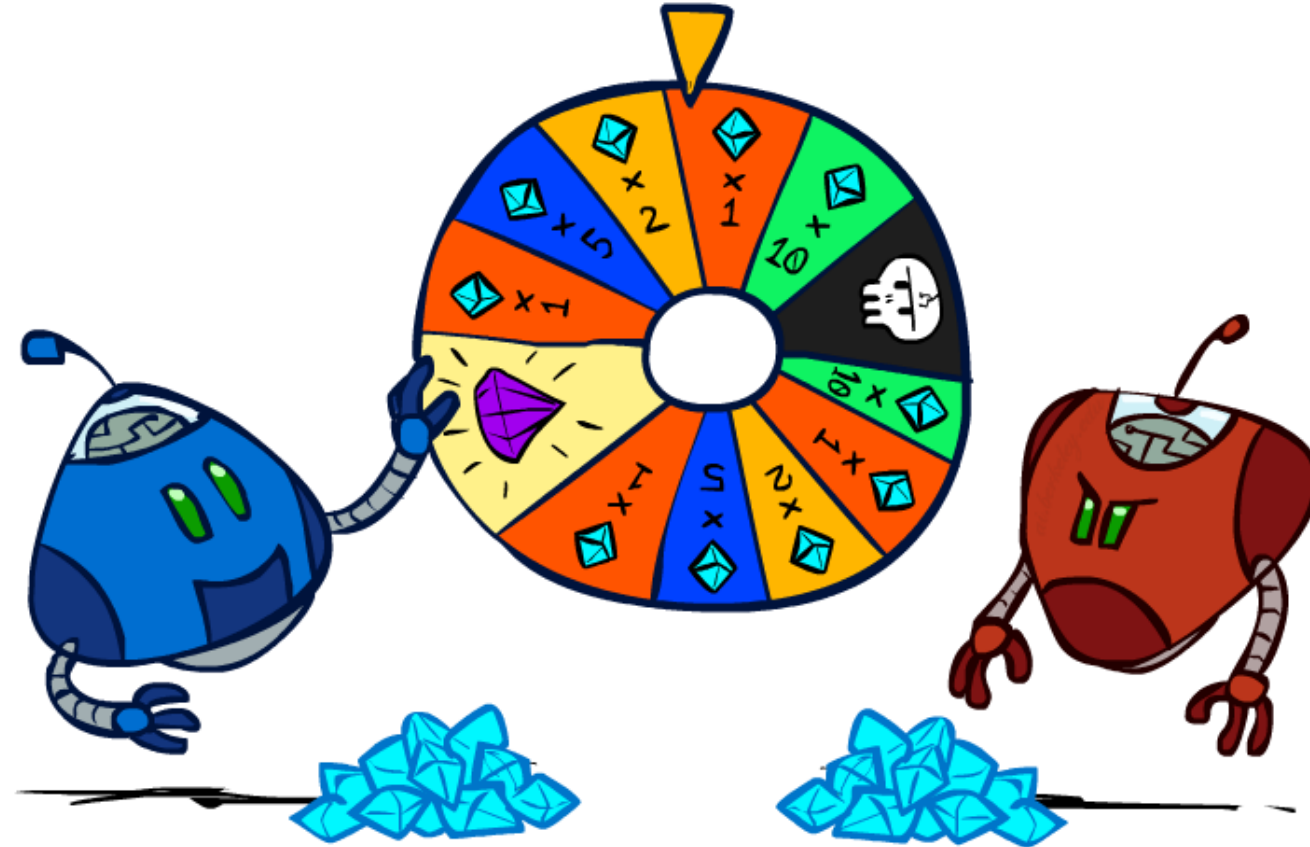


# Artificial Intelligence: Basics & Applications

## Search with Other Agents II (Uncertainty)



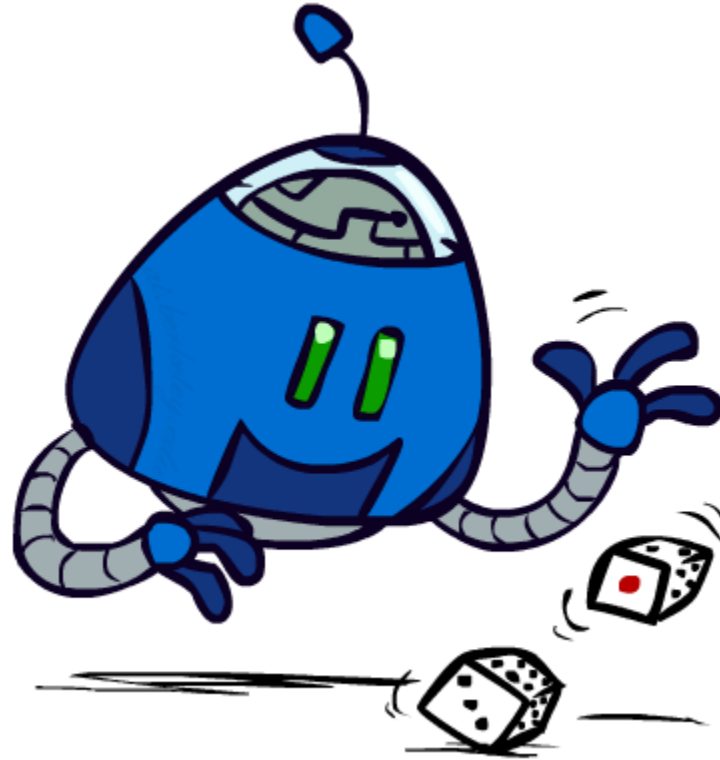
Instructor: *Mahdi Javanmardi*

Amirkabir University of Technology

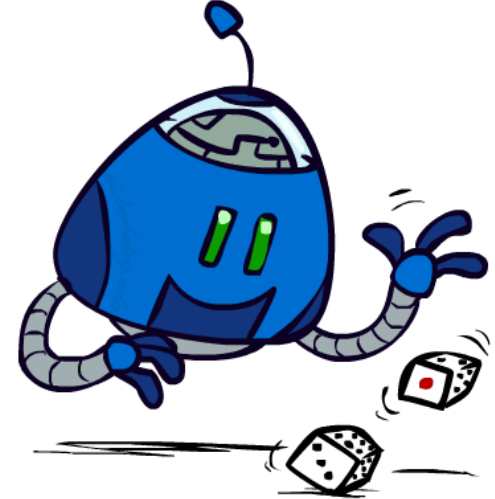
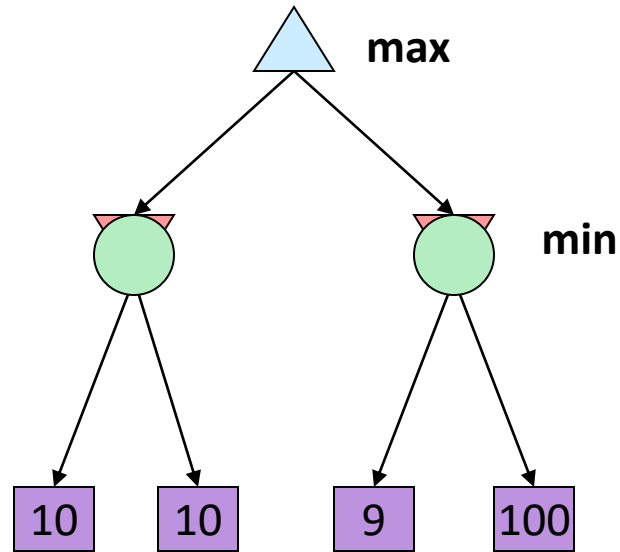
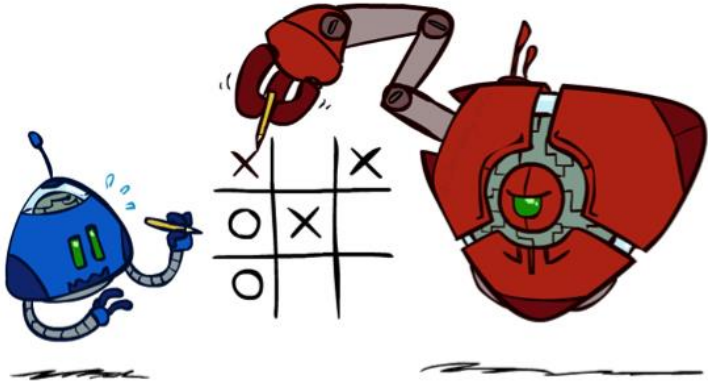
[slides adapted with permission from Dan Klein, Pieter Abbeel, Anca Dragan @ai.berkeley.edu]

# Uncertain Outcomes

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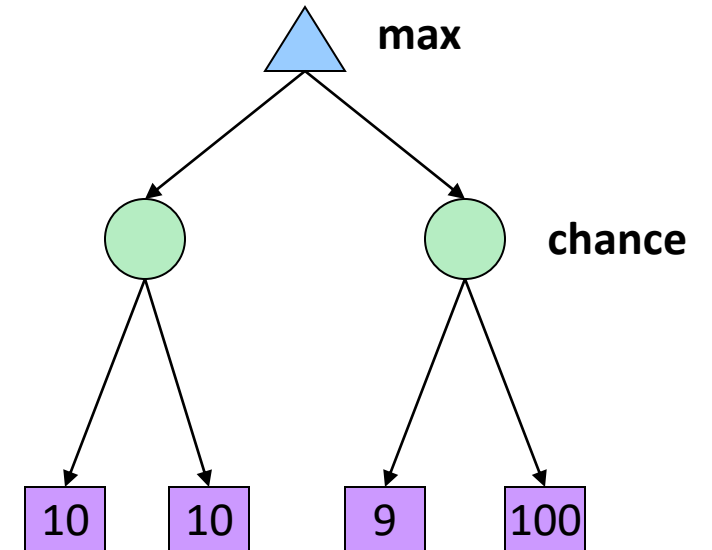
# Worst-Case vs. Average Case



Idea: Uncertain outcomes controlled by chance, not an adversary!

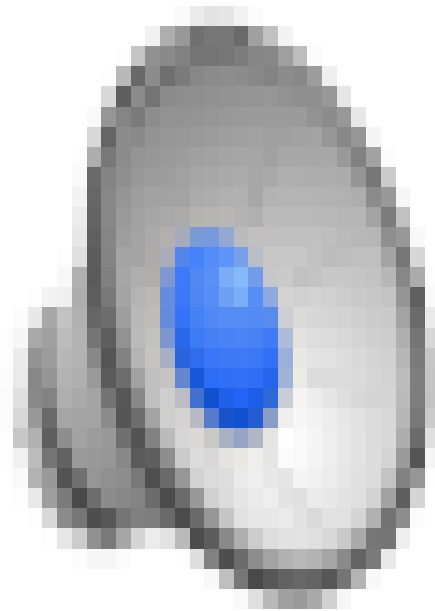
# Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes
- **Expectimax search**: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their **expected utilities**
