Q.1 $\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z}\, dx\,dy\,dz$

$$\int_0^2 \int_0^x e^{x+y} \left(e^{2x+2y} - e^0\right) dy\,dx$$

$$\int_0^2 \left(e^{3x} \cdot \frac{e^{3y}}{3} - e^x \cdot e^y\right)_0^x$$

$$\int_0^2 \left(\frac{e^{6x}}{3} - e^{2x} - \frac{e^{3x}}{9} + e^x\right)_0^2$$

$$\frac{e^{12}}{18} - \frac{e^4}{2} - \frac{e^6}{9} + e^2 - \left(\frac{1}{18} - \frac{1}{2} - \frac{1}{9} - 1\right)$$

$$\frac{e^{12}}{18} - \frac{e^4}{2} - \frac{e^4}{2} + e^2 - \frac{4}{9}$$

Ans:

2. $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \dfrac{dx\,dy\,dz}{(1+x+y+z)^3}$

$$\Rightarrow \int_0^1 \int_{y=0}^{y=1-x} (1+x+y+z)^{-3}\, dx\,dy\,dz$$

$$\int_0^1 \int_0^{y=1-x} \left[\frac{(1+x+y+z)^{-2}}{-2}\right]_0^{1-x-y} dy\,dx$$

$$\int_0^1 \int_0^{y=1-x} \left[\frac{(1+\cancel{x}+\cancel{y}+1-\cancel{x}-\cancel{y})^{-2}}{-2} - \frac{(1+x+y+0)^{-2}}{2}\right]$$

$$= \frac{-1}{2}\int_0^1 \int_0^{1-x} \left[\frac{1}{4} - \frac{1}{(1+x+y)^2}\right] dy\,dx$$

$$\Rightarrow \frac{-1}{2}\int_0^1 \left[\frac{y}{4} + \frac{1}{(1+x+y)}\right]_0^{1-x} dx$$

$$\Rightarrow \frac{-1}{2}\int_0^1 \left[\frac{1-x}{4} + \frac{1}{1+\cancel{x}+1-\cancel{x}} - \frac{1}{1+x}\right] dx$$

$$\Rightarrow \frac{-1}{2}\int_0^1 \left[\frac{1-x}{2} + \frac{1}{2} - \frac{1}{1+x}\right] dx$$

$$\Rightarrow \cancel{\frac{-1}{2}\left[\frac{(1-x)^2}{(-1)2} + \frac{x}{2} - \log(1+x)\right]_0^1}$$

$$-\frac{1}{2}\int_0^1 \left[\frac{1}{2} - \frac{x}{2} + \frac{1}{2} - \frac{1}{1+x}\right] dx$$

$$-\frac{1}{2}\left[\frac{x}{2} - \frac{x^2}{4} + \frac{x}{2} - \log(1+x)\right]_0^1$$

$$-\frac{1}{2}\left[\frac{1}{2} - \frac{1}{4} + \frac{1}{2} - \log 2\right]$$

$$\frac{1}{2}\left[\log 2 - \frac{1}{8} + 1\right]$$

$$\frac{1}{2}\left[\log 2 - \frac{5}{8}\right]$$

Q. 3.

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} (x+y+z)\, dx\,dy\,dz$$

$$\int_0^1 \int_0^{1-x} \left[z(x+y) + \frac{z^2}{2} \right]_0^{1-x-y} dx\,dy$$

$$\int_0^1 \int_0^{1-x} \left[(1-x-y)(x+y) + \frac{(1-x-y)^2}{2} \right] dx\,dy$$

$$\int_0^1 \int_0^{1-x} (1-x-y) \left[\frac{2x+2y+1-x-y}{2} \right] dx\,dy$$

$$\int_0^1 \int_0^{1-x} (1-x-y) \left[\frac{1+x+y}{2} \right] dx\,dy$$

$$\int_0^1 \int_0^{1-x} \frac{[1-(x+y)][1+(x+y)]}{2}\, dx\,dy.$$

$$\int_0^1 \int_{y=0}^{1-x} \left[\frac{1^2}{2} - \frac{(x+y)^2}{2} \right] dx\,dy$$

