Tutorial No- 4

Limits & Continuity

(1) $\lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{x - y}$

$$\frac{(x-y)(x^2 - xy + y^2)}{(x-y)}$$

$$\lim_{(x,y)\to(0,0)} (x^2 - yx + y^2) = 0$$

(2) $\lim_{(x,y)\to(2,0)} \frac{x \sin y}{x^2 + 1}$

$$\lim_{(x,y)\to(2,0)} \frac{2}{4+1} \times \left(\frac{\sin y}{y}\right) \times y = 0$$

$$\lim_{y\to 0} \frac{\sin y}{y} = 0$$

(3) $\lim_{(x,y)\to(0,0)} \dfrac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

$$\frac{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})+2(\sqrt{x}+\sqrt{y})}{\sqrt{x}-\sqrt{y}}$$

$$= \sqrt{x}+\sqrt{y}+2$$

On applying limit

$$= 2$$

(4) $\lim_{(x,y)\to(0,0)} \dfrac{xy^3}{x^2+y^6}$

Put $y=0$ then $f(x,0) = \dfrac{0}{x^2} = 0$

Put $x=0$ then $f(0,y) = \dfrac{0}{y^2} = 0$

Hence limit exists.

(5) $\lim_{(x,y)\to(1,0)} \frac{y}{x+y-1}$

Put $y=0$ then $f(x,0) = \frac{0}{x-1} = 0$

Put $y=0$ then $f(0,y) = \frac{0\cdot y}{0+y-1} = \frac{y}{y-1}$

∴ Limit doesnot exists.

(6) $f(x,y) = \frac{x^3}{x^3+y^2-2x}$

Put $x=0$ $f(0,y) = \frac{0}{y^2} = 0$

Put $y=0$ $f(x,0) = \frac{0\cdot x^3}{x^3-2x} = \frac{x^2}{x^2-2}$

∴ Limit doesnot exist.

Q.7

(i) $f(x,y) = \frac{2xy}{x^2+y^2}$

Put $y = mx$, $f(x,y) = \frac{2mx^2}{x^2+m^2x^2}$

$= \frac{2m}{1+m^2}$

∴ limit depends on different value of m.

∴ limit does not exist.

(ii) $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$

Put $y = mx$

$f(x,y) = \frac{mx^2}{\sqrt{x^2+m^2x^2}} = \frac{mx}{\sqrt{1+m^2}}$

∴ $f(x,y)$ depends on x

It is continous $f(x,y)$

(iii) $f(x,y)$ $\begin{cases} \frac{x^3-y^3}{x-y} \\ 0 \end{cases}$

$\lim_{(x,y)\to(0,0)} = x^2 + xy + y^2 = 0$

So $f(x,y)$ is continous at $(x,y)=(0,0)$

(iv) $f(x,y) = \frac{x^2 y}{x^2+y^2}$

Put $y = mx$

$= \frac{x^2 \times mx}{x^2 + m^2x^2} = \frac{mx}{1+m^2}$

$f(0,0) = f(x,y) = 0$

$\therefore$ It is continous.