CSE218

Offline 3

Topic: Integration

A rocket is going vertically up and expels fuel at a velocity 2000m/s at a consumption rate of 2100 kg/s. The initial mass of the rocket is 140,000 kg. If the rocket starts from rest at $t = 0$ seconds, how can one calculate the vertical distance covered by the rocket from $t = 8$ to $t = 30$ seconds?

Applying rules of physics, one can estimate that the distance covered by the rocket from time $t = t_0$ to $t = t_1$ is:

$$x = \int_{t_0}^{t_1} \left[u \log_e \left(\frac{m_0}{m_0 - qt} \right) - gt \right] dt$$

where

m_0 = initial mass of the rocket at $t = 0$

q = rate at which fuel is expelled (kg/s)

u = velocity at which fuel is being expelled (m/s)

g = 9.8 m/s^2

Programming tasks:

1. Write a python program to evaluate the distance covered by the rocket from $t = 8$ to $t = 30$ seconds using trapezoid rule. Your program should accept the number of sub-intervals n as a parameter from the user and use the trapezoid rule by partitioning the given interval into n equally spaced sub-intervals. Print the calculated values for $n = 1$ to 5 and show the absolute approximate relative errors. (10 Marks)
2. Solve the same problem given in (1) using Simpson's 1/3 rule. For this case, partition the given interval into $2n$ number of equally spaced sub-intervals and use the single application of a Simpson's rule n times. (10 Marks)