  - I.e. take weighted average (expectation) of children
- Later, we'll learn how to formalize the underlying uncertain-result problems as **Markov Decision Processes**



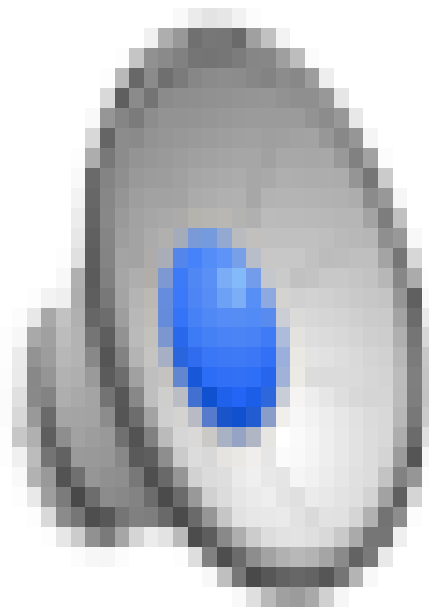
# Video of Demo Minimax vs Expectimax (Min)

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# Video of Demo Minimax vs Expectimax (Exp)

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# Expectimax Pseudocode

```
def value(state):
```

if the state is a terminal state: return the state's utility

if the next agent is MAX: return max-value(state)

if the next agent is EXP: return exp-value(state)

```
def max-value(state):
```

initialize  $v = -\infty$

for each successor of state:

$v = \max(v, \text{value}(\text{successor}))$

return  $v$

```
def exp-value(state):
```

initialize  $v = 0$

for each successor of state:

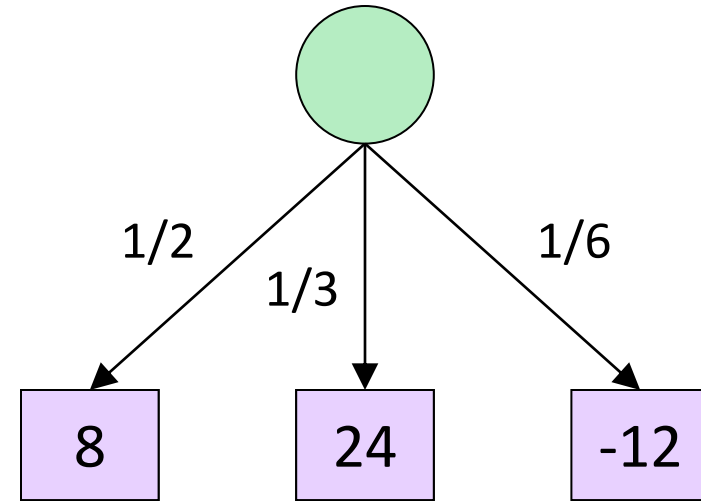
$p = \text{probability}(\text{successor})$

