Carleton University

Modelling of Integrated Devices

ELEC 4700

Assignment 2

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1 Non Resistive Region

Laplace's Equation The first problem solved in this assignment is to solve the electrostatic potential defined by equation (1) below, where the boundary conditions are as follow,

$$\nabla^2 V = 0 \tag{1}$$

The simulation model the problem as an orthogonal resistor network with resistance values of 1.

1.1 1D Solution

The first simulation was ran with the simple case,

$$V = V_0 \text{ at } x = 0 \quad \text{and } V = 0 \text{ at } x = L$$

Figure 1 below shows the results of the simulation, where the region was set as a rectangle with a voltage of 1V at x=0, and grounded at x=L. The y boundary's of the region were set as free.

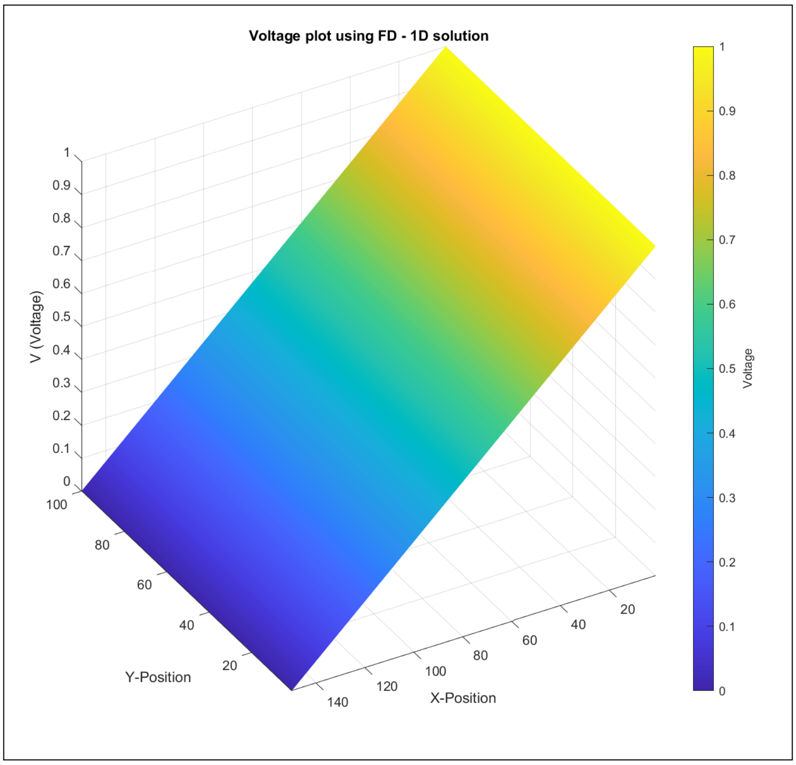


Figure 1: 1D Simulation result - Y bounds free.

1.2 2D Simulation and analytical solution

Finite Difference The next case tested for was the 2D finite difference, where the boundary conditions are as follow,

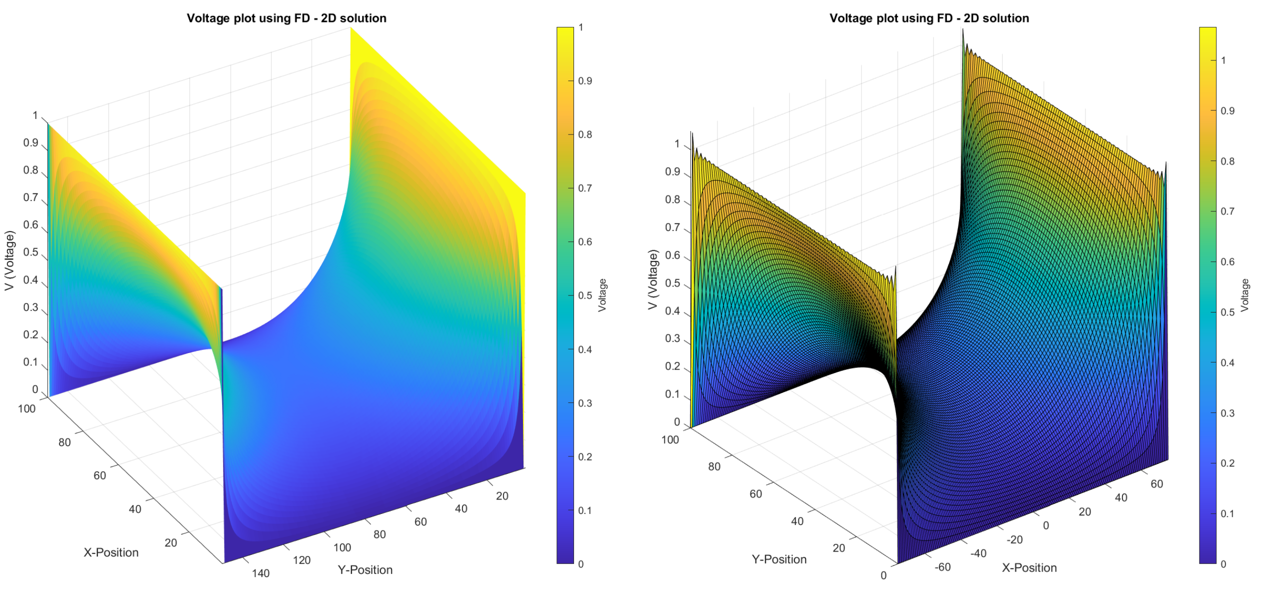
$$V = V_0 \text{ at } x = 0 \quad \text{and } V = 0 \text{ at } x = L \quad \text{and } V = 0 \text{ at } y = 0 \quad \text{and } y = W.$$

The Y dimensions of the same rectangular region as in **(A)** were now simulated as grounded, where the simulation results are shown in Figure 2.1 below.

