C(S_2 - S_1)$ then, 'C' is equal to

(A) 4 (B) 3 (C) 2 (D) 1

25. If three positive number a, b, c are in H.P. then

(A) $b^2 < ac$ (B) $a^n + c^n > 2b^n$ (C) Both correct (D) None correct

26. If $x + y + z = 1$ and x, y, z are positive numbers such that $(1 - x)(1 - y)(1 - z) \geq kxyz$, then greatest value of k is equal to

(A) 2 (B) 4 (C) 8 (D) 16

27. The value of $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... \infty\right)}$ is equal to

(A) 4 (B) 6 (C) 8 (D) 2

28. The sum of the first three terms of an increasing G.P. is 21 and the sum of their squares is 189. Then the sum of its first n terms is

(A) $3(2^n - 1)$ (B) $12\left(1 - \frac{1}{2^n}\right)$ (C) $6\left(1 - \frac{1}{2^n}\right)$ (D) $6(2^n - 1)$

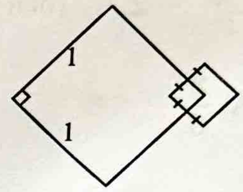
29. If $a \neq 1$ and $\ln a^2 + (\ln a^2)^2 + (\ln a^2)^3 + ... = 3\left(\ln a + (\ln a)^2 + (\ln a)^3 + (\ln a)^4 + ...\right)$ then 'a' is equal to

(A) $e^{1/5}$ (B) $\sqrt{e}$ (C) $\sqrt[3]{e}$ (D) $\sqrt[4]{e}$

30. Starting with a unit square, a sequence of square is generated. Each square in the sequence has half the side length of its predecessor and two of its sides bisected by its predecessor's sides as shown. This process is repeated indefinitely. The total area enclosed by all the squares in limiting situation, is

(A) $\frac{5}{4}$ sq. units (B) $\frac{79}{64}$ sq. units

(C) $\frac{75}{64}$ sq. units (D) $\frac{1}{12}$ sq. units



31. The sum of the first 100 terms common to the series 17, 21, 25, ... and 16, 21, 26, ... is
(A) 101100 (B) 111000 (C) 110010 (D) 100101

32. In a geometric progression the sixth term is eight times the third term and sum of the seventh and the eight term is 192, then
(A) the sum of the fifth to eleventh terms (including both) is 2032.
(B) the sum of the sixth and the ninth term is 188.
(C) the first term of the G.P. is 2.
(D) the common ratio of the G.P. is 1/2.

33. The sum of the product of the integers 1, 2, 3, ..., n taken two at a time is

(A) $\frac{n(n+1)}{24}(3n^2-n-2)$ (B) $\frac{n^2(n+1)^2}{4}(n^2+1)$

(C) $\frac{n(n+1)}{18}(n^2-n-2)$ (D) $\frac{n(n+1)(n+2)(n+3)}{24}$

34. If 1, $\log_3\sqrt{3^{1-x}+2}$, $\log_3(4.3^x-1)$ are in A.P. then x equals. **[AIEEE 2002]**
(A) $\log_3 4$ (B) $1-\log_3 4$ (C) $1-\log_4 3$ (D) $\log_4 3$

35. Sum of infinite number of terms in G.P. is 20 and sum of their square is 100. The common ratio of G.P. is- **[AIEEE 2002]**
(A) 5 (B) 3/5 (C) 8/5 (D) 1/5

36. Fifth term of a G.P. is 2, then the product of its 9 terms is- **[AIEEE 2002]**
(A) 256 (B) 512 (C) 1024 (D) None of these

37. The sum of the series $1^3-2^3+3^3-.....+9^3=$ **[AIEEE 2002]**
(A) 300 (B) 125 (C) 425 (D) 0

38. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m,n, $m\neq n$, $T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then a – d equals **[AIEEE 2004]**

