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Tutorial - 01 Calculas

(1)

→ $\log\left[\frac{x^2+6}{5x}\right]$ on $[2, 3]$

$$f(x) = \log\left[\frac{x^2+6}{5x}\right]$$

$$f(x) = \log(x^2+6) - \log 5x$$

$$f'(x) = \frac{2x}{x^2+6} - \frac{5}{5x}$$

$$f(2) = f(2)$$

$$\therefore f'(c) = 0$$

$$\frac{2c}{6+c^2} - \frac{5}{5c} = 0$$

$$\frac{2c}{c^2+6} = \frac{1}{c} \Rightarrow \boxed{c = \pm\sqrt{6}}$$

but $\boxed{c = \sqrt{6}}$

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(ii) $1 - \sqrt[3]{(x-1)^2}$ on $[0, 2]$

$f(x) = 1 - \sqrt[3]{(x-1)^2}$

$f'(x) = \dfrac{-2}{3(x-1)^{1/3}}$

$\because$ 1 lies in $[0, 2]$ where $f(x)$ is not defined

$\therefore$ Rolle's theorem not verified.

(3) $\cos^2 x$ on $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$f(x) = \cos^2 x$

$\because$ it is differentiable it is continous

$f'(x) = 2\cos x(-\sin x)$

$\because f\left(\frac{-\pi}{4}\right) = f\left(\frac{\pi}{4}\right)$

$\therefore f(c)$ exist & $f'(c) = 0$

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$\sin 2c = 0$

$2c = \sin^{-1} 0$

$\boxed{c = 0}$

(4) $|\cos x|$ on $[0, \pi]$

$$f(x) = \begin{cases} \cos x & 0 < x < \pi/2 \\ -\cos x & \pi/2 < x < \pi \end{cases}$$

∵ If is differentiable, So continious as well

$f(0) = \cos 0 = 1$
$f(\pi) = -\cos \pi = 1$
$f(\pi) = f(0) = 1$

$\lim\limits_{x \to \frac{\pi}{2}^-} = f_1'(x) = \lim\limits_{x \to \pi/2} (-\sin x) = -1$

$\lim\limits_{x \to \frac{\pi}{2}^+} = f_2'(x) = \lim\limits_{x \to \pi/2^+} \sin x = 1$

$\therefore f_1'(x) \neq f_2'(x)$

Rolles theorm not verified

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2)

$$f(x) = e^{-x}(\sin x - \cos x) \text{ in } \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$$

$\because$ $\sin x$ and $\cos x$ both are continous and differentiable also.

Now,

$$f'(x) = e^{-x}(\cos x + \sin x) - e^{-x}(\sin x - \cos x)$$

$$f'(x) = e^{-x}[\cos x + \cancel{\sin x} - \cancel{\sin x} + \cos x]$$

$$f'(x) = 2e^{-x}\cos x$$

Now,

$$f\left(\frac{\pi}{4}\right) = f\left(\frac{5\pi}{4}\right)$$

$\therefore$ Rolle's theorem verified

$$f'(c) = 0$$

$$2 \cdot e^{-c}\cos c = 0$$

$$\cos c = 0 \Rightarrow c = \cos^{-1} 0$$

$$\boxed{c = \pi/2}$$

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3)

Given :

$f(x) = x(x+1)(x+2)(x+3)$

$f(x) = x^4 + 5x^3 + 6x^2 + x^3 + 5x^2 + 6x$

$f(x) = x^4 + 6x^3 + 11x^2 + 6x$

Now

$f'(x) = 4x^3 + 18x^2 + 22x + 6$

$\because$ f(x) is differentiable so continuous.

~~we have interval~~

Put $f(x) = 0$

So we have

$f(x) = x(x+1)(x+2)(x+3) = 0$

$\therefore$ We get $x = -1, -2, -3, 0$

We have 3 Real roots as $-1, -2, -3$

Q.4

=) Given: $y = x^2 + 2k_1 x + k_2$, $x = a$, $x = b$

$\frac{dy}{dx} = 2x + 2k_1$ is equation of tangent to curve but ~~than~~ tangent is parallel to chord.

$x = a$, $x = b$

Tangent to curve $= a + b + 2k_1$

$2x + \cancel{2k_1} = a + b + \cancel{2k_1}$

$$\boxed{x = \frac{a+b}{2}}$$

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$\because f'(c) = \dfrac{1}{\sqrt{1-c^2}}$

$$\frac{1}{} < \frac{\sin^{-1}x}{x} < \frac{1}{\sqrt{1-x^2}}$$

Hence proved.

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$\therefore f'(c) = \dfrac{1}{\sqrt{1-c^2}}$

$$\frac{1}{\quad} < \frac{\sin^{-1}x}{x} < \frac{1}{\sqrt{1-x^2}}$$

Hence proved.

2.6

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Q.6

Let

$f(x) = \log(1+x)$ where $x > 0$

$\because$ $f(x)$ satisfy LMVT $[0, x]$

$\therefore$ $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$f'(x) = \dfrac{1}{(1+x)}$

$\boxed{f'(c) = \dfrac{1}{1+c}} = \dfrac{\log(1+x) - 0}{x}$

Now,

c is b/w $[0, x]$

$0 < c < x$

$1 < c+1 < x+1$

$1 > \dfrac{1}{1+c} > \dfrac{1}{1+x}$

$\therefore$ $\boxed{\dfrac{1}{1+c} = \dfrac{\log(1+x)}{x}}$

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$$1 > \frac{\log(1+x)}{x} > \frac{1}{1+x}$$

$$x > \log(1+x) > \frac{x}{1+x}$$

$$\boxed{\frac{1}{x} < \frac{1}{\log(1+x)} < \frac{x+1}{x}}$$

Hence proved

Q. 7

Given:

$f(x) = \sqrt{x}$

$g(x) = \frac{1}{\sqrt{x}}$

$\because$ It verifies CMVT

$$\therefore \frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$g'(x) = (x)^{-1/2} = \frac{-1}{2} x^{-3/2} \Rightarrow \frac{-1}{2 \cdot (x^{1/2})^3}$$

$$\Rightarrow \frac{-1}{2(\sqrt{x})^3}$$

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$$f'(c) = \frac{1}{2\sqrt{c}}$$

$$g'(c) = \frac{-1}{2(\sqrt{c})^3}$$

$$\frac{f'(c)}{g'(c)} = \frac{1}{\cancel{2}\cancel{\sqrt{c}}} \times \frac{-\cancel{2}(\sqrt{c})^{\cancel{3}\,2}}{}$$

$$= -(\sqrt{c})^2 = -c$$

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{\sqrt{b} - \sqrt{a}}{\frac{1}{\sqrt{b}} - \frac{1}{\sqrt{a}}}$$

$$-c = \frac{\sqrt{b} - \sqrt{a}}{\frac{\sqrt{a} - \sqrt{b}}{\sqrt{ab}}}$$

$$-c = \cancel{\sqrt{b} - \sqrt{a}} \times \left(\frac{-\sqrt{ab}}{\cancel{\sqrt{b} - \sqrt{a}}}\right)$$

$$c = \sqrt{ab}$$

$$\boxed{c^2 = ab}$$