

CS 188: Artificial Intelligence

Constraint Satisfaction Problems II

Instructor: *Mahdi Javanmardi*

Amirkabir University of Technology



Today

- Efficient Solution of CSPs
- Iterative CSP

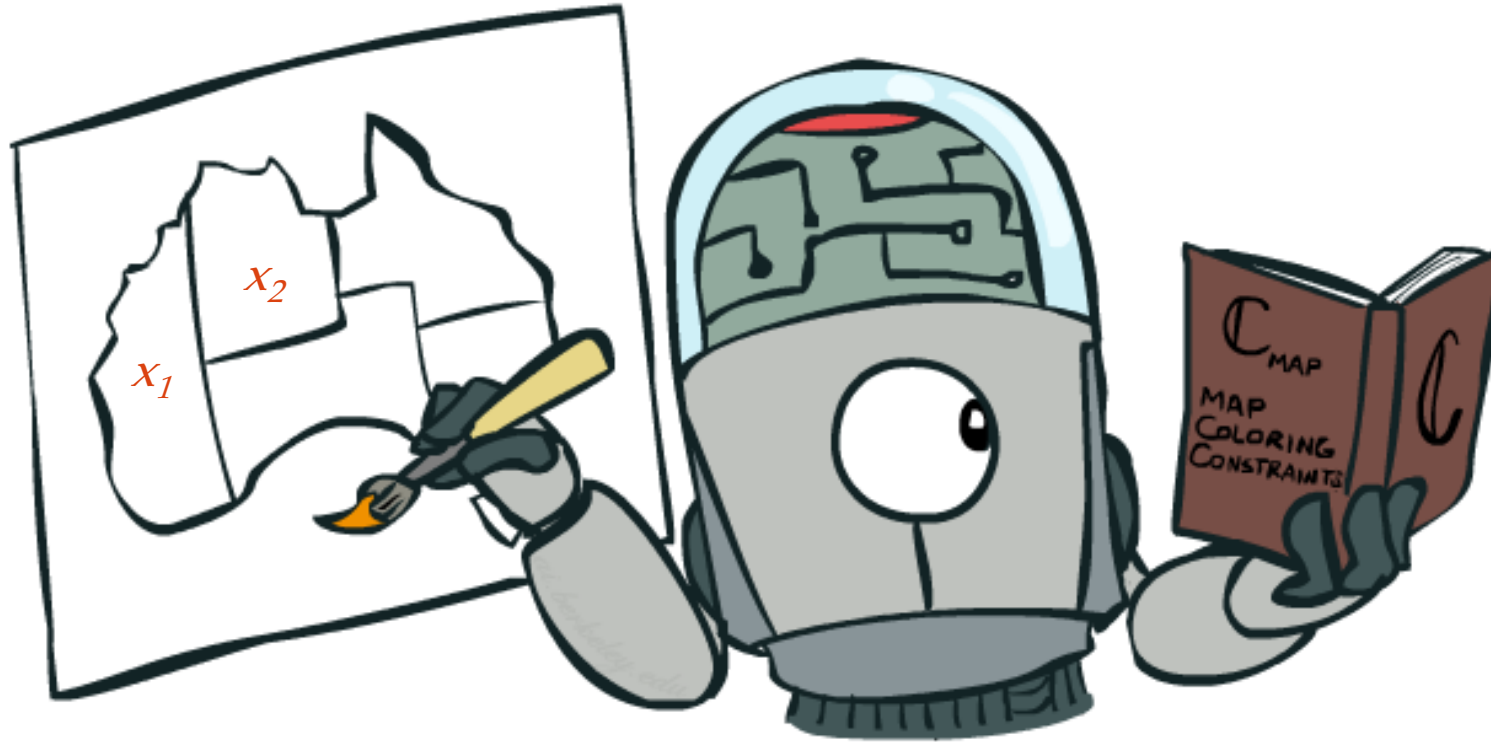


Constraint Satisfaction Problems

N variables

domain D

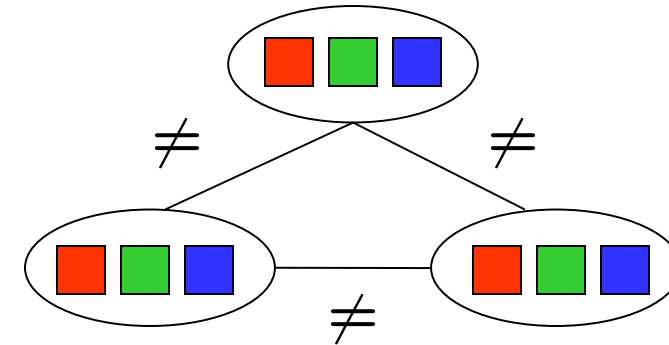
constraints



Reminder: CSPs

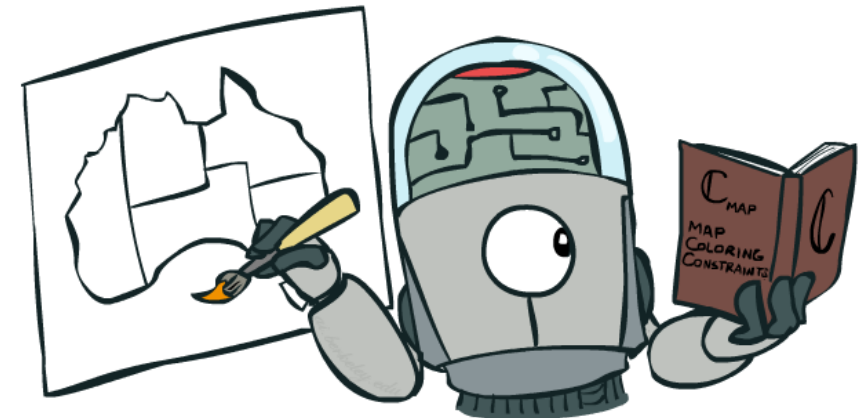
- CSPs:

- Variables
- Domains
- Constraints
 - Implicit (provide code to compute)
 - Explicit (provide a list of the legal tuples)
 - Unary / Binary / N-ary