Analytical solution The analytical solution was obtained by using equation (2) as shown below,

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} \frac{\cosh\left(\frac{n\pi x}{a}\right)}{\cosh\left(\frac{n\pi b}{a}\right)} \sin\left(\frac{n\pi y}{a}\right) \tag{2}$$

Once again using the Matlab Surf command to plot the voltage, the final results obtained are as shown in Figure 2.2.



(2.1) 2D Simulation result - Laplace's Equation via FD (2.2) 2D Simulation result - Analytical series solution

Figure 2: 2D Simulation result - Y bounds set to 0V.

Comparing the two figures, it is clear to see that both figures have some explicit error. Both solutions show a high level of correlation in most of the solution, at the Y bounds of the region there is a discrepancy. Furthermore, during the simulation of the analytical solution the solution converges quickly, then slows down as the solution gets close to the full solution. This determines that is there is a trade off between accuracy and speed.

Meshing As mentioned above, meshing is a powerful technique to solve problems via geometry. Different mesh can have different associated complexities (time required for computation inside of a mesh), and depending on the geometry they cover one mesh can be better than another. A mesh that is dense (smaller individual mesh size) can provide more accurate results, but will take more simulation time then a larger (more inaccurate) mesh. It is fundamentally up to the user to determine the accuracy-time trade off required to complete the analysis.

2 Current Flow Through Bottle Neck

2.1 Conductivity of region $\sigma(\mathbf{x}, \mathbf{y})$

Using the Finite Difference method, a bottle neck is now introduced into the rectangular region. The current flow in this region is given by,

$$\nabla(\sigma_{x,y} \nabla V) = 0 \tag{3}$$

Wherein the boxes are highly resistive, having a very low $\sigma_{x,y}$ value. Using these conditions the conductivity map of the region is generated as shown in Figure 3.

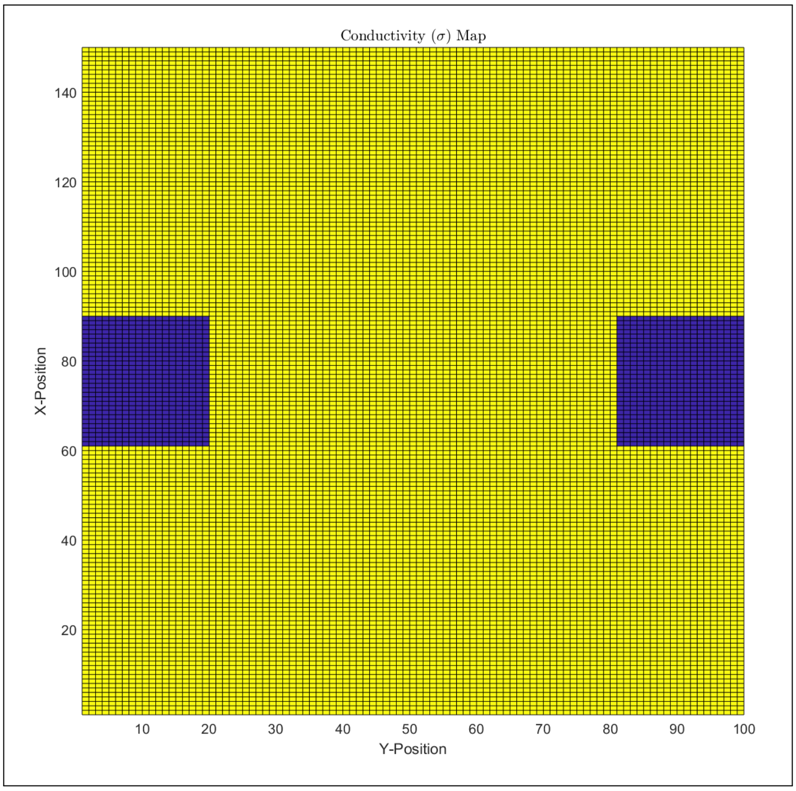


Figure 3: Conductivity map of the simulation region

2.2 Voltage across region $\mathbf{V(x,y)}$

Using the same G matrix mapping as previously (with the inclusion of the consideration towards the sigma) the voltage of the region was obtained, as shown in Figure 4. The results of this simulation show the effect of introducing a bottle neck, wherein uniformity of the region is lost.

