Section A : Control Theoretic Delay limit.

Given a plasma, its instabilities grow at a rate of $1/\tau$

$x(t) \propto e^{t/\tau}$, we call this λ.

Standard MHD theory says that Z-pinch yields unstable modes with :

$$\gamma \sim \frac{V_A}{a}, \quad V_A = \frac{B}{\sqrt{\mu_0 \rho}}, \quad \tau = \frac{a}{V_A}$$

; τ = alvén transit time
$V_A \simeq$ alvén speed
a = pinch radius

Given an unstable system.

$$\dot{x}(t) = \lambda x(t)$$

we can establish a delayed control depending on a slightly delayed state $x(t-T_d)$

$$\dot{x}(t) = \lambda x(t) + K x(t-T_d)$$

we need to find the maximum T_d that allows us to control this system in terms of λ.

using laplace transform, taking $x(0) = 0$ for simplicity.

$$sX(s) = \lambda X(s) + Ke^{-sT_d} X(s)$$

$$s = \lambda + Ke^{-sT_d}$$

important : $Re(s)$ must be < 0 or 0...
why?

$$s = \sigma + j\omega.$$

$$e^{st} = e^{\sigma} e^{j\omega}$$

if $\sigma > 0$, then this system will grow unstable.
$\sigma = 0$, then the system oscillates, we want less than 0 so that instabilities die out.

But, to find max T_d, let us take $\sigma = 0$ (and for simplicity)

$$j\omega = \lambda + Ke^{-j\omega T_d}$$

Real part: $0 = \lambda + K\cos(\omega T_d)$

imag part: $j\omega = -jK\sin(\omega T_d)$

$$\frac{\omega}{\lambda} = \tan(\omega T_d).$$

$$\cos(\omega T_d) = \frac{-\lambda}{K}$$

(2)

$$\sin(\omega T_d) = -\frac{\omega}{K}$$

$\theta = \omega T_d$

$$\omega = \frac{\theta}{T_d}$$

$$\frac{\theta}{T_d} = \lambda \tan(\theta)$$

$$T_d = \frac{\theta}{\lambda \tan\theta}$$

$\lambda > 0$ is constant

$$f(\theta) = \frac{\theta}{\tan\theta} = \theta\cot\theta$$

$$f'(\theta) = \cot\theta - \theta\,\mathrm{cosec}^2\theta$$

$$f'(\theta) = \frac{\cos\theta}{\sin\theta} - \frac{\theta}{\sin^2\theta}$$

$$= \frac{\sin\theta\cos\theta - \theta}{\sin^2\theta}$$ over 0 to $\pi/2$, $\sin^2\theta > 0$

$$\frac{\sin 2\theta - 2\theta}{2\sin^2\theta}$$ which is always negative

T_d is strictly decreasing, max occurs at left endpoint

$$\sup f(\theta) = \lim_{\theta \to 0^+} \frac{\theta}{\tan\theta} = 1$$

$\sup T_d(\theta) = \frac{1}{\lambda}$; $T_d < \frac{1}{\lambda}$ for asymptotic stability;

if $T_d = \frac{1}{\lambda}$, root at $s = 0$ (loss of controllability) (3)

Conclusion : There is a strict upper limit for our control theoretic delay limit of $1/\lambda$ or τ_A

$$T_d < \tau_A$$

Thus according to data from Zap and FuZe, we have only (taking lower limit) (20 - margin) ns to react to any changes.

$$\theta_{max} \sim O(<1)\tau_A$$

Section B

Latency budget : (Preliminary , based on rough estimates)

Sensors :

B-dot : BW : 2 GHz (usable) DOI : 10.1109/JSEN.2016.2530841
4 GHz (theoretically possible)
Rogowski : 20 MHz TIDA-01063 [impractical]
Photodiode : 1-10 GHz InGaS wp.optics.arizona.edu.
70 GHz possible

Cable :

Coax (1m) $v \approx 0.66c$: 5 ns

Gate driver : 200 MHz
Actuator field : 200 MHz

Analog filtering and combarator: 300MHz and 500 MHz respectively

B-dot to actuator time:

$$\theta_{tot}: \underbrace{0.5nS}_{\text{B-dot}} + \underbrace{5nS}_{\text{Coax}} + \underbrace{2nS}_{\text{Comparator}} + \underbrace{3nS}_{\text{filter}} + \underbrace{5ns}_{\text{gate d-l-}} + \underbrace{5ns}_{\text{Actuator}} = 20.5nS$$

Rogowiski pushes this to 24ns
Photodiode : 20ns.

Any EMI induced retries or thermal detuning pushes us over the limits

Despite the above latency table being based on loose estimates and does not take into account detuning and induced retries as this only makes the loop longer. It can be seen that we are exceeding our minimum alfén time of 20nS.

ADCs, FPGA's and other digital components only add to the delay loop and are thus infeasible.

Conclusion: Given $\tau_A \sim 20nS$, a conventional closed-loop stabalization system is unlikely to be robustly feasible. However, an ultra-local analog reflex loop may still reduce growth rate enough to extend the linear regime, enabling slower supervisory control or passive mechanisms.