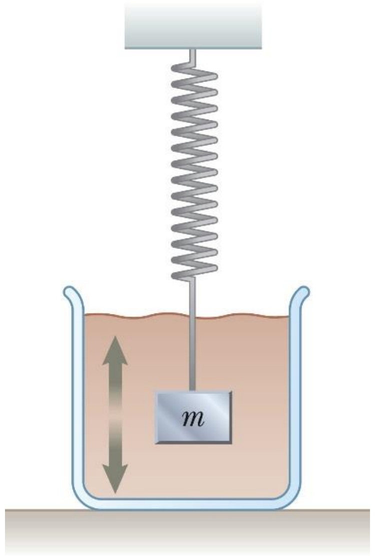
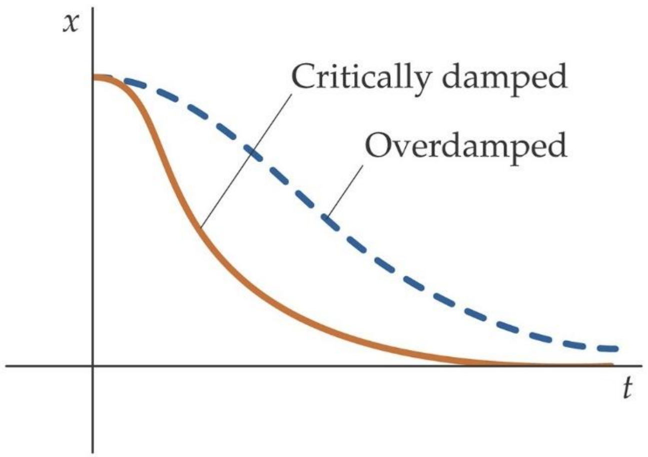
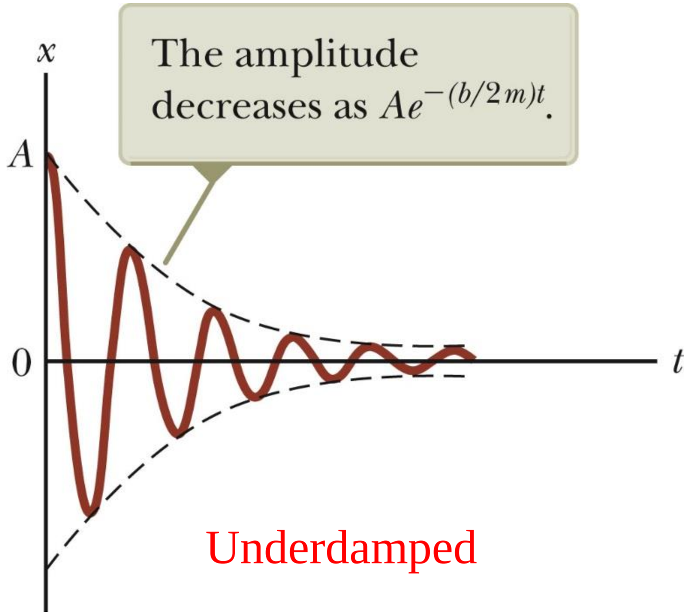
Physics (PH 1007)

Forced oscillation

Recap- Damped Oscillations







- Damped system: System stops oscillating because the mechanical energy is dissipated by friction/viscous drag etc
- Overdamped: Damping is very large (speed approaches zero as the object approaches the equilibrium position).
- Underdamped: Damping is very small (system oscillates with a amplitude that decreases slowly with time) Example: child on a playground swing when a parent stops providing a push each cycle.
- Critically damped: Motion with the minimum damping for non-oscillatory motion.

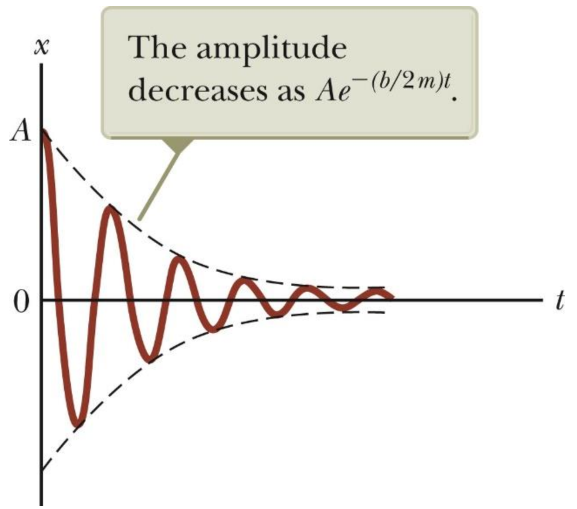
Damping

Damping in an oscillating system causes

(a) the amplitude, and

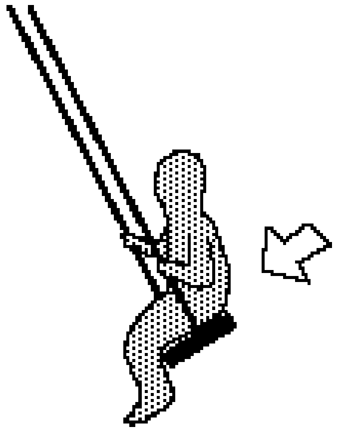
(b) the energy of the system to decrease

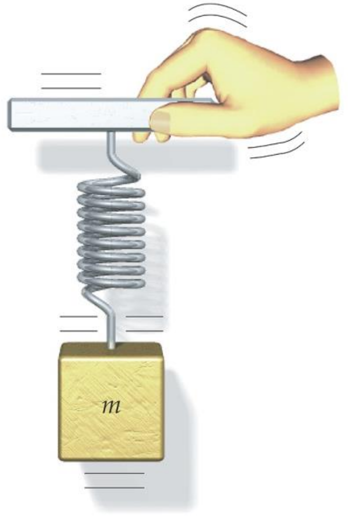
(c) the *frequency*, **to reduce slightly**.



To enable an oscillating system to go on continuously, an **external force** must be applied to the system.

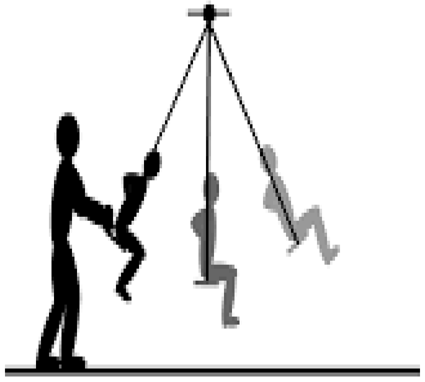
The external force supplies energy to the system.
Such a motion is called a **Forced or Driven Oscillation**.