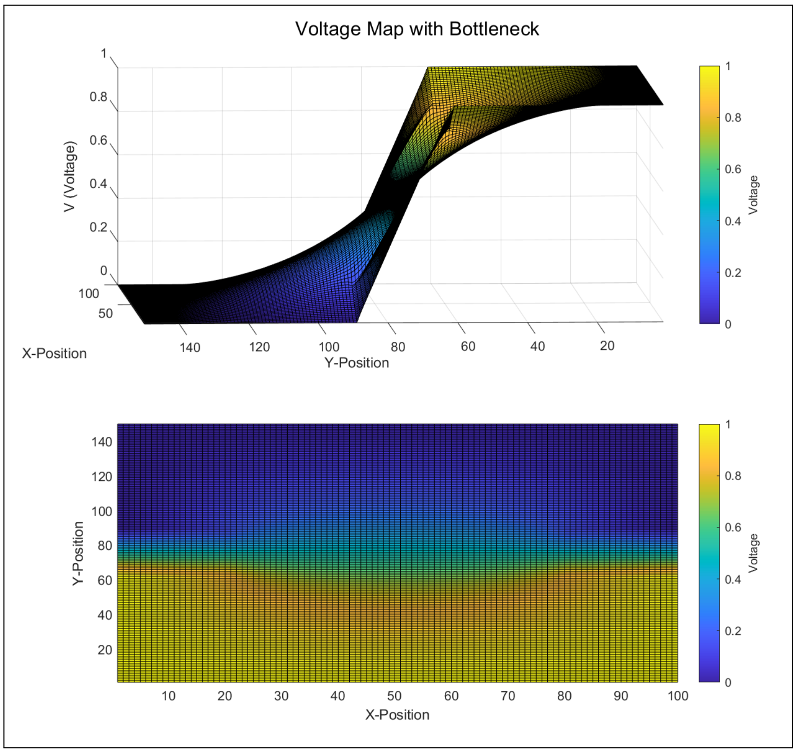


Figure 4: Voltage map across rectangular region with bottle neck

Electric field Using the classical electrostatic definition of a time independent electric field $\mathbf{E}$,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{4}$$

Which implies that the electric field is the gradient of the electric potential,

$$-\mathbf{E} = \nabla V \tag{5}$$

Equation 5 can be applied very simply in Matlab using the *gradient*() function, where both the E_x and E_y components can be derived in 1 line. Figure 5 on the following page shows the quiver plot of the electric field across the entire rectangular region.

2.3 Electric field along x $(\vec{E_x})$

Figure 6.1 shows the surface plot of the electric field along the x axis. The results present in this graph agree with those found in Figure 4, wherein along the x axis there is not a large change in voltage. The peaks of this surface plot represent the boundary's of the box, where the difference in conductivity effects the electric field.

2.4 Electric field along y $(\vec{E_y})$

Figure 6.2 shows the surface plot of the electric field along the y axis. Once again this result agrees with those found in Figure 4, where along the y axis there is a large delta period when moving across the boxes. The peaks of this surface plot represent the effects of the change in conductivity when moving from the region outside of the box into the region inside box (difference in sigma).

