ECE 514 Mini-project Fall 2021

Background

This mini-project explores the use of confidence intervals to quantify the accuracy of channel estimation for a wired, digital communications system. The system sends a sequence of known (+1) and unknown (+1 or -1) symbols. After appropriate filtering and sampling, the real-valued received sample for the *i*th symbol period can be modeled as

$$X_i = C + N_i, \qquad (1)$$

where C is the channel coefficient to be estimated (deterministic, real-valued) and N_i is a sequence of i.i.d. Gaussian noise r.v.'s with zero mean and variance σ_N^2. It is assumed that there are *n* pilot symbols, producing *n* received samples available for channel estimation. A channel coefficient estimate is needed to recover the set of unknown symbols (demodulation).

Project description

In this project, we will use MATLAB® (ML) to simulate channel estimation and quantify its performance. The true channel coefficient is $C = 10$. At the receiver, the channel coefficient is estimated using the sample mean from *n* received samples. Let's define the received signal quality in terms of signal-to-noise-ratio (SNR), which is the ratio of desired signal power to noise power. In our case,

$$SNR = |C|^2/\sigma_N^2 \qquad (2)$$

We are interested in providing confidence intervals for different values of *n* and different approaches to computing confidence intervals.

Simulate *m* trials. For each trial, generate *n* received values and use the sample mean of X to estimate C. Use the *randn* function in ML. Initialize the random number generator using *rng(seed,'twister')*, where *seed* is set to an integer determined by certain vowels in your first and last name (A=16, E=64, I=256, O = 512, U = 1024) by summing the numbers corresponding to the vowels present. For example, "Jane Jones" would give the seed 16 (a) + 64 (e) + 512 (o) + 64 (e) = 656 (include in report). This will ensure each person gets different results.

Consider three ways for computing the confidence intervals:

- Determine the 90% confidence interval using the known variance of X (Recommendation: use norminv in ML and verify using Table 6.2). This will give a lower and upper confidence interval boundary per trial. From the *m* trials, determine the fraction of trials for which the true channel coefficient falls within the confidence interval. For the first 10 trials, plot a) the sample mean of X, b) the lower confidence interval boundary, c) the upper confidence interval boundary, and d) the true channel coefficient (same for all trials). The x axis is trial number. Label the x axis and provide a legend to indicate which curve is which. Ensure all text is readable (large enough font). Hint: use *subplot(2,1,1),…* command in ML.
- Repeat using the estimated (sample) variance to estimate the confidence interval, but do not assume (do not take advantage) of the fact that the measurements are Gaussian (i.e., use the same $y_{\alpha/2}$ as for the case of known variance)
- Repeat using the estimated (sample) variance but assume (take advantage) of the fact that the measurements are Gaussian (Recommendation: compute $y_{\alpha/2}$ using tinv in ML and verify using Table 6.3).

Note: all 3 cases should be run in parallel so they use exactly the same data samples. The plots should all have the same x and y axis ranges for ease of comparison (see *axis* command in ML).

Use *m*=1 000, SNR = -6 dB (0.25) and two cases: *n* = 10 and *n* = 100. Don't forget to change $y_{\alpha/2}$ when *n* changes. From (2), the noise should have variance

$$\sigma_N^2 = |C|^2/SNR = 100/0.25 = 400, \quad \text{(the standard deviation is 20).}$$

Deliverable

Provide a short report (3-4 pages excluding appendix) in PDF that includes the following sections.

- 1. Background: A restatement of the problem in your own words

- 2. Experimental setup. A description of how you performed the simulation and how you computed the confidence intervals, including formulas. Indicate the random seed you used. Also, provide the following tables.
    - A table with a row for each *n* value and columns for the three $y_{\alpha/2}$ values you used for the three cases. Comment on how using the Student *t* distribution affects the width of the confidence interval.
    - A table with a row for each *n* value providing the theoretical standard deviation for the sample mean ($\sigma_x/\sqrt{n}$), i.e., the value used to compute the confidence interval for case 1.
- 3. Results.
    - A table with the fraction of trials for which the true channel coefficient (true mean) fell within the confidence interval. A row for each value of *n* and columns for each of the 3 cases. Provide a paragraph discussing the results of the table. Was it what you expected? How should case 3 compare to case 2?
    - Three plots as described above corresponding to the three cases and to *n* = 10. Provide a paragraph discussing these plots. All three plots should have the same y axis range for comparison.
- 4. Conclusion. A paragraph or two describing what you learned from this exercise. How does different values of *n* affect estimation accuracy? When using sample variance to form a confidence interval, how does performance compare between the three ways of computing confidence intervals?
- 5. Appendix. A printout of the source code you wrote.

Note: only provide what is asked for. For this exercise, the presence or absence of additional work has no grade impact.

Grading rubric: if you follow the instructions and provide all that was asked for in a clear manner, you get 100%. Points will be deducted if components are missing or wording is unclear.

Advice: carefully make a list of each item requested within this document (not all of them are in the deliverable section). In industry and academia, when responding to a request-for-proposal (RFP), this is called doing a “shred” of the RFP. Make sure each item is addressed in your report.