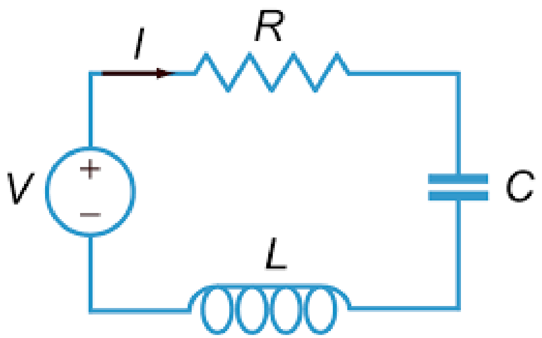


Forced Oscillation Examples

- A swing in a park where the moving swing is periodically tapped by a person standing next to the swing. However, the moving swing without periodic "tap" may be considered as a "DHO".
- Similarly, an RLC circuit as shown in the diagram can be considered as an example of a driven oscillator. **[You will learn it in your BEE course]**

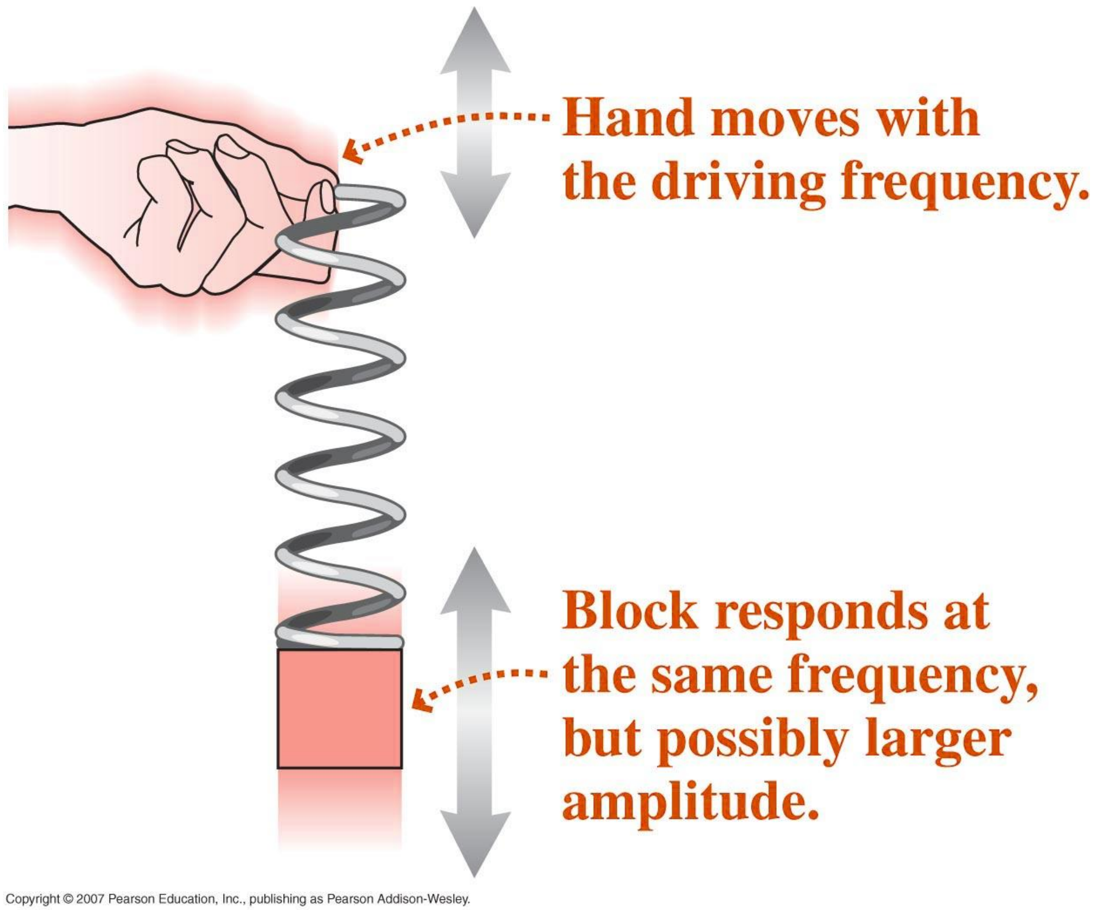


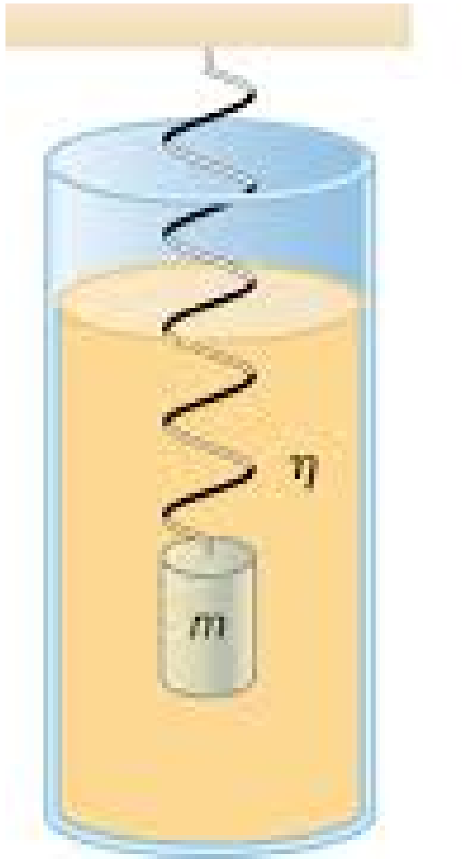
Series RLC circuit



https://encrypted-tbn0.gstatic.com/images?q=tbn%3AANd9GcSwJ_cVqzy91PiUh0p9dadCtKFQfj8nF7Ig_g&usqp=CAU

From damped to driven oscillations





Picture courtesy: http://www.physics.louisville.edu/cldavis/phys298/notes/resonance

Driving Force & forced/driven oscillation

In addition to a **damping force which is a function of velocity** (v) [$F_d = -bv$] *b is a* positive constant.

- A driving force is needed which is a periodic external force (**$F = F_0$ *sin pt***) acting on the oscillator (e.g. a regular tapping during the motion of the swing).

- Typically for weak damping case the damping effect dies out quickly with time & the system starts to oscillate with the frequency of the periodic external force.

Equation of Motion of Forced oscillation

Let x be the instantaneous displacement at time t, then various forces acting on the body under driven condition are:

1. **Restoring Force (F_R)**: This acts opposite to the displacement and tries to bring the body back to its equilibrium position.

$$F_R \propto -x \Rightarrow F_R = -kx$$

2. **Damping Force (F_D)**: The damping force is in the opposite direction of the velocity and is proportional to velocity.

$$F_D \propto v \Rightarrow F_D = -bv \Rightarrow F_D = -b\frac{dx}{dt}$$

3. **Inertial Force (F_I)**: Each moving particle has inertia force which is proportional to its acceleration.

$$F_I \propto \frac{d^2x}{dt^2} \Rightarrow F_I = m\frac{d^2x}{dt^2}$$

4. **External Force (F_I)**: A periodic external force that drives the oscillation.

$$F = F_0 \sin pt$$

Equation of Motion (cont.):

The force of inertia along with the external periodic force balances the restoring and the damping force, i.e.,

$$F_I = F_R + F_D + F_0 \sin pt$$

$$\Rightarrow m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx + F_0 \sin pt$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{b}{m}\frac{dx}{dt} - \frac{k}{m}x + \frac{F_0}{m}\sin pt$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + 2r\frac{dx}{dt} + \omega^2 x = f_0 \sin pt}, \text{ where } f_0 = \frac{F_0}{m}$$

Second order differential equation of motion for forced harmonic oscillator (FHO)

Here, b = damping constant, r = b/2m = damping coefficient

$\omega = \sqrt{\frac{k}{m}}$ = angular frequency of the undamped oscillator.

Solution for the Equation of Motion:

$$\frac{d^2x}{dt^2} + 2r\frac{dx}{dt} + \omega^2 x = f_0 \sin pt$$

Let the general solution be $x(t) = x_C(t) + x_P(t)$

Let's find the complementary solution, $x_C(t)$ for an underdamped oscillator.
For a weak-damped driven osccilator, we can start with

$$\frac{d^2x}{dt^2} + 2r\frac{dx}{dt} + \omega^2 x = 0$$

$$\Rightarrow \boxed{x_c(t) = Ce^{-rt}\sin(\omega_1 t + \varphi)}$$

Solution to an underdamped oscillator with frequency

$$\omega_1 = \sqrt{\omega^2 - r^2}$$

At sufficiently long time i.e. $t >> 1/\omega_1$, $x_C(t)$ vanishes, and the general solution reduces to a **steady-state solution** given by particular integral ($x_P(t)$). We will be interested in this **steady-state solution**.