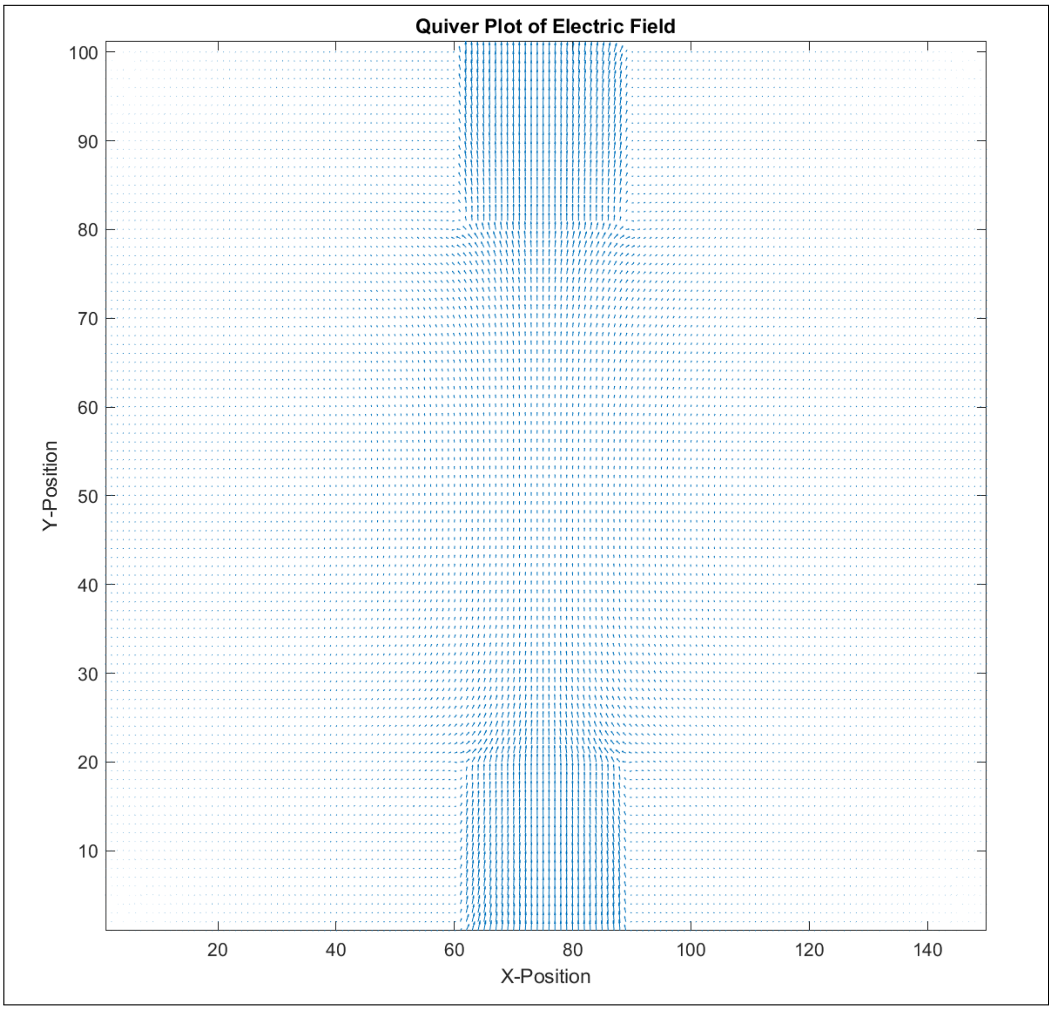
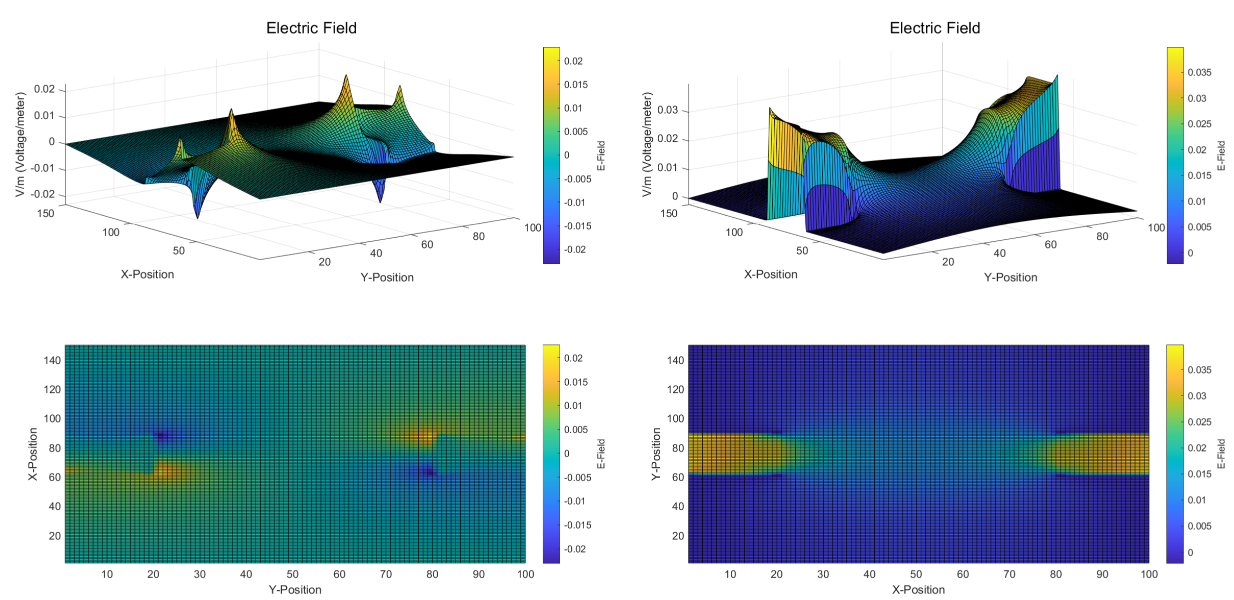


Figure 5: Electric field across region - Quiver plot



(6.1) Electric field along x - E_x

(6.2) Electric Field along y - E_y

Figure 6: Electric field simulation results via Finite Difference

2.5 Current density $\vec{J}(x, y)$

Using the common approximation of the current density, such that the current is proportional to the electric field,

$$J = \sigma \mathbf{E} \tag{6}$$

The current density of the rectangular region was obtained as shown in Figure 7 below.