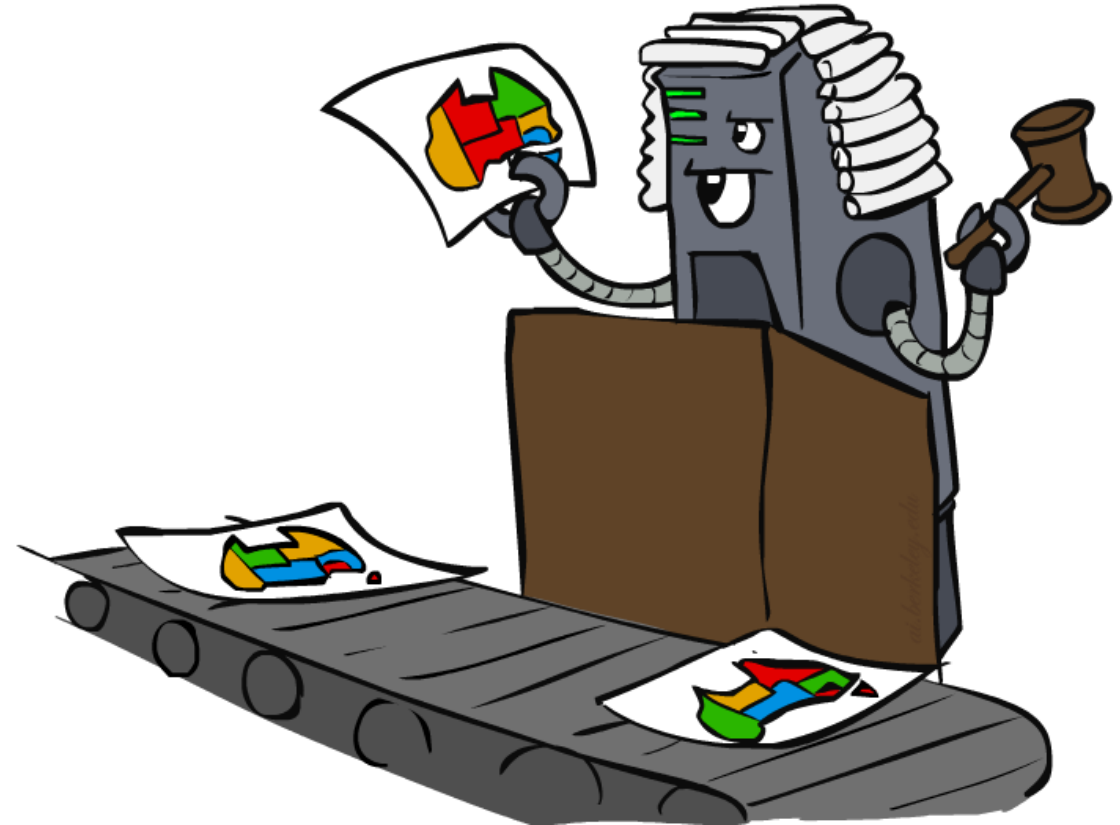
- Goals:

- Here: find any solution
- Also: find all, find best, etc.

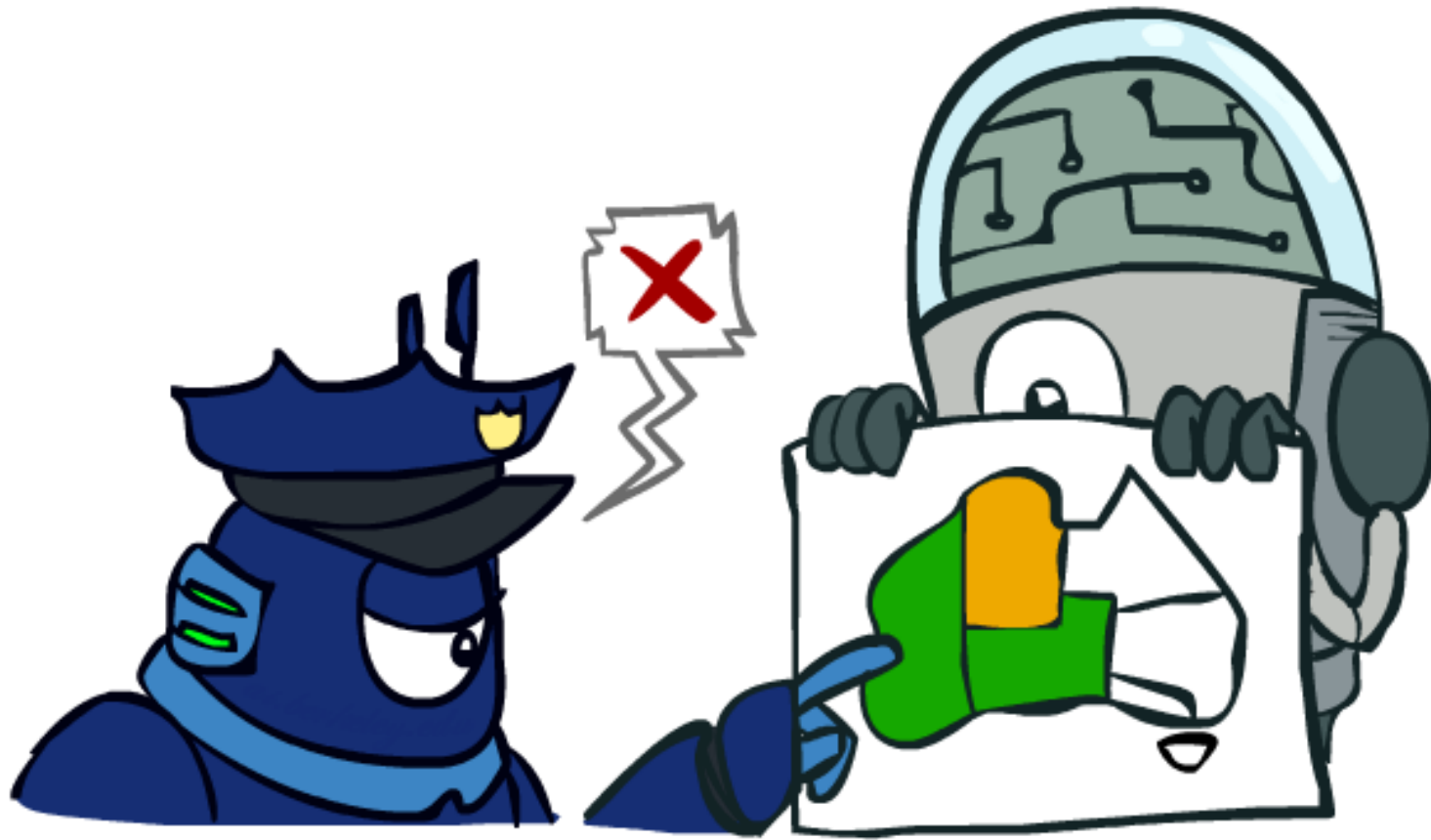


Standard Search Formulation

- Standard search formulation of CSPs
- States defined by the values assigned so far (partial assignments)
 - Initial state: the empty assignment, $\{\}$
 - Successor function: assign a value to an unassigned variable
 - Goal test: the current assignment is complete and satisfies all constraints
- We started with the straightforward, naïve approach, then improved it