Particular integral or steady state behavior

Here the external force plays a key role, We seek a solution (trial) as shown below with the same frequency but a different phase to that of the driving force ($F_0 \sin pt$).

$$x = x_p(t) = A\sin(pt - \theta)$$

With unknown A & θ, which we have to determine so that this is a solution to the inhomogeneous equation

$$\Rightarrow \frac{dx}{dt} = pA\cos(pt - \theta)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -p^2 A\sin(pt - \theta)$$

$$\Rightarrow -p^2 A\sin(pt-\theta) + 2prA\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f_0 \sin pt$$

$$\Rightarrow -p^2 A\sin(pt-\theta) + 2prA\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f_0 \sin[(pt-\theta)+\theta]$$

Particular integral or steady state behavior, contd..

Expanding all the <u>*sine*</u> and <u>*cosine*</u> terms, and comparing the coefficients of *sine* and *cosine* terms one can easily obtain,

$$A(\omega^2 - p^2) = f_0 \cos\theta \qquad\qquad 2rpA = f_0 \sin\theta$$

$$\because f_0^{\,2} = \{f_0 \cos\theta\}^2 + \{f_0 \sin\theta\}^2$$

$$\Rightarrow f_0^{\,2} = A^2\{(\omega^2 - p^2)^2 + 4p^2r^2\}$$

$$\Rightarrow \boxed{A = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}}}$$

Thus, the *amplitude of forced/driven oscillations* is therefore a function of the *natural angular frequency* of oscillation ω, the *damping coefficient*, r and the *frequency 'p' of the external force*, p.

phase difference is given as a tangent of the angle

$$\boxed{\tan\theta = \left(\frac{2rp}{\omega^2 - p^2}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{2rp}{\omega^2 - p^2}\right)}$$

The general Solution for the Equation of Motion:

$$Now, x_p(t) = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}} \sin(pt - \theta)$$

$$\Rightarrow x(t) = x_p(t) + x_c(t)$$

$$\boxed{\Rightarrow x(t) = \left\{ \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}} \right\} \sin(pt - \theta) \ + Ce^{-rt} \sin(\omega_1 t + \varphi)}$$

Steady-state solution

$$\boxed{x(t) = \left\{ \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}} \right\} \sin(pt - \theta)}$$

Steady-state solution

$$x(t) = A\sin(pt-\theta), \text{where } A = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}} \text{ is the Amplitude}$$

- We have met a number of frequencies in this discussion, so it might be helpful to list them in one place to help keep them straight.

$$\omega = \sqrt{k/m} = \text{natural frequency of undamped oscillator}$$

$$\omega_1 = \sqrt{\omega^2 - r^2} = \text{frequency of damped oscillator}$$

$$p = \text{frequency of driving force}$$

- To find out the maximum amplitude of a particular driven oscillator, just let $p \sim \omega$ in

$$A^2 = \frac{f_o^2}{\left(\omega^2 - p^2\right)^2 + 4r^2 p^2}.$$

i.e. $$A_{\max} = \frac{f_o}{2r\,\omega}.$$

- From this you can see that the amplitude goes as r^{-1}.

Summary:

- We have discussed Forced oscillation (i.e a DHO with an external periodic force).
- We have derived the equation of motion for a Forced harmonic oscillator (FHO).
- We have also solved the second order inhomogeneous differential equation of FHO to find the Steady State Solution.

Next Class:

- We shall discuss about Resonance phenomena with examples.
- We shall see mathematically how to arrive at conditions of Amplitude resonance with different cases of damping.

Physics (PH 1007)

Forced oscillation & Resonance

Quick re-cap of FHO

- We have discussed Forced oscillation (i.e a DHO with an external periodic force).
- We have derived the equation of motion for a Forced harmonic oscillator (FHO).
- We have also solved the second order inhomogeneous differential equation of FHO to find the Steady State Solution.

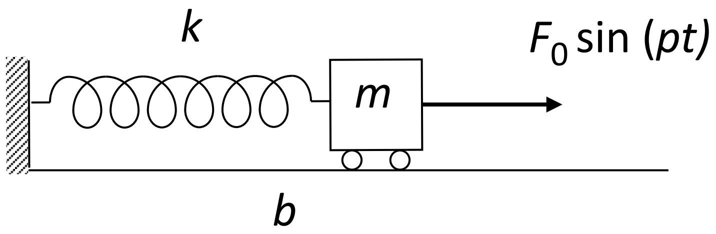
This Class:

- We shall discuss about Resonance phenomena with examples.
- We shall see mathematically how to arrive at conditions of Amplitude resonance with different cases of damping.

Recap – Last Class

Forced Harmonic Oscillator

$$r = \frac{b}{2m}; \quad \omega = \sqrt{\frac{k}{m}}$$



$$m\frac{d^2x}{dt^2} + 2r\frac{dx}{dt} + kx = F_0 \sin(pt)$$

General Solution:

$$x(t) = x_c(t) + \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}} \sin(pt - \theta)$$

$$\theta = \tan^{-1}\left(\frac{2rp}{\omega^2 - p^2}\right).$$

General solution of associated homogeneous equation
(Transient Term)
(Decays to zero as $t \to \infty$)

Particular solution of inhomogeneous equation
(Steady State Term)

Recap – Last Class

In special case of underdamped oscillation ($r < \omega$)

$$x(t) = Ce^{-rt}\sin(\omega_1 t + \varphi) + \left\{ \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}} \right\} \sin(pt - \theta)$$

Transient Term

Steady State Term

$$\theta = \tan^{-1}\left(\frac{2rp}{\omega^2 - p^2} \right)$$

$\omega = \sqrt{\frac{k}{m}}$	$\rightarrow$ Natural frequency of free oscillation
$\omega_1 = \sqrt{\omega^2 - r^2}$	$\rightarrow$ Frequency of damped oscillation
p	$\rightarrow$ Frequency of external driving force

Conclusion: Given enough time, oscillation associated with natural frequency (i.e., ω, ω_1) dies out and the system oscillates with frequency of external force (i.e., p).