(A) 0 (B) 1 (C) $\frac{1}{mn}$ (D) $\frac{1}{m}+\frac{1}{n}$

39. If AM and GM of two roots of a quadratic equation are 9 and 4 respectively, then this quadratic equation is- **[AIEEE 2004]**
(A) $x^2-18x+16=0$ (B) $x^2+18x-16=0$
(C) $x^2+18x+16=0$ (D) $x^2-18x-16=0$

40. If $x=\sum_{n=0}^{\infty} a^n$, $y=\sum_{n=0}^{\infty} b^n$, $z=\sum_{n=0}^{\infty} C^n$ where a, b, c are in A.P. and $|a|<1, |b|<1, |c|<1$ then x, y, z are in- **[AIEEE 2005]**

(A) HP (B) Arithmetic - Geometric Progression

(C) AP (D) GP

41. Let $a_1, a_2, a_3, \ldots\ldots$ be terms of an A.P. If $\frac{a_1+a_2+\ldots.+a_p}{a_1+a_2+\ldots\ldots+a_q}=\frac{p^2}{q^2}$, $p \neq q$ then $\frac{a_6}{a_{21}}$ equals-**[AIEEE-2006]**

(A) $\frac{2}{7}$ (B) $\frac{11}{41}$ (C) $\frac{41}{11}$ (D) $\frac{7}{2}$

42. If $a_1, a_2, \ldots\ldots, a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + \ldots\ldots + a_{n-1}a_n$ is equal to- **[AIEEE-2006]**

(A) na_1a_n (B) $(n-1)a_1a_n$ (C) $n(a_1 - a_n)$ (D) $(n-1)(a_1 - a_n)$

43. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals- **[AIEEE-2007]**

(A) $\frac{1}{2}\sqrt{5}$ (B) $\sqrt{5}$ (C) $\frac{1}{2}(\sqrt{5}-1)$ (D) $\frac{1}{2}(1-\sqrt{5})$

44. Let $p, q, r \in R^+$ and $27\,pqr \geq (p+q+r)^3$ and $3p + 4q + 5r = 12$ then $p^3 + q^4 + r^5$ is equal to -

(A) 2 (B) 6 (C) 3 (D) None of these

45. If $a_1, a_2, \ldots\ldots a_n \in R^+$ and $a_1.a_2\ldots\ldots a_n = 1$ then the least value of $(1+a_1+a_1^2)(1+a_2+a_2^2)\ldots..(1+a_n+a_n^2)$ is

(A) 3^n (B) $n3^n$ (C) 3^{3n} (D) data inadequate

46. Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is $\frac{3}{4}$, then - **[JEE 2000, Screening, 1+1M out of 35]**

(A) $a=\frac{7}{4}$, $r=\frac{3}{7}$ (B) $a = 2$, $r=\frac{3}{8}$ (C) $a=\frac{3}{2}$, $r=\frac{1}{2}$ (D) $a = 3$, $r=\frac{1}{4}$

47. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation -

(A) $0 \leq M \leq 1$ (B) $1 \leq M \leq 2$ (C) $2 \leq M \leq 3$ (D) $3 \leq M \leq 4$

48. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integer values of p and q respectively, are - **[JEE 2001 Screening 1+1+1M out of 35]**

(A) –2, –32 (B) –2, 3 (C) –6, 3 (D) –6, –32

49. If the sum of the first 2n terms of the A.P. 2, 5, 8 .... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ... then n equals -

(A) 10 (B) 11 (C) 12 (D) 13

50. Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are

(A) not in A.P./G.P./H.P. (B) in A.P. (C) in G.P. (D) in H.P.

51. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a+b+c=\frac{3}{2}$, then the value of a is

[JEE 2002 (Screening), 3M]

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

52. The first term of an infinite geometric progression is x and its sum is 5. Then - **[JEE 2004]**

(A) $0 \le x \le 10$ (B) $0 < x < 10$ (C) $-10 < x < 0$ (D) $x > 10$

53. Let $a_1, a_2, \ldots\ldots, a_{10}$ be in A.P. & $h_1, h_2, \ldots\ldots h_{10}$ be in H.P. . If $a_1 = h_1 = 2$ & $a_{10} = h_{10} = 3$ then a_4h_7 is-

(A) 2 (B) 3 (C) 5 (D) 6

54. We inscribe a square in a circle of unit radius and shade the region between them. Then we inscribe another circle in the square and another square in the new circle and shade the region between the new circle and the square. If the process is repeated infinitely many times, the area of the shaded region.

