

CARLETON UNIVERSITY

MODELLING OF INTEGRATED DEVICES
ELEC 4700

Assignment 2

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1 Non Resistive Region

Laplace's Equation The first problem solved in this assignment is to solve the electrostatic potential defined by equation (1) below, where the boundary conditions are as follow,

$$\nabla^2 V = 0 \quad (1)$$

The simulation model the problem as an orthogonal resistor network with resistance values of 1.

1.1 1D Solution

The first simulation was ran with the simple case,

$$V = V_0 \text{ at } x = 0 \quad \text{and} \quad V = 0 \text{ at } x = L$$

Figure 1 below shows the results of the simulation, where the region was set as a rectangle with a voltage of 1V at $x=0$, and grounded at $x=L$. The y boundary's of the region were set as free.

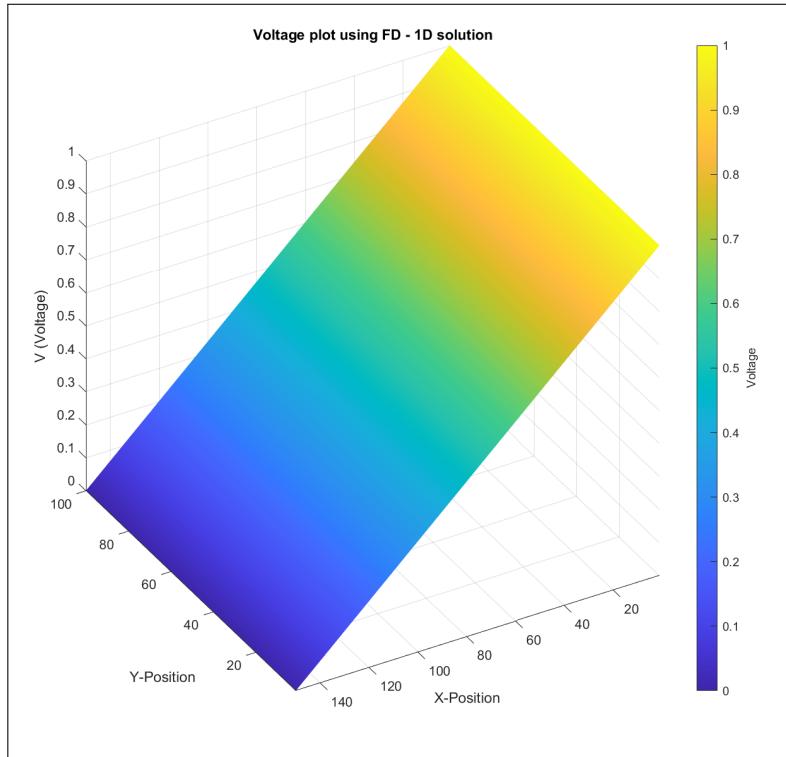


Figure 1: 1D Simulation result - Y bounds free.

1.2 2D Simulation and analytical solution

Finite Difference The next case tested for was the 2D finite difference, where the boundary conditions are as follow,

$$V = V_0 \text{ at } x = 0 \quad \text{and } V = 0 \text{ at } x = L \quad \text{and } V = 0 \text{ at } y = 0 \quad \text{and } y = W.$$

The Y dimensions of the same rectangular region as in (A) were now simulated as grounded, where the simulation results are shown in Figure 2.1 below.

Analytical solution The analytical solution was obtained by using equation (2) as shown below,

$$V(x, y) = \frac{4V_0}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \frac{\cosh(\frac{n\pi x}{a})}{\cosh(\frac{n\pi b}{a})} \sin\left(\frac{n\pi y}{a}\right) \quad (2)$$

Once again using the Matlab Surf command to plot the voltage, the final results obtained are as shown in Figure 2.2.