Steady State Solution

$$x(t) = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}} \sin(pt - \theta)$$

$$x(t) = A\sin(pt - \theta), \quad A = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}}$$

(Amplitude for a driven/forced oscillator)

$$\theta = \tan^{-1}\left(\frac{2rp}{\omega^2 - p^2}\right)$$

(Phase constant for a driven oscillator)

- The displacement and the driving force oscillate with the same frequency, but they differ in phase by θ. When the driving frequency p approaches zero, θ approaches zero.
- When $\omega = p$, $\theta = 90^o$, and when p is much greater than ω, θ approaches 180^o. The phase of a driven oscillator always lags behind the phase of the driving force.
- The velocity of the object in the steady state is

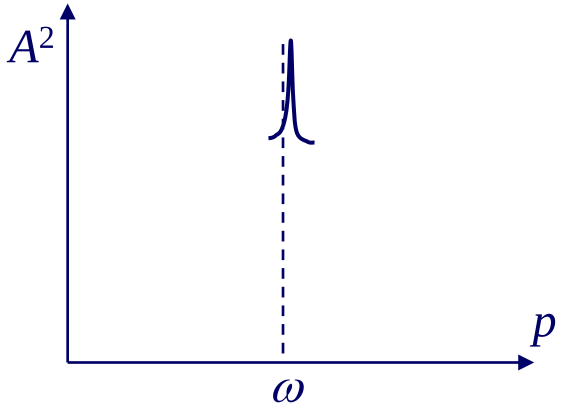
$$V = \frac{dx}{dt} = pA\cos(pt - \theta)$$

For $\omega = p$ (i.e. $\theta = 90^o$), $\quad V\big|_{p=\omega} = \frac{dx}{dt} = pA\cos(pt - 90) = pA\sin pt$

Resonance

$$A^2 = \frac{f_o^2}{\left(\omega^2 - p^2\right)^2 + 4r^2\, p^2}.$$

- When the frequency of the driving force is near the natural frequency (p ~ ω) an increase in amplitude occurs. This dramatic increase in the amplitude is called **resonance**

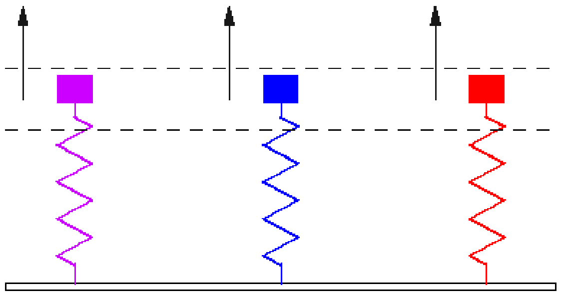


- The amplitude expression is interesting, because it says that as you drive a system at frequency *p*, its amplitude depends on both how far off you are from the resonant frequency ω, and also on how big the damping is.

- If there is no damping at all (bit unrealistic), and you drive the system at the resonant frequency, then both terms $\left(\omega^2 - p^2\right)^2 = 0$ and $4r^2\, p^2 = 0.$

- In this case, the amplitude goes to infinity! The way to think about this is that the driving force pumps energy into the oscillator (like pushing a child on a swing), and if there is no dissipation, there is no loss of energy and the energy grows to become infinite.

- If you tune the driver frequency (variable *p*) for a given oscillator (fixed ω), what is the value of ω for which A^2 is maximum? This is condition for **Amplitude Resonance**.



f = $0.4f_0$ f = $1.1f_0$ f = $1.6f_0$

natural freq. = f_0

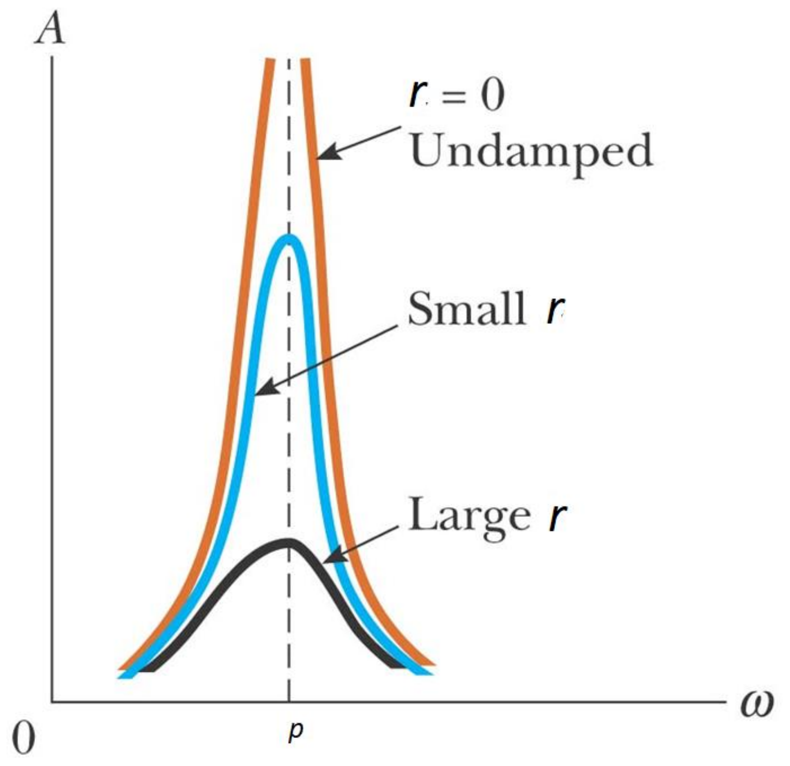
Condition for Amplitude Resonance

$$A = \frac{f_o}{\sqrt{\left(\omega^2 - p^2\right)^2 + 4r^2 p^2}}$$

- Amplitude of *the driven oscillator* is maximum when the denominator is a minimum

$$\frac{d}{dp}\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2} = 0$$

$$\Rightarrow \boxed{p = \sqrt{\omega^2 - 2r^2}}$$



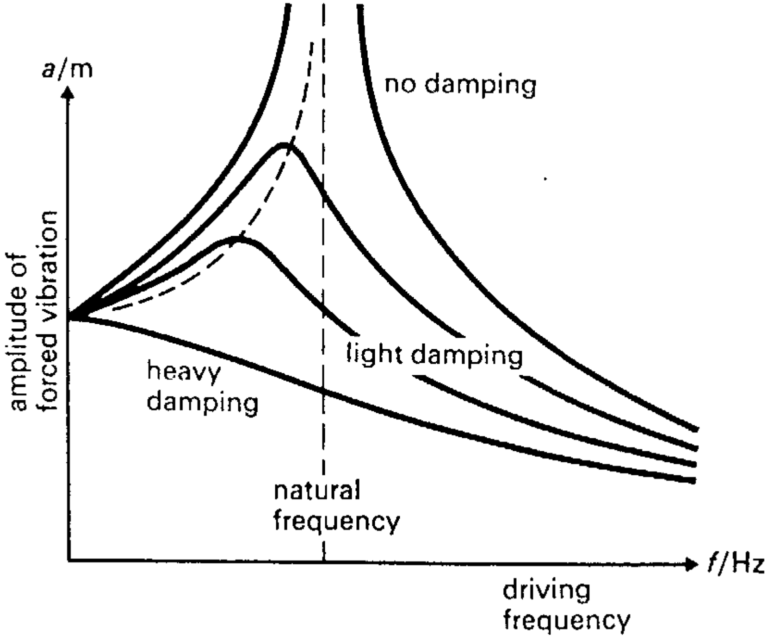
- When ω ≈ p, the amplitude becomes maximum, and this condition is known as Amplitude resonance. At resonance,

$$\Rightarrow A = A_{max} = \frac{f_0}{2rp}$$

- Thus, the amplitude (at resonance) goes as r^{-1}
- Smaller the r, the larger is the resonance amplitude and sharper is the resonance curve.
- In principle for r = 0, A= ∞, though it is practically unattainable.

Resonance, contd.