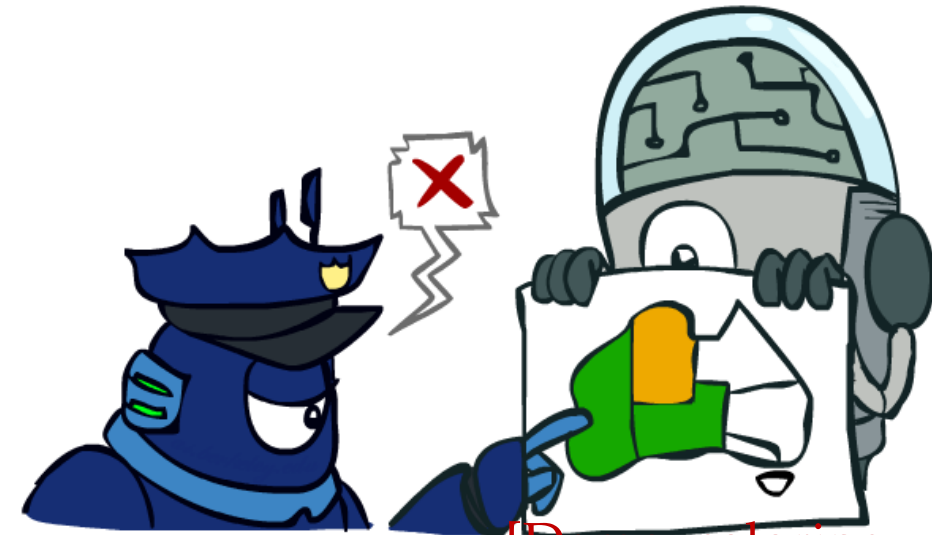
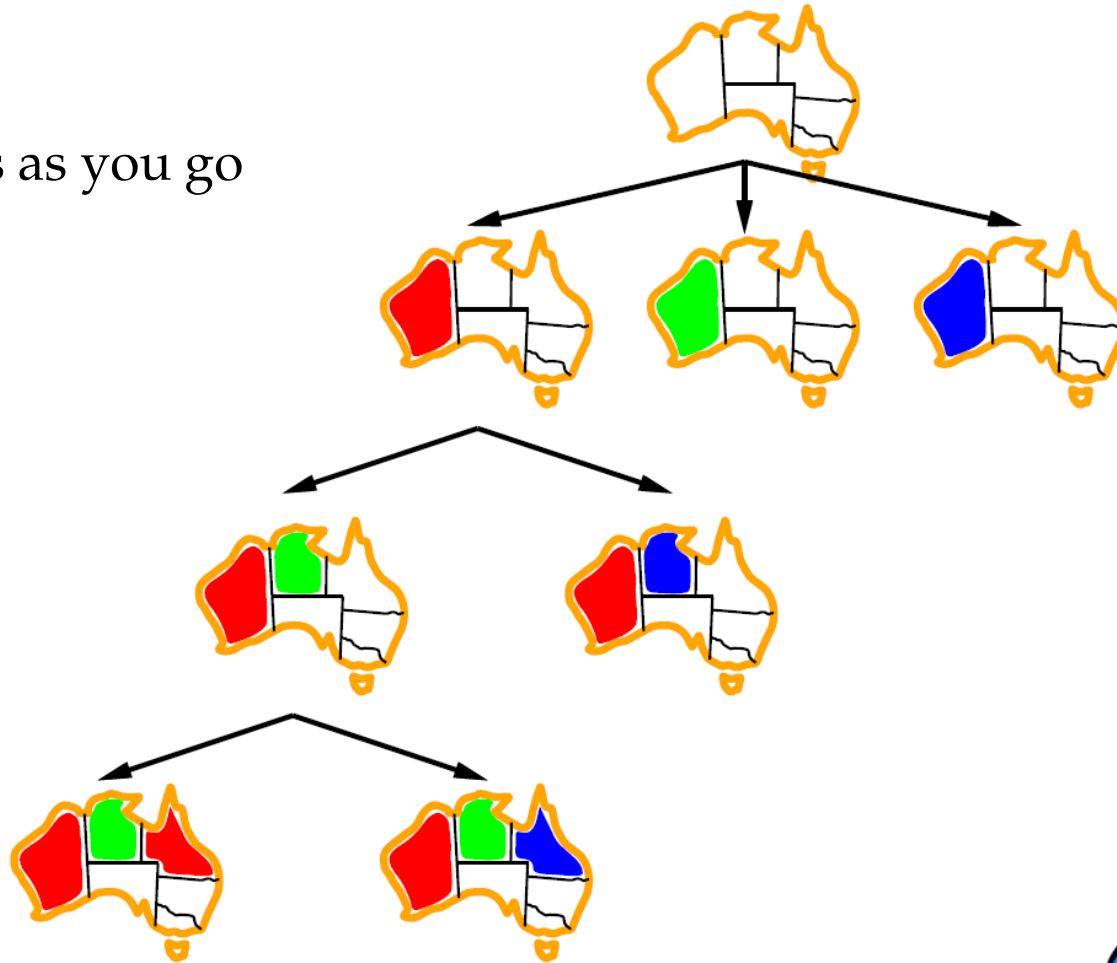


Backtracking Search



Backtracking Search

1. fix ordering
2. check constraints as you go



[Demo: coloring --
backtracking]

Explain it to your rubber duck!

Why is it ok to fix the ordering of variables?

Why is it good to fix the ordering of variables?