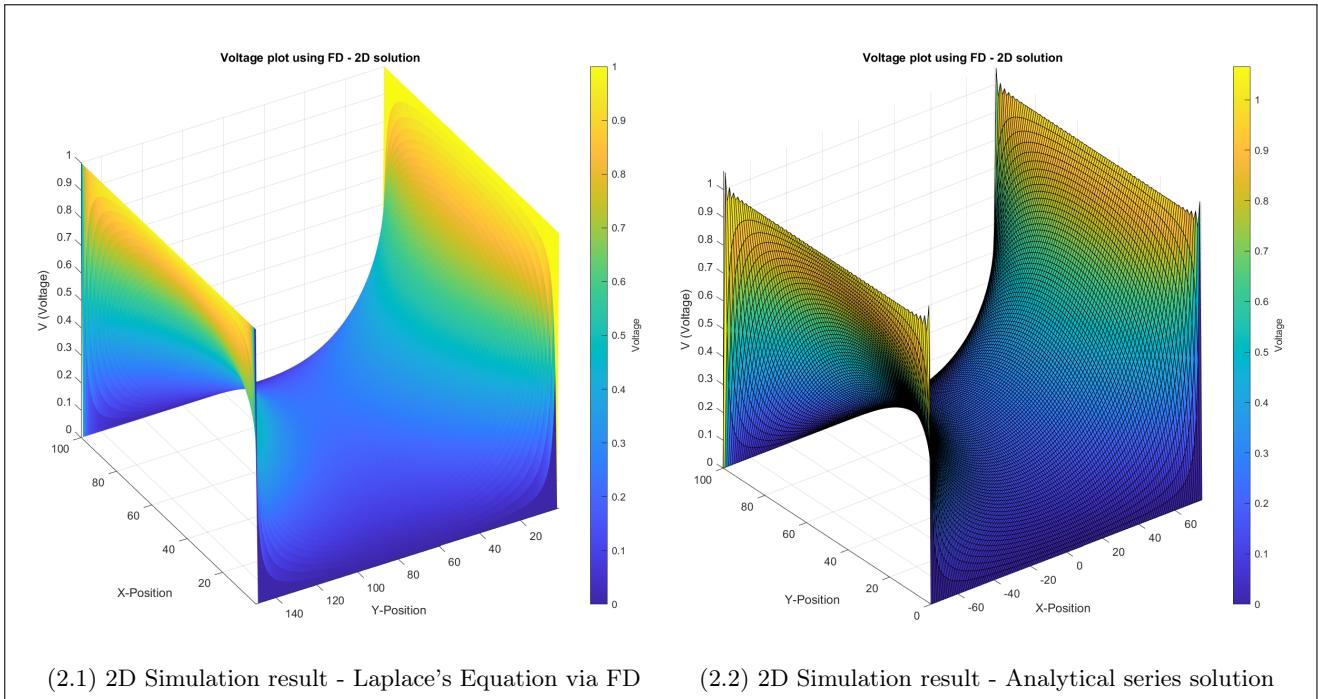


Figure 2: 2D Simulation result - Y bounds set to 0V.

Comparing the two figures, it is clear to see that both figures have some explicit error. Both solutions show a high level of correlation in most of the solution, at the Y bounds of the region there is a discrepancy. Furthermore, during the simulation of the analytical solution the solution converges quickly, then slows down as the solution gets close to the full solution. This determines that there is a trade off between accuracy and speed.

Meshing As mentioned above, meshing is a powerful technique to solve problems via geometry. Different mesh can have different associated complexities (time required for computation inside of a mesh), and depending on the geometry they cover one mesh can be better than another. A mesh that is dense (smaller individual mesh size) can provide more accurate results, but will take more simulation time than a larger (more inaccurate) mesh. It is fundamentally up to the user to determine the accuracy-time trade off required to complete the analysis.

2 Current Flow Through Bottle Neck

2.1 Conductivity of region $\sigma(x, y)$

Using the Finite Difference method, a bottle neck is now introduced into the rectangular region. The current flow in this region is given by,

$$\nabla(\sigma_{x,y} \nabla V) = 0 \quad (3)$$

Wherein the boxes are highly resistive, having a very low $\sigma_{x,y}$ value. Using these conditions the conductivity map of the region is generated as shown in Figure 3.

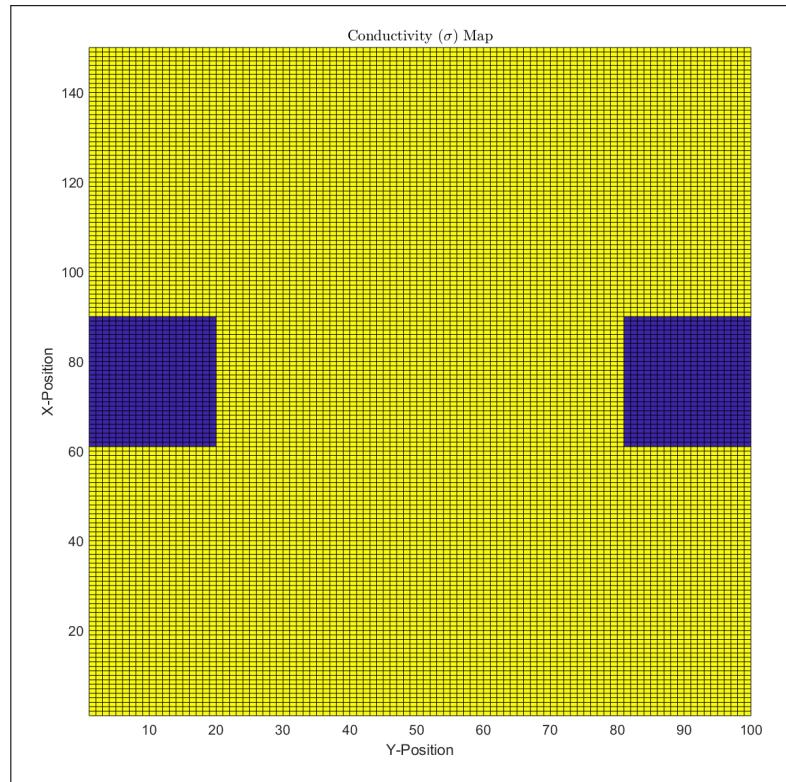


Figure 3: Conductivity map of the simulation region

2.2 Voltage across region $V(x, y)$

Using the same G matrix mapping as previously (with the inclusion of the consideration towards the sigma) the voltage of the region was obtained, as shown in Figure 4. The results of this simulation show the effect of introducing a bottle neck, wherein uniformity of the region is lost.