- Resonance (maximum peak in amplitude, velocity, power etc) occurs when driving frequency is nearly equal to the natural frequency
- The amplitude increases with decreased damping
- The curve broadens as the damping increases
- The shape of the resonance curve depends on damping constant *b*

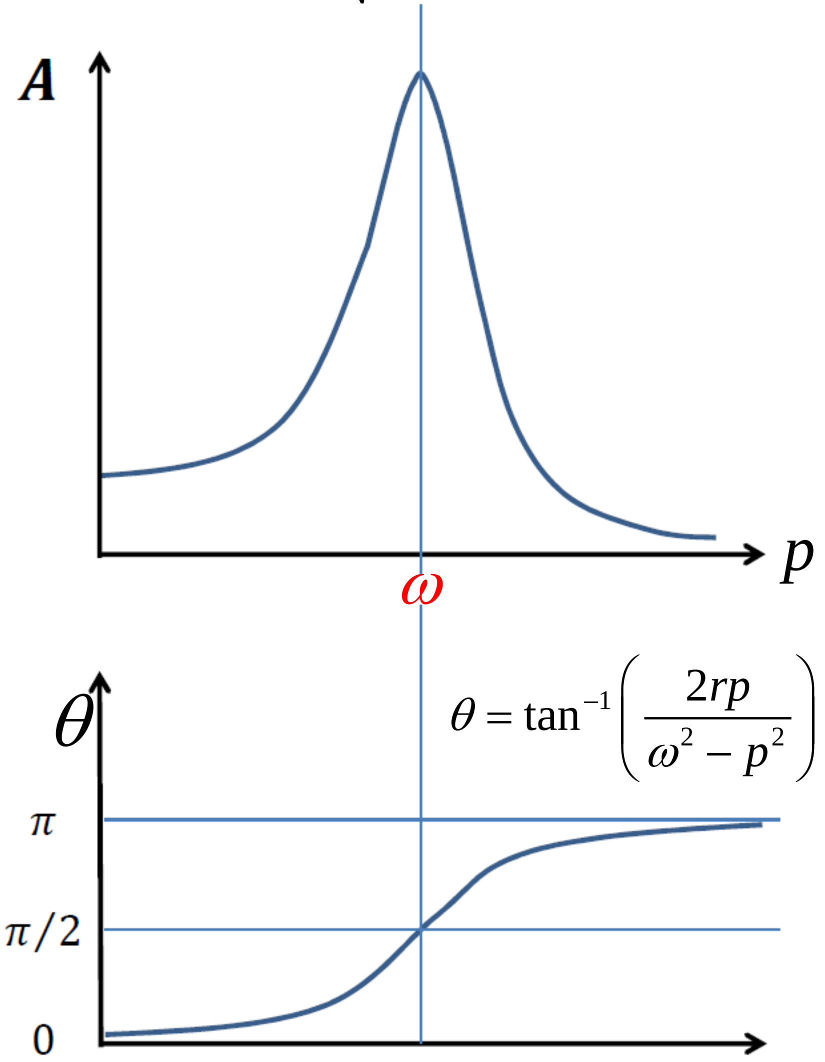


- *Amplitude & energy* of oscillator decreases rapidly as the frequency increases orr decreases from either side of the resonant frequency (**ω_R**).
- The rate of decrease of amplitude/energy on either side of *resonant frequency* defines *the sharpness of resonance*
- *Smaller the r*, or weaker damping, *sharper is the resonance* & larger the damping flatter is the resonance.

More on Resonance: The Phase

$$A = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}}$$

- The The peak occurs at a frequency that is close to ω, but not exactly equal to ω displacement and the driving force oscillate with the same frequency, but they differ in phase by θ.
- The phase of a driven oscillator always lags behind the phase of the driving force.
- At resonance (i.e. $\omega \sim p$), phase shift θ=90°. The force pushes the mass in the direction it is already moving adding energy to the system
- At resonance, the applied force is in phase with the velocity (dx/dt, with θ = 90) object is always moving in the direction of the driving force and thus the power transferred to the oscillator is a maximum

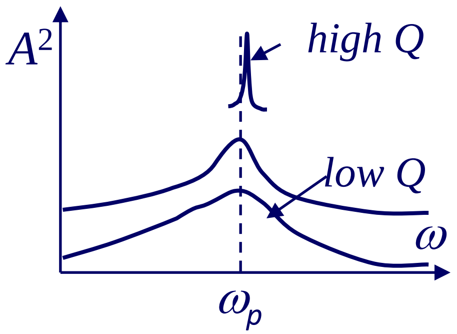


$$\theta = \tan^{-1}\left(\frac{2rp}{\omega^2 - p^2}\right)$$

Quality Factor (Q)

- There is a quantitative measure of how sharp the resonance is, the **quality factor**, or *Q*, a dimensionless quantity. It measures the strength of response of the oscillator to an external driver at the resonant frequency.
- It is just the ratio of the resonance peak frequency ω to the peak width $2r$.

$$Q = \frac{\omega}{2r}.$$



- In the case of a pendulum, which you might think is a good resonator (keeps good time), the Q might be about 100. For a quartz clock, on the other hand, Q may reach 10000.

- An alternative way of thinking about the Q of a resonator is to consider it as

$$Q = \pi \frac{\text{decay time}(\tau)}{\text{period}} = \pi \frac{1/r}{2\pi/\omega} = \frac{\omega}{2r}.$$

Applications of Resonance

The **loudness of music** produced by musical instruments such as the trumpet and flute is the result of resonance in the air.

Applications of Resonance

- Electricity, tuning a radio
  - The natural frequency of the radio circuit is made equal to the incoming electromagnetic wave by changing its capacitance
  - The electrons in the circuit will oscillate with the incoming electromagnetic wave.
  - The electric current will oscillate and this can be turned into sound, through a speaker

Applications of Resonance

- **Quartz Oscillators**

  -A quartz feels a force if placed in an electric field and will oscillate when removed.

  -Appropriate electronics are added to generate an oscillating voltage from the mechanical movements of the crystal and this is used to drive the crystal at its own natural frequency.

  -These devices provide accurate clocks for microprocessor systems.

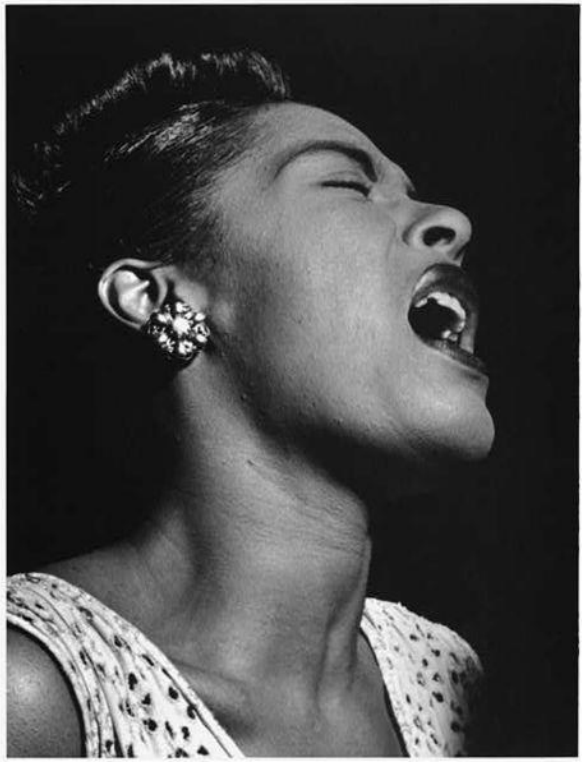
Resonance Effects

- A Driving force at resonance increases the oscillations, sometimes this is unwanted
- Structures
  - Tacoma Narrows bridge, this bridge was destroyed as the wind (driving force) was at the same as the natural frequency. The bridge vibrated and shook itself apart

ON NOVEMBER 7, 1940, THE ACCLAIMED TACOMA NARROWS BRIDGE COLLAPSED DUE TO OVERWHELMING RESONANCE.

