Electrical Engineering

Passive Filter Design

EE2100 – Circuits & Fields

Group Members

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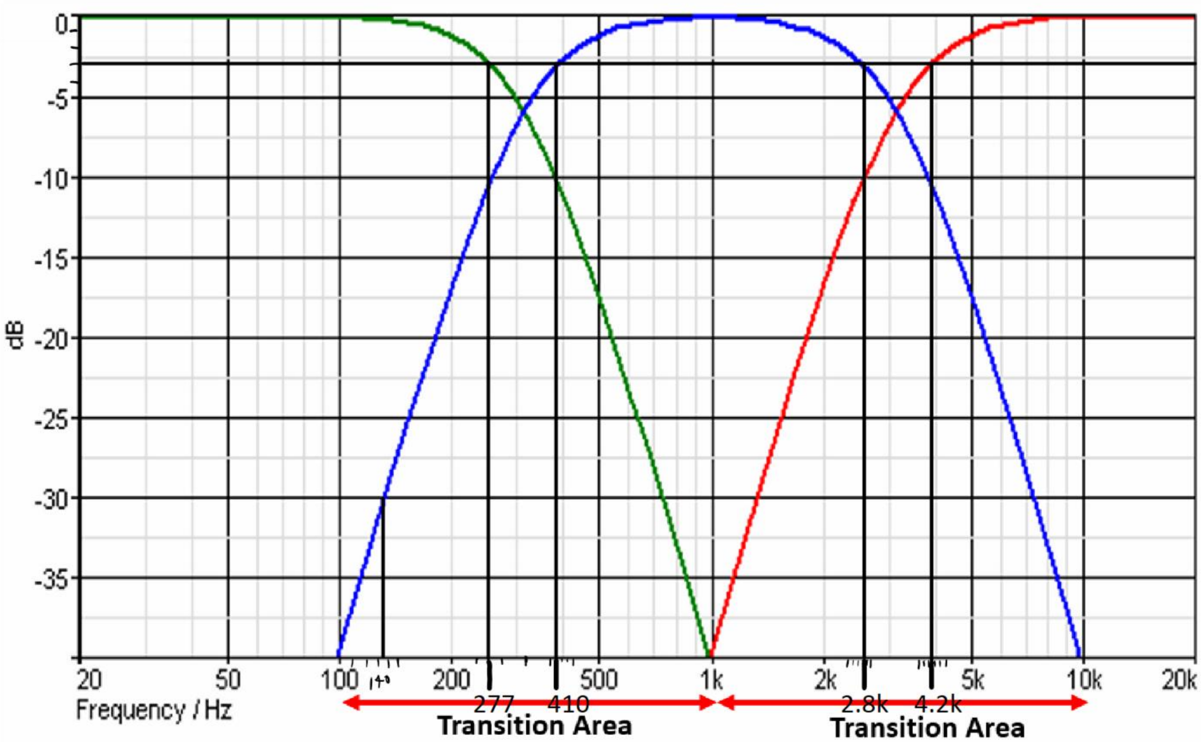
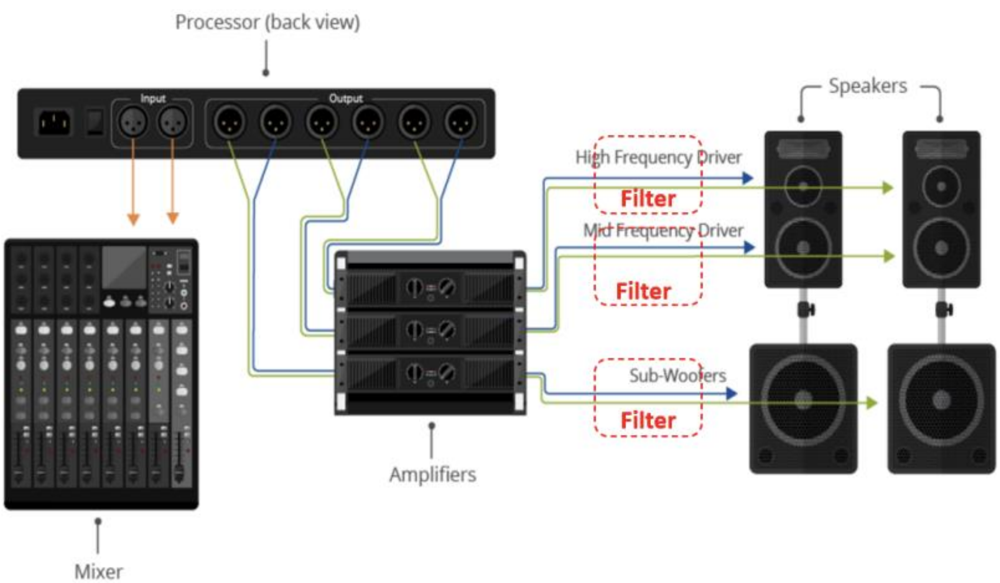
Data sheet of the old speaker system





Q1. What is the ***order*** of the filter that they have used in the old filter design?

Project - Passive Filter Design

Question 01:-

* The high-frequency roll-off of an n^{th}-order filter is 20 n dB / decade.

For Low-Pass filter :-

Let's consider the roll-off from 100 Hz - 1 kHz.

$$20n = 40$$
$$n = 2$$

$\therefore$ Order of the Low-Pass Filter = 2 //

For High-Pass Filter:-

Considering the roll-off from 1 kHz - 10 kHz

$$20n = 40$$
$$n = 2$$

$\therefore$ Order of the High-Pass Filter = 2 //

For Band-Pass Filter :-

We can consider the Decibel plot of the band-pass filter as a combination of a Low-Pass Filter and a High-Pass Filter.

It has two high frequency roll-offs same as the above Low-Pass and High Pass Filters.

$\therefore$ Order of the Band-Pass Filter = 2×2 = 4

Q2. What are the ***cutoff frequencies*** of each filter that they have used in the old filter design? Approximate your answer to a one-decibel point.?

Question 02:-

* To find the cutoff frequencies of each filter we have to draw a horizontal line at -3 dB and get the corresponding frequency values of the intersecting points of the plots and the -3dB horizontal line.

Cut off Frequencies of;

Low-Pass Filter = 277 Hz

High-Pass Filter = 4.15 KHz

Band-Pass Filter = 410 Hz and 2.8 KHz

Q3. Design and synthesize maximally flat filters (all three) with the passband tolerance of $\boldsymbol{3dB}$ and frequency attenuation of $\boldsymbol{90dB}$ at **(5 ×** $\boldsymbol{cuttoff\ frequency}$**).** (a) Determine the transfer function of the (b) Using LTspice software, determine the bode plot of the filter. (c) Compare the bode plot with the old filter bode plot and determine the transition area reduction in Hz. **Assume the** $\boldsymbol{source\ resistance}$ **=** $\boldsymbol{load\ resistance}$ **= 50Ω.**