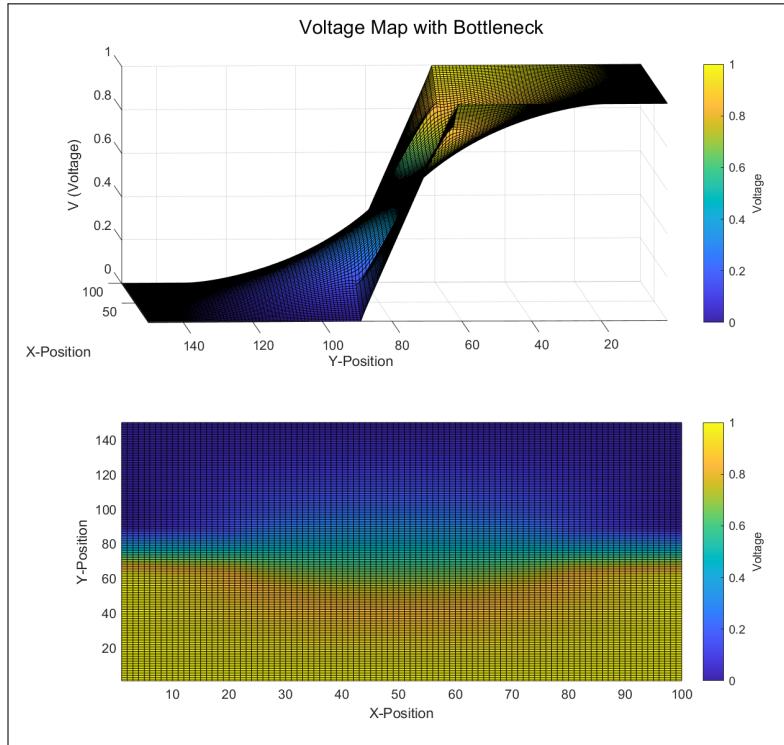


Figure 4: Voltage map across rectangular region with bottle neck

Electric field Using the classical electrostatic definition of a time independent electric field \mathbf{E} ,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = 0 \quad (4)$$

Which implies that the electric field is the gradient of the electric potential,

$$-\mathbf{E} = \nabla V \quad (5)$$

Equation 5 can be applied very simply in Matlab using the *gradient()* function, where both the E_x and E_y components can be derived in 1 line. Figure 5 on the following page shows the quiver plot of the electric field across the entire rectangular region.

2.3 Electric field along x (\vec{E}_x)

Figure 6.1 shows the surface plot of the electric field along the x axis. The results present in this graph agree with those found in Figure 4, wherein along the x axis there is not a large change in voltage. The peaks of this surface plot represent the boundary's of the box, where the difference in conductivity effects the electric field.

2.4 Electric field along y (\vec{E}_y)

Figure 6.2 shows the surface plot of the electric field along the y axis. Once again this result agrees with those found in Figure 4, where along the y axis there is a large delta period when moving across the boxes. The peaks of this surface plot represent the effects of the change in conductivity when moving from the region outside of the box into the region inside box (difference in sigma).

