Tutorial 1

Question 1

Suppose *a* and *b* are real numbers, not both *0*. Find real number *c* and *d* such that

$$1/(a+bi) = c + di$$

Solution

To Prove

1/(a +bi) $\overset{\Delta}{=}$ c + di

Now

$$\begin{aligned} & \frac{1}{a+bi} = c + di \\ or, \quad & \frac{1(a-bi)}{(a+bi)(a-bi)} = c + di \\ or, \quad & \frac{a-bi}{a^2-b^2} = c + bi \ (\because i^2 = -1) \\ or, \quad & \frac{a}{a^2-b^2} - \frac{b}{a^2-b^2} i = c - di \\ \therefore c = & \frac{a}{a^2-b^2}, \ d = \frac{-b}{a^2-b^2} \end{aligned}$$

Question 2

Show that $\frac{-1+\sqrt{3i}}{2}$ is a cube root of 1 (meaning that its cube equals 1).

Solution

To prove

$$(\frac{-1+\sqrt{3i}}{2})^3 = 1$$

Let

$$x = \frac{-1+\sqrt{3i}}{2}$$

Now

$$x^2 = (\frac{-1+\sqrt{3i}}{2})^2$$

$$or, \quad x^2 = \frac{1-2\sqrt{3i}-3}{4}$$

$$or, \quad x^2 = 2(\frac{-1-\sqrt{3i}}{4})$$

$$\therefore \quad x^2 = \frac{-1-\sqrt{3i}}{2}$$

Then

$$x^3 = x.x^2$$

$$or, \quad x^3 = (\frac{-1+\sqrt{3i}}{2}).(\frac{-1-\sqrt{3i}}{2})$$

Taking '-' sign common on both ends.

$$x^3 = (\frac{1-\sqrt{3i}}{2}).(\frac{1+\sqrt{3i}}{2})$$

$$or, \quad x^3 = \frac{1^2-\sqrt{3i}^2}{2^2}$$

$$or, \quad x^3 = \frac{1+3}{4} = 1$$

$$\because \quad x^3 = 1$$

$$\therefore \quad (\frac{-1+\sqrt{3i}}{2})^3 = 1$$

Question 4

Show that $\alpha + \beta = \beta + \alpha$ for all $\alpha, \beta \in$ C.

Solution

Let us suppose

$$\alpha = a_1 + b_1 i$$
$$and, \beta = a_2 + b_2 i$$

Now

$$\alpha + \beta = (a_1 + b_1 i) + (a_2 + b_2 i)$$
$$= (a_1 + a_2) + (b_1 + b_2)i - - - - - I$$

Then

$$\beta + \alpha = (a_2 + b_2 i) + (a_1 + b_1 i)$$
$$= (a_2 + a_1) + (b_2 + b_1)i$$
$$= (a_1 + a_2) + (b_1 + b_2)i - - - - - II$$
$$(\because a + b = b + a \; where \; a, \; b \in \mathbb{R})$$

$\because eq^n\ I$ and $eq^n\ II$ are same we can say $\alpha\ +\ \beta\ =\ \beta\ +\ \alpha\,\forall\,\alpha,\ \beta\ \in$ C.

Question 5

Show that $(\alpha+\beta)+\lambda\ =\ \alpha+(\beta+\lambda)\,\forall\,\alpha,\ \beta,\ \lambda\ \in$ C.

Let us Suppose

$$\begin{aligned}
&\alpha = a_1 + b_1 i\\
&\beta = a_2 + b_2 i\\
&\lambda = a_3 + b_3 i\\
&where\ a_1,\ b_1,\ a_2,\ b_2,\ a_3,\ b_3 \in \mathbb{R}\ and\ i\ is\ imaginary\ unit.
\end{aligned}$$

Now

$$\begin{aligned}
(\alpha+\beta)\lambda &= ((a_1+b_1 i)+(a_2+b_2 i))+(a_3+b_3 i)\\
&= ((a_1+a_2)+(b_1+b_2)i)+(a_3+b_3 i)\\
&= (a_1+a_2+a_3)+(b_1+b_2+b_3)i - - - - - I
\end{aligned}$$

Then

$$\begin{aligned}
\alpha+(\beta\lambda) &= (a_1+b_1 i)+((a_2+b_2 i)+(a_3+b_3 i))\\
&= (a_1+b_1 i)+((a_2+a_3)+(b_2+b_3)i)\\
&= (a_1+a_2+a_3)+(b_1+b_2+b_3)i - - - - - II
\end{aligned}$$

$\because eq^n I$ and $eq^n II$ are same, $(\alpha+\beta)+\lambda\ =\ \alpha+(\beta+\lambda)\,\forall\,\alpha,\ \beta,\ \lambda\ \in$ C is proved.

Question 7

Show that $\forall\ \alpha \in$ C, $\exists$ a unique $\beta \in$ C : $\alpha+\beta=0$.

Solution

Let us Suppose

$$\alpha = x + yi$$

$$and$$

$$\alpha + \beta = 0 - - - I \; and \; \alpha + \lambda = 0 - - - II$$

$$from \; eq^n I$$

$$\alpha + \beta = 0$$

$$or, \; \beta = -\alpha$$

$$or, \; \beta = -(x + yi)$$

$$or, \; \beta = -x - yi$$

$$again \; from \; eq^n II$$

$$\alpha + \lambda = 0$$

$$or, \; \lambda = -\alpha$$

$$or, \; \lambda = -(x + yi)$$

$$or, \; \lambda = -x - yi$$

$\because \beta = -x - yi = \lambda, \; \beta = \lambda$

$\therefore \beta$ and λ are same number, we can say that $\forall \; \alpha \in$ C $\exists$ a unique $\beta \in$ C : $\alpha + \beta = 0$.