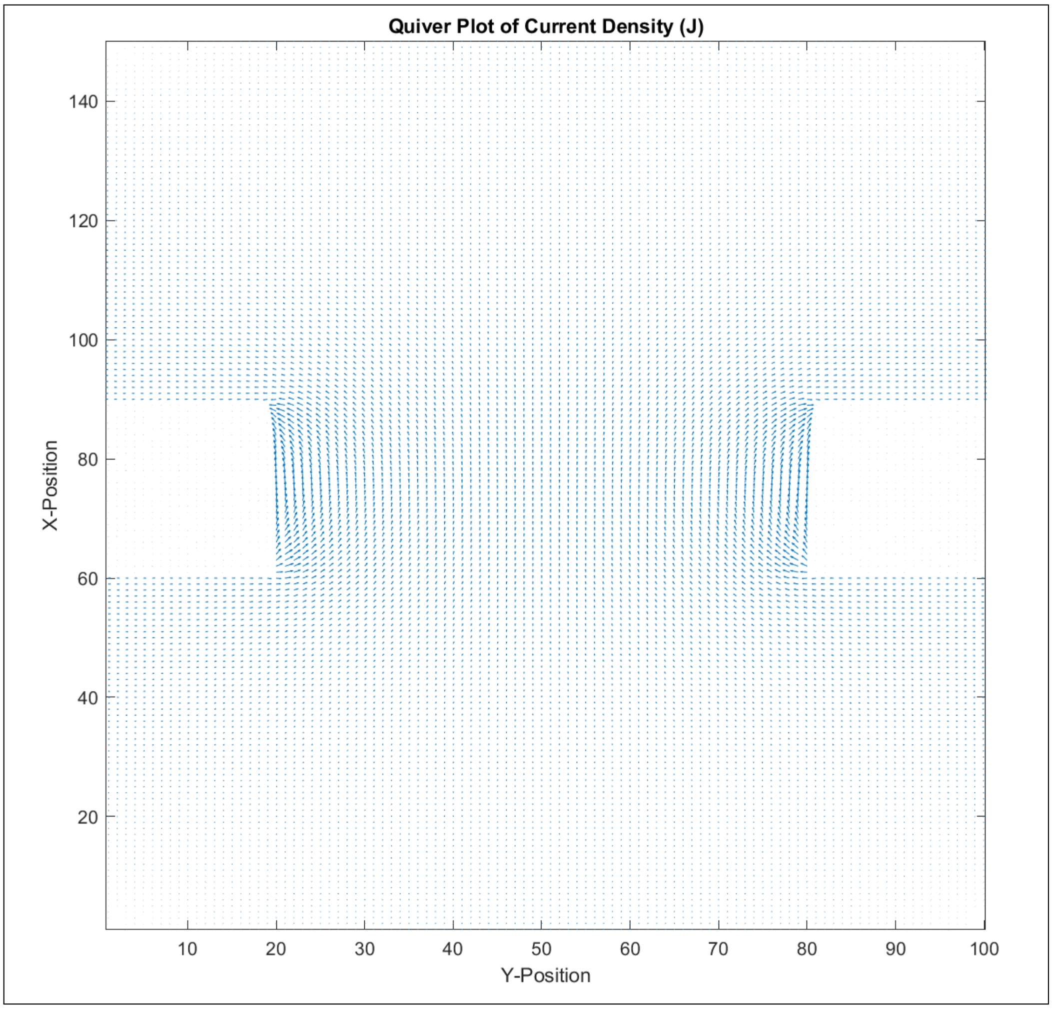


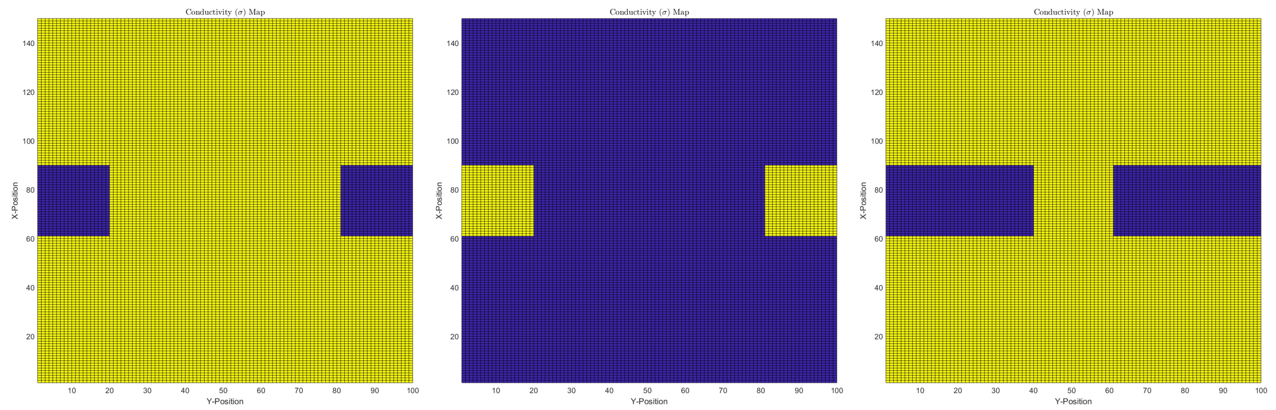
Figure 7: Current density across region - Quiver plot

2.6 Variations in the simulation

Figure 8.1 on the next page shows the 'base' case for the simulation, where the sigma inside of the box is significantly lower then outside, and the bottle neck is wide. Figure 8.2 shows the simulation case where the sigma inside of the box is larger then outside, but the bottle neck is the same as the control case. Finally, Figure 8.3 shows the simulation case where the sigma of the box is the same as in the control, but the bottle neck is narrow.