(A) 2π (B) $3(\pi-2)$ (C) $2(\pi-2)$ (D) None of these

SECTION - 2 : MULTIPLE CHOICE CORRECT QUESTIONS

55. If $(1 + 3 + 5 + \ldots + a) + (1 + 3 + 5 + \ldots + b) = (1 + 3 + 5 + \ldots + c)$, where each set of parentheses contains the sum of consecutive odd integers as shown such that - (i) $a + b + c = 21$, (ii) $a > 6$ If $G = \text{Max}\{a, b, c\}$ and $L = \text{Min}\{a, b, c\}$, then -

(A) $G - L = 4$ (B) $b - a = 2$ (C) $G - L = 7$ (D) $a - b = 2$

56. Let a, x, b be in A.P. ; a, y, b be in G.P. and a, z, b be in H.P. If $x = y + 2$ and $a = 5z$ then -

(A) $y^2 = xz$ (B) $x > y > z$ (C) $a = 9, b = 1$ (D) $a = \frac{9}{4}, b = \frac{1}{4}$

57. The p^{th} term T_p of H.P. is $q(q + p)$ and q^{th} term T_q is $p(p + q)$ when $p > 1, q > 1$, then -

(A) $T_{p+q} = pq$ (B) $T_{pq} = p + q$ (C) $T_{p+q} > T_{pq}$ (D) $T_{pq} > T_{p+q}$

SECTION - 3 : COMPREHENSION BASED QUESTIONS

Comprehension–1

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$.

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1,2,\ldots\ldots$

58. The sum $V_1 + V_2 + \ldots + V_n$ is : **[JEE 2007, 4M]**

(A) $\frac{1}{12}n(n + 1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n + 1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

59. T_r is always : **[JEE 2007, 4M]**

(A) an odd number (B) an even number (C) a prime number (D) a composite number

Comprehension–2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively:

60. Which one of the following statements is correct ? **[JEE 2007, 4M]**

(A) $G_1 > G_2 > G_3 > \ldots$
(B) $G_1 < G_2 < G_3 < \ldots$
(C) $G_1 = G_2 = G_3 = \ldots$
(D) $G_1 < G_2 < G_3 < \ldots$ and $G_4 > G_5 > G_6 > \ldots$

61. Which one of the following statements is correct ? **[JEE 2007, 4M]**

(A) $A_1 > A_2 > A_3 > \ldots$
(B) $A_1 < A_2 < A_3 < \ldots$
(C) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$
(D) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

62. Which one of the following statements is correct ? **[JEE 2007, 4M]**

(A) $H_1 > H_2 > H_3 > \ldots$
(B) $H_1 < H_2 < H_3 < \ldots$
(C) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 > \ldots$
(D) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

SECTION - 4 : MATRIX - MATCH QUESTIONS

63. Match the Column-I with Column-II

	Column-I	Column-II
(A)	a, b, c are positive real numbers different from 1. If $\log_a 100$, $2\log_b 10$, $2\log_c 5 + \log_c 4$, are in HP then a, b, c are in	(P) AP
(B)	a, b, c are different positive real numbers such that $a > b > c$. If $2\log(a - c)$, $\log(a^2 - c^2)$, $\log(a^2 + 2b^2 + c^2)$ are in AP, then a, b, c are in	(Q) GP
(C)	a, b, c are positive real numbers such that a, b, c are in AP and a^2, b^2, c^2 are in HP then, a^3, b^3, c^3, are in	(R) HP
(D)	If a, b, c are in AP, b, c, d are GP, c, d, e are in HP then a, c, e are in	(S) AGP