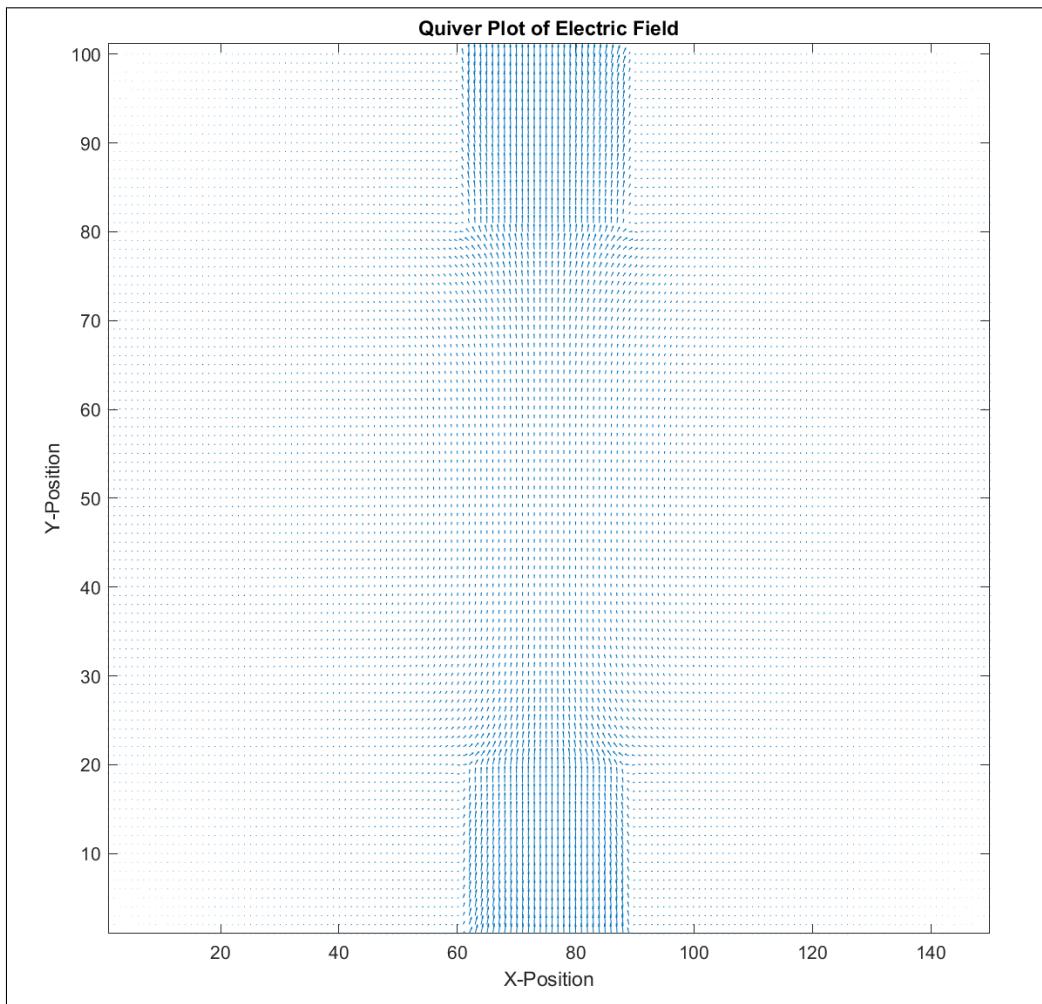


Figure 5: Electric field across region - Quiver plot

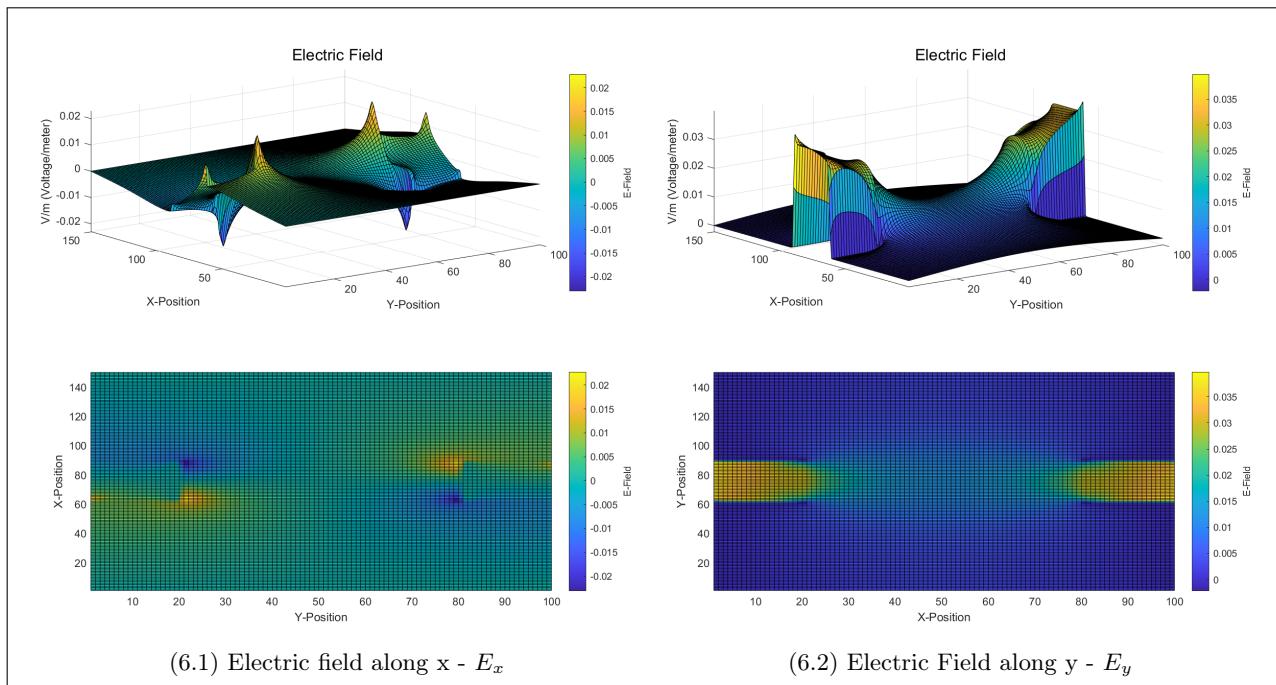


Figure 6: Electric field simulation results via Finite Difference

2.5 Current density $\vec{J}(x, y)$

Using the common approximation of the current density, such that the current is proportional to the electric field,

$$\mathbf{J} = \sigma \mathbf{E} \quad (6)$$

The current density of the rectangular region was obtained as shown in Figure 7 below.