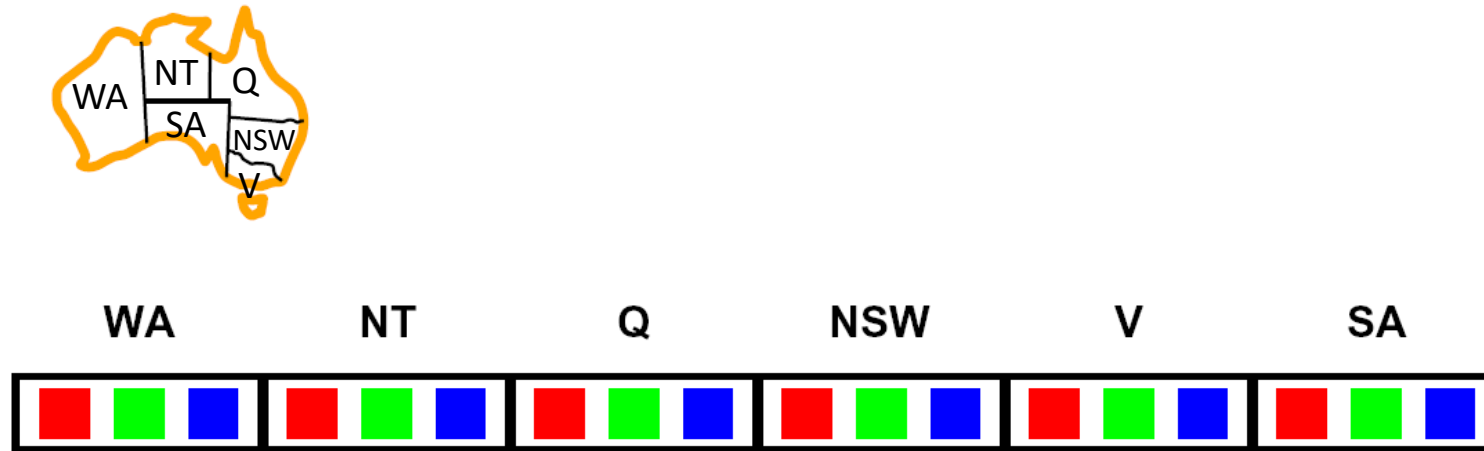
Filtering



Keep track of domains for unassigned variables and cross off bad options

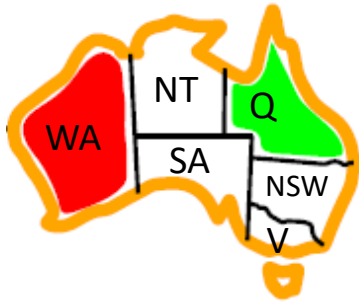
Filtering: Forward Checking

- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment



Filtering: Constraint Propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

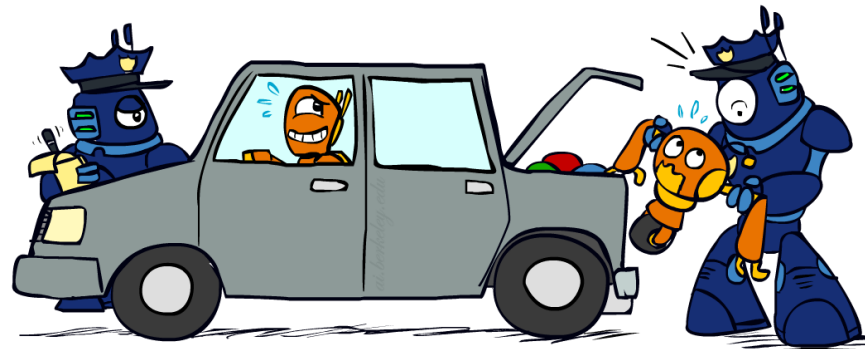
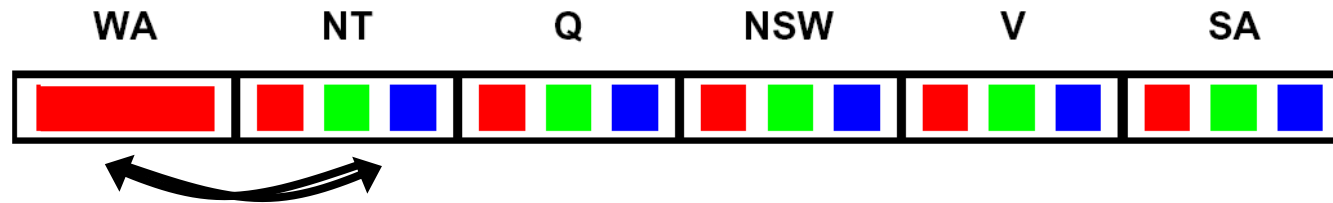
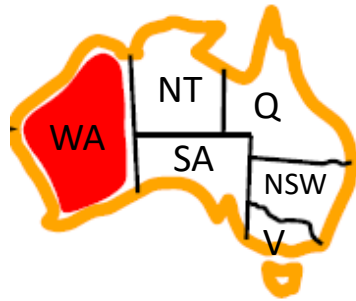


WA	NT	Q	NSW	V	SA
<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>
<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>
<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div></div>

- NT and SA cannot both be blue!
- Why didn't we detect this yet?
- Constraint propagation*: reason from constraint to constraint

Consistency of A Single Arc

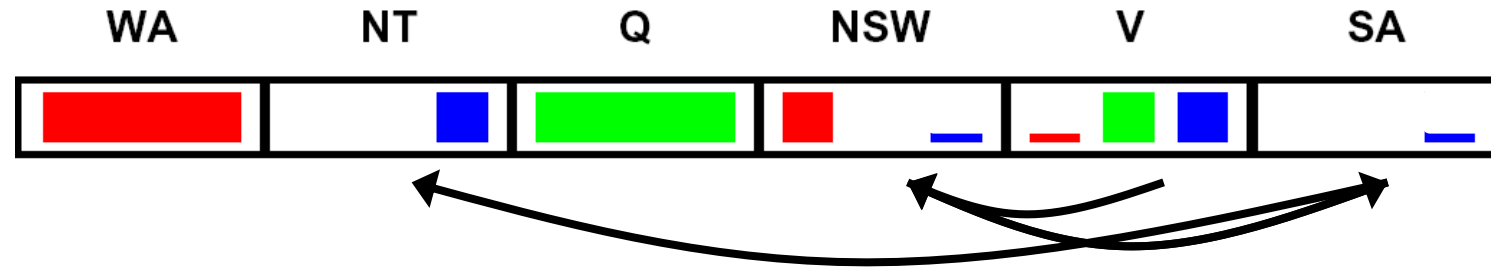
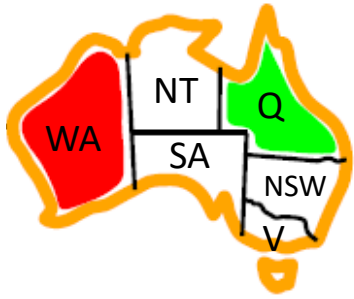
- An arc $X \rightarrow Y$ is **consistent** iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint



Delete from the tail!

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure **all** arcs are consistent:



- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

*Remember: Delete
from the tail!*

Enforcing Arc Consistency in a CSP

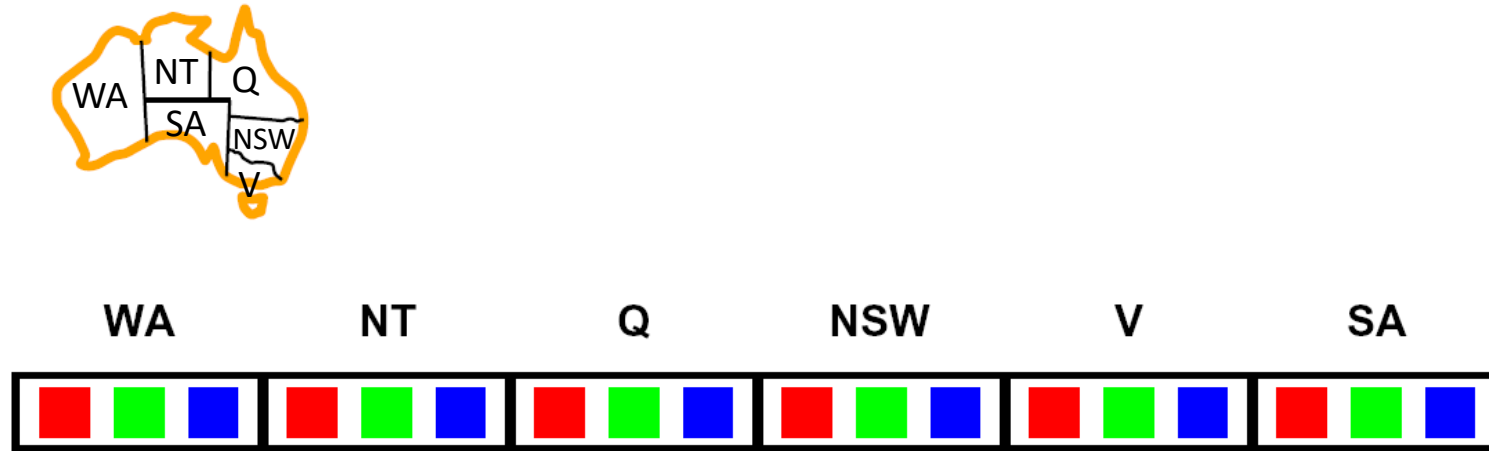
```
function AC-3(cs) returns the CSP, possibly with reduced domains
  inputs: cs, a binary CSP with variables  $\{X_1, X_2, \dots, X_n\}$ 
  local variables queue, a queue of arcs, initially all the arcs in cs
  while queue is not empty do
     $(X_i, X_j) \leftarrow \text{REMOVE-FIRST}(\textit{queue})$ 
    if REMOVE-INCONSISTENT-VALUES( $X_i, X_j$ ) then
      for each  $X_k$  in NEIGHBORS[ $X_i$ ] do
        add  $(X_k, X_i)$  to queue

function REMOVE-INCONSISTENT-VALUES( $X_i, X_j$ ) returns true iff succeeds
  removed  $\leftarrow$  false
  for each  $x$  in DOMAIN[ $X_i$ ] do
    if no value  $y$  in DOMAIN[ $X_j$ ] allows  $(x, y)$  to satisfy the constraint  $X_i \leftrightarrow X_j$ 
      then delete  $x$  from DOMAIN[ $X_i$ ]; removed  $\leftarrow$  true
  return removed
```

- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- ... but detecting all possible future problems is NP-hard – why?

Forward Checking – how does it relate?

- Forward checking: Cross off values that violate a constraint when added to the existing assignment

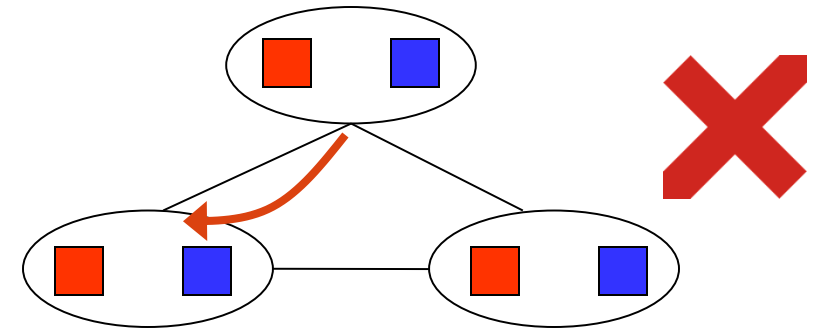
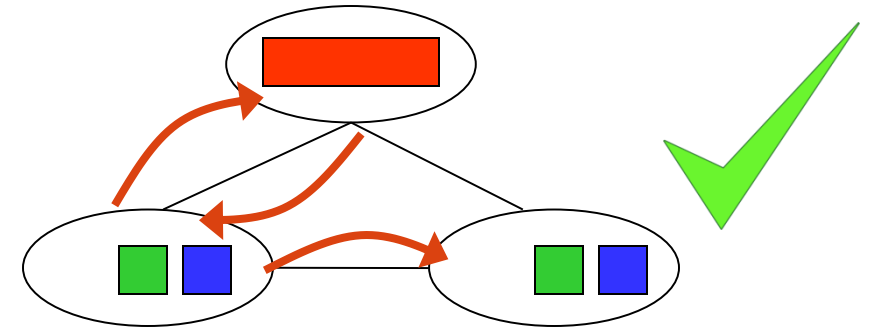


Explain it to your neighbor!

- Forward checking is a special type of enforcing arc consistency, in which we only enforce the arcs pointing into the newly assigned variable.

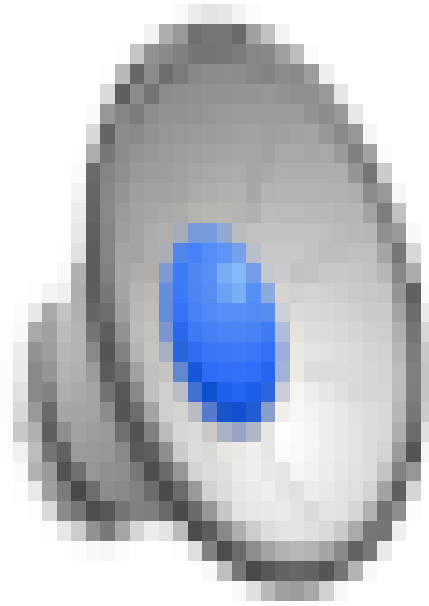
Limitations of Arc Consistency

- After enforcing arc consistency:
 - Can have one solution left
 - Can have multiple solutions left
 - Can have no solutions left (and not know it)
- Arc consistency still runs inside a backtracking search!

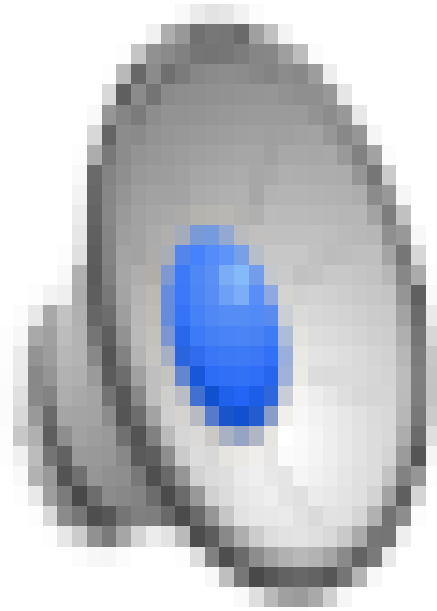


[Demo: coloring -- forward checking]
[Demo: coloring -- arc consistency]