64. Match the statement given in Column I with the values in column II

	Column I	Column II
(A)	If $a_1, a_2, \ldots\ldots a_n$ are in A.P. and $a_4 = 5$ then the value of $\sum_{i=1}^{7} a_i$ is	(P) 19
(B)	If $a_1, a_2, a_3, \ldots\ldots a_n$ are in G.P. and $a_3 = 2$ then the value of $\prod_{i=1}^{5} a_i$ is	(Q) 24
(C)	If the roots of cubic equation $x^3 - kx^2 + 114x - 216 = 0$, are in G.P. then the value of k is	(R) 35
(D)	Let sum of infinite geometric progression with non zero common ratio is 4 Then the sum of all possible value of its first term is	(S) 32

SECTION - 5 : NUMERICAL ANSWER BASED QUESTIONS

65. If $x \in R$ and the numbers $(5^{1+x} + 5^{1-x})$, $a/2$, $(25^x + 25^{-x})$ form an A.P. then 'a' must lie in the interval ________.

66. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots$ upto $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = $ ________.

67. When 9^{th} term of an A.P. is divided by its 2^{nd} term the quotient is 5 & when 13^{th} term is divided by the 6^{th} term, the quotient is 2 and remainder is 5. The first term and the common difference of the A.P. are ________ & ________ respectively.

68. Find the sum of the series, $7 + 77 + 777 + \ldots\ldots$ to n terms.

69. Find three numbers a, b, c between 2 & 18 which satisfy following conditions :
(i) their sum is 25
(ii) the numbers 2, a, b are consecutive terms of an AP &
(iii) the numbers b, c, 18 are consecutive terms of a GP.

70. Find the sum of the first n terms of the series : $1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + 4\left(1+\frac{1}{n}\right)^3 + \ldots\ldots\ldots$

71. Find all x such that $\sum_{k=1}^{\infty} k \cdot x^k = 20$.

72. Sum the following series to n terms and to infinity :

(a) $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \ldots\ldots\ldots$ (b) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$ (c) $\sum_{r=1}^{n} \frac{1}{4r^2-1}$

73. Find the value of the sum $\sum_{r=1}^{n}\sum_{s=1}^{n} \delta_{rs} 2^r 3^s$ where δ_{rs} is zero if $r \neq s$ & δ_{rs} is one if $r = s$.

74. Find the sum $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} 1$

75. The value of $x + y + z$ is 15, if a, x, y, z, b are in A.P. while the value of; $(1/x)+(1/y)+(1/z)$ is 5/3 if a, x, y, z, b are in H.P. Find a & b.

76. Prove that the sum of the infinite series $\frac{1.3}{2} + \frac{3.5}{2^2} + \frac{5.7}{2^3} + \frac{7.9}{2^4} + \ldots\ldots\ldots \infty = 23$.

77. For an arithmetic progression with n^{th} term as a_n and sum to n terms as S_n. Given 9, $\frac{81}{2}$, $\frac{851}{2}$ as the first, tenth and the last terms. Find the value of $\frac{4}{n} \cdot S_n + 2 \cdot a_{70}$.

78. If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k^{th} match are given by $k. 2^{n+1-k}$, where $1 \leq k \leq n$. Find n.

[JEE-05, Mains-2M out of 60]

79. The odd positive numbers are written in the form of a triangle

1

3 5

7 9 11

13 15 17 19

....

....

find the sum of terms in n^{th} row.

80. Find the nth term and the sum to 'n' terms of the series :

(a) $1+5+13+29+61+......$ (b) $6+13+22+33+......$

SECTION - 6 : SUBJECTIVE QUESTIONS

81. Given that $a^x = b^y = c^z = d^u$ & a, b, c, d are in GP, show that x, y, z, u are in HP.

82. If the p^{th}, q^{th} & r^{th} terms of an AP are in GP. Show that the common ratio of the GP is $\frac{q-r}{p-q}$.

83. If one AM 'a' & two GM's p & q be inserted between any two given numbers then show that $p^3 + q^3 = 2apq$.

84. Let $a_1, a_2, a_3 ...... a_n$ be an AP. Prove that :

$$\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + ... + \frac{1}{a_n a_1} = \frac{2}{a_1 + a_n}\left[\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + ... + \frac{1}{a_n}\right]$$

85. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either $a = b = c$ or $a, b, -\frac{c}{2}$ form a G.P.