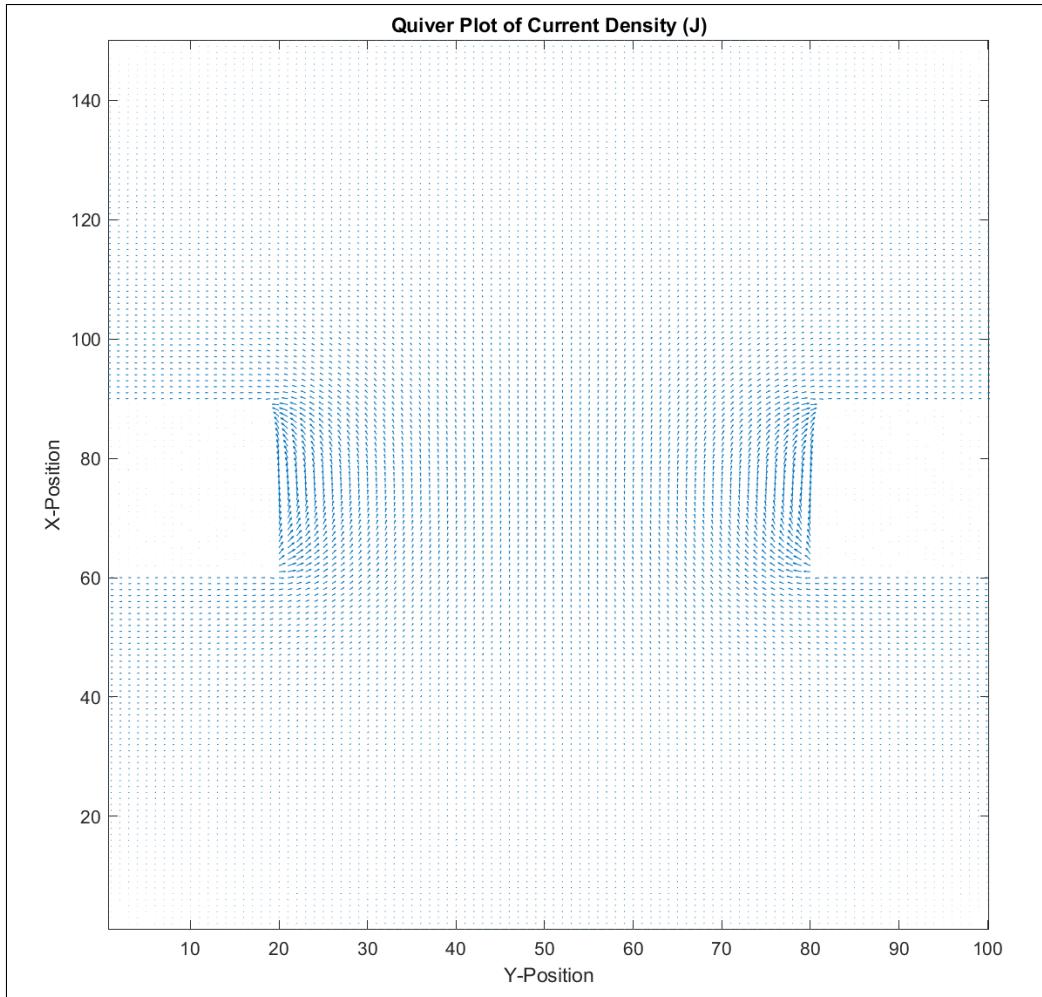


Figure 7: Current density across region - Quiver plot

2.6 Variations in the simulation

Figure 8.1 on the next page shows the ‘base’ case for the simulation, where the sigma inside of the box is significantly lower than outside, and the bottle neck is wide. Figure 8.2 shows the simulation case where the sigma inside of the box is larger than outside, but the bottle neck is the same as the control case. Finally, Figure 8.3 shows the simulation case where the sigma of the box is the same as in the control, but the bottle neck is narrow.

Varying box conductivity Figure 9.2 shows the effects on the voltage map where the conductivity inside the box is higher than outside. This plot shows a vastly linear relation of the voltage, with the exception of the small region where the boxes exist, causing the flat portion. Figure 10.2 shows an interesting result, comparing to Figure 10.1 shows an ‘inversion’. This is a artifact of the change in conductivity, where the current flow is directed to the two boxes instead of the rest of the rectangular region.

Narrowing the bottle neck Figure 9.3 shows the effects on the voltage map where the bottle neck is narrowed. This plot shows a parallel result to that of Figure 9.1, where simply the slope at the bottle neck is larger. The results of Figure 10.3 also agree with the previous statement, wherein due to the narrowed bottle neck the magnitude of the current density at the bottle neck is increased.