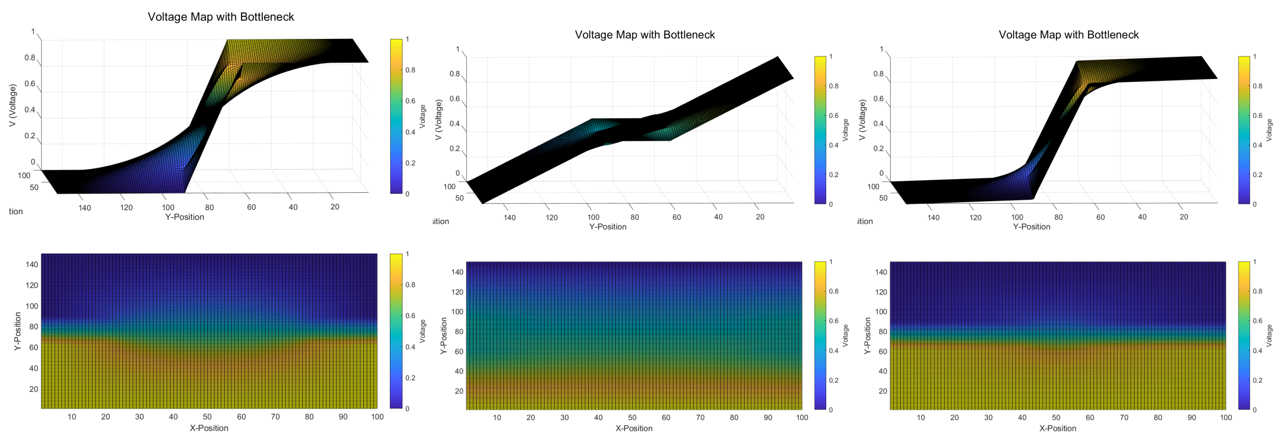
Varying box conductivity Figure 9.2 shows the effects on the voltage map where the conductivity inside the box is higher then outside. This plot shows a vastly linear relation of the voltage, with the exception of the small region where the boxes exist, causing the flat portion. Figure 10.2 shows an interesting result, comparing to Figure 10.1 shows an 'inversion'. This is a artifact of the change in conductivity, where the current flow is directed to the two boxes instead of the rest of the rectangular region.

Narrowing the bottle neck Figure 9.3 shows the effects on the voltage map where the bottle neck is narrowed. This plot shows a parallel result to that of Figure 9.1, where simply the slope at the bottle neck is larger. The results of Figure 10.3 also agree with the previous statement, wherein due to the narrowed bottle neck the magnitude of the current density at the bottle neck is increased.



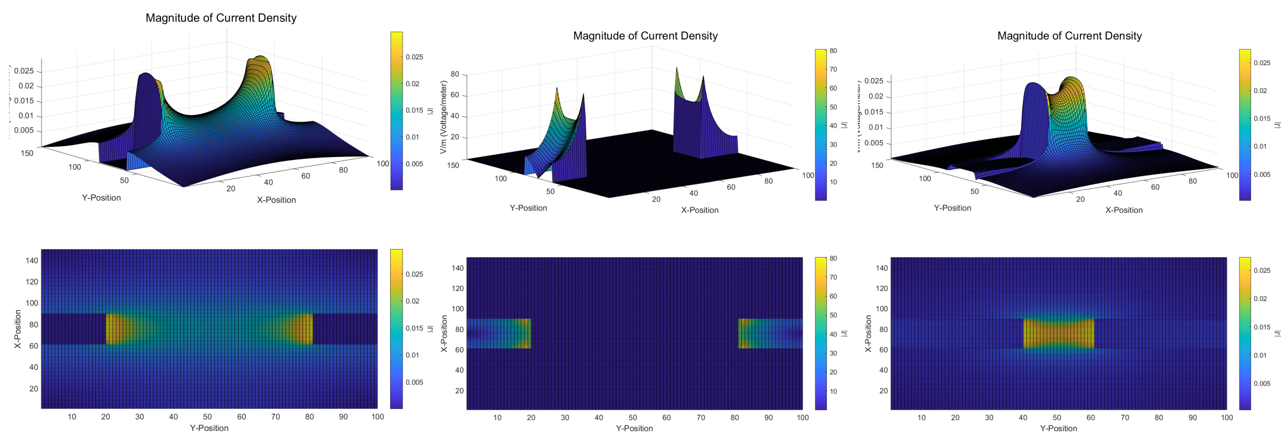
(8.1) Base simulation case (8.2) Delta sigma simulation case (8.3) Delta bottle neck simulation case

Figure 8: Three simulation cases - Conductivity map



(9.1) Base simulation case (9.2) Delta sigma simulation case (9.3) Delta bottle neck simulation case

Figure 9: Three simulation cases - Voltage map



(10.1) Base simulation case (10.2) Delta sigma simulation case (10.3) Delta bottle neck simulation case

Figure 10: Three simulation cases - Magnitude of current density map

3 Appendix

3.1 Free Y dimension code

```
%Initialize size of mesh
nx = 150;
ny = 100;
%Initialize G and V matrix
G = sparse(nx*ny,nx*ny);
V = zeros(nx*ny,1);
V0=1;
%The G matrix is filled in using a loop using the FD method
for i = 1:nx
    for j = 1:ny
        n = j + (i-1)*ny;
        if i == 1
            G(n,:) = 0;
            G(n,n) = 1;
            V(n) = 1;
        elseif i == nx
            G(n,:) = 0;
            G(n,n) = 1;
            V(n) = 0;
        elseif j == 1
            G(n, :) = 0;
            G(n, n) = -3;
            G(n, n+1) = 1;
            G(n, n+ny) = 1;
            G(n, n-ny) = 1;
        elseif j == ny
            G(n, n) = -3;
            G(n, n-1) = 1;
            G(n, n+ny) = 1;
            G(n, n-ny) = 1;
        else
            G(n, n) = -4;
            G(n, n+1) = 1;
            G(n, n-1) = 1;
            G(n, n+ny) = 1;
            G(n, n-ny) = 1;
        end
    end
end

V = G\V;
Soln = zeros(nx,ny,1);
for i=1:nx
    for j=1:ny
        Soln(i,j) = V(j+(i-1)*ny);
    end
end
figure('Name','1D FD Solution')
surf(Soln,'edgecolor','none')
title('Voltage plot using FD - 1D solution')
xlabel('X-Position')
ylabel('Y-Position')
zlabel('V (Voltage)')
view(-120,35)
```