Resonance Effects

Cracking of wine glass



Applications of Resonance

An additional unwanted resonance would be

- Tower blocks, the same effect as the bridge the wind, or earthquakes, can cause vibrations to destroy the buildings
- Vibrations in machinery, if the driving force equals the natural frequency the amplitude may get dangerously high. Ex. At a particular speed in a truck's rear view mirror can be seen to vibrate

This can be stopped by designing the building with heavy damping

- High stiffness
- Large mass
- Shape
- Good at absorbing energy

Summary:

- We have discussed about amplitude variation in forced oscillation as a function of frequency.
- We have derived the expressions for different cases depending on r, p and ω.
- We observed the maximum amplitude for FHO when the frequency of FO is nearly same as that of the driving force

Next Class:

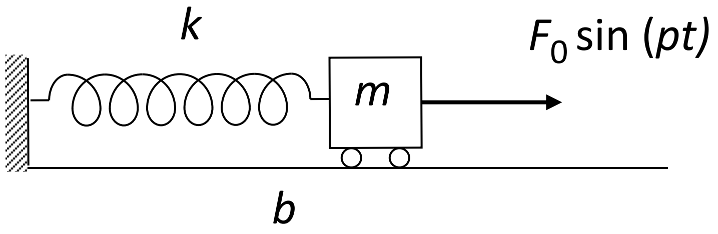
- We shall also obtain bandwidth of resonance.
- We will then discuss the sharpness of resonance and the Q-factor.

Physics (PH 1007)

Forced oscillation & Resonance [Extra, don't bother about the Math]

Recap-Steady State Solution

$$x(t) = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}} \sin(pt - \theta)$$



$$x(t) = A\sin(pt - \theta), \qquad A = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}}$$

(Amplitude for a driven/forced oscillator)

$$\theta = \tan^{-1}\left(\frac{2rp}{\omega^2 - p^2}\right)$$

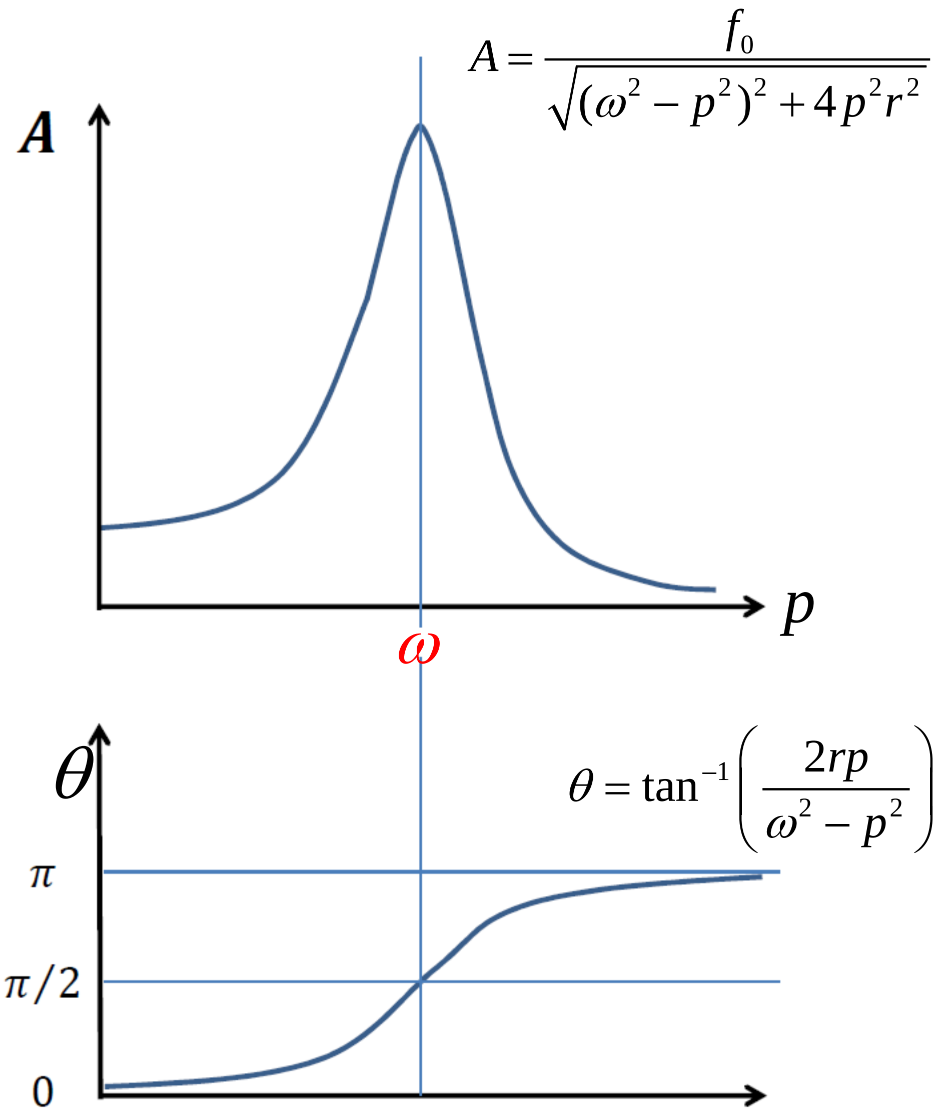
(Phase constant for a driven oscillator)

- The displacement $\left(x\right)$ and the driving force $\left(F\right)$ oscillate with the same frequency, but differ in phase by θ.
  - When the driving frequency p approaches zero, θ approaches zero.
- for $p = \omega$, $\theta = 90^o$, and $p >> \omega$, $\theta \sim 180^o$. The phase of a driven oscillator always lags behind the phase of the driving force.
- The velocity of the object in the steady state is

For $\omega = p$ (*i.e.* $\theta = 90^o$), $V\big|_{p=\omega} = \frac{dx}{dt} = pA\cos(pt - 90) = pA\sin pt$

More on Resonance: The Phase

- The peak of the *amplitude* occurs at a frequency $p = \omega_R$, such that $\omega_R \approx \omega$ & $\omega_R \neq \omega$. The oscillator amplitude (A) *falls on either side of* the peak @ω_R; the driving force (F) oscillate with the same frequency, but A & F differ in phase by θ.
- The phase of a driven oscillator, θ always lags behind the phase of the driving force.
- At resonance (i.e. $\omega \sim p$):
  - Phase shift $\theta = 90^{o}$
  - The applied force is **in phase with** the velocity
  - The *driving force* pushes the mass in the direction it is already moving *adding energy* to the system.
  - As object is moving *in the direction of the driving force*, the power transferred to the oscillator is a maximum.



Quality Factor (Q)

Instead of using r = b/2m and $\omega = \sqrt{k/m}$, it is convenient to describe the shape of the resonance curve using the variables ω and Q = ω /2r .