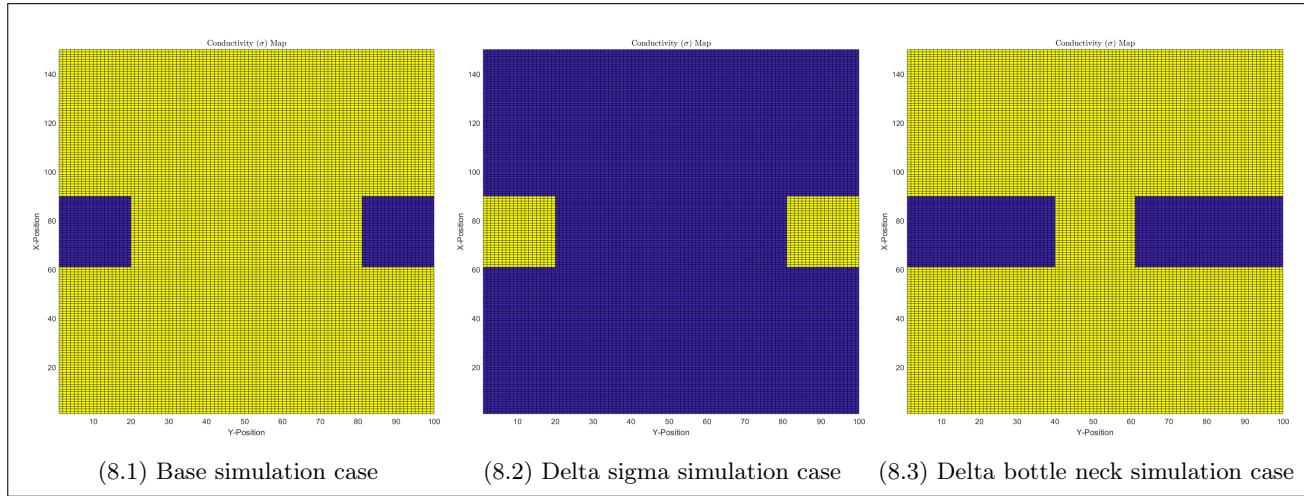


Figure 8: Three simulation cases - Conductivity map

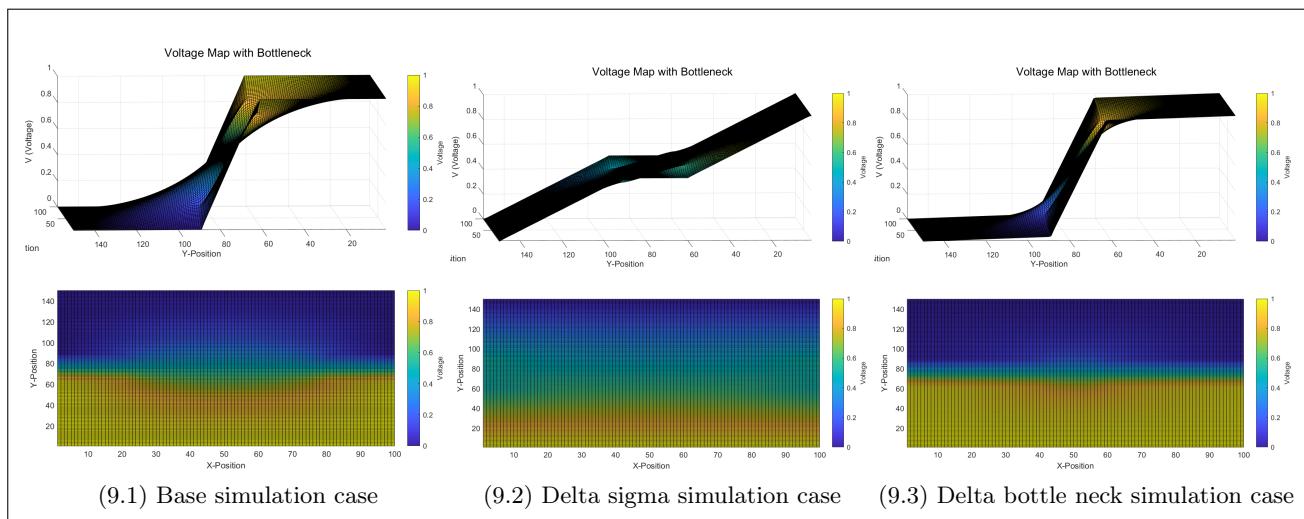


Figure 9: Three simulation cases - Voltage map

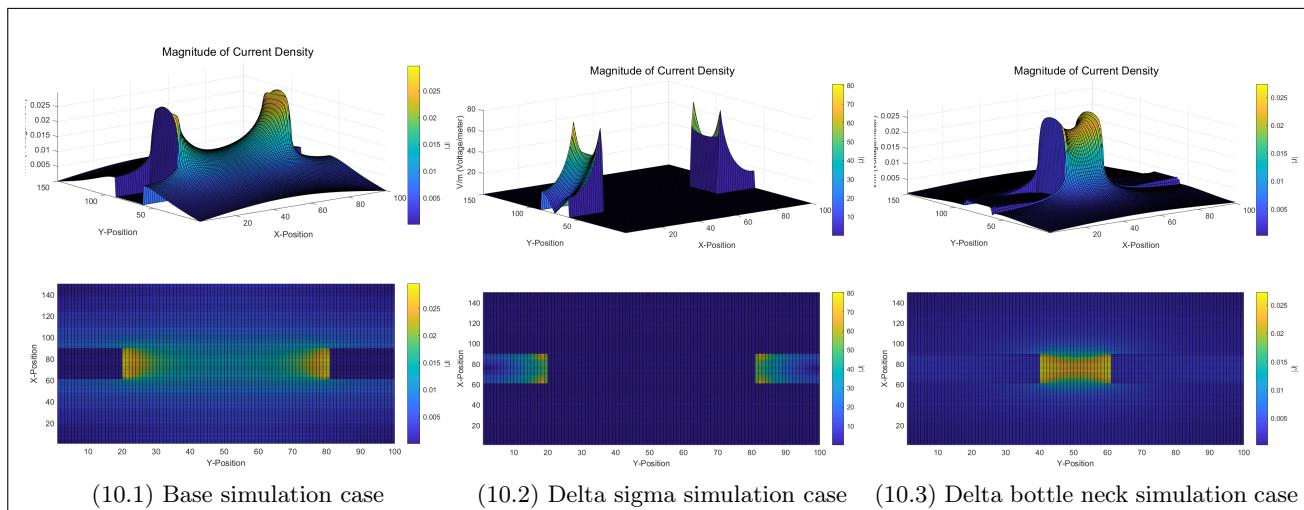


Figure 10: Three simulation cases - Magnitude of current density map

3 Appendix

3.1 Free Y dimension code

```

11 %Initialize size of mesh
12 nx = 150;
13 ny = 100;
14 %Initialize G and V matrix
15 G = sparse(nx*ny,nx*ny);
16 V = zeros(nx*ny,1);
17 VO=1;
18 %The G matrix is filled in using a loop using the FD method
19 for i = 1:nx
20     for j = 1:ny
21         n = j + (i-1)*ny;
22         if i == 1
23             G(n,:) = 0;
24             G(n,n) = 1;
25             V(n) = 1;
26         elseif i == nx
27             G(n,:) = 0;
28             G(n,n) = 1;
29             V(n) = 0;
30         elseif j == 1
31             G(n, :) = 0;
32             G(n, n) = -3;
33             G(n, n+1) = 1;
34             G(n, n+ny) = 1;
35             G(n, n-ny) = 1;
36         elseif j == ny
37             G(n, n) = -3;
38             G(n, n-1) = 1;
39             G(n, n+ny) = 1;
40             G(n, n-ny) = 1;
41         else
42             G(n, n) = -4;
43             G(n, n+1) = 1;
44             G(n, n-1) = 1;
45             G(n, n+ny) = 1;
46             G(n, n-ny) = 1;
47         end
48     end
49 end
50
51 V = G\V;
52 Soln = zeros(nx,ny,1);
53 for i=1:nx
54     for j=1:ny
55         Soln(i,j) = V(j+(i-1)*ny);
56     end
57 end
58 figure('Name','1D FD Solution')
59 surf(Soln,'edgecolor','none')
60 title('Voltage plot using FD - 1D solution')
61 xlabel('X-Position')
62 ylabel('Y-Position')
63 zlabel('V (Voltage)')
64 view(-120,35)