$v += p * \text{value}(\text{successor})$

return  $v$

# Expectimax Pseudocode

```
def exp-value(state):  
    initialize v = 0  
    for each successor of state:  
        p = probability(successor)  
        v += p * value(successor)  
    return v
```

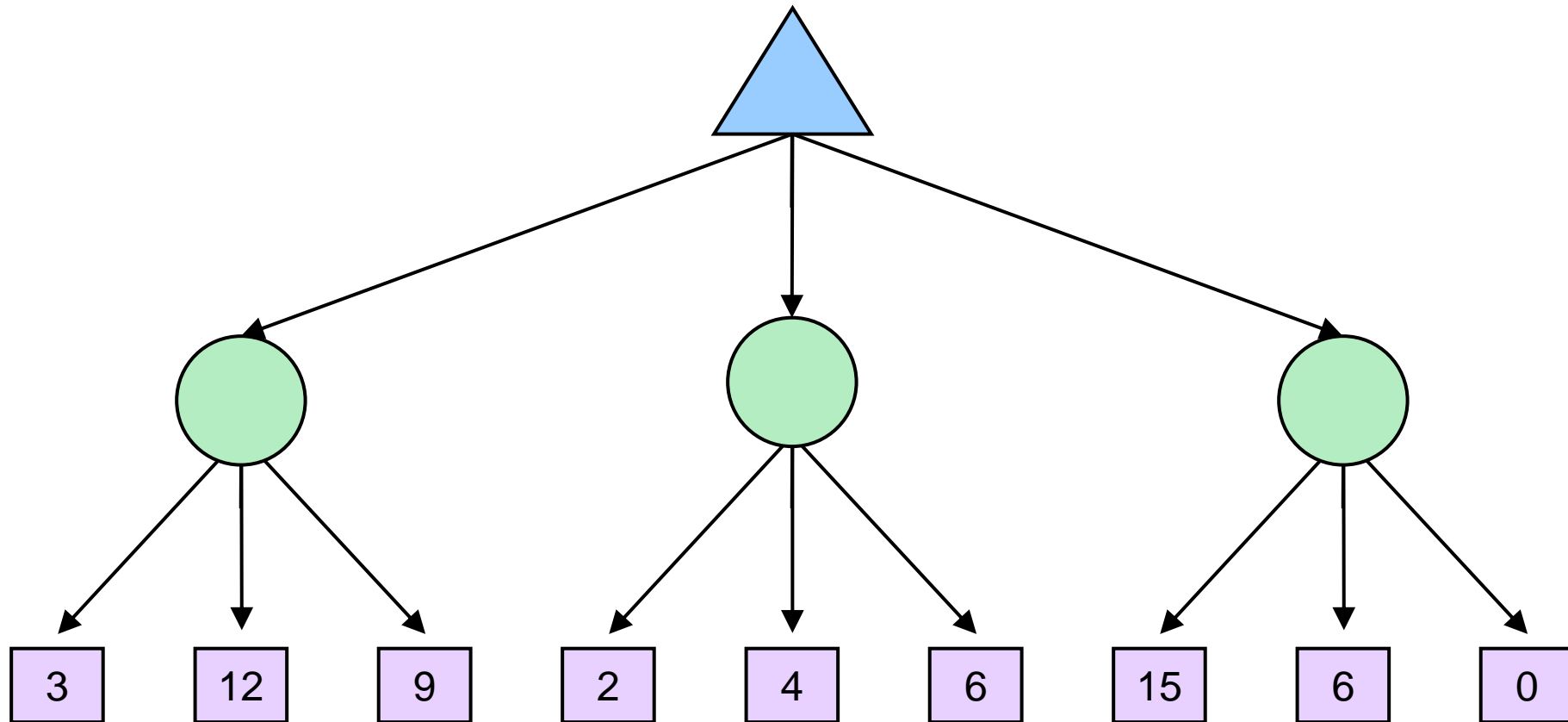


$$v = (1/2) (8) + (1/3) (24) + (1/6) (-12) = 10$$

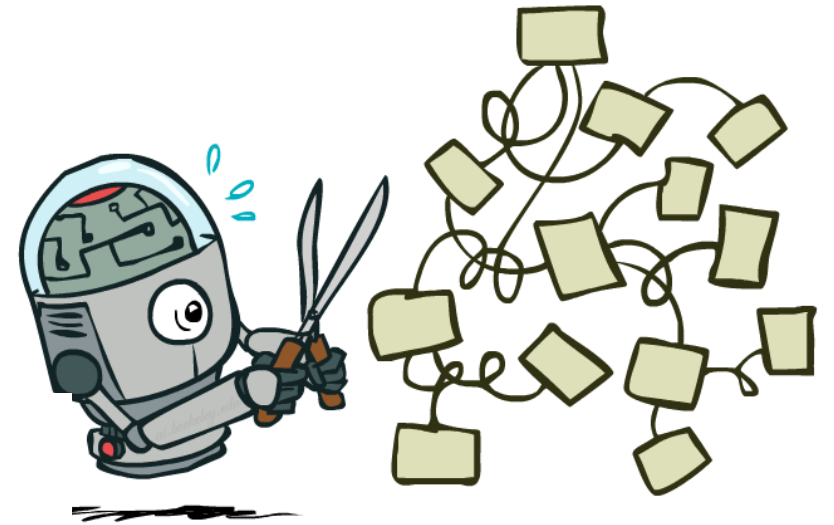
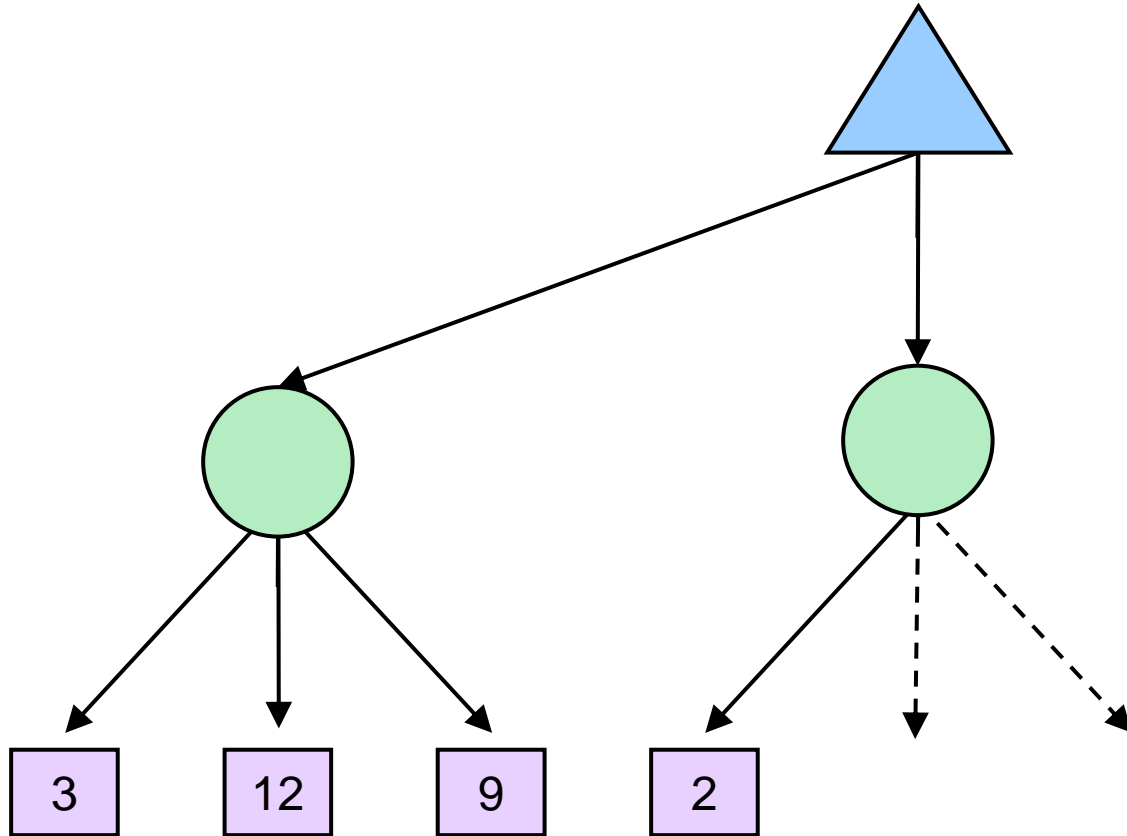