1.) <u>Butterworth Low Pass Filter</u>

Question 03

Designing a maximally flat Low-Pass filter :-

Passband tolerance = 3 dB

High frequency attenuation = 90 dB

at (5x 277 Hz) = 1385 Hz

$R_s = R_\ell = 50\ \Omega$

$$K_o = \frac{4R_1R_2}{(R_1+R_2)^2} = 1$$

$$10 \log_{10}(1+\varepsilon^2) = 3$$

$$\log_{10}(1+\varepsilon^2) = 0.3$$

$$\varepsilon^2 = 10^{0.3} - 1 = 0.995 \simeq 1$$

$$10 \log_{10}\left[1 + \varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2n}\right] = 90$$

$$\log_{10}\left[1 + \left(\frac{1385}{277}\right)^{2n}\right] = 9$$

$$5^{2n} = 10^9 - 1$$

$$2n \log_{10} 5 = 9$$

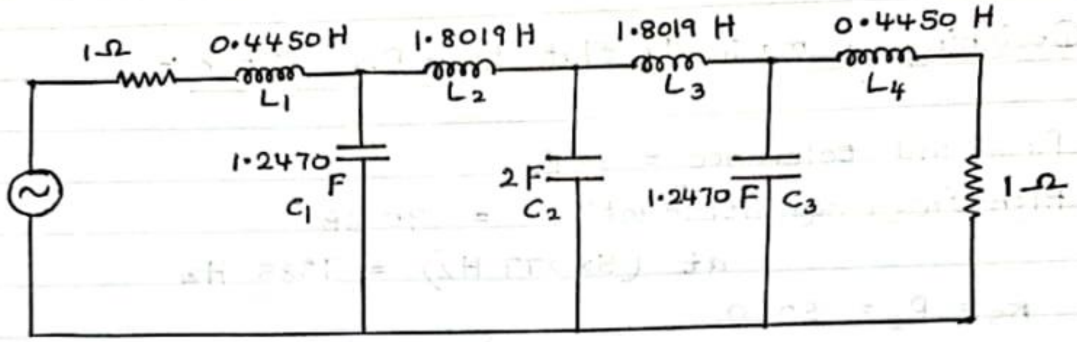
$$n = \frac{9}{2\log_{10} 5} = 6.44$$

∴ Order of the Low-Pass Maximally flat filter = 7

Transfer Function ⇒

$$|H(s)|^2 = \frac{1}{1+\left(\frac{s}{j}\right)^{14}} = \frac{1}{1-s^{14}}$$

Normalized Low-Pass filter Synthesize



De-Normalization :-

For L_1 and L_4 ⇒

$$L = \frac{0.4450 \times 50}{277 \times 2\pi} = 12.7841 \text{ mH}$$

For L_2 and L_3 ⇒

$$L = \frac{1.8019 \times 50}{277 \times 2\pi} = 51.7656 \text{ mH}$$

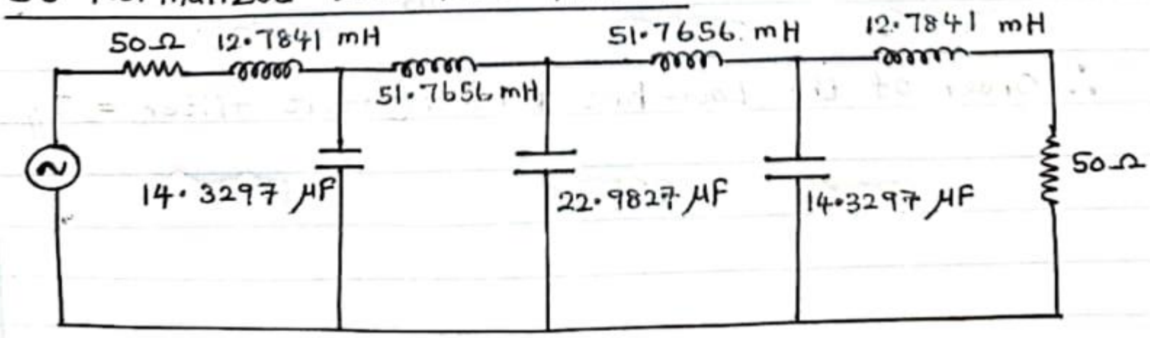
For C_1 and C_3 ⇒

$$C = \frac{1.2470}{50 \times 277 \times 2\pi} = 14.3297 \ \mu F$$

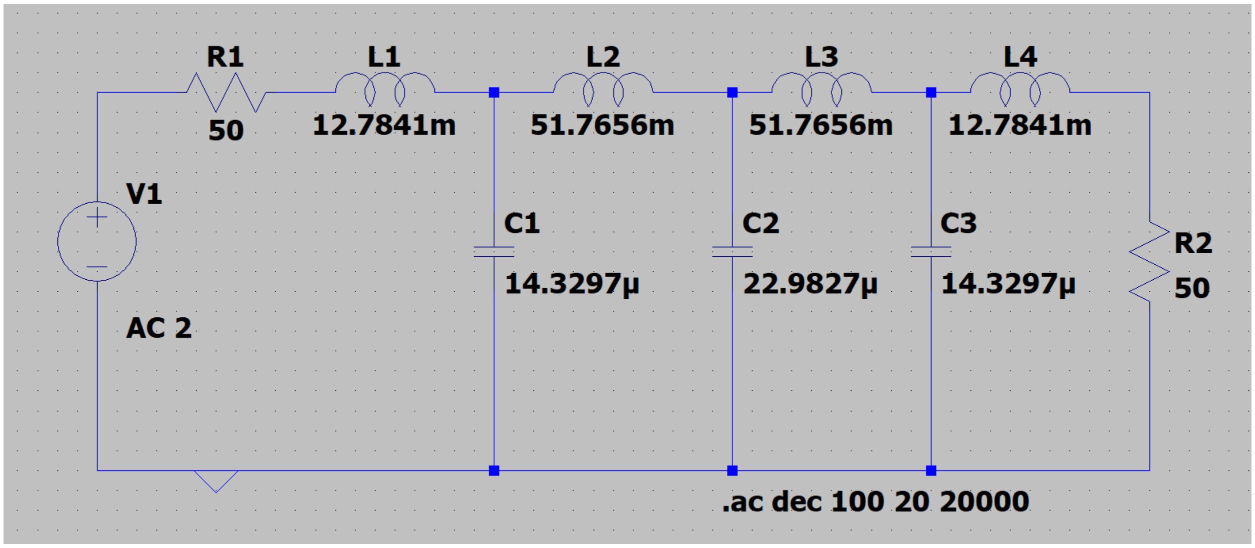
For C_2 ⇒

$$C = \frac{2}{50 \times 277 \times 2\pi} = 22.9827 \ \mu F$$

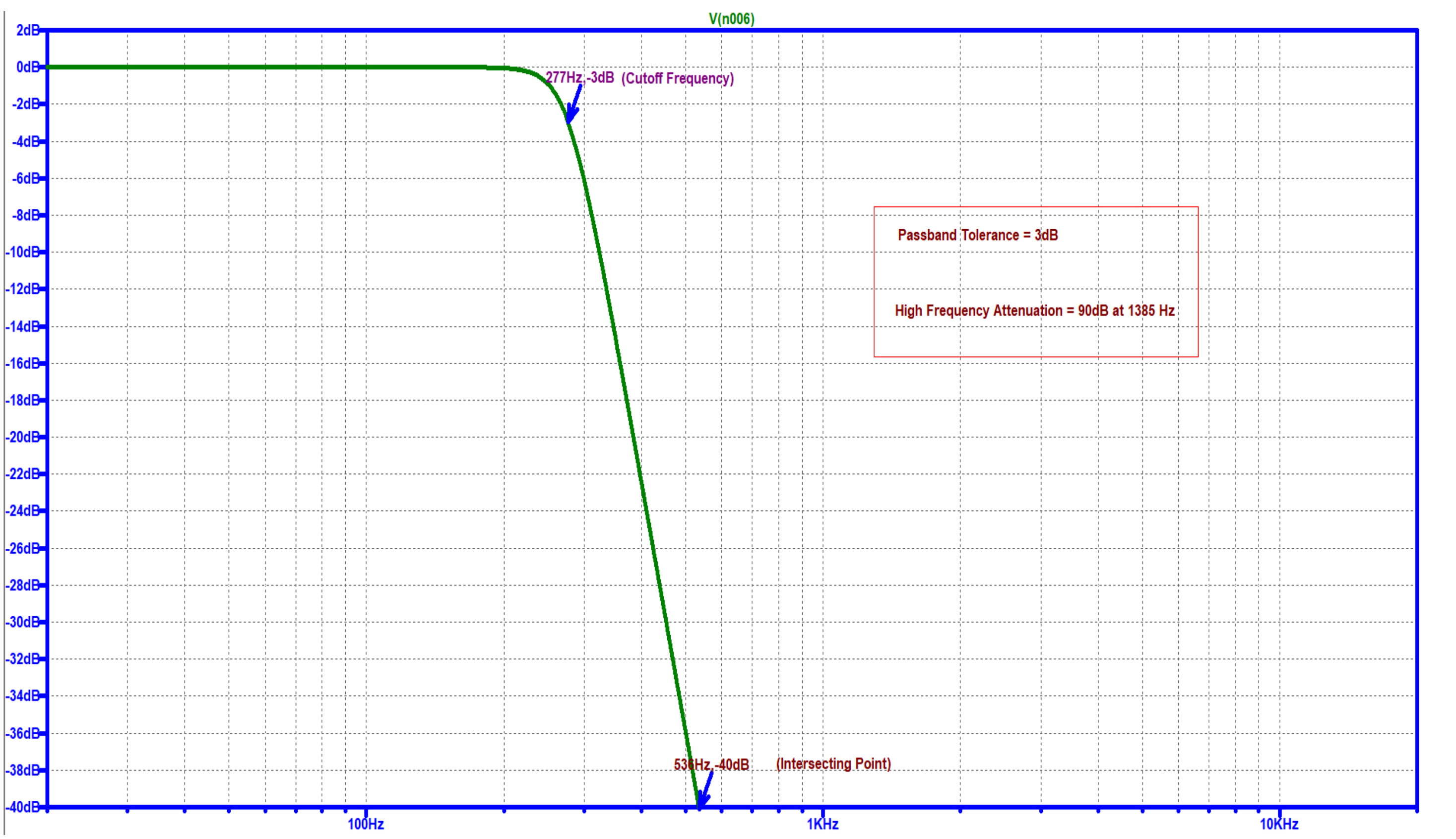
De-Normalized Low-Pass Filter :-



De-Normalized Low-Pass filter circuit



De-Normalized Butterworth Low-Pass filter Bode plot



