Section A : Control Theoretic Delay limit.

Given a plasma, its instabilities grow at a rate of $1/\tau$

$x(t) \propto e^{t/\tau}$, we call this λ.

Standard MHD theory says that Z-pinch yields unstable modes with :

$$\gamma \sim \frac{V_A}{a}, \quad V_A = \frac{B}{\sqrt{\mu_0 \rho}}, \quad \tau = \frac{a}{V_A}$$

; τ = alvén transit time
$V_A \simeq$ alvén speed
a = pinch radius

Given an unstable system.

$$\dot{x}(t) = \lambda x(t)$$

we can establish a delayed control depending on a slightly delayed state $x(t-T_d)$

$$\dot{x}(t) = \lambda x(t) + K\hat{x}(t-T_d)$$

we need to find the maximum T_d that allows us to control this system in terms of λ.

using laplace transform, taking $x(0)=0$ for simplicity.

$$sX(s) = \lambda X(s) + Ke^{-sT_d}X(s)$$

$$s = \lambda + Ke^{-sT_d}$$

important : $Re(s)$ must be <0 or 0...
why?

$$s = \sigma + j\omega.$$

$$e^{st} = e^{\sigma}e^{j\omega}$$

if $\sigma > 0$, then this system will grow unstable.
$\sigma = 0$, then the system oscillates, we want less than 0 so that instabilities die out.

But, to find max T_d, let us take $\sigma = 0$ (and for simplicity)

$$j\omega = \lambda + Ke^{-j\omega T_d}$$

Real part: $0 = \lambda + K\cos(\omega T_d)$

imag part: $j\omega = -jK\sin(\omega T_d)$

$$\frac{\omega}{\lambda} = \tan(\omega T_d).$$

$$\cos(\omega T_d) = \frac{-\lambda}{K}$$

(2)

$$\sin(\omega T_d) = -\frac{\omega}{K}$$

$\theta = \omega T_d$

$$\omega = \frac{\theta}{T_d}$$

$$\frac{\theta}{T_d} = \lambda \tan(\theta)$$

$$T_d = \frac{\theta}{\lambda \tan\theta}$$

$\lambda > 0$ is constant

$$f(\theta) = \frac{\theta}{\tan\theta} = \theta\cot\theta$$

$$f'(\theta) = \cot\theta - \theta\operatorname{cosec}^2\theta$$

$$f'(\theta) = \frac{\cos\theta}{\sin\theta} - \frac{\theta}{\sin^2\theta}$$

$$= \frac{\sin\theta\cos\theta - \theta}{\sin^2\theta}$$ over 0 to $\pi/2$, $\sin^2\theta > 0$

$$\frac{\sin 2\theta - 2\theta}{2\sin^2\theta}$$ which is always negative

T_d is strictly decreasing, max occurs at left endpoint.

$$\sup f(\theta) = \lim_{\theta \to 0^+} \frac{\theta}{\tan\theta} = 1$$

$\sup T_d(\theta) = \frac{1}{\lambda}$; $T_d < \frac{1}{\lambda}$ for asymptotic stability;

if $T_d = \frac{1}{\lambda}$, root at $s = 0$ (loss of controllability) (3)

Conclusion : There is a strict upper limit for our control theoretic delay limit of $1/\lambda$ or τ_A

$$T_d < \tau_A$$

Thus according to data from ZaP and FuZe, we have only (taking lower limit) (20 - margin) ns to react to any changes.