86. If a, b, c are positive real numbers, then prove that $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$.

87. Prove that : $(ab + xy)(ax + by) \geq 4abxy$ where $a, b, x, y \in R^+$

88. If $a, b, c \in R^+$ & $a + b + c = 1$; then show that $(1 - a)(1 - b)(1 - c) \geq 8abc$

89. If a, b, c are sides of a scalene triangle then show that $(a + b + c)^3 > 27 (a + b - c)(b + c - a)(c + a - b)$

90. For positive number a, b, c show that $\frac{bc}{a} + \frac{ac}{b} + \frac{ab}{c} \geq a + b + c$

91. If a, b, c are three positive real number then prove that : $\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} \geq \frac{3}{2}$

92. In a A.P. & an H.P. have the same first term, the same last term & the same number of terms; prove that the product of the r^{th} term from the beginning in one series & the r^{th} term from the end in the other is independent of r.

93. If there be 'm' A.P's beginning with unity whose common difference is 1, 2, 3 .... m. Show that the sum of their n^{th} terms is $\left(\frac{m}{2}\right)(mn - m + n + 1)$.

94. If a, b, c are in H.P., b, c, d are in G.P. & c, d, e are in A.P., then Show that $e = \frac{ab^2}{(2a-b)^2}$.

95. If a, b, c be in G.P. & $\log_c a, \log_b c, \log_a b$ be in A.P., then show that the common difference of the A.P. must be $\frac{3}{2}$.

96. Find the sum to n terms :

(a) $\frac{1}{x+1} + \frac{2x}{(x+1)(x+2)} + \frac{3x^2}{(x+1)(x+2)(x+3)} + ...$

(b) $\frac{a_1}{1+a_1} + \frac{a_2}{(1+a_1)(1+a_2)} + \frac{a_3}{(1+a_1)(1+a_2)(1+a_3)} + ...$

97. The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. **[JEE 2000, Mains, 4M out of 100]**

98. Let a_1, a_2.......... be positive real numbers in G.P.. For each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of a_1, a_2, a_3,.......... a_n. Find an expression for the G.M. of G_1, G_2,............G_n in terms of A_1, A_2, ...A_n, H_1, H_2,..........H_n **[JEE 2001 (Mains) ; 5M]**

99. Let a, b be positive real numbers. If a, A_1, A_2, b are in A.P. ; a, G_1, G_2, b are in G.P. and a, H_1, H_2, b are in H.P., show that $\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2} = \frac{(2a+b)(a+2b)}{9ab}$. **[JEE 2002, Mains, 5M out of 60]**

SECTION - 7 : ASSERTION–REASON QUESTIONS

Assertion & Reason

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

100. **Statement-I–** If a, b, c are three distinct positive number in H.P., then $\left(\frac{a+b}{2a-b}\right) + \left(\frac{c+b}{2c-b}\right) > 4$

Because

Statement-II– Sum of any number and it's reciprocal is always greater than or equal to 2.

(A) A (B) B (C) C (D) D

101. Statement-I– If $x^2y^3 = 6 (x, y > 0)$, then the least value of $3x + 4y$ is 10

Because

Statement-II– If $m_1, m_2 \in N$, $a_1, a_2 > 0$ then $\frac{m_1a_1 + m_2a_2}{m_1 + m_2} \geq (a_1^{m_1} a_2^{m_2})^{\frac{1}{m_1+m_2}}$ and equality holds when $a_1 = a_2$.