2.) Butterworth High Pass Filter

Designing a maximally flat High-Pass filter :-

Passband tolerance = 3 dB

High frequency attenuation = 90 dB

$$\text{at} \left(\frac{4150}{5} \text{Hz} \right) = 830 \text{ Hz}$$

$R_S = R_L = 50\Omega$ $\qquad K_0 = \frac{4R_1R_2}{(R_1+R_2)^2} = 1$

$$10 \log_{10} (1+\varepsilon^2) = 3$$

$$\log_{10} (1+\varepsilon^2) = 0.3$$

$$\varepsilon^2 = 10^{0.3} - 1 = 0.995 \doteqdot 1$$

$$10 \log_{10} \left[1 + \varepsilon^2 \left(\frac{\omega_c}{\omega} \right)^{2n} \right] = 90$$

$$\log_{10} \left[1 + \left(\frac{4150}{830} \right)^{2n} \right] = 9$$

$$5^{2n} = 10^9 - 1$$

$$2n \ln 5 = 9 \ln 10$$

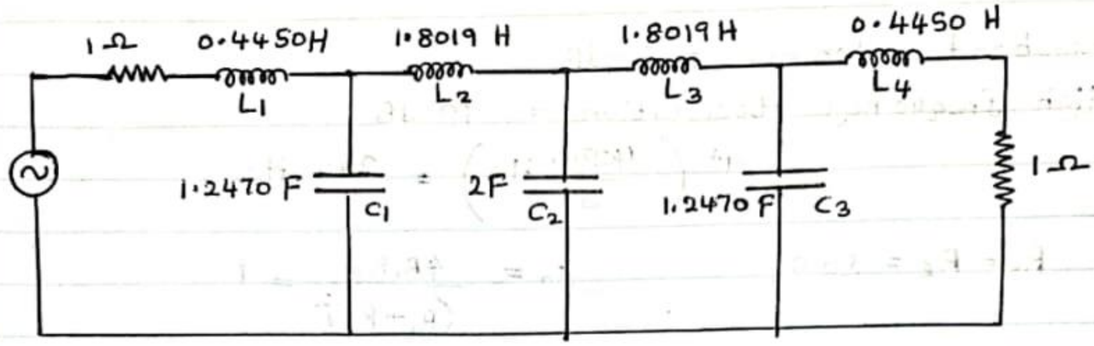
$$n = \frac{9 \ln 10}{2 \ln 5} = 6.44$$

$\therefore$ Order of the High-Pass Maximally flat filter = 7

Transfer Function $\Rightarrow$

$$|H(s)|^2 = \frac{1}{1 + \frac{1}{(s/j)^{14}}} = \frac{1}{1 - \frac{1}{s^{14}}} = \frac{s^{14}}{s^{14} - 1}$$

Normalized Low-Pass filter Synthesize



De-Normalization to a High-Pass Filter :-

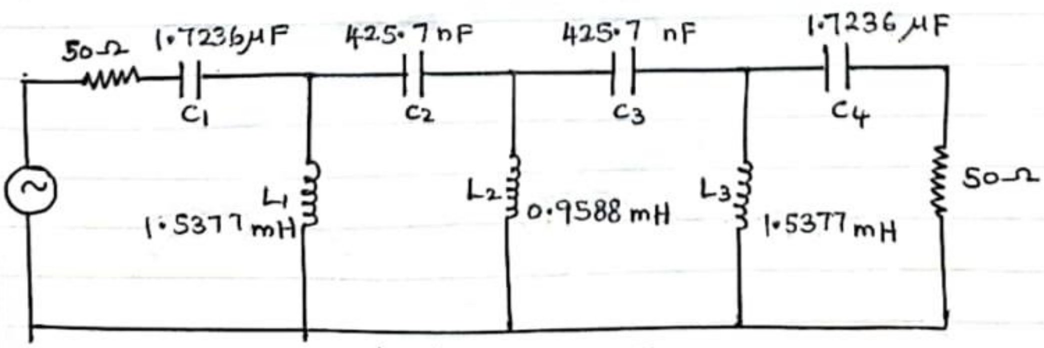
* L_1, L_2, L_3 and L_4 become capacitors and C_1, C_2 and C_3 becomes inductors when transforming into a High-Pass Filter.

For C_1 and C_4 ⇒ $\dfrac{1}{2\pi \times 4150 \times 0.4450 \times 50} = 1.7236\ \mu F$

For C_2 & C_3 ⇒ $\dfrac{1}{2\pi \times 4150 \times 1.8019 \times 50} = 425.7\ nF$

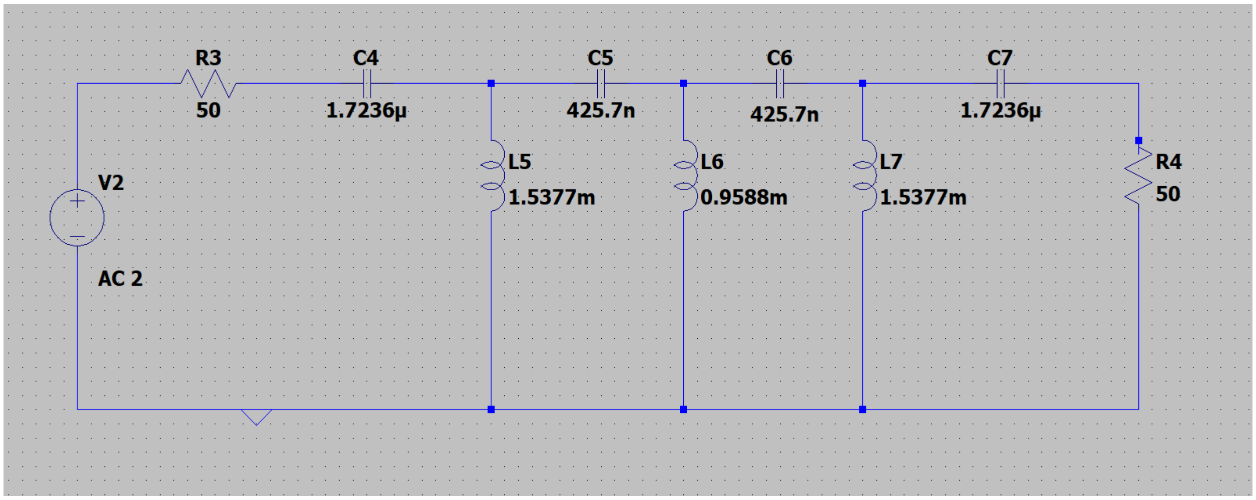
For L_1 and L_3 ⇒ $\dfrac{50}{4150 \times 2\pi \times 1.2470} = 1.5377\ mH$

