

Math section

$$\ddot{x}(t) = \lambda x(t) - m x(t - T_d) \quad \dots (1)$$

where from our non linearity calculation.

$$\xi(t) = \xi_0 e^{\lambda t} \quad ; \quad G = \xi(t) / \xi_0$$

$$t_{\text{non-lin}} = \frac{\ln G}{\lambda} \quad ; \quad \text{we placed } G \text{ between } 10 \text{ and } 100$$

$$\text{giving } t_{\text{non-lin}} \in [2.303 z_A, 4.606 z_A]$$

so $T_{d_{\text{max}}}$ for linear regime is $\leq 2.303 z_A$ to $4.606 z_A$
acc. to given parameters of G

Why linear regime? [will get to this]

$$s = \lambda - m e^{-s T_d}$$

$$\text{Re}(s) = \lambda - m e^{-\text{Re}(s) T_d} \cos(\text{Im}(s) T_d)$$

$$\text{Im}(s) = m e^{-\text{Re}(s) T_d} \sin(\text{Im}(s) T_d)$$

Now $\text{Re}(s) = c\lambda$, c : reduction factor

$$c\lambda = \lambda - m e^{-c\lambda T_d}$$

T_d is in the form $F z_A = \frac{F}{\lambda}$

$\Rightarrow \lambda - \lambda c = m e^{-cF}$... we can plot this.

$$L_B < F < 4.606 z_A$$

L_B is our lower bound based on our hardware latency budget.

λ is constant

m is assumed constant for this math. albeit m 's strength depends on the latency of our control law.

m , likely an inductor will ramp up from 0 to m

c our reduction factor will be determined for various values of F

$$\ln(\lambda) + \ln(1-c) = \ln(m) - cF$$

$$cF + \ln(1-c) = \ln(m) - \ln(\lambda)$$

$$cF + \ln(1-c) = \ln\left(\frac{m}{\lambda}\right)$$

$$\ln(e^{cF}(1-c)) = \ln\left(\frac{m}{\lambda}\right)$$

$$e^{cF}(1-c) = \frac{m}{\lambda} = \text{constant } \beta$$

$$e^{cF}(1-c) = \beta$$