Video of Demo Coloring – Backtracking with Forward Checking – Complex Graph

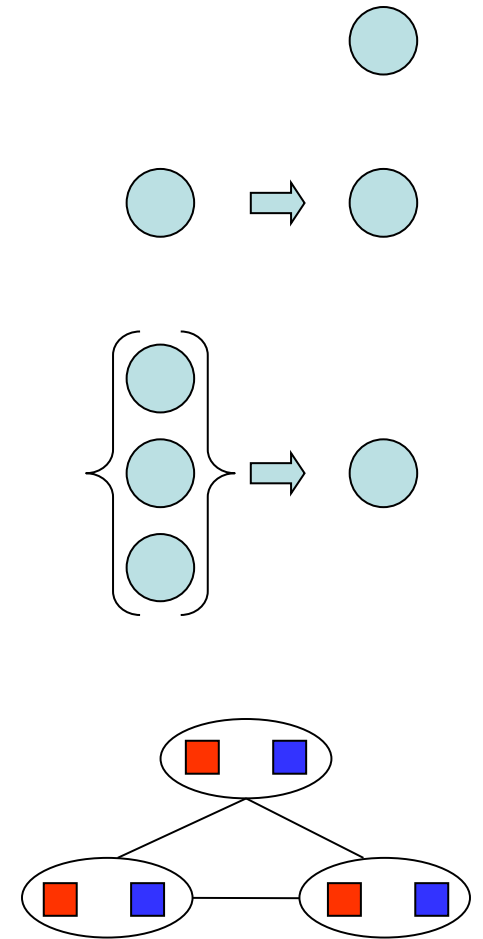


Video of Demo Coloring – Backtracking with Arc Consistency – Complex Graph



K-Consistency

- Increasing degrees of consistency
 - 1-Consistency (Node Consistency): Each single node's domain has a value which meets that node's unary constraints
 - 2-Consistency (Arc Consistency): For each pair of nodes, any consistent assignment to one can be extended to the other
 - K-Consistency: For each k nodes, any consistent assignment to k-1 can be extended to the kth node.
- Higher k more expensive to compute
- (You need to know the k=2 case: arc consistency)



Strong K-Consistency

- Strong k -consistency: also $k-1, k-2, \dots, 1$ consistent
- Claim: strong n -consistency means we can solve without backtracking!
- Why?
 - Choose any assignment to any variable
 - Choose a new variable
 - By 2-consistency, there is a choice consistent with the first
 - Choose a new variable
 - By 3-consistency, there is a choice consistent with the first 2
 - ...
- Lots of middle ground between arc consistency and n -consistency! (e.g. $k=3$, called path consistency)

Ordering

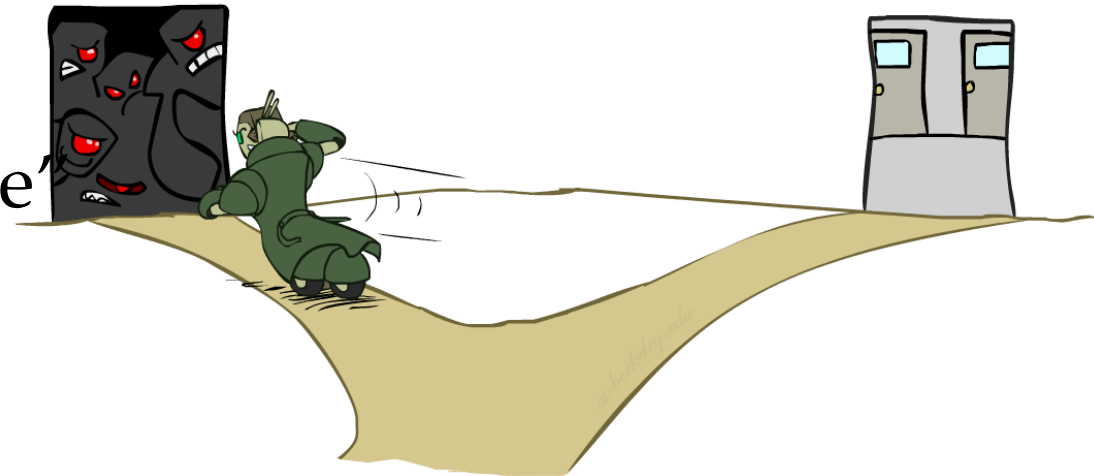


Ordering: Minimum Remaining Values

- Variable Ordering: Minimum remaining values (MRV):
 - Choose the variable with the fewest legal left values in its domain

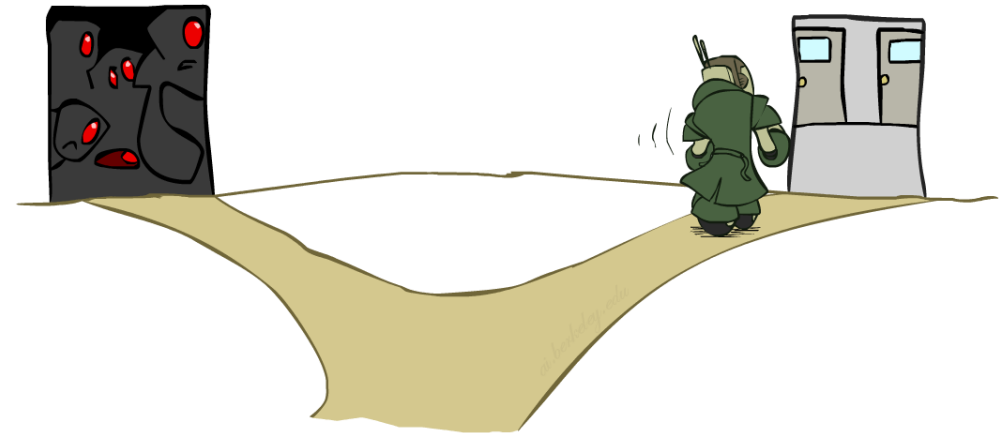
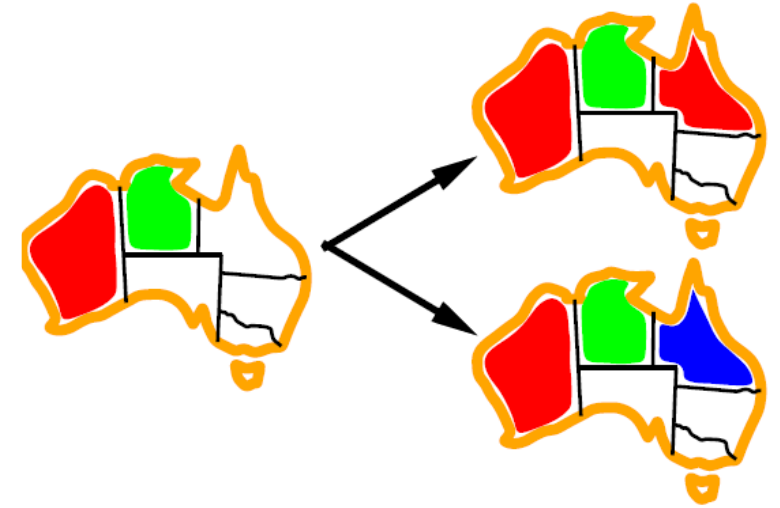


- Why min rather than max?
- Also called “most constrained variable”
- “Fail-fast” ordering



Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
 - Given a choice of variable, choose the *least constraining value*
 - I.e., the one that rules out the fewest values in the remaining variables
 - Note that it may take some computation to determine this! (E.g., rerunning filtering)
- Why least rather than most?
- Combining these ordering ideas makes 1000 queens feasible