For L_2 ⇒ $\dfrac{50}{4150 \times 2\pi \times 2} = 0.9588\ mH$

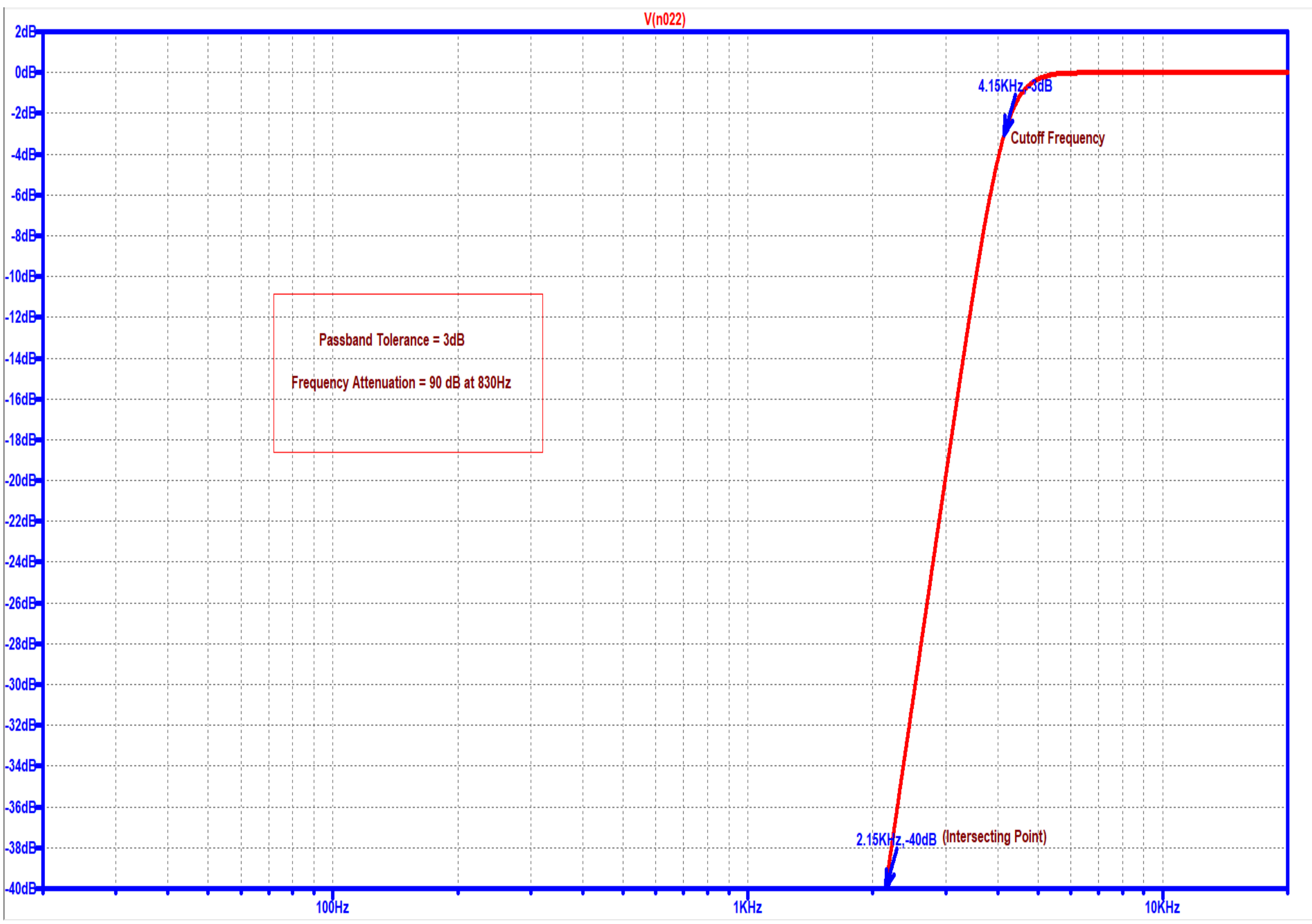


De-Normalized High-Pass Filter

De-Normalized High-pass filter circuit



De-Normalized Butterworth High-pass filter plot



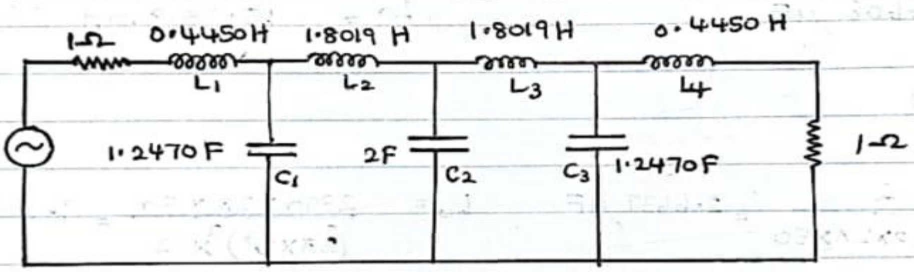
3.) <u>**Butterworth Band Pass Filter**</u>

Designing a maximally flat Band-Pass Filter :-

* According to the symmetrical shape of the Decible Bode Plot of the Band-Pass Filter; we can directly take the order of the filter as ;

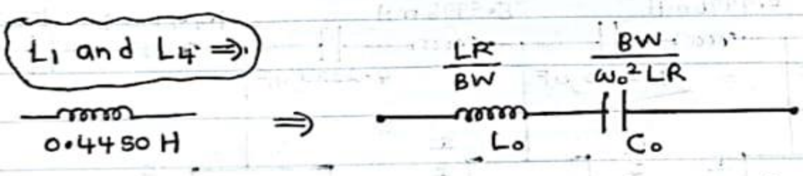
New order of the Band-Pass Maximally flat filter $= 7 \times 2 = \underline{\underline{14}}$

Corresponding Normalized Low-Pass filter Synthesize :-



De-Normalization to a Band-Pass filter :-

- L1 and L4 ⇒



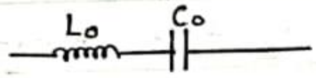
Band with $= (2800 - 410)\text{ Hz} = (2390\text{ Hz}) \times 2\pi$

ω_0 (center frequency) $= 2\pi \times 1000$ Hz

$$L_0 = \frac{0.4450 \times 50}{2390 \times 2\pi} = 1.4817 \text{ mH}$$

$$C_0 = \frac{2390 \times 2\pi}{(2\pi \times 10^3)^2 \times 0.4450 \times 50} = 17.0957\ \mu F$$

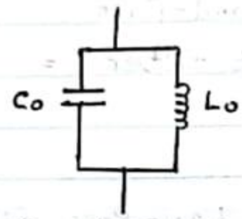
For L_2 and L_3



$$L_o = \frac{1.8019 \times 50}{2390 \times 2\pi} = 5.9996 \text{ mH}$$

$$C_o = \frac{2390 \times 2\pi}{(2\pi \times 10^3)^2 \times 1.8019 \times 50} = 4.2220 \ \mu F$$