# Expectimax Example

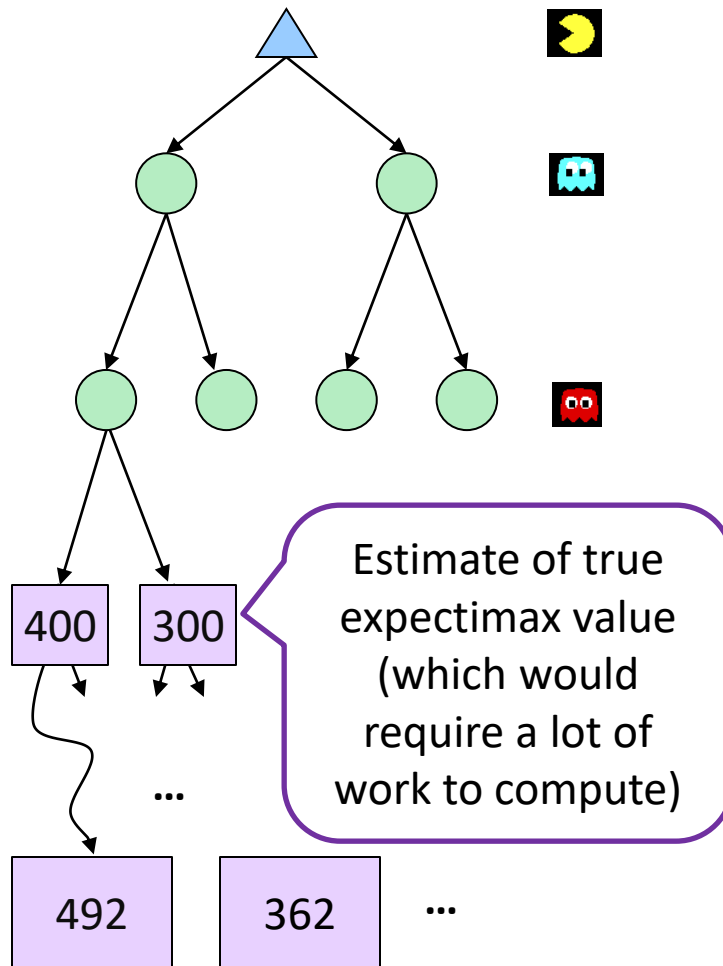
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# Expectimax Pruning?

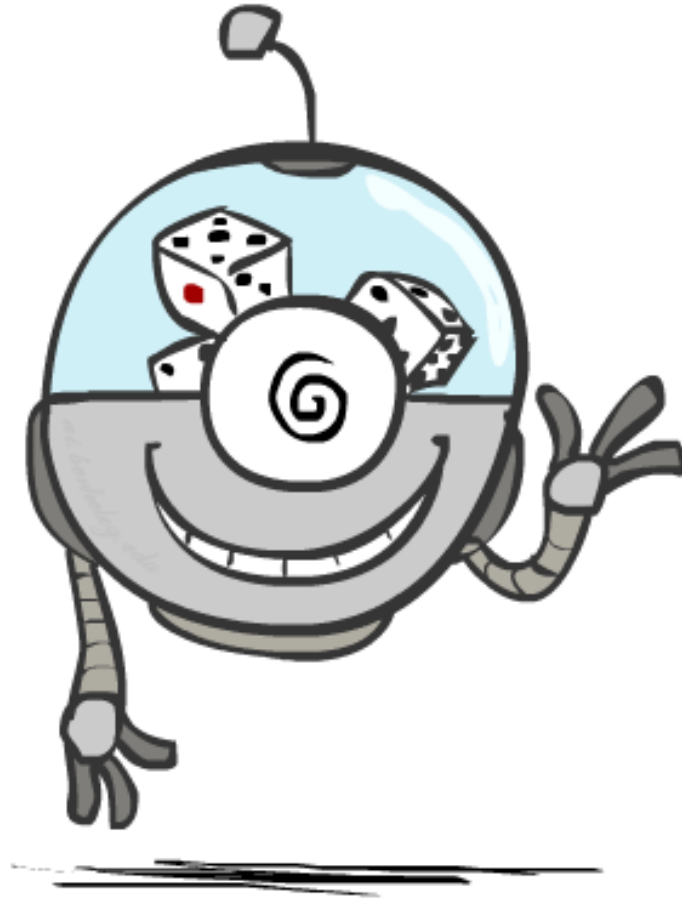


# Depth-Limited Expectimax



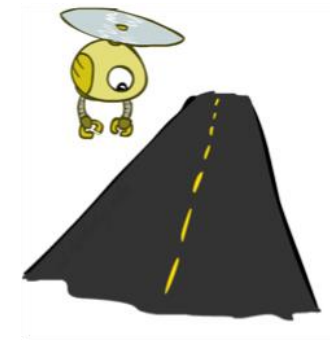
# Probabilities

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# Reminder: Probabilities

- A **random variable** represents an event whose outcome is unknown
- A **probability distribution** is an assignment of weights to outcomes
- Example: Traffic on freeway
  - Random variable:  $T$  = whether there's traffic
  - Outcomes:  $T$  in {none, light, heavy}
  - Distribution:  $P(T=\text{none}) = 0.25$ ,  $P(T=\text{light}) = 0.50$ ,  $P(T=\text{heavy}) = 0.25$
- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one
- As we get more evidence, probabilities may change:
  - $P(T=\text{heavy}) = 0.25$ ,  $P(T=\text{heavy} \mid \text{Hour}=8\text{am}) = 0.60$
  - We'll talk about methods for reasoning and updating probabilities later



