Homework 11

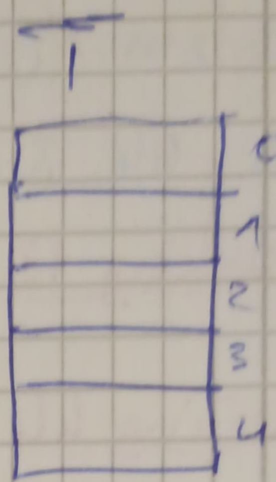
* Problem 11.1:

a) sequence: 3, 10, 2, 4.

$h_1(k) = k \mod 5$

$h_2(k) = 7k \mod 8$

$h(k,i) = (h_1(k) + i\, h_2(k)) \mod m$
$(m = 5)$



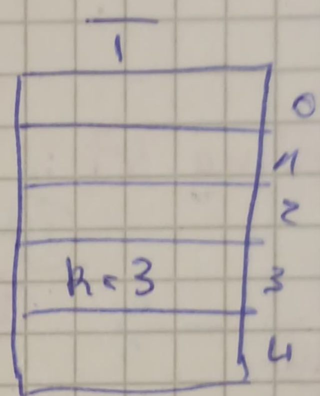
→ inserting 3:

$h_1(k) = 3 \mod 5 = 3$

we first probe with $i = 0$:

$h(3,0) = h_1(3) \mod 5$
$= 3 \mod 5$
$= 3$

So, $k = 3$ is inserted in position 3



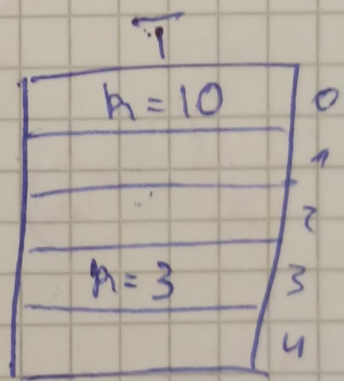
→ inserting 10:

$h_1(10) = 0$, ~~$h_2(10)$~~ =

we first probe with $i = 0$

$h(10,0) = h_1(10) \mod 5$
$= 0 \mod 5$
$= 0$

$k = 10$ inserted in position 0

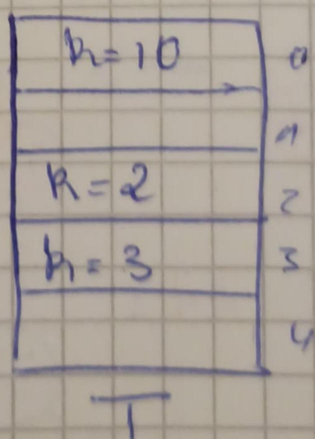


→ inserting 2:

$h_1(2) = 2$

we first probe with $i = 0$

$h(2,0) = h_1(2) \mod 5$
$= 2 \mod 5 = 2$



→ inserting 4.

$h_1(4) = 4$

we first probe with $i = 0$

$h(4, 0) = h_1(4) \bmod 5$

$= 4$

so $k = 4$ is inserted in position 4

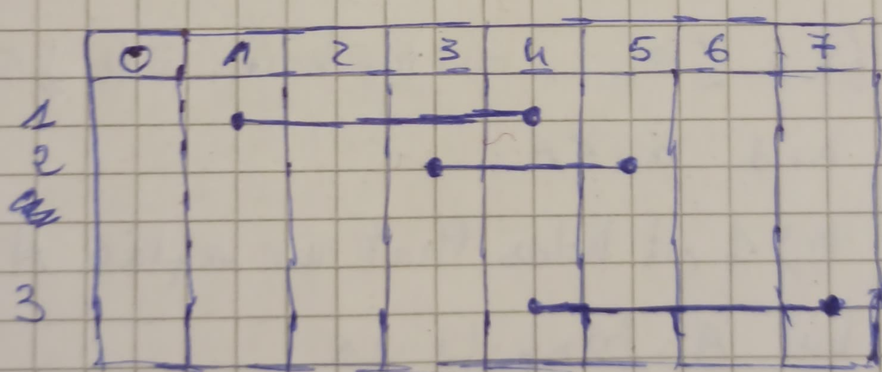
T	
$k_2 = 10$	0
	1
$k_1 = 2$	2
$k_1 = 3$	3
$k_1 = 4$	4

b) See implementation with test cases.

Problem 11.2:

a) To show that making the greedy decision of selecting the activity with shortest duration may fail, we can give a counter example

if we consider 3 activities with start/finish time shown in the table:



with the greedy choice stated earlier our solution set would only contain activity 2, which is not a globally optimal solution.

Selecting activity 1 and 2 is the global optimum solution.

b) idea of the greedy algorithm.

1) select activity with latest starting time

2) check all remaining elements and pick the element with ~~large~~

- largest finish time less than last added activity

$\boxed{\text{and}}$

- largest starting time

Lemma: The greedy choice of picking element with largest start time as first choice is optimal.

Proof:

- assume a_m elem with largest starting time s_m in S.
- " A globally optimal solution for S,
- $a_k \in A$ element with largest start time in A, s_k.
- $a_{k-1} \in A$ predecessor of a_k in A, with finish time f_{k-1}

since A global optimal solution.

$f_{k-1} \leq s_k$; and we have $s_m \geq s_k$

so if $k = 1$ $a_m \in A$ ✓

if $k > 1$, it holds that we replace A

by $A \setminus \{a_k\} \cup a_m$

since we have shown

that $f_{k-1} \leq s_m$.

by Construction, the alg will lead to an optimally global solution since we can consider it as "naive", However very costly in terms of operations.