```

Source Code 1: Part 1 - Simple 1D case code

3.2 Grounded Y dimension code

```

74 G2 = sparse(nx*ny,nx*ny);
75 V2 = zeros(nx*ny,1);
76
77 %The G matrix is filled in using a loop using the FD method
78 for x = 1:nx
79     for y = 1:ny
80         n = y + (x-1)*ny;
81
82         if x == 1
83             G2(n,:) = 0;
84             G2(n, n) = 1;
85             V2(n) = 1;
86         elseif x == nx
87             G2(n,:) = 0;
88             G2(n, n) = 1;
89             V2(n) = 1;
90         elseif y == 1
91             G2(n,:) = 0;
92             G2(n, n) = 1;
93         elseif y == ny
94             G2(n,:) = 0;
95             G2(n, n) = 1;
96         else
97             G2(n, n) = -4;
98             G2(n, n+1) = 1;
99             G2(n, n-1) = 1;
100            G2(n, n+ny) = 1;
101            G2(n, n-ny) = 1;
102        end
103    end
104 end
105
106 V2 = G2\W2;
107 Soln2 = zeros(nx,ny,1);
108 for i=1:nx
109     for j=1:ny
110         Soln2(i,j) = V2(j+(i-1)*ny);
111     end
112 end
113 figure('Name','2D FD Solution')
114 surf(Soln2,'edgecolor','none')
115 title('Voltage plot using FD - 2D solution')
116 xlabel('X-Position')
117 ylabel('Y-Position')
118 zlabel('V (Voltage)')
119 cb=colorbar;
120 cb.Label.String = 'Voltage';
121 cb.Location = 'eastoutside';
122 axis tight
123 view(-120,35)

```

Source Code 2: Part 1 - Simple 2D case code

3.3 Part 1 - Analytical Solution Code

```

126 %Analytical solution
127 x = linspace(-nx/2,nx/2,nx);
128 y = linspace(0,ny,ny);
129 [i,j] = meshgrid(x,y);
130 AnalyticalSoln = sparse(ny,nx);
131 for n = 1:2:300
132     AnalyticalSoln = AnalyticalSoln + cosh(n*pi*i/ny).*sin(n*pi*j/ny)./(n*cosh(n*pi*nx/2/ny));
133     figure(3)
134     surf(x,y,(4/pi)*AnalyticalSoln)
135     title('Voltage plot using FD - 2D solution')
136     xlabel('X-Position')
137     ylabel('Y-Position')
138     zlabel('V (Voltage)')
139     cb=colorbar;
140     cb.Label.String = 'Voltage';
141     cb.Location = 'eastoutside';
142     axis tight
143     view(3)
144     pause(0.1)
145 end

```

Source Code 3: Part 1 - Simple 2D case analytical series solution code

3.4 Part 2 - Current Flow Through Bottle Neck Code

```

9 nx = 150;
10 ny = 100;
11 G = sparse((nx *ny), (nx*ny));
12 V = zeros(nx *ny);
13 Sigma_Outside_Box = 1;
14 Sigma_In_Box = 10^-2;
15 %% Create boxes
16 Lower_Box = [(nx* 2/5), (nx* 3/5), ny , (ny *4/5)];
17 Top_Box = [(nx* 2/5), (nx* 3/5),0, (ny *1/5)];
18 %% Generate and plot the Conductivity map
19 Cond_map = ones(nx,ny);
20 for i = 1:nx
21     for j = 1:ny
22         if (i < Lower_Box(2) && i >Lower_Box(1) && ((j < Top_Box(4)) || (j > Lower_Box(4))))
23             Cond_map(i,j) = Sigma_In_Box;
24         end
25     end
26 end
27 figure('name','Conductivity Map');
28 surf(Cond_map);
29 xlabel('Y-Position')
30 ylabel('X-Position')
31 axis tight
32 grid on;
33 view(2)
34 title('Conductivity ($\sigma$) Map','interpreter','latex');
35 %saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
36 %→ 2\Figures','Part2ConductivityMap.png'), 'png')
37 %% Generate G and V matrices
38 for i = 1:nx
39     for j = 1: ny
40         n = j+ (i-1)*ny;

```

```

41     if (i == 1)
42         G(n,:) = 0;
43         G(n,n) = 1;
44         V(n) = 1;
45     elseif (i == nx)
46         G(n,:) = 0;
47         G(n,n) = 1;
48     elseif ((i > 1) && (i < nx) && (j==1))
49         G(n, n+ny) = Cond_map(i+1,j);
50         G(n,n) = -(Cond_map(i,j+1)+Cond_map(i-1,j)+Cond_map(i+1,j));
51         G(n,n-ny) = Cond_map(i-1,j);
52         G(n,n-1) = Cond_map(i,j+1);
53     elseif ((j == ny) && (i < nx) && (i >1))
54         G(n, n+ny) = Cond_map(i+1,j);
55         G(n,n) = -(Cond_map(i-1,j)+Cond_map(i+1,j)+Cond_map(i,j-1));
56         G(n,n-ny) = Cond_map(i-1,j);
57         G(n,n+1) = Cond_map(i,j-1);
58     else
59         G(n, n+ny) = Cond_map(i+1,j);
60         G(n,n) = -(Cond_map(i-1,j)+Cond_map(i+1,j)+Cond_map(i,j+1)+Cond_map(i,j-1));
61         G(n,n-ny) = Cond_map(i-1,j);
62         G(n,n-1) = Cond_map(i,j+1);
63         G(n,n+1) = Cond_map(i,j-1);
64     end
65   end
66 end
67 %%
68 solution1 = G\V;
69 Voltage = zeros(nx,ny);
70 for i =1:nx
71     for j = 1:ny
72         Voltage(i,j) = solution1(j+(i-1) * ny);
73     end
74 end
75 %%
76 %plot the voltage map
77 figure('name','Voltage map (Including bottle neck)');
78 %3D Plot
79 subplot(2,1,1)
80 surf(Voltage);
81 view(-92.9556,10.2554)
82 xlabel('X-Position')
83 ylabel('Y-Position')
84 zlabel('V (Voltage)')
85 axis tight
86 grid on;
87 cb=colorbar;
88 cb.Label.String = 'Voltage';
89 cb.Location = 'eastoutside';
90 %2D plot
91 subplot(2,1,2)
92 surf(Voltage);
93 axis tight
94 grid on;
95 view(2)
96 xlabel('X-Position')
97 ylabel('Y-Position')
98 %Title and colorbar
99 sgtitle('Voltage Map with Bottleneck');
100 cb=colorbar;
101 cb.Label.String = 'Voltage';
102 cb.Location = 'eastoutside';
103 %saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
104 → 2\Figures','Part2VoltageMapBottleNeck.png'), 'png')
```