For C_1 and C_3



$$C_o = \frac{1.2470}{2390 \times 2\pi \times 50}$$

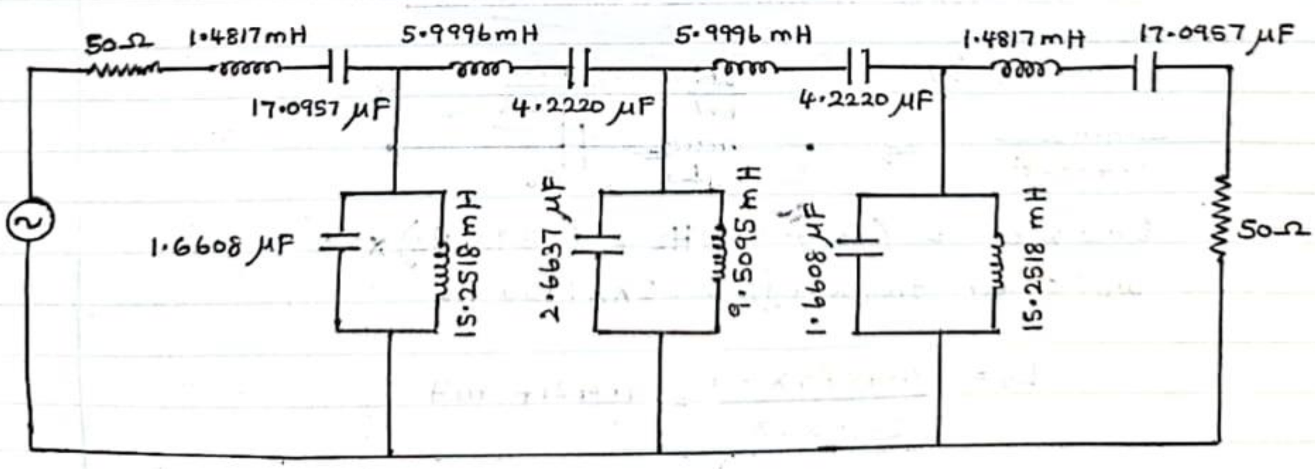
$$C_o = 1.6608 \ \mu F$$

$$L_o = \frac{2390 \times 2\pi \times 50}{(2\pi \times 10^3)^2 \times 1.2470}$$

$$L_o = 15.2518 \text{ mH}$$

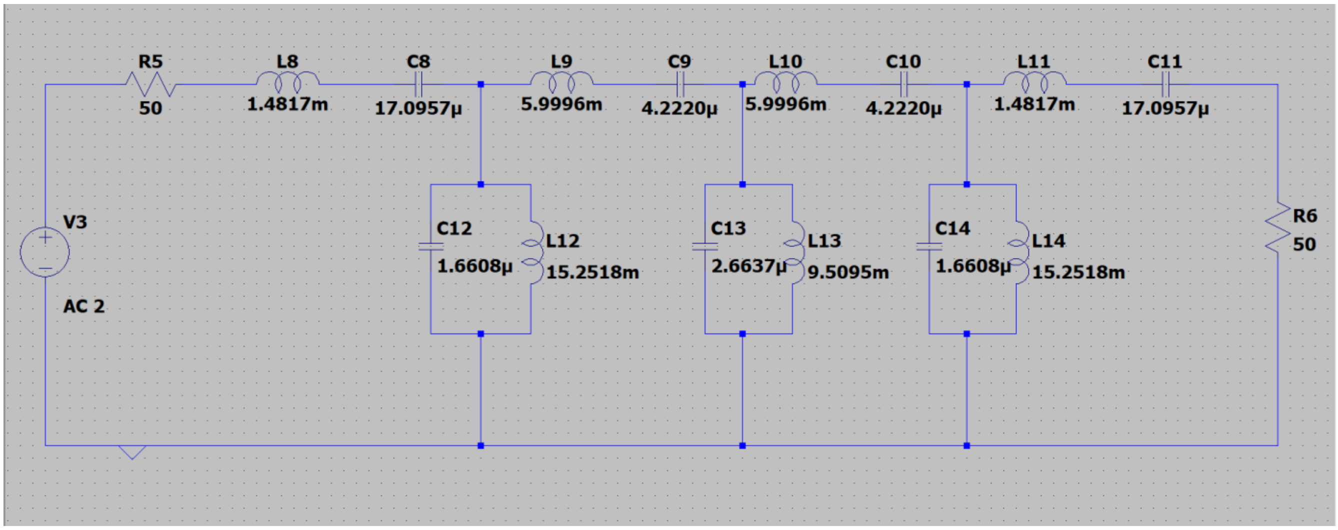
For C_2

$$C_o = \frac{2}{2390 \times 2\pi \times 50} = 2.6637 \ \mu F \qquad L_o = \frac{2390 \times 2\pi \times 50}{(2\pi \times 10^3)^2 \times 2} = 9.5095 \text{ mH}$$

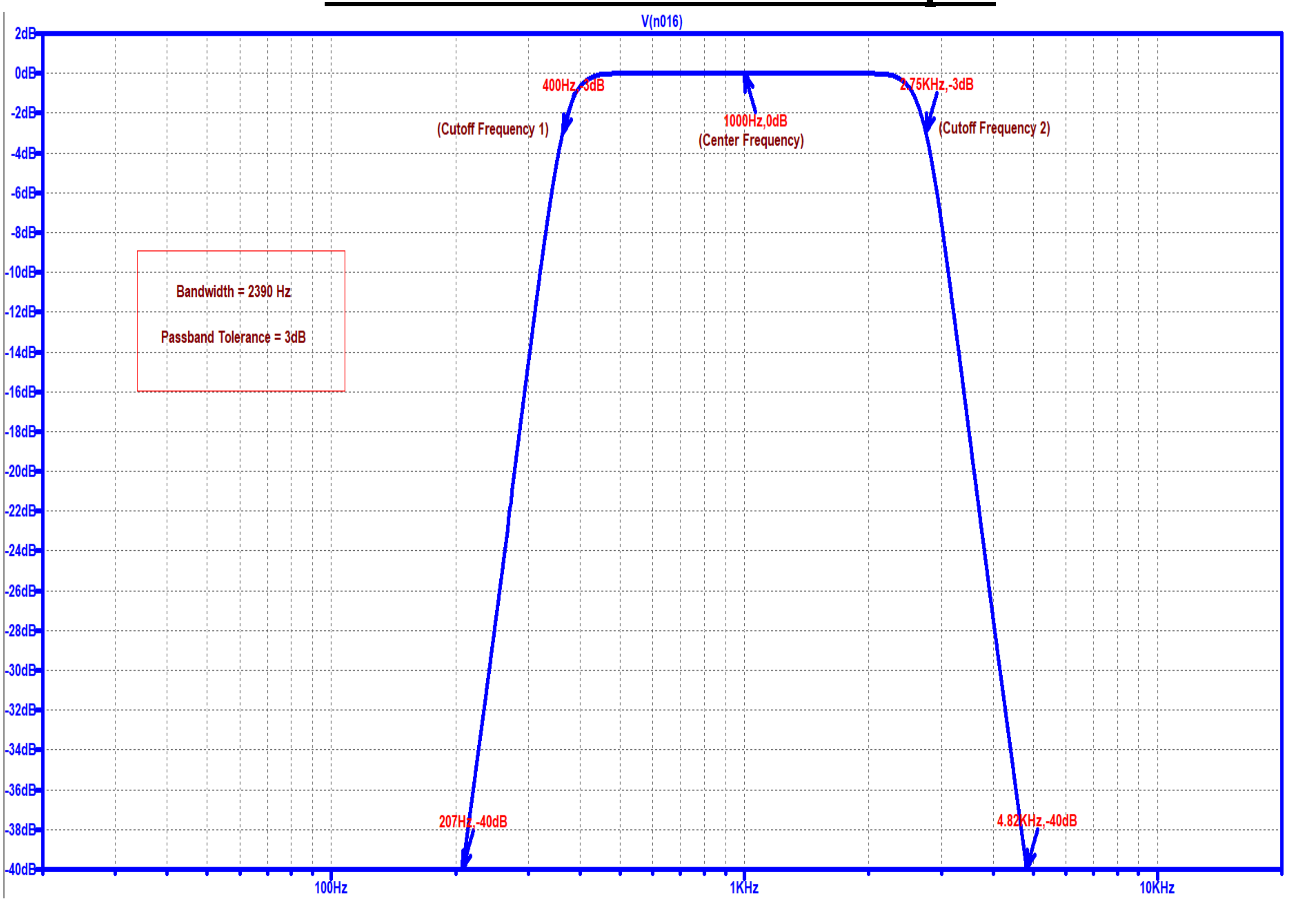


De-Normalized Band-Pass Filter.