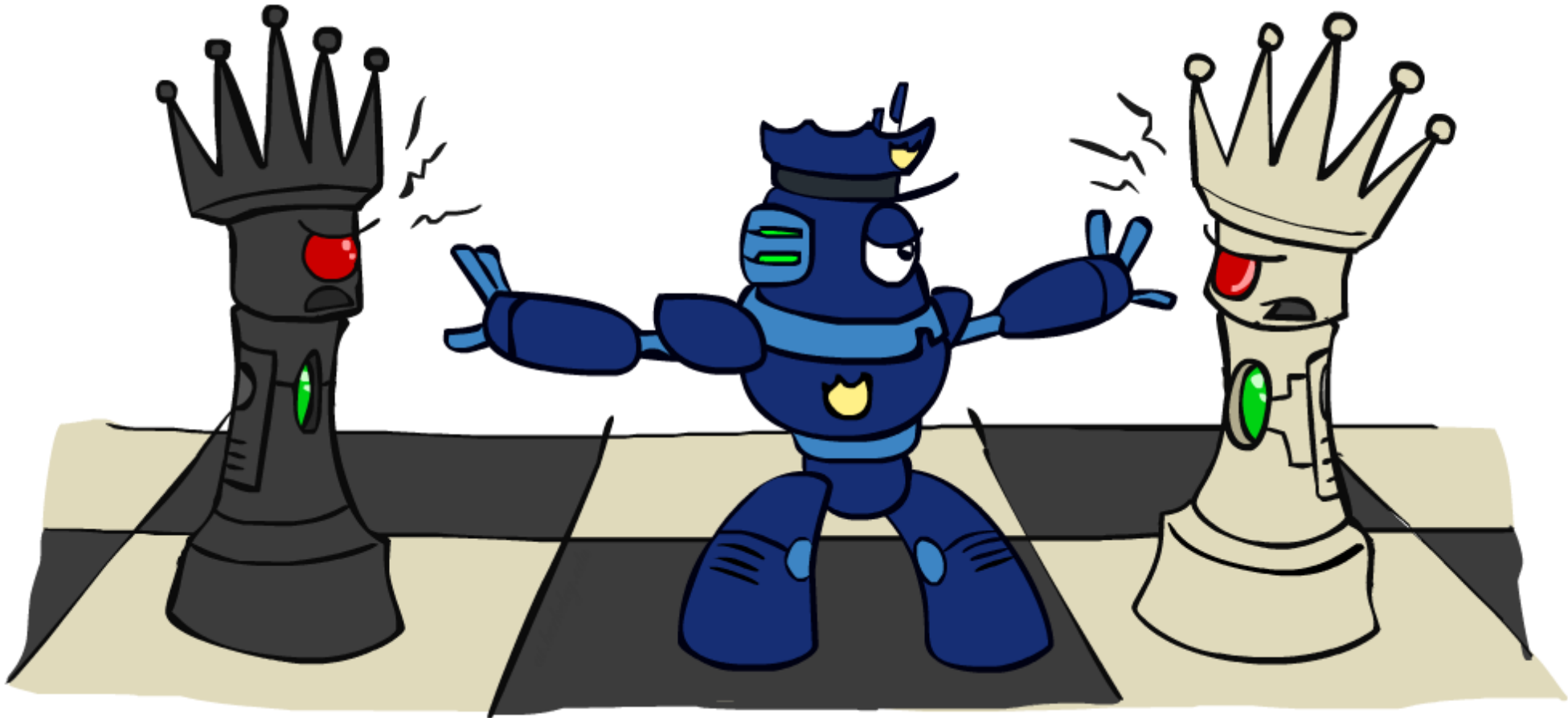


Demo: Coloring -- Backtracking + Forward Checking + Ordering

Summary

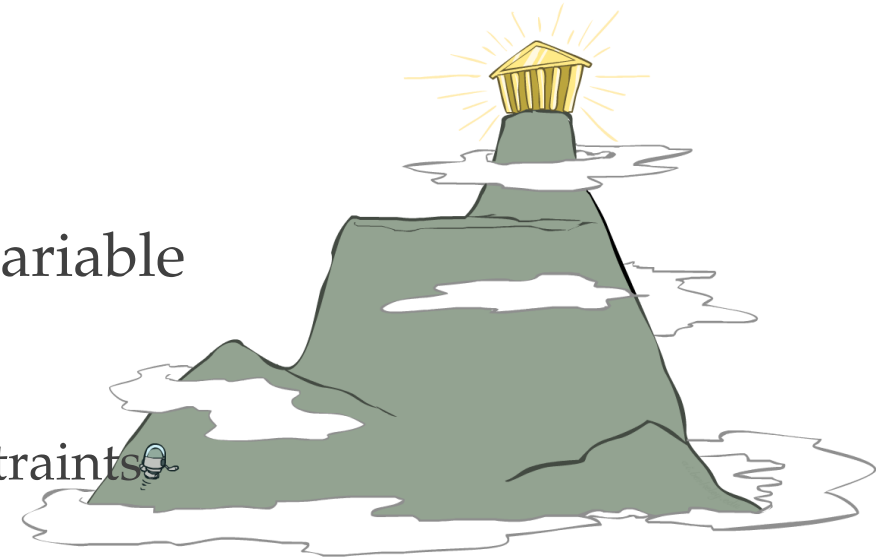
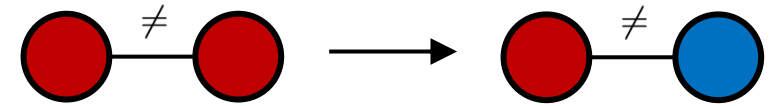
- Work with your rubber duck to write down:
 - How we order variables and why
 - How we order values and why