0.25



0.50



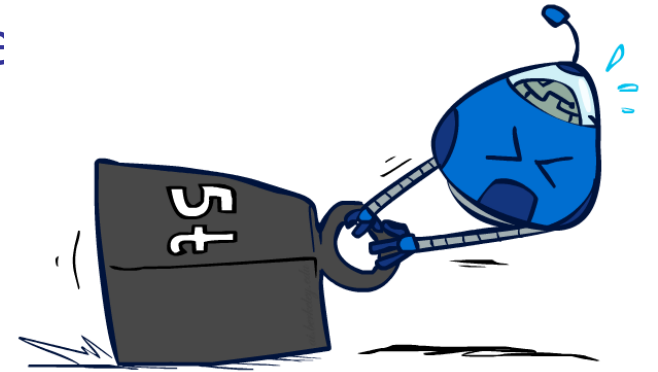
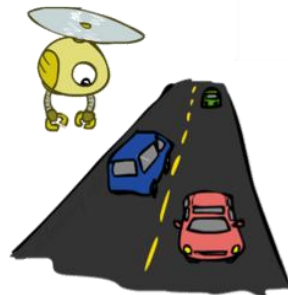
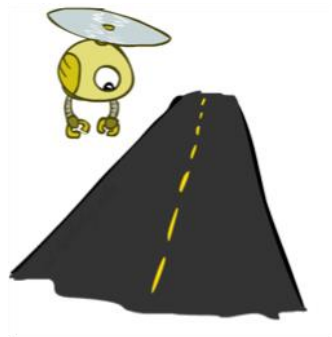
0.25

# Reminder: Expectations

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes
- Example: How long to get to the airport?

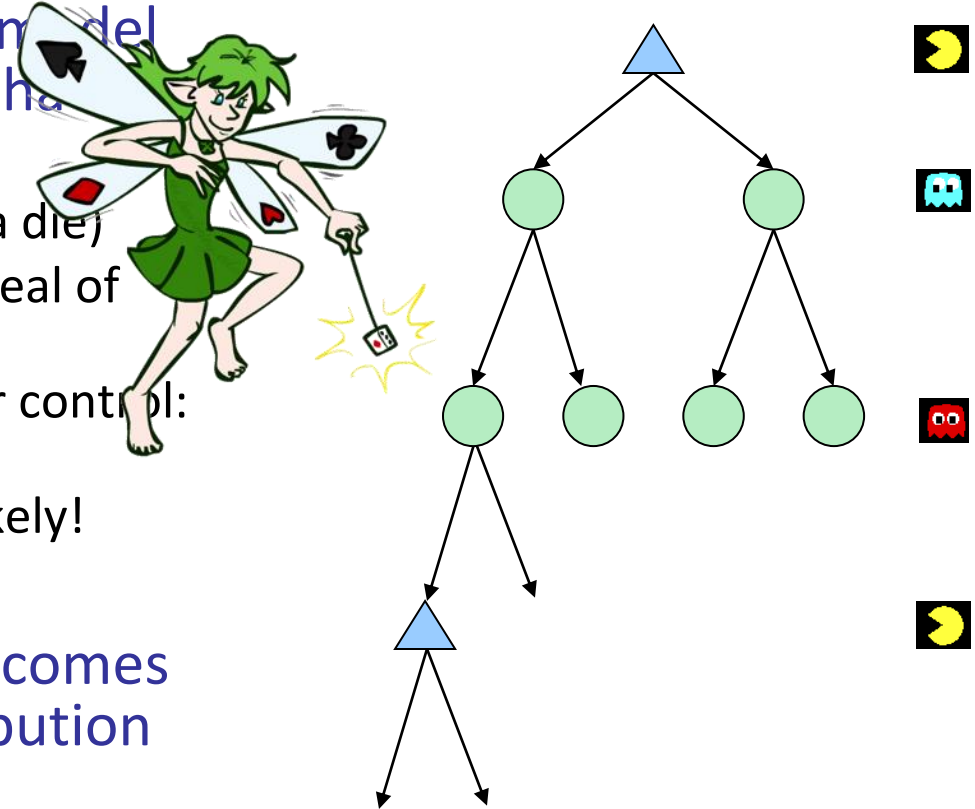
Time:	20 min		30 min		60 min
	x	+	x	+	x
Probability:	0.25		0.50		0.25

→ 35 min



# What Probabilities to Use?

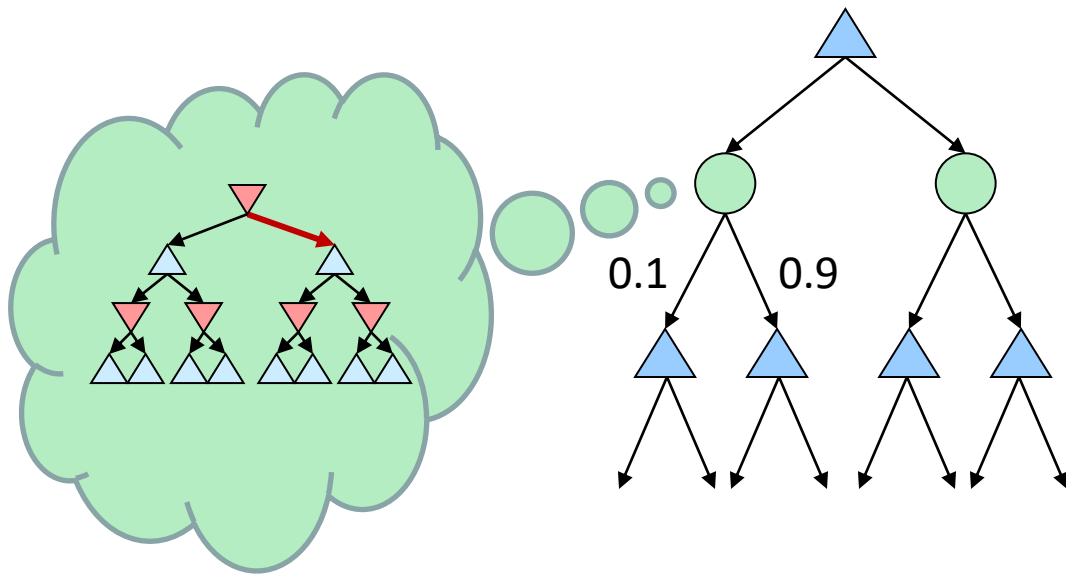
- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!
- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes



*Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!*

# Quiz: Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?



- Answer: Expectimax!
  - To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you...
  - ... except for minimax, which has the nice property that it all collapses into one game tree



# Modeling Assumptions

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# The Dangers of Optimism and Pessimism

## Dangerous Optimism

Assuming chance when the world is adversarial

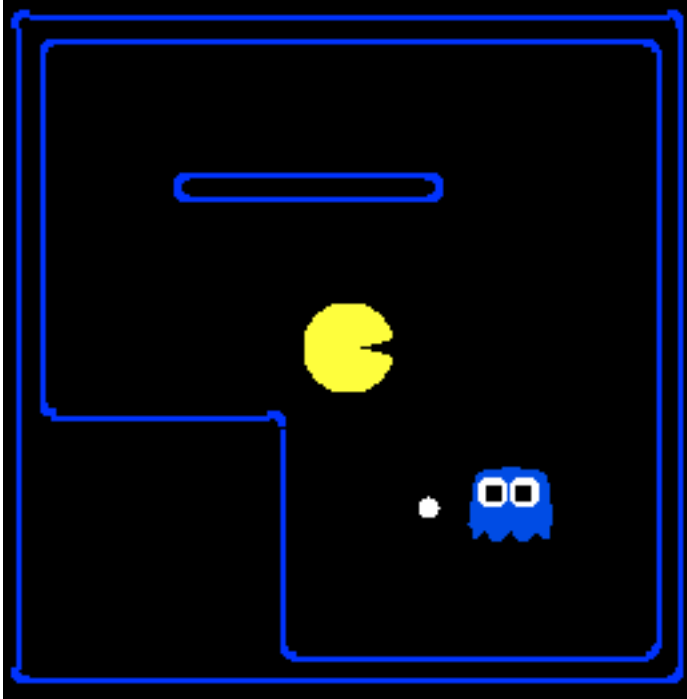


## Dangerous Pessimism

Assuming the worst case when it's not likely



# Assumptions vs. Reality



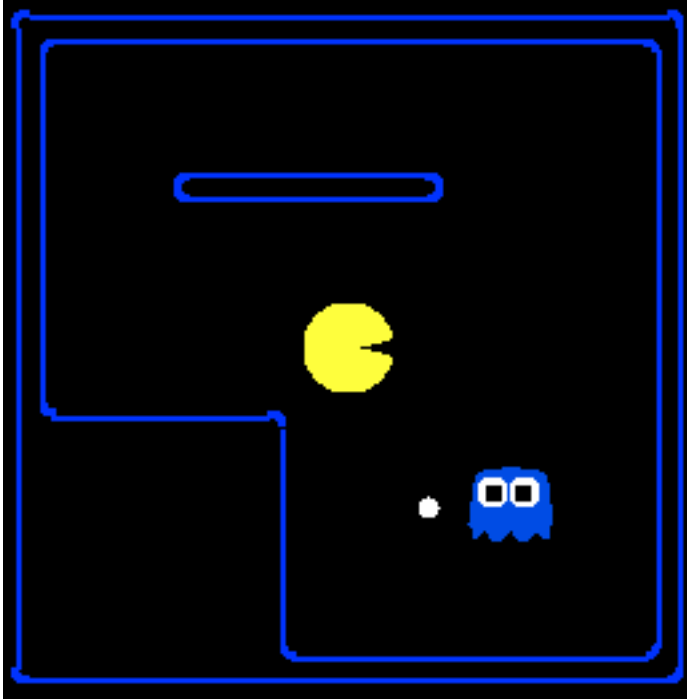
	Adversarial Ghost	Random Ghost
Minimax Pacman		
Expectimax Pacman		

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble  
Ghost used depth 2 search with an eval function that seeks Pacman

[Demos: world assumptions (L7D3,4,5,6)]

# Assumptions vs. Reality



	Adversarial Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 483	Won 5/5 Avg. Score: 493
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503

Results from playing 5 games

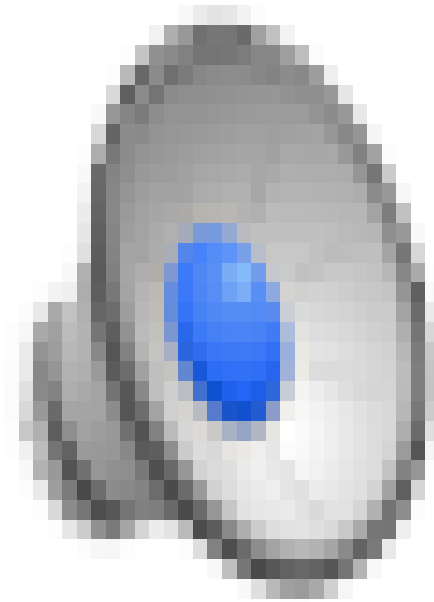
Pacman used depth 4 search with an eval function that avoids trouble  
Ghost used depth 2 search with an eval function that seeks Pacman

[Demos: world assumptions (L7D3,4,5,6)]

# Video of Demo World Assumptions

## Random Ghost – Expectimax Pacman

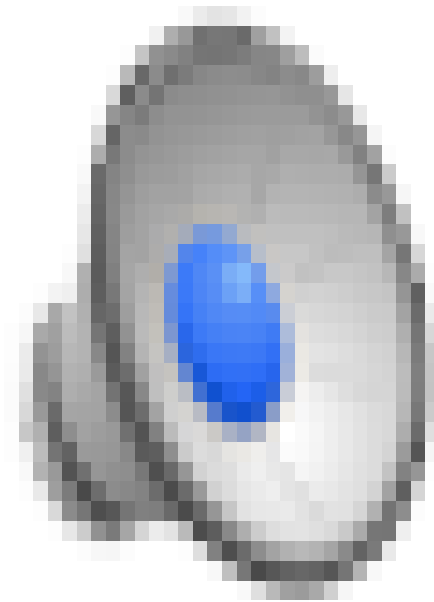
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# Video of Demo World Assumptions