Source Code 1: Part 1 - Simple 1D case code

3.2 Grounded Y dimension code

```
G2 = sparse(nx*ny,nx*ny);
V2 = zeros(nx*ny,1);

%The G matrix is filled in using a loop using the FD method
for x = 1:nx
    for y = 1:ny
        n = y + (x-1)*ny;

        if x == 1
            G2(n,:) = 0;
            G2(n, n) = 1;
            V2(n) = 1;
        elseif x == nx
            G2(n,:) = 0;
            G2(n, n) = 1;
            V2(n) = 1;
        elseif y == 1
            G2(n,:) = 0;
            G2(n, n) = 1;
        elseif y == ny
            G2(n,:) = 0;
            G2(n, n) = 1;
        else
            G2(n, n) = -4;
            G2(n, n+1) = 1;
            G2(n, n-1) = 1;
            G2(n, n+ny) = 1;
            G2(n, n-ny) = 1;
        end
    end
end

V2 = G2\V2;
Soln2 = zeros(nx,ny,1);
for i=1:nx
    for j=1:ny
        Soln2(i,j) = V2(j+(i-1)*ny);
    end
end
figure('Name','2D FD Solution')
surf(Soln2,'edgecolor','none')
title('Voltage plot using FD - 2D solution')
xlabel('X-Position')
ylabel('Y-Position')
zlabel('V (Voltage)')
cb=colorbar;
cb.Label.String = 'Voltage';
cb.Location = 'eastoutside';
axis tight
view(-120,35)
```

Source Code 2: Part 1 - Simple 2D case code

3.3 Part 1 - Analytical Solution Code

```
%Analytical solution
x = linspace(-nx/2,nx/2,nx);
y = linspace(0,ny,ny);
[i,j] = meshgrid(x,y);
AnalyticalSoln = sparse(ny,nx);
for n = 1:2:300
   AnalyticalSoln = AnalyticalSoln + cosh(n*pi*i/ny).*sin(n*pi*j/ny)./(n*cosh(n*pi*nx/2/ny));
   figure(3)
   surf(x,y,(4/pi)*AnalyticalSoln)
   title('Voltage plot using FD - 2D solution')
   xlabel('X-Position')
   ylabel('Y-Position')
   zlabel('V (Voltage)')
   cb=colorbar;
   cb.Label.String = 'Voltage';
   cb.Location = 'eastoutside';
   axis tight
   view(3)
   pause(0.1)
end
```

Source Code 3: Part 1 - Simple 2D case analytical series solution code

3.4 Part 2 - Current Flow Through Bottle Neck Code

```
nx = 150;
ny = 100;
G = sparse((nx *ny), (nx*ny));
V = zeros(nx *ny);
Sigma_Outside_Box = 1;
Sigma_In_Box = 10^-2;
%% Create boxes
Lower_Box = [(nx* 2/5), (nx* 3/5), ny , (ny *4/5)];
Top_Box = [(nx* 2/5), (nx* 3/5),0, (ny *1/5)];
%% Generate and plot the Conductivity map
Cond_map = ones(nx,ny);
for i = 1:nx
    for j = 1:ny
        if (i < Lower_Box(2) && i >Lower_Box(1) && ((j < Top_Box(4)) || (j > Lower_Box(4))))
            Cond_map(i,j) = Sigma_In_Box;
        end
    end
end
figure('name','Conductivity Map');
surf(Cond_map);
xlabel('Y-Position')
ylabel('X-Position')
axis tight
grid on;
view(2)
title('Conductivity ($\sigma$) Map','interpreter','latex');
%saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
↪ 2\Figures','Part2ConductivityMap.png'),'png')

%% Generate G and V matrices
for i = 1:nx
    for j = 1: ny
        n = j+ (i-1)*ny;
```

```
        if (i == 1)
            G(n,:) = 0;
            G(n,n) = 1;
            V(n) = 1;
        elseif (i == nx)
            G(n,:) = 0;
            G(n,n) = 1;
        elseif ((i > 1) && (i < nx) && (j==1))
            G(n, n+ny) = Cond_map(i+1,j);
            G(n,n) = -(Cond_map(i,j+1)+Cond_map(i-1,j)+Cond_map(i+1,j));
            G(n,n-ny) = Cond_map(i-1,j);
            G(n,n-1) = Cond_map(i,j+1);
        elseif ((j == ny) && (i < nx) && (i >1))
            G(n, n+ny) = Cond_map(i+1,j);
            G(n,n) = -(Cond_map(i-1,j)+Cond_map(i+1,j)+Cond_map(i,j-1));
            G(n,n-ny) = Cond_map(i-1,j);
            G(n,n+1) = Cond_map(i,j-1);
        else
            G(n, n+ny) = Cond_map(i+1,j);
            G(n,n) = -(Cond_map(i-1,j)+Cond_map(i+1,j)+Cond_map(i,j+1)+Cond_map(i,j-1));
            G(n,n-ny) = Cond_map(i-1,j);
            G(n,n-1) = Cond_map(i,j+1);
            G(n,n+1) = Cond_map(i,j-1);
        end
    end
end
%%
solution1 = G\V;
Voltage = zeros(nx,ny);
for i =1:nx
    for j = 1:ny
        Voltage(i,j) = solution1(j+(i-1) * ny);
    end
end
%%
%plot the voltage map
figure('name','Voltage map (Including bottle neck)');
%3D Plot
subplot(2,1,1)
surf(Voltage);
view(-92.9556,10.2554)
xlabel('X-Position')
ylabel('Y-Position')
zlabel('V (Voltage)')
axis tight
grid on;
cb=colorbar;
cb.Label.String = 'Voltage';
cb.Location = 'eastoutside';
%2D plot
subplot(2,1,2)
surf(Voltage);
axis tight
grid on;
view(2)
xlabel('X-Position')
ylabel('Y-Position')
%Title and colorbar
sgtitle('Voltage Map with Bottleneck');
cb=colorbar;
cb.Label.String = 'Voltage';
cb.Location = 'eastoutside';
%saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
↪ 2\Figures','Part2VoltageMapBottleNeck.png'),'png')
```

```
%%
```