Iterative Improvement

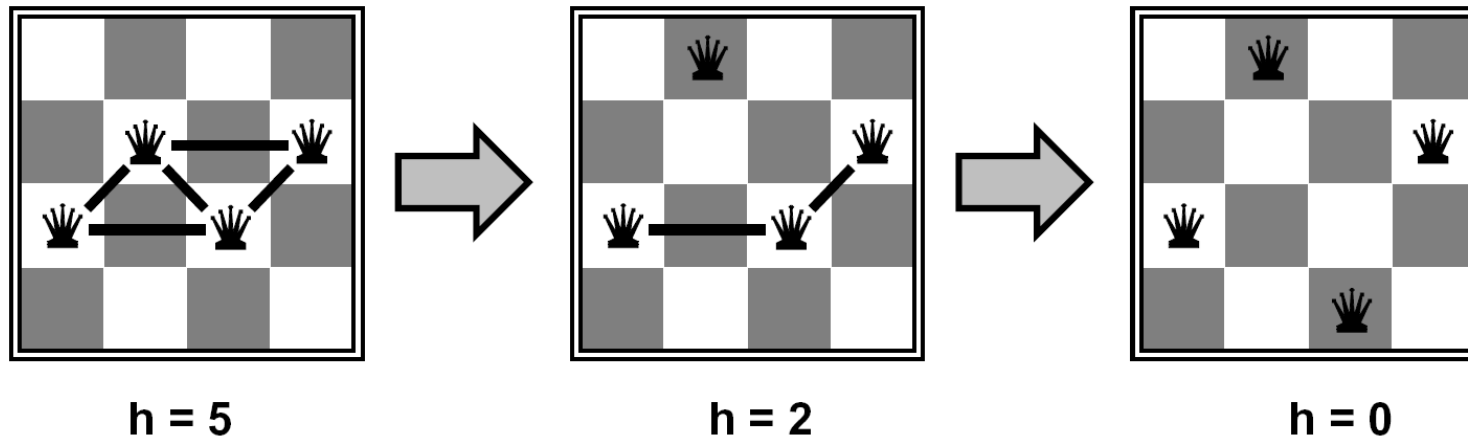


Iterative Algorithms for CSPs

- Local search methods typically work with “complete” states, i.e., all variables assigned
- To apply to CSPs:
 - Take an assignment with unsatisfied constraints
 - Operators *reassign* variable values
 - No fringe! Live on the edge.
- Algorithm: While not solved,
 - Variable selection: randomly select any conflicted variable
 - Value selection: min-conflicts heuristic:
 - Choose a value that violates the fewest constraints
 - I.e., hill climb with $h(x)$ = total number of violated constraints

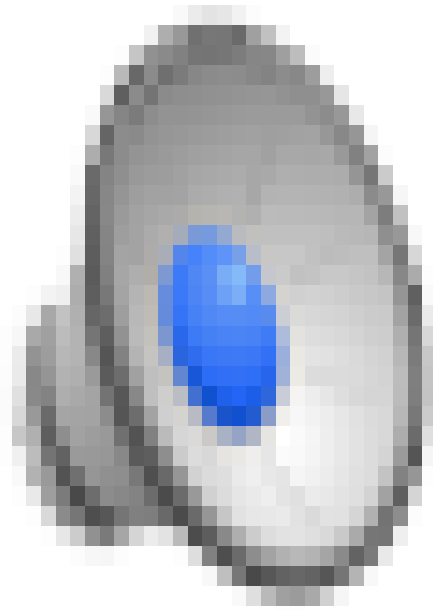


Example: 4-Queens

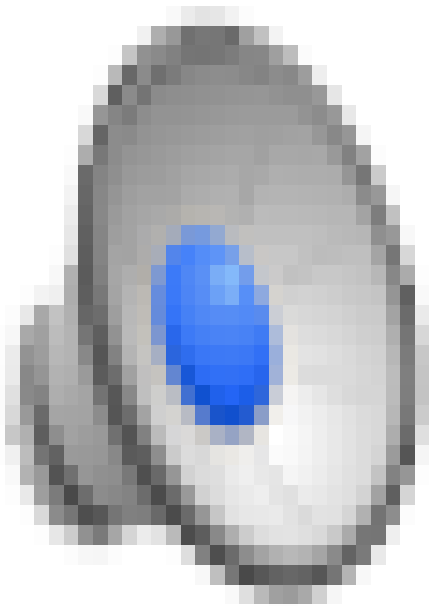


- States: 4 queens in 4 columns ($4^4 = 256$ states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: $c(n)$ = number of attacks

Video of Demo Iterative Improvement – n Queens



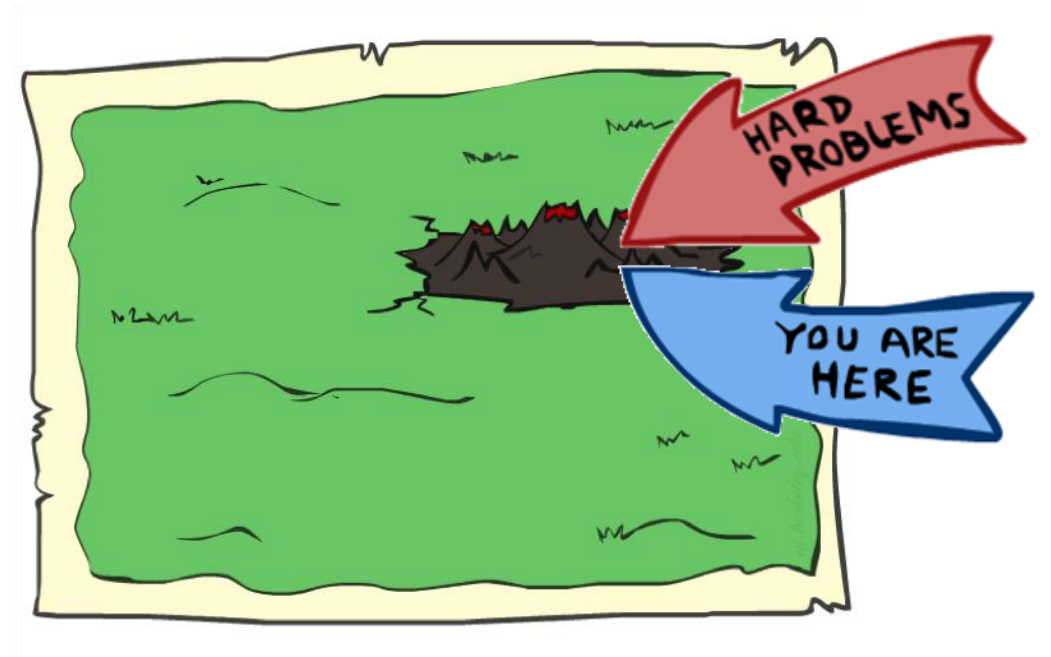
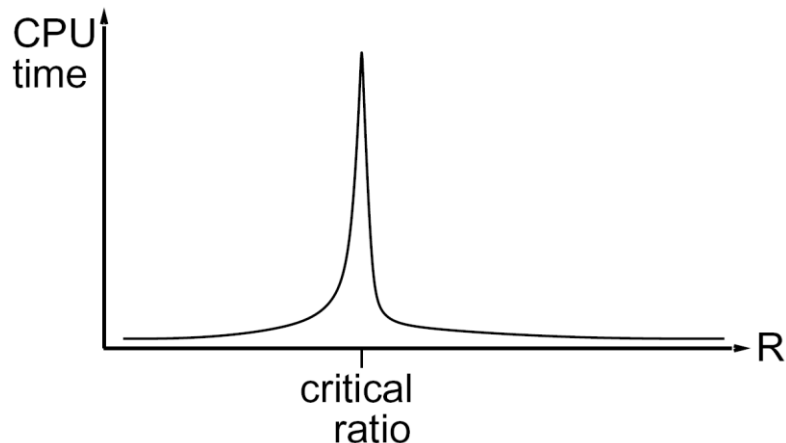
Video of Demo Iterative Improvement – Coloring



Performance of Min-Conflicts

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)!
- The same appears to be true for any randomly-generated CSP *except* in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$



Summary: CSPs

- CSPs are a special kind of search problem:
 - States are partial assignments
 - Goal test defined by constraints
- Basic solution: backtracking search
- Speed-ups:
 - Ordering
 - Filtering
 - Structure – turns out trees are easy:
- Iterative min-conflicts is often effective in practice

