consider an
assymmetric $m = "0"$ (sausage) instability in a current carrying cylindrically symmetric Z pinch plasma column.

straight plasma with radius a, effective unstable axial length L (length over which the mode has coherent structure), with mass density ρ, driven by an axial current I, producing an azumithal magnetic field $B_\theta(r)$ with edge field $B_\theta(a)$

We treat the instability in the ideal MHD linear regime so that

mode is characterized by k (fundamental or dominant axial wavenumber)
oscillation frequency scale ω
exponential growth rate λ which will be estimated from Alvénic time scales.

Alvén speed: use relevant equilibrium magnetic field magnitude $B \simeq B_\theta(a)$

$$V_A = \frac{B}{(\mu_0 \rho)^{1/2}}$$

Dominant oscillation frequency

$$k \sim \frac{\pi}{L} \; ; \quad \omega \sim k V_A \sim \frac{V_A \pi}{L}$$ (fundamental axial node as starting point)

growth rate for ideal MHD

$$\lambda \sim \frac{V_A}{a}$$

Defining scope of Linear Scale.

note: $k \sim \pi/L$ comes from boundary conditions on Z-pinch plasma column. rigid ends at $z=0, L$ forces a standing wave for sausage mode perturbation. $n(z) \propto \sin(kz)$ where lowest energy longest lived axial node

$$\lambda T_d \sim \frac{V_A}{a} T_d \quad ; \quad \omega T_d \sim \frac{V_A \pi}{L} T_d$$

we cannot have either $\gg 1$ otherwise, instability galore.

λ: exponential growth rate
ω: oscillation frequency

- Restoring force comes from magnetic pressure gradient
- Balance b/w internal plasma pressure and $\frac{B_\theta^2}{2\mu_0}$

For kink ($m=1$)

Restoring force $F_{tension} \sim \frac{(B.\nabla)B}{\mu_0}$

$$\lambda_{m=1} \sim \frac{\pi V_A}{L}$$

Characteristic time is set by alvén propagation along the column.

$m=0$, instability exists in a pure z pinch
$m=1$, stability depends on field line twist

since $L >> a$, $m=1$ instabilities propogate a lot slower than $m=0$.

In this paper, we will be looking at $m=0$ instabilities.

Section B

$v_A = \frac{B_\theta(a)}{\sqrt{\mu_0 \rho}}$ $\lambda \sim \frac{v_A}{a}$

Linear MHD assumes $\|\xi\| << a$, radial displacement amplitude << pinch radius

radius a
perturbation ξ

local radius: $a + \xi$

magnetic pressure $\propto \frac{I^2}{(a+\xi)^2}$

Magnetic Pressure comes from Maxwell Stress Tensor.

$$F = \nabla \cdot T$$

$$T_{ij} = \frac{1}{\mu_0}\left(B_i B_j - \underbrace{\frac{1}{2}\delta_{ij} B^2}_{\text{isotropic magnetic pressure}}\right)$$

$$P_B = \frac{B^2}{2\mu_0}$$ Freidburg, Ideal MHD

$$B_\theta(r) = \frac{\mu_0 I}{2\pi r}$$

$$P_B = \frac{[B_\theta(r)]^2}{2\mu_0}$$

$$P_B = \frac{\mu_0^2 I^2}{2\mu_0 r^2}$$

$$= \frac{\mu_0 I^2}{2(a+\xi)^2}$$

$\frac{1}{(a+\xi)^2}$ has a geometric expansion

$$\frac{1}{a^2}\left(1 - 2\frac{\xi}{a} + 3\frac{\xi^2}{a^2} + \dots\right)$$

if $\frac{\xi}{a} < 0.01$, then $\left(\frac{\xi}{a}\right)^{n>1}$ can be ignored practically and we can assume the system to be linear

$\xi(t) = \xi_0 e^{\lambda t}$

Linear stability does not prescribe an initial amplitude. For engineering estimation we assume an initial geometric assymnetry. of order 10^{-3} consistent with milimeter scale fabrication tolerances in centimeter scale pinch geometries

But, we can also use $T_{non\,lin} = \frac{1}{\lambda} \ln\left(\frac{\alpha a}{\xi_0}\right)$, plotting $T_{non\,lin}$ across

$(10^{-5}$ to $10^{-2}) \rightarrow$ for ξ_0/a

$G = \frac{10^{-2}}{10^{-3}} = 10$, $\frac{10^{-2}}{10^{-4}} = 100$, $\frac{10^{-2}}{10^{-5}} = 100$

$T_{non\,lin} = \frac{\ln G}{\lambda} = \frac{\ln 10}{\lambda} = 2.303\ \tau_A$

$\frac{\ln(100)}{\lambda} = 4.606\ \tau_A$

46– 92 nS

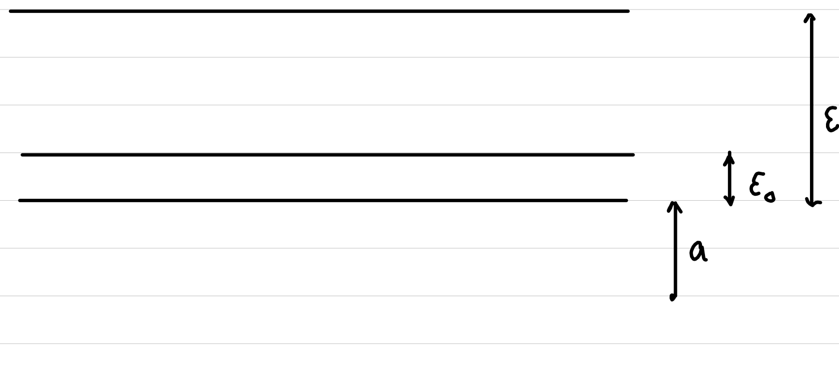
$T_{nonlinear} = \frac{1}{\lambda} \ln G$

$G = \frac{\alpha a}{\xi_0}$

$\frac{P_B(\varepsilon)}{P_{B0}} = \frac{1}{(1+\varepsilon)^2}$

$G = 3$ means the your final pertubation is 3 times the initial one.

$\frac{\xi}{a} : \frac{\xi_0}{a} = G$



ε_0: noise and base perturbation
ε: $(m=0)$ instability perturbation

$\frac{\varepsilon}{\varepsilon_0} = G$; $\frac{\varepsilon}{a}$ influences magnetic pressure

$\varepsilon(t) = \varepsilon_0 e^{\lambda t}$ $\qquad$ $T_{nonlinear} = \frac{1}{\lambda} \ln G$

we don't want $\frac{\varepsilon}{a}$ to pass a certain threshold so that the magnetic pressure remains linear. as well as the growth rate of the plasma instability remains linear

assuming δ_{crit} to be 0.01 to 0.1 and manufacturing accuracy to be b/w 10^{-4} and 10^{-3}, we get G between: 10 to 1000. But it is more likely to be 10 to 100

So, our circuit has b/w 2.3 and 4.6 τ_A to react to linearly growing instabilities. So our T_d has to fit b/w 2.3 τ_A and 4.6 τ_A or 46 to 92 ns acc. to the ZaP/FuZE reactor.