Source Code 4: Part 2 - Solving G matrix and plotting Voltage Map

```
%Plot the Electric field using Maxwell's equation
% Using the classical E = -GradV equation
[E_x,E_y] = gradient(Voltage);
% Applying the '-'
E_x = -E_x;
E_y = -E_y;
figure('name','Electric Field')
quiver(E_x',E_y');
axis tight
xlabel('X-Position')
ylabel('Y-Position')
title('Quiver Plot of Electric Field')
%saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment 2\Figures','Part2QuiverPlotEf.png'),'png')
%%
```

Source Code 5: Part 2 - Plotting electric field map (Quiver)

```
%%
%Plot the current density
% Using Maxwells equation for the current
% J = sigma E
Jx = E_x .*Cond_map;
Jy = E_y .*Cond_map;
figure('name','Current Density')
quiver(Jx,Jy);
axis tight
xlabel('Y-Position')
ylabel('X-Position')
title('Quiver Plot of Current Density (J)')
%saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment 2\Figures','Part2CCurrentDensity.png'),'png')
```

Source Code 6: Part 2 - Plotting Current density map (Quiver)

```
%%
%Plot the Magnitude of the current density
% Simply getting the magnitude of the current density
magnitude_current_density = sqrt(((Jx .^2) + (Jy .^2)));
% Plot the magnitude of the current density
figure('name','|Current Density|')
%3D Plot
subplot(2,1,1)
surf(magnitude_current_density)
axis tight
grid on;
xlabel('X-Position')
ylabel('Y-Position')
zlabel('V/m (Voltage/meter)')
cb=colorbar;
cb.Location = 'eastoutside';
cb.Label.String = '|J|';
%2D Plot
subplot(2,1,2)
surf(magnitude_current_density)
```

```
axis tight
view(2)
grid on;
xlabel('Y-Position')
ylabel('X-Position')
cb=colorbar;
cb.Location = 'eastoutside';
cb.Label.String = '|J|';
sgtitle('Magnitude of Current Density')
%saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
↪ 2\Figures','Part2MagCurrentDensity.png'),'png')
```

Source Code 7: Part 2 - Plotting the Magnitude of the current density

```
%%
%Plot the Electric field in X
figure('name','Electric Field in X')
%3D Plot
subplot(2,1,1)
surf(E_x)
axis tight
grid on;
xlabel('Y-Position')
ylabel('X-Position')
zlabel('V/m (Voltage/meter)')
cb=colorbar;
cb.Location = 'eastoutside';
cb.Label.String = 'E-Field';
%2D Plot
subplot(2,1,2)
surf(E_x)
axis tight
grid on;
view(2)
sgtitle('Electric Field along the X direction')
xlabel('Y-Position')
ylabel('X-Position')
cb=colorbar;
cb.Location = 'eastoutside';
cb.Label.String = 'E-Field';
sgtitle('Electric Field')
%saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
↪ 2\Figures','Part2EfieldX.png'),'png')
```

Source Code 8: Part 2 - Plotting electric field in x

```
%%
%plot the Electric Field in Y
figure('name','Electric Field in Y')
%3D Plot
subplot(2,1,1)
surf(E_y)
axis tight
grid on;
xlabel('Y-Position')
ylabel('X-Position')
zlabel('V/m (Voltage/meter)')
cb=colorbar;
cb.Location = 'eastoutside';
cb.Label.String = 'E-Field';
%2D Plot
```

```
subplot(2,1,2)
surf(E_y)
axis tight
grid on;
view(2)
xlabel('X-Position')
ylabel('Y-Position')
sgtitle('Electric Field along the Y direction')
cb=colorbar;
cb.Location = 'eastoutside';
cb.Label.String = 'E-Field';
sgtitle('Electric Field')
%saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
↪ 2\Figures','Part2EfieldY.png'),'png')
```

Source Code 9: Part 2 - Plotting electric field in y