## Adversarial Ghost – Minimax Pacman

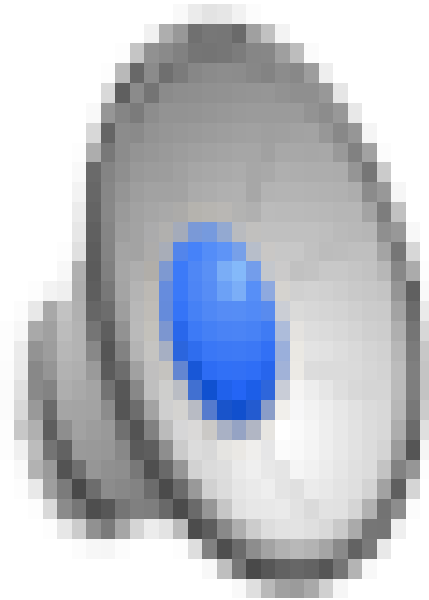
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# Video of Demo World Assumptions

## Adversarial Ghost – Expectimax Pacman

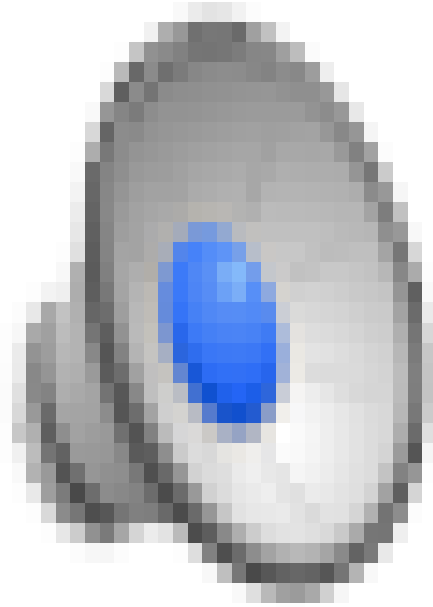
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# Video of Demo World Assumptions

## Random Ghost – Minimax Pacman

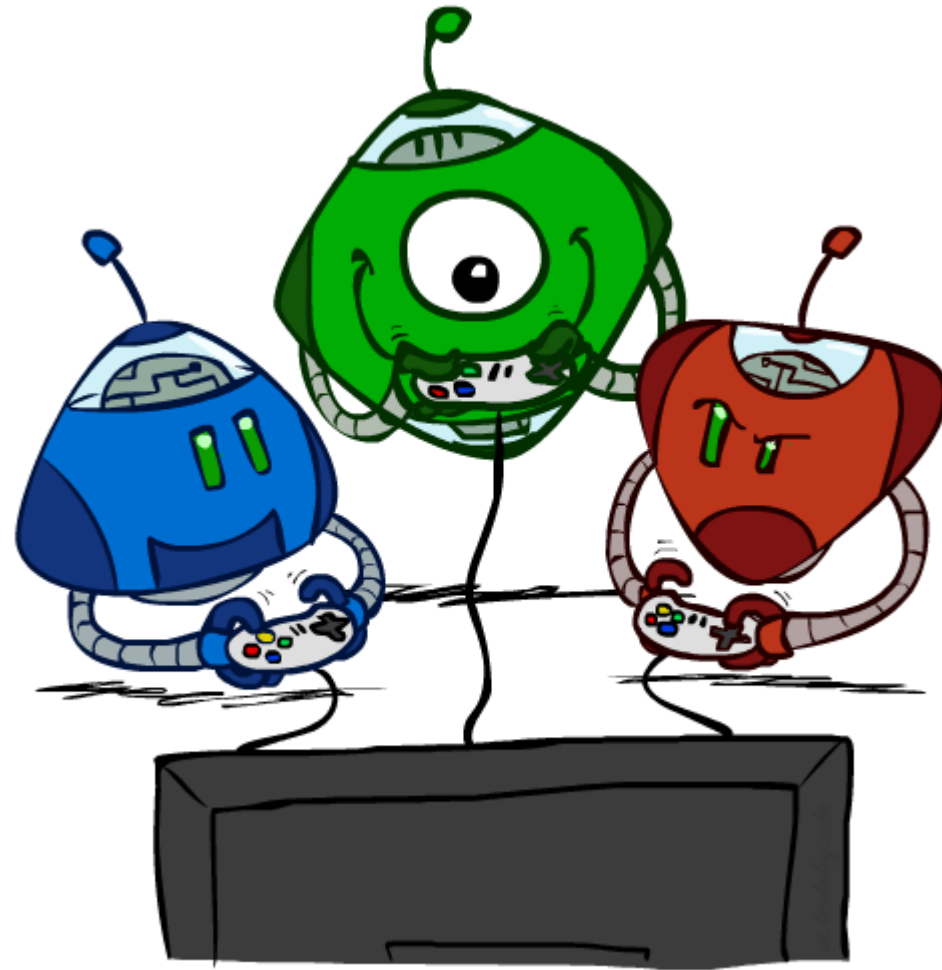
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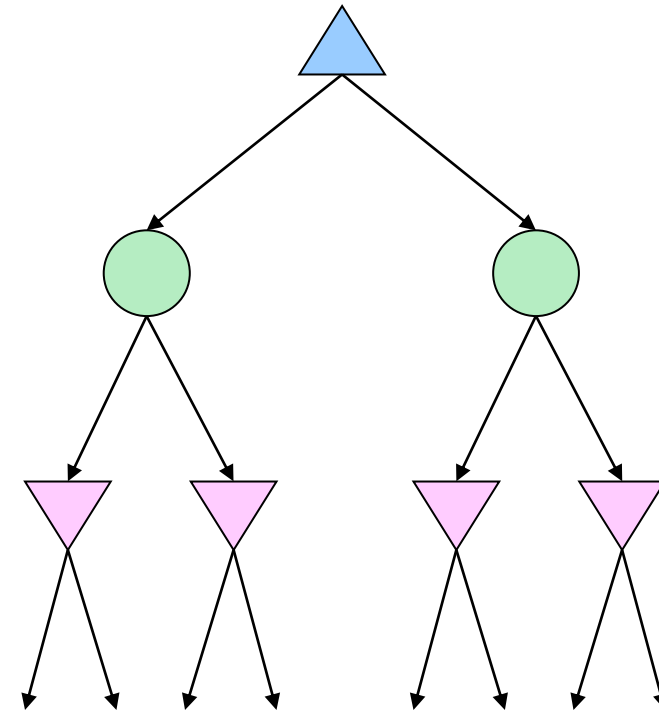
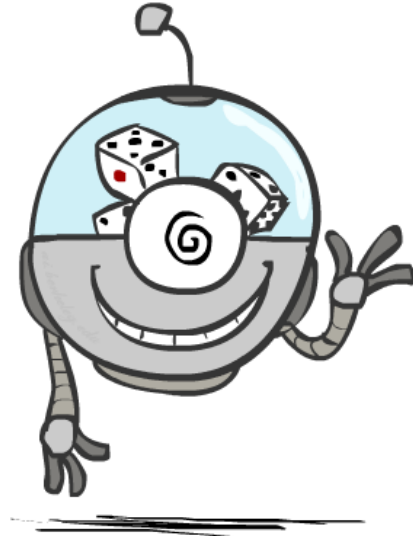
# Other Game Types

