Section A : Can a fast inner loop reduce the effective instability growth rate from λ to λ' thereby permitting a slower outer loop with delay T_s' to stabilize the system

$$\dot{x}(t) = \lambda x(t) + b u(t)$$

$$U_f(t) = (\text{fast inner loop}) = -K_f x(t - T_f)$$

closed loop characteristic equation

$$\dot{x}(t) = \lambda x(t) - b K_f x(t - T_f)$$

$$s = \lambda - b K_f e^{-sT_f}$$

Stability boundary (Hopf Condition); $s = \pm j\omega$

$$j\omega = \lambda - b K_f e^{-j\omega T_f}$$

real $= -\lambda + b K_f \cos(\omega T_f) = 0$
Imaginary : $\omega - b K_f \sin(\omega T_f) = 0$

$$\omega = \lambda \tan(\omega T_f)$$

$$\theta = \omega T_f$$
$$\omega = \frac{\theta}{T_f}$$

(1)

$$\frac{\theta}{T_f} = \lambda \tan\theta$$

$$s = a + j\omega$$

$$\lambda T_f = \frac{\theta}{\tan\theta}$$

$$e^{st}$$

$\left(\boxed{-a}\right)$

$$\frac{1}{\lambda'} > \frac{1}{\lambda}$$

$$\lambda T_f < 1 \quad \text{for stability}$$

$$\underline{a > 0}$$

Assume $\exists \; \lambda' < \lambda$

$$a + j\omega = \lambda - me^{-aT_f} e^{-j\omega T_f}$$

$$a + j\omega = \lambda - me^{-aT_f}\cos(\omega T_f) + me^{-aT_f}\sin(\omega T_f)i$$

$$a = \lambda - me^{-aT_f}\cos(\omega T_f)$$

$$\omega = me^{-aT_f}\sin(\omega T_f)$$

this is good ↗

$$\lambda - a = me^{-aT_f}\cos(\omega T_f)$$
$$\omega = me^{-aT_f}\sin(\omega T_f)$$

$$\cos(\omega T_f) > 0 \;, \text{ so } \quad a < \lambda$$
$$\tfrac{\pi}{2} > \omega T_f > 0$$

$$\frac{\omega}{\lambda - a} = \tan(\omega T_f)$$

$$\lambda - \alpha = me^{-aT_f}\cos(\omega T_f) \leq me^{-aT_f} < m$$

$$\omega \leq me^{-aT_f}$$

$$\lambda - \alpha \leq m$$
$$\lambda - m < \alpha \;\; \rightarrow \text{ in the best case, you}$$

$$\text{for } a \geq 0 \;, \;\; \omega \leq m \quad \downarrow$$
can't reduce the growth more than the control law

$$\underline{\lambda > \alpha > \lambda - m}$$

$$(\lambda - a)^2 + \omega^2 = (m\bar{e}^{aT_d})^2$$

$$m = \left[(\lambda-a)^2 + \omega^2\right]^{1/2} e^{aT_d}$$

↓

contrd law required.

ω is angular frequency of Node oscillation.

to get max damping, we want $\cos(\omega T_d) \to 1$.

λ : $\overset{\text{old}}{\text{growth rate}}$

a : new growth rate

m : control law strength → controllable

T_d : time delay → controllable

ω : angular offsetandwe want it to be as close to 0 for max damping as ω is between 0 and $\pi/2$.

∴ .

open loop plasmas

$$a = \lambda - me^{-aTd} \cos(\omega Td)$$
$$\omega = me^{-aTd} \sin(\omega Td)$$

$S = \lambda_o + j\omega_o \rightarrow$ natural oscillation frequency $\qquad$ (in open loop)

$\downarrow$

intrinsic
growth rate

ω_o is influenced by V_A, a, mode number K

$$\omega_o = KV_A$$

closed loop means that there is delay.
delay means phase change. and thus. oscillation.

$$\omega = me^{-aTd} \sin(\omega T)$$

large a "mutes" the oscillations, yet contributes to growth rate.
smaller a could make mode $m=1$ oscillations worse.

$$\cos(\omega t) > 0, \quad \sin(\omega t) = \omega t$$

$\omega T \ll 1$, much smaller than a radian
we want an extremely tiny phase lag.

$$\omega \sim k v_A, \quad v_A = \frac{B}{\sqrt{\mu_0 \rho}} \qquad\qquad k \sim \frac{\pi}{L}$$

$$a = c\lambda$$

from non linear theory

$$T_d \leq 4.606 \, z_A \qquad\qquad \lambda T_d = \text{factor } F$$

$$0 < F < 5$$

$$a = \lambda - m e^{-a T_d} \cos(\omega T_d)$$

$$\dot{a} = \lambda - m e^{-a T_d}$$

$$c\lambda = \lambda - m e^{-c\lambda T_d}$$

$$\lambda(1-c) = m e^{-cF}$$

$$\lambda(1-c) = m e^{-cF}$$