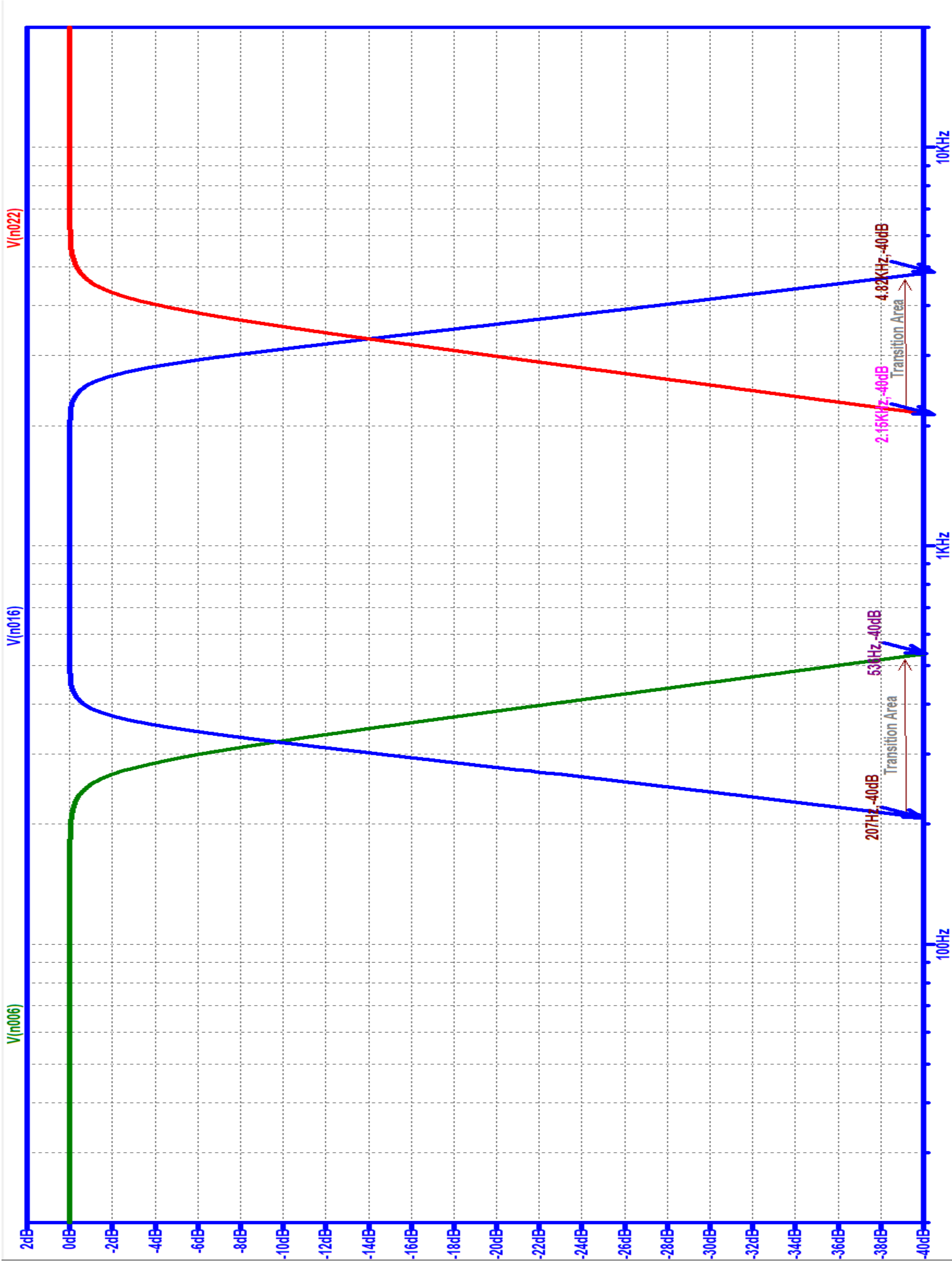
De-Normalized Band-Pass filter circuit



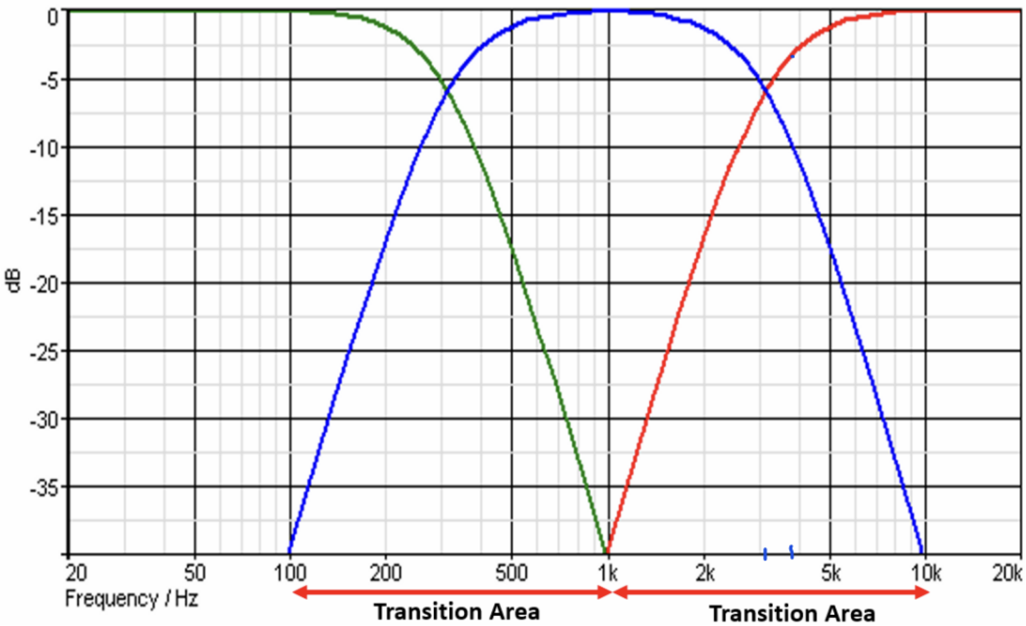
De-Normalized Band-Pass filter plot



Filter Design



Old Transition Areas



- **Old Transition Area 1 = 1000 Hz – 100 Hz = 900 Hz**
- **Old Transition Area 2 = 10 kHz - 1 kHz = 9 kHz**

New Transition Areas (According to the bode plot at page 13)

- **New Transition Area 1 = 536Hz - 207Hz = 329Hz**
- **New Transition Area 2 = 4.82kHz - 2.15kHz = 2.67kHz**

Transition Area Reductions

Transition Area reduction (1) = 900 Hz – 329 Hz = 571 Hz

Transition Area reduction (2) = 9 kHz – 2.67 kHz = 6.33 kHz

Q4. Design and synthesize a Chebyshev filter (Type 1) with a passband tolerance of $\mathbf{0.5dB}$ and frequency attenuation of $\mathbf{90dB}$ at **(5 ×** $\boldsymbol{cuttoff\ frequency}$**) only for the sub-woofer.** (a) Determine the transfer function of the filter (no need to simplify it). (b) Using LTspice software, determine the bode plot of the filter. **Assume the** $\boldsymbol{source\ resistance}$ **=** $\boldsymbol{load\ resistance}$ **= 50Ω**