- Q = ω /2r is called the "quality factor".
- Written in terms ω & Q , the amplitude (A) is

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - p^2)^2 + 4p^2r^2}}$$

$$\Rightarrow A = \frac{F_0/m}{\sqrt{(\omega^2 - p^2)^2 + (\omega p)^2/Q^2}}$$

$$\Rightarrow \boxed{A = \frac{F_0}{k}\frac{\omega/p}{\sqrt{\left(\frac{\omega}{p} - \frac{p}{\omega}\right)^2 + \frac{1}{Q^2}}}}$$

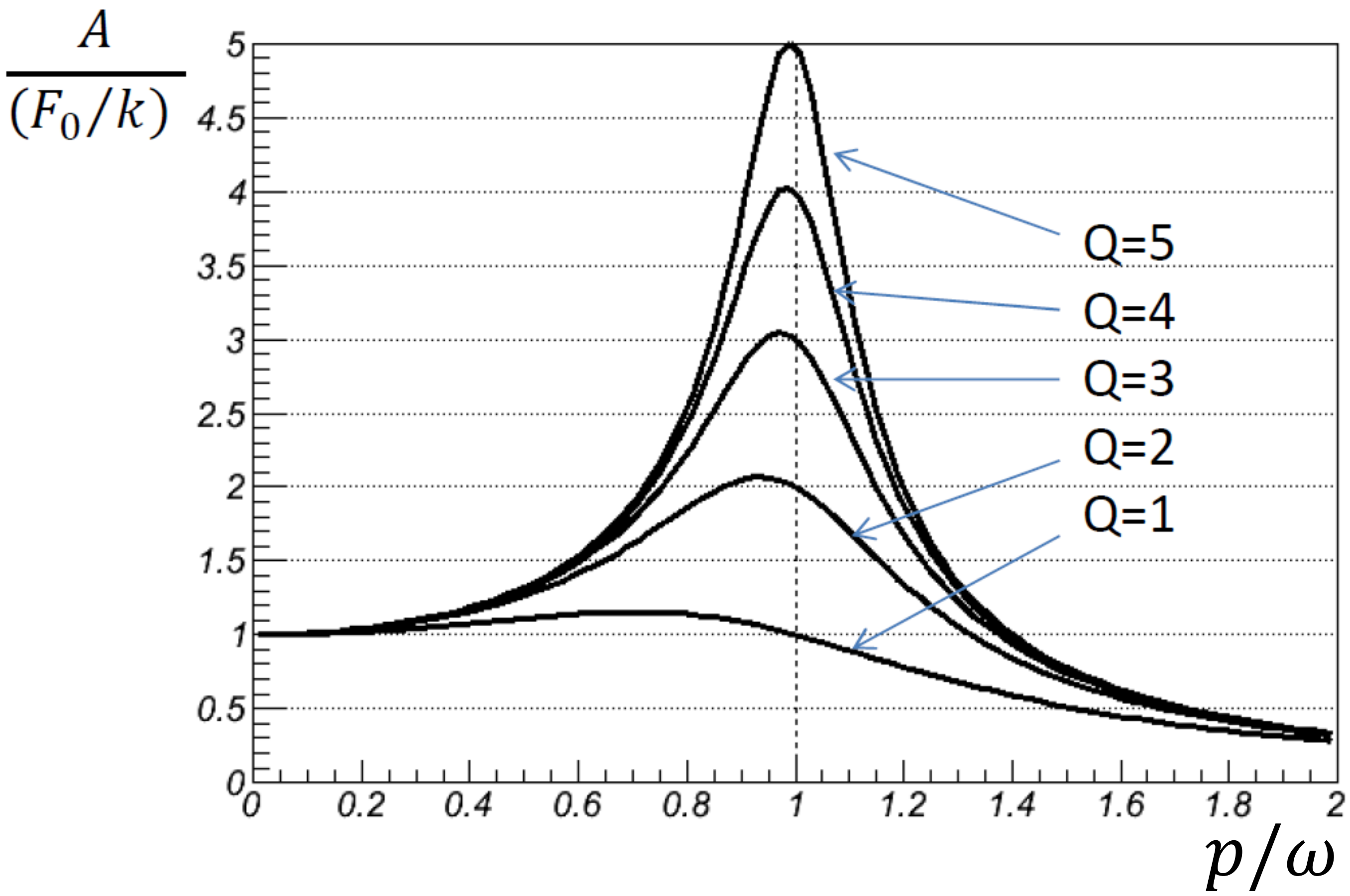
Quality Factor (Q)

$$\boxed{A(\omega) = \frac{F_0}{k} \frac{\omega/p}{\sqrt{\left(\frac{\omega}{p} - \frac{p}{\omega}\right)^2 + \frac{1}{Q^2}}}}$$

Why is this a convenient form?

- ❑Dimensionless quantities are easier to analyze
- ❑The scale of the amplitude is determined by F_0/k
- ❑The shape of the curve is determined by the dimensionless quantities $\frac{p}{\omega}$ & Q

Quality Factor (Q)



$$A(\omega) = \frac{F_0}{k} \frac{\omega/p}{\sqrt{\left(\frac{\omega}{p} - \frac{p}{\omega}\right)^2 + \frac{1}{Q^2}}}$$

- The normalized height is approximately Q
- The maximum occurs when $\frac{p}{\omega} \approx 1$
- At resonance, the motion is amplified by the factor Q.

Energy/Power

- An oscillator stores energy
- The driving force adds energy to the system
- The damping force dissipates energy
- Instantaneous rate at which energy is added:

$$P = \frac{dw}{dt} = F\frac{dx}{dt},$$

$$x(t) = A\sin(pt-\theta)$$

$$F(t) = F_0\ \sin(pt)$$

$$\frac{dx}{dt} = pA\cos(pt-\theta)$$

$$P = F_0\ pA\sin(pt)\ \cos(pt-\theta)$$

$$P_{av} = \frac{1}{T}\int_0^T F_0\ pA\sin(pt)\ \cos(pt-\theta)dt$$

$$\Rightarrow \boxed{P_{av} = \frac{1}{2}F_0 Ap\sin\theta = \frac{1}{2}bA^2p^2}, \quad where\ \ \sin\theta = \frac{2rpmA}{F_0}, \quad r = \frac{b}{2m}$$

Maximal when $\theta = \pi/2$

Average power & power dissipation in forced oscillation

- The ***steady state*** condition in a FHO is achieved *at* ***the expnse*** **of** ***the energy absorbed*** from the *driving force* which is utilized in overcoming the effect of damping.
- The ***average power, P_{av}*** given by the external force in each cycle is:

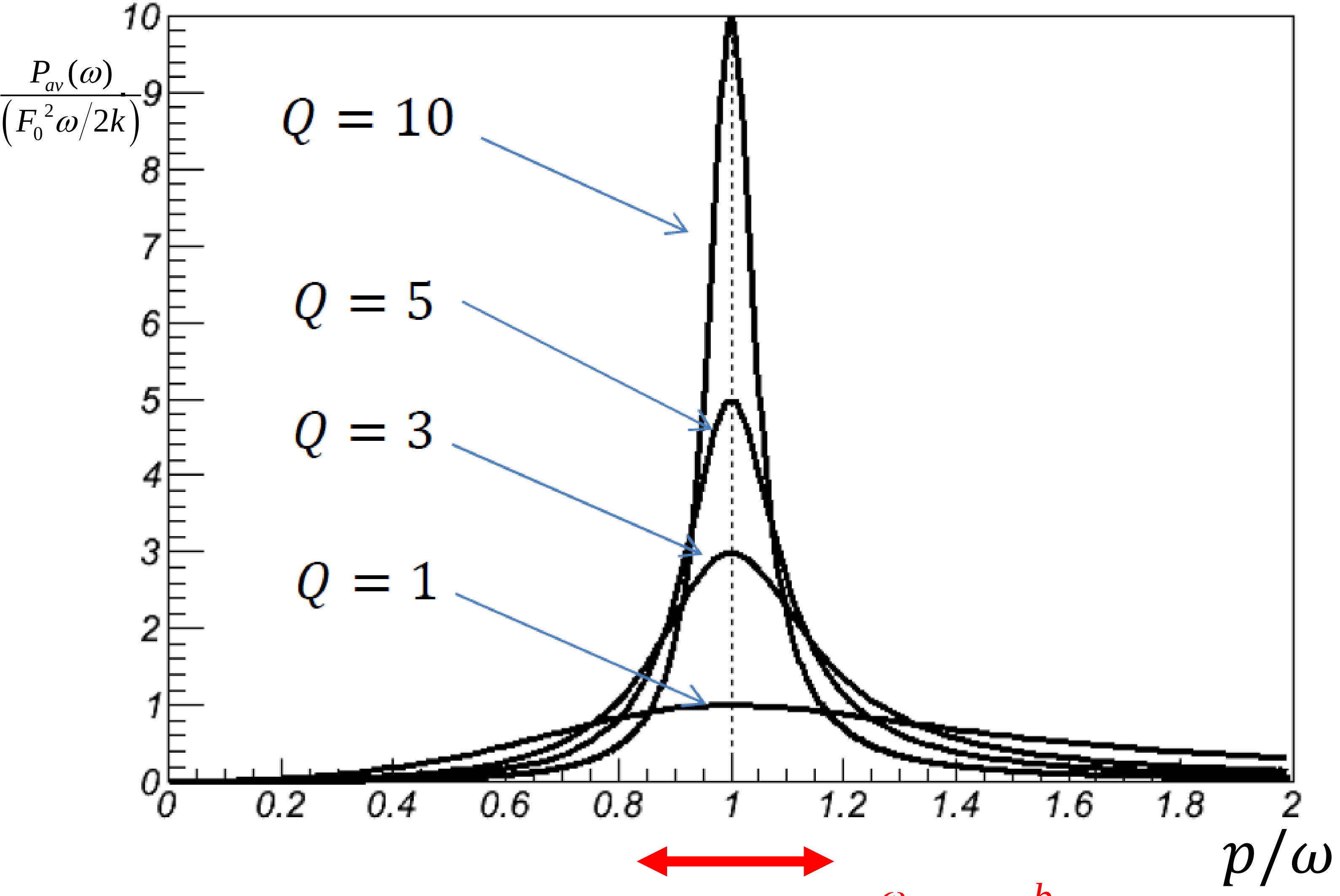
$$P_{av} = \frac{F_0 Ap}{2}\sin\theta = \frac{1}{2}bA^2p^2$$

- *In a cycle of driven oscillation*, power supplied by the driving force is not stored rather dissipated via work done against damping force. Thus the *average power disspated across a cycle* is

$$dP_{dissipated} = F_d\frac{dx}{dt} = (b\frac{dx}{dt})\frac{dx}{dt}$$

$$\Rightarrow P_{dissipated} = \int dP_{dissipated} = \frac{1}{T}\int_0^T b(\frac{dx}{dt})^2 dt = \frac{1}{2}bA^2p^2$$

Power Resonance Shape



$$\frac{P_{av}(\omega)}{(F_0{}^2\omega/2k)} = \frac{1}{Q}\frac{1}{\left(\frac{\omega}{p}-\frac{p}{\omega}\right)^2+\frac{1}{Q^2}}$$

Bandwidth (FWHM) $= \Delta\omega = \frac{\omega}{Q} = 2r = \frac{b}{m}$

Average power & power dissipation in a cycle in forced oscillation

- The *average power, P_{av} absorbed or dissipated* in a cycle in the FHO, is thus $\Rightarrow$ $\boxed{P_{av} = P_{dissipated} = \frac{1}{2} b A^2 p^2}$

$$\Rightarrow (P_{av})_{\max} = \frac{1}{2} b A_{\max}{}^2 p^2$$

- The *maximum absorbed power* can be given as

$$\because A_{\max} = \frac{F_0}{2prm} = \frac{F_0}{bp}$$

$$\Rightarrow (P_{av})_{\max} = \frac{F_0{}^2}{2b} = \frac{F_0{}^2}{4brm} \Rightarrow (P_{av})_{\max} = \frac{mf_0{}^2}{4r}$$

- The power thus depends on A^2 . A graph plotted between average power and frequency of driving force, p is called as the absorption curve, which follows a similar pattern as *amplitude* of FHO.

- The power is maximum at the resonant frequency i.e. *when $p \approx \omega$*

- ***Average power*** falls on either side of the resonant frequency, **(ω_R),** & the sharpness of the absorption curve is decided from the frequency range over which the P_{av} falls *to half* of its peak value.

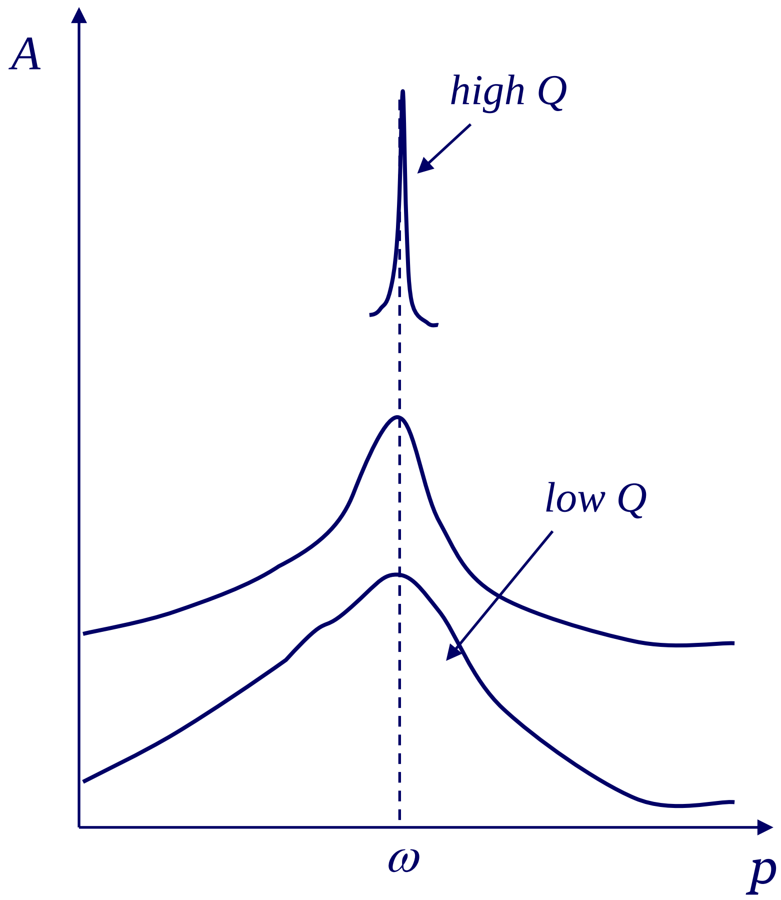
- "2*r*" is the bandwidth (FWHM) of frequency range over which the *average power falls to half the peak value on* either side of the resonant frequency, **(ω_R),** & represents the sharpness of the absorption curve.

Quality Factor (Q)

- **Quality factor**, or *Q, which is a* measure of sharpness of resonance can be defined in terms of absorption band width as,

$$Q = \frac{\omega}{\omega_2 - \omega_1} = \frac{\omega}{2r}$$

$$\Rightarrow Q = \frac{m\omega}{b}$$



- Weaker the damping better the Q-value & higher would be the energy transfer/resonance.

- A pendulum has a Q-value of ~100, where as for the seismic wave during earth quake, the Q ~1000. For a quartz clock, Q may reach 10000.

Summary: Resonance Curves

General properties:

- **Amplitude** at resonance: Static **displacement x *Q***
- **FWHM** power bandwidth: $2r = \omega/Q$
- When **Q is large**, a **small force** at the resonant frequency produces **large oscillations**
- Large amplitudes persist only when the **driving force is near the oscillation frequency**