(A) A (B) B (C) C (D) D

102. **Statement-I–** For $n \in N$, $2^n > 1 + n\left(\sqrt{(2^{n-1})}\right)$

Because

Statement-II– G.M. > H.M. and (AM) (HM) = $(GM)^2$

(A) A (B) B (C) C (D) D

103. **Statement-I–** If a, b, c are three positive numbers in G.P., then $\left(\frac{a+b+c}{3}\right)\cdot\left(\frac{3abc}{ab+bc+ca}\right) = \left(\sqrt[3]{abc}\right)^2$

Because

Statement-II– (A.M.) (H.M.) = $(G.M.)^2$ is true for any set of positive numbers.

(A) A (B) B (C) C (D) D

104. **Statement-I–** n^{th} term (T_n) of the sequence (1, 6, 18, 40, 75, 126,....) is $an^3 + bn^2 + cn + d$, and $6a + 2b - d$ is $= 4$.

Because

Statement-II– If the second successive differences (Differences of the differences) of a series are in A.P., then T_n is a cubic polynomial in n.

(A) A (B) B (C) C (D) D

105. **Statement-I–** The format of n^{th} term (T_n) of the sequence ($\ell n2$, $\ell n4$, $\ell n32$, $\ell n1024$.......) is $an^2 + bn + c$.

Because

Statement-II– If the second successive differences between the consecutive terms of the given sequence are in G.P., then $T_n = a + bn + cr^{n-1}$, where a, b, c are constants and r is common ratio of G.P.

(A) A (B) B (C) C (D) D

ANSWER KEY

YEAR LONG REVISION EXERCISE

SECTION - 1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	D	A	C	B	A	B	B	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	A	C	B	C	A	C	A	C
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	A	B	C	C	A	A	D	A
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	A	A	A	B	B	B	C	A	A	A
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	B	B	C	C	A	D	A	A	B	D
Que.	51	52	53	54						
Ans.	D	B	D	C						

SECTION - 2

Que.	55	56	57
Ans.	AD	ABC	ABC

SECTION - 3

Que.	58	59	60	61	62
Ans.	B	D	C	A	B

SECTION - 4

63. (A) $\rightarrow$ (Q), (B) $\rightarrow$ (Q), (C) $\rightarrow$ (PQRS), (D) $\rightarrow$ (Q)

64. (A) $\rightarrow$ (R), (B) $\rightarrow$ (S), (C) $\rightarrow$ (P), (D) $\rightarrow$ (Q)

SECTION - 5

65. $[12, \infty)$

66. $\frac{\pi^2}{8}$

67. $a = 3 \quad d = 4$

68. $S=\left(\frac{7}{81}\right)(10^{n+1}-9n-10)$

69. $a = 5, b = 8, c = 12$

70. n^2

71. $\frac{1}{20}$

72. **(a)** $\frac{1}{24}-\frac{1}{6(3n+1)(3n+4)}, \frac{1}{24}$

(b) $\frac{n(n+1)(n+2)(n+3)(n+4)}{5}$

(c) $\frac{n}{2n+1}, \frac{1}{2}$

73. $\frac{6}{5}(6^n-1)$

74. $\frac{[n(n+1)(n+2)]}{6}$

75. $a = 1, b = 9$ or $b = 1, a = 9$

77. 1370

78. 7

79. n^3

80. **(a)** $2^{n+1}-3; 2^{n+2}-4-3n;$

(b) $n^2 + 4n + 1 ; \frac{1}{6}n(n+1)(2n+13)+n$

SECTION - 6

96. **(a)** $1-\dfrac{x^n}{(x+1)(x+2)..........(x+n)}$ **(b)** $1-\dfrac{1}{(1+a_1)(1+a_2)..........(1+a_n)}$

97. **(i)** $\left[\dfrac{a+b+3}{2}\right]$ **(ii)** $\dfrac{a+b+2}{2}$

98. $\left[(A_1,A_2,....A_n)(H_1,H_2,....H_n)\right]^{\frac{1}{2n}}$

SECTION - 7

100. (C) **101.** (A) **102.** (C) **103.** (C)
104. (A) **105.** (B)