Question 04:-

Design and synthesize a Chebyshev filter for the Sub-woofer

Passband tolerance = 0.5 dB $\qquad f_c = 277$ Hz
Frequency attenuation = 90 dB at $(5 \times 277\ Hz) = 1385$ Hz
$R_S = R_L = 50\ \Omega$

$$K_0 = (1+\varepsilon^2)\frac{4R_S R_L}{(R_S+R_L)^2} = 1.122$$

$$10\log_{10}(1+\varepsilon^2) = 0.5$$
$$\varepsilon^2 = 10^{0.05} - 1$$
$$\varepsilon^2 = 0.122 \quad —①$$

$$10\log_{10}(1+\varepsilon^2 T_n^2(y)) = 90$$
$$T_n^2(y) = \frac{10^9 - 1}{0.122}$$
$$T_n(y) = 90535.746$$

$$y = \frac{\omega}{\omega_c} = \frac{1385}{277} = 5 > 1$$

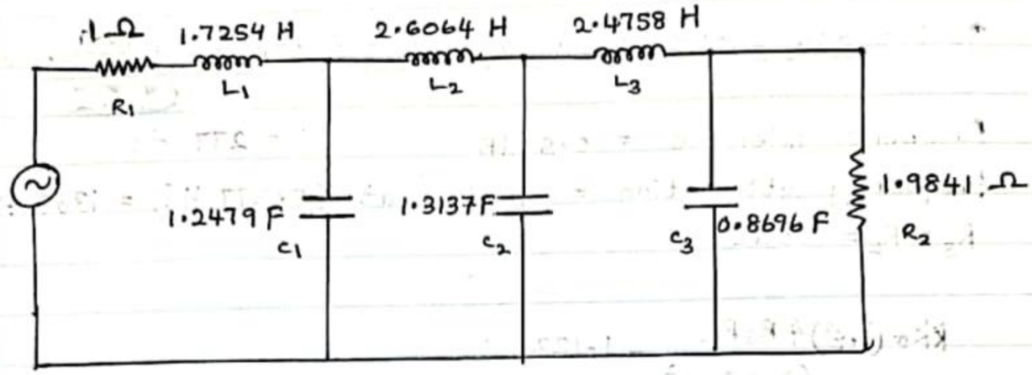
$$\therefore T_n(y) = \cosh(n \times \cosh^{-1} y)$$

$$n = \frac{\cosh^{-1}(90535.746)}{\cosh^{-1}(5)}$$

$$n = 5.28 \qquad T_6(y) = 32y^6 - 48y^4 + 18y^2 - 1$$

$\therefore$ Order of the Chebyshev (Type I) Low Pass filter for the sub-woofer $= \underline{\underline{6}}$

Normalized Low-Pass filter Synthesize



De-Normalization -

For $L_1 \Rightarrow$ $L_1 = \dfrac{1.7254 \times 50}{2\pi \times 277} = 49.5679\,mH$

For $L_2 \Rightarrow$ $L_2 = \dfrac{2.6064 \times 50}{2\pi \times 277} = 74.8775\,mH$

For $L_3 \Rightarrow$ $L_3 = \dfrac{2.4758 \times 50}{2\pi \times 277} = 71.1256\,mH$

For $C_1 \Rightarrow$ $C_1 = \dfrac{1.2479}{50 \times 2\pi \times 277} = 14.3400\,\mu F$

For $C_2 \Rightarrow$ $C_2 = \dfrac{1.3137}{50 \times 2\pi \times 277} = 15.0962\,\mu F$

For $C_3 \Rightarrow$ $C_3 = \dfrac{0.8696}{50 \times 2\pi \times 277} \doteq 9.9929\,\mu F$

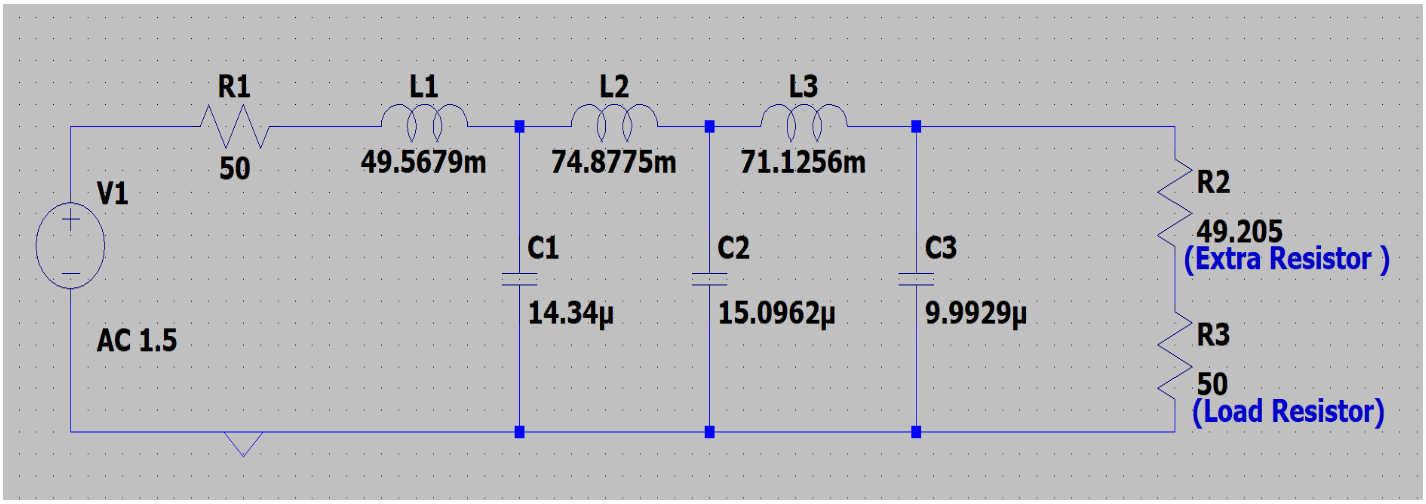
$R_1 = 50\,\Omega$ $\quad$ $R_2 = 50 \times 1.9841 = 99.205\,\Omega$

Transfer function $\Rightarrow$

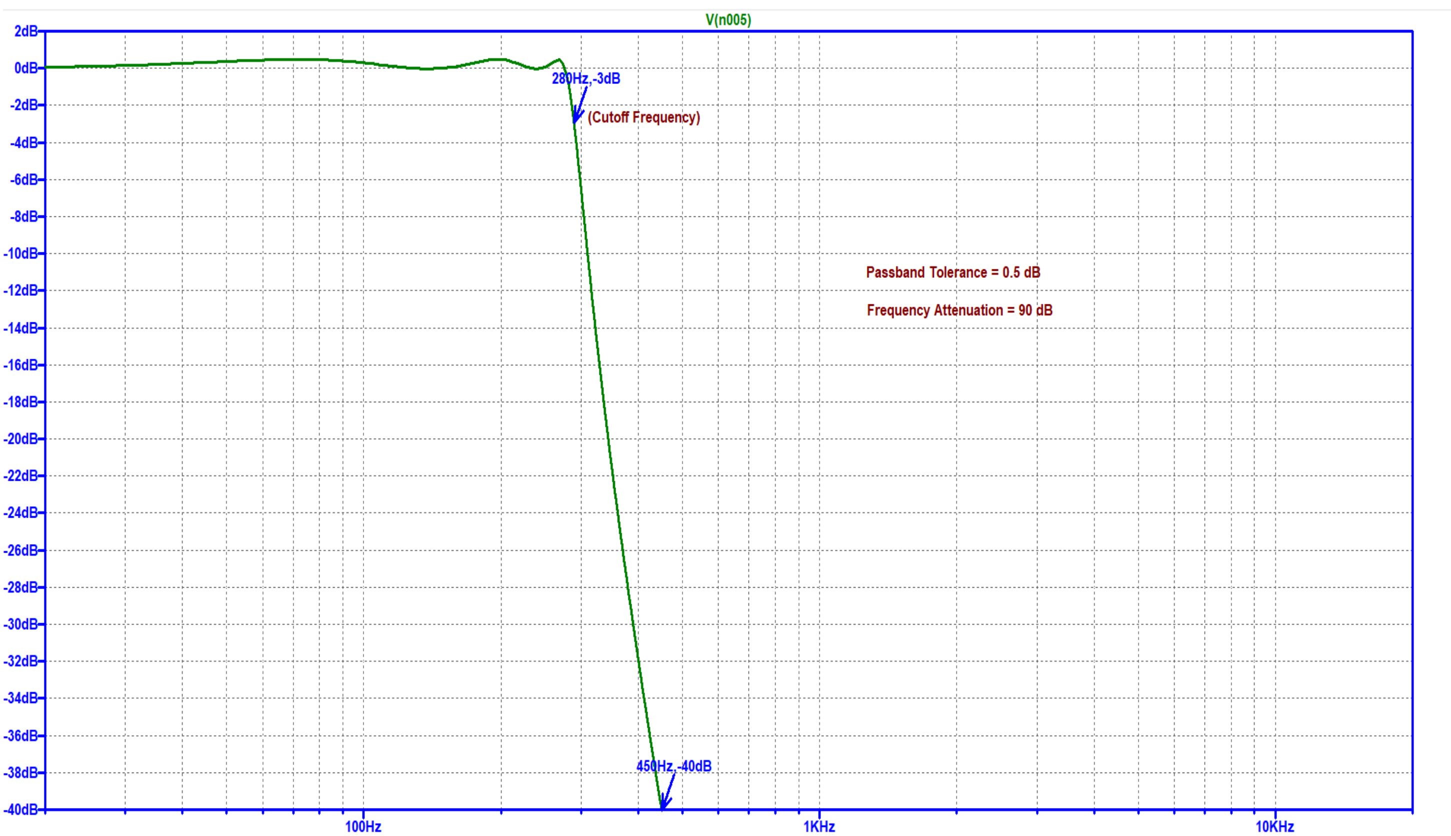
$$|H(j\omega)|^2 = \frac{1.122}{1 + (0.122)\left[32y^6 - 48y^4 + 18y^2 - 1\right]^2}$$