$$\int_0^1 \left[\frac{y}{2} - \frac{(x+y)^3}{6}\right]_0^{1-x} dx$$

$$\int_0^1 \left[\frac{1-x}{2} - \frac{(x+1-x)^3}{6} + \left(\frac{x^3}{6}\right)\right] dx$$

$$\int_0^1 \left[\frac{1-x}{2} - \frac{1}{6} + \frac{x^3}{6}\right] dx$$

$$\cancel{\int_0^1 \frac{1-x}{2} - \frac{x^3}{3}}$$

$$\int_0^1 \left[\frac{1}{2} - \frac{x}{2} - \frac{1}{6} + \frac{x^3}{6}\right] dx$$

$$\left[\frac{x}{2} - \frac{x^2}{4} - \frac{x}{6} + \frac{x^4}{24}\right]_0^1$$

$$\frac{1}{2} - \frac{1}{4} - \frac{1}{6} + \frac{1}{24}$$

$$\frac{12-6-4+1}{24} = \frac{12-9}{24} = \frac{1}{8}$$

Q. 4

$$\iiint (x^2y^2+y^2z^2+z^2x^2)\, dx\, dy\, dz$$

$$x^2+y^2+z^2=a^2$$

$x = r\sin\theta\cos\phi$; $y = r\sin\theta\sin\phi$,

$z = r\cos\theta$, $dx\,dy\,dz = r^2\sin\theta\, dr\, d\theta\, d\phi$

$$I = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{a} (r^4\sin^4\theta\sin^2\phi\cos^2\phi + r^4\sin^2\theta\cos^2\theta \sin^2\phi + r^4\sin^2\theta\cos^2\theta\cos^2\phi) \cdot r^2\sin\theta\, dr\, d\theta\, d\phi .$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} (\sin^4\theta\sin^2\phi\cos^2\phi + \sin^2\theta\cdot\cos^2\theta)\sin\theta\, d\theta\, d\phi \cdot \int_0^{a} r^6\, dr$$

$$\Rightarrow \frac{a^7}{7}\left[\int_0^{\pi/2}\int_0^{\pi/2} \sin^5\theta \sin^2\phi\cos^2\phi\, d\theta\, d\phi + \int_0^{\pi/2}\int_0^{\pi/2} \sin^3\theta\cos^2\theta\, d\theta\, d\phi\right]$$

$$\frac{a^7}{7}\left[\int_0^{\pi/2} \sin^5\theta \, d\theta \int_0^{\pi/2} \sin^2\phi \cos^2\phi \, d\phi + \int_0^{\pi/2} \sin^3\theta \cos^2\theta \, d\theta \int_0^{\pi/2} d\phi\right]$$

$$\Rightarrow \frac{a^7}{7}\left[\frac{4\cdot 2}{5\cdot 3} \times \frac{\pi}{8\times 2} + \frac{2\pi}{30}\right]$$

$$\Rightarrow \frac{a^7}{7} \times \left[\frac{1}{30} + \frac{1}{15}\right] = \frac{a^7}{7} \times \frac{\pi}{10}$$

$$\Rightarrow 4 \cancel{8} \times \frac{a^7 \times \pi}{7 \times \cancel{10}_{5}} \Rightarrow \frac{4a^7\pi}{35}$$

Q. 5

$$\iiint \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{3/2}}$$

$$8\int_0^{\pi/2}\int_0^{\pi/2}\int_4^5 \frac{r^2 \sin\theta \cdot dr\,d\theta\,d\phi}{(r^2)^{3/2}}$$

$$I = \int_0^{\pi/2}\int_0^{\pi/2}\int_4^5 \frac{1}{r} \sin\theta\, dr\, d\theta\, d\phi$$

$$I = 8\int_0^{\pi/2}\int_0^{\pi/2} \left[\log r\right]_4^5 \sin\theta\, d\theta\, d\phi$$

$$I = 8\int_0^{\pi/2}\int_0^{\pi/2} \log\frac{5}{4} \sin\theta\, d\theta\, d\phi$$

$$I = 8\int_0^{\pi/2} \left[-\cos\theta\right]_0^{\pi/2} \log\frac{5}{4}\, d\phi$$

$$I = 8\int_0^{\pi/2} \log\frac{5}{4}\, d\phi$$

$$I = 4\pi \log\frac{5}{4}$$