104 %%

Source Code 4: Part 2 - Solving G matrix and plotting Voltage Map

```

103 %Plot the Electric field using Maxwell's equation
104 % Using the classical E = -GradV equation
105 [E_x,E_y] = gradient(Voltage);
106 % Applying the '-'
107 E_x = -E_x;
108 E_y = -E_y;
109 figure('name','Electric Field')
110 quiver(E_x',E_y');
111 axis tight
112 xlabel('X-Position')
113 ylabel('Y-Position')
114 title('Quiver Plot of Electric Field')
115 %saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
→ 2\Figures','Part2QuiverPlotEf.png'), 'png')
116 %%
```

Source Code 5: Part 2 - Plotting electric field map (Quiver)

```

117 %%
118 %Plot the current density
119 % Using Maxwell's equation for the current
120 % J = sigma E
121 Jx = E_x .*Cond_map;
122 Jy = E_y .*Cond_map;
123 figure('name','Current Density')
124 quiver(Jx,Jy);
125 axis tight
126 xlabel('Y-Position')
127 ylabel('X-Position')
128 title('Quiver Plot of Current Density (J)')
129 %saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
→ 2\Figures','Part2CCurrentDensity.png'), 'png')
```

Source Code 6: Part 2 - Plotting Current density map (Quiver)

```

130 %%
131 %Plot the Magnitude of the current density
132 % Simply getting the magnitude of the current density
133 magnitude_current_density = sqrt(((Jx .^2) + (Jy .^2)));
134 % Plot the magnitude of the current density
135 figure('name','|Current Density|')
136 %3D Plot
137 subplot(2,1,1)
138 surf(magnitude_current_density)
139 axis tight
140 grid on;
141 xlabel('X-Position')
142 ylabel('Y-Position')
143 zlabel('V/m (Voltage/meter)')
144 cb=colorbar;
145 cb.Location = 'eastoutside';
146 cb.Label.String = '|J|';
147 %2D Plot
148 subplot(2,1,2)
149 surf(magnitude_current_density)
```

```

150 axis tight
151 view(2)
152 grid on;
153 xlabel('Y-Position')
154 ylabel('X-Position')
155 cb=colorbar;
156 cb.Location = 'eastoutside';
157 cb.Label.String = '|J|';
158 sgttitle('Magnitude of Current Density')
159 %saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
→ 2\Figures','Part2MagCurrentDensity.png'),'png')

```

Source Code 7: Part 2 - Plotting the Magnitude of the current density

```

160 %%
161 %Plot the Electric field in X
162 figure('name','Electric Field in X')
163 %3D Plot
164 subplot(2,1,1)
165 surf(E_x)
166 axis tight
167 grid on;
168 xlabel('Y-Position')
169 ylabel('X-Position')
170 zlabel('V/m (Voltage/meter)')
171 cb=colorbar;
172 cb.Location = 'eastoutside';
173 cb.Label.String = 'E-Field';
174 %2D Plot
175 subplot(2,1,2)
176 surf(E_x)
177 axis tight
178 grid on;
179 view(2)
180 sgttitle('Electric Field along the X direction')
181 xlabel('Y-Position')
182 ylabel('X-Position')
183 cb=colorbar;
184 cb.Location = 'eastoutside';
185 cb.Label.String = 'E-Field';
186 sgttitle('Electric Field')
187 %saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
→ 2\Figures','Part2EfieldX.png'),'png')

```

Source Code 8: Part 2 - Plotting electric field in x

```

188 %%
189 %plot the Electric Field in Y
190 figure('name','Electric Field in Y')
191 %3D Plot
192 subplot(2,1,1)
193 surf(E_y)
194 axis tight
195 grid on;
196 xlabel('Y-Position')
197 ylabel('X-Position')
198 zlabel('V/m (Voltage/meter)')
199 cb=colorbar;
200 cb.Location = 'eastoutside';
201 cb.Label.String = 'E-Field';
202 %2D Plot

```

```
203 subplot(2,1,2)
204 surf(E_y)
205 axis tight
206 grid on;
207 view(2)
208 xlabel('X-Position')
209 ylabel('Y-Position')
210 sgtitle('Electric Field along the Y direction')
211 cb=colorbar;
212 cb.Location = 'eastoutside';
213 cb.Label.String = 'E-Field';
214 sgtitle('Electric Field')
215 %saveas(gcf,fullfile('D:\School Work\ELEC 4700\My 4700 Code\Assignment
→ 2\Figures','Part2EfieldY.png'), 'png')
```

Source Code 9: Part 2 - Plotting electric field in y