$$|H(s)|^2 = \frac{1.122}{1 + 0.122\left(-32s^6 + 48s^4 - 18s^2 - 1\right)^2}$$

De-Normalized Low-Pass Chebyshev filter circuit



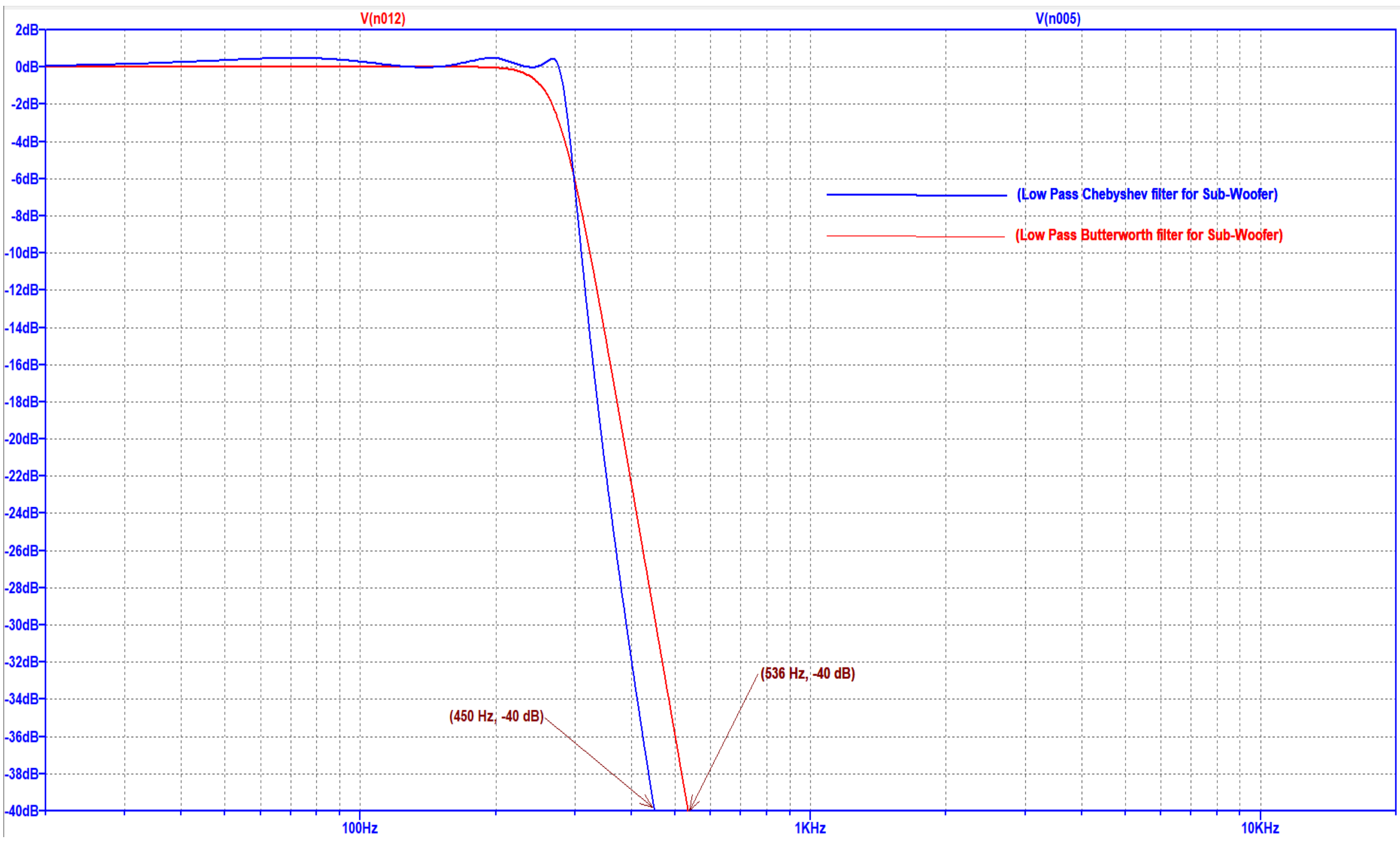
De-Normalized Low-Pass Chebyshev filter plot



Q5. Based on the results from both type of sub-woofer filters, which filter you will select for the application mentioned above.

Explain why you have chosen this filter.

Low Pass Filter Bode Plots for both Butterworth and Chebyshev Approximations



Comparison

Characteristic	Butterworth	Chebyshev
Passband Response	Maximally flat	Ripple
Stopband Response	Slow roll-off	Sharp roll-off
Distortion	Low	Higher

Conclusion

Butterworth filter has a maximally flat passband response, meaning that the frequency response is as flat as possible within the passband. This makes them ideal for applications where a linear phase response is important, such as audio playback. Butterworth filter also has a relatively **slow roll-off rate**, meaning that they do not attenuate frequencies outside the passband as quickly as some other filter types.

Chebyshev filter has a **sharper roll-off** rate than the Butterworth filter, meaning that it can attenuate frequencies outside the passband more quickly. This makes it ideal for applications where it is important to reduce the transition area between the passband and stopband, such as subwoofer crossover filters. However, the Chebyshev filter also has a **ripple** in the passband response, which can cause distortion.

The figure at page 18 shows the bode plots of Butterworth and Chebyshev type 1 filters. We can see that the transition area of the Chebyshev filter is lesser than the Butterworth filter. It is because Chebyshev filters have shaper roll-off rate than Butterworth filters as mentioned in the above paragraph. Since our main task is to reduce the transition area, Chebyshev filter is a better option at that point of view. Although both the filters have transition areas less than the older one, we can obtain a lesser transition area in Chebyshev filter with an order of **6** while the Butterworth filter required an order of **7** to do the same task.

Furthermore, the number of inductors in the Chebyshev type 1 filter is less than the Butterworth filter. So, if we chose the Chebyshev type 1 filter for the sub-woofer, we could save the space as well.

- Considering all the factors mentioned above, **Chebyshev Type 1 Low Pass filter** is the best option for the Sub-Woofer.