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# Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children



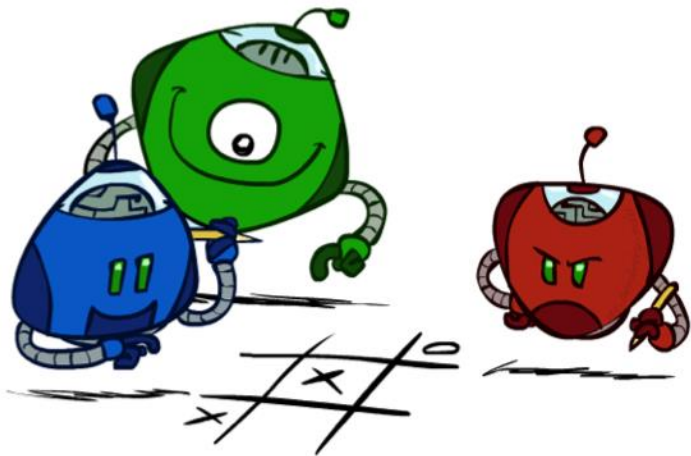
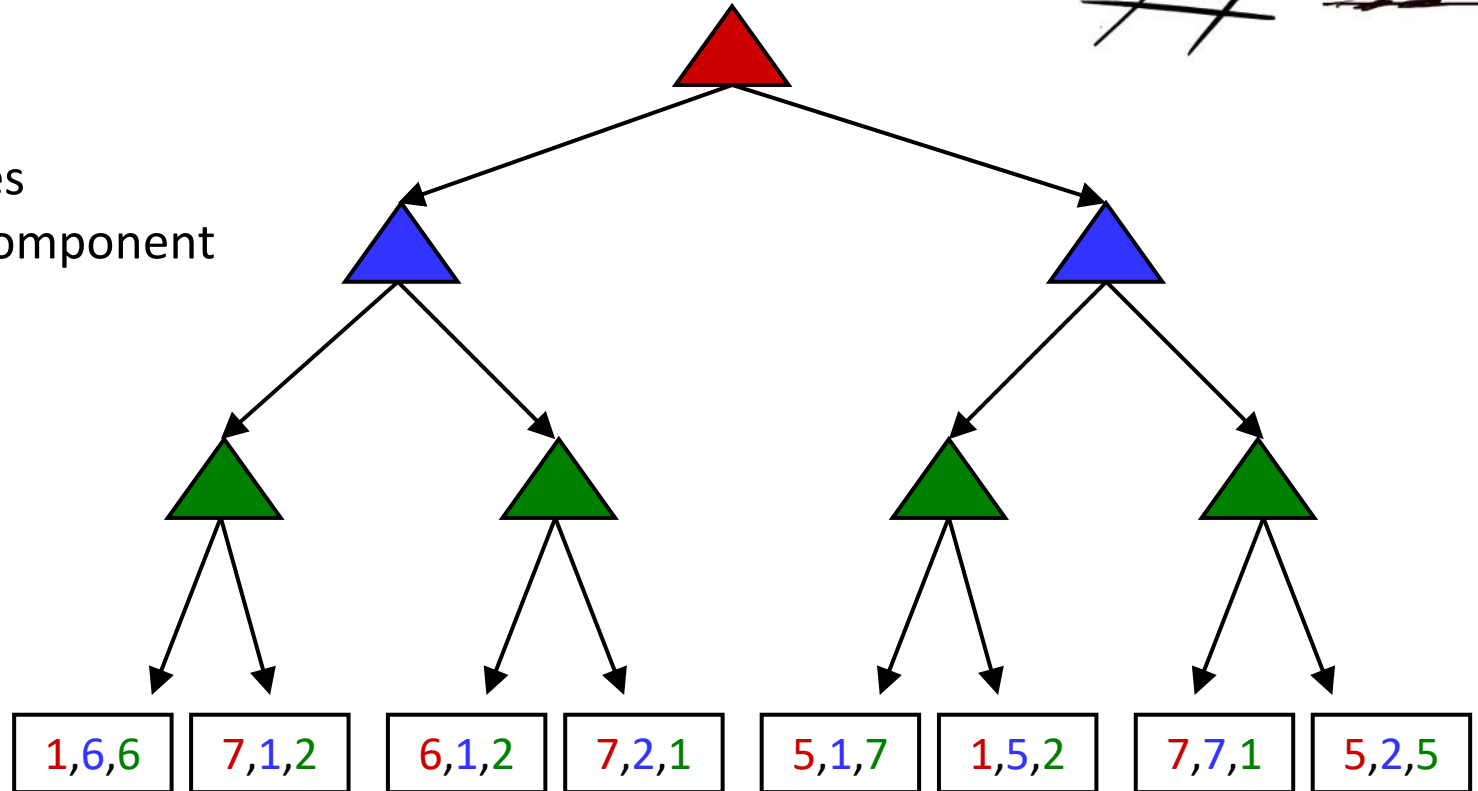
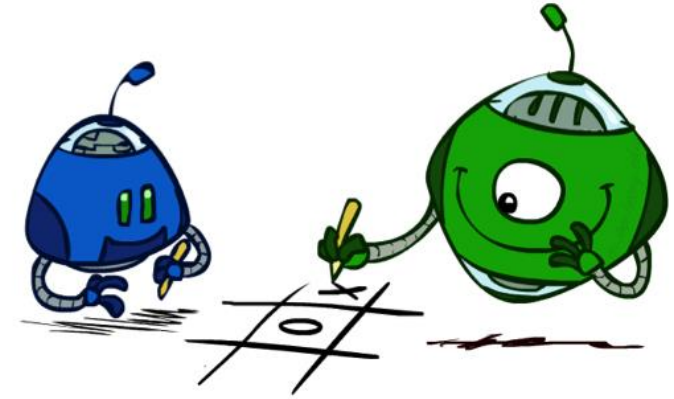
# Example: Backgammon

- Dice rolls increase  $b$ : 21 possible rolls with 2 dice
  - Backgammon  $\approx 20$  legal moves
  - Depth 2 =  $20 \times (21 \times 20)^3 = 1.2 \times 10^9$
- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier...
- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play
- 1<sup>st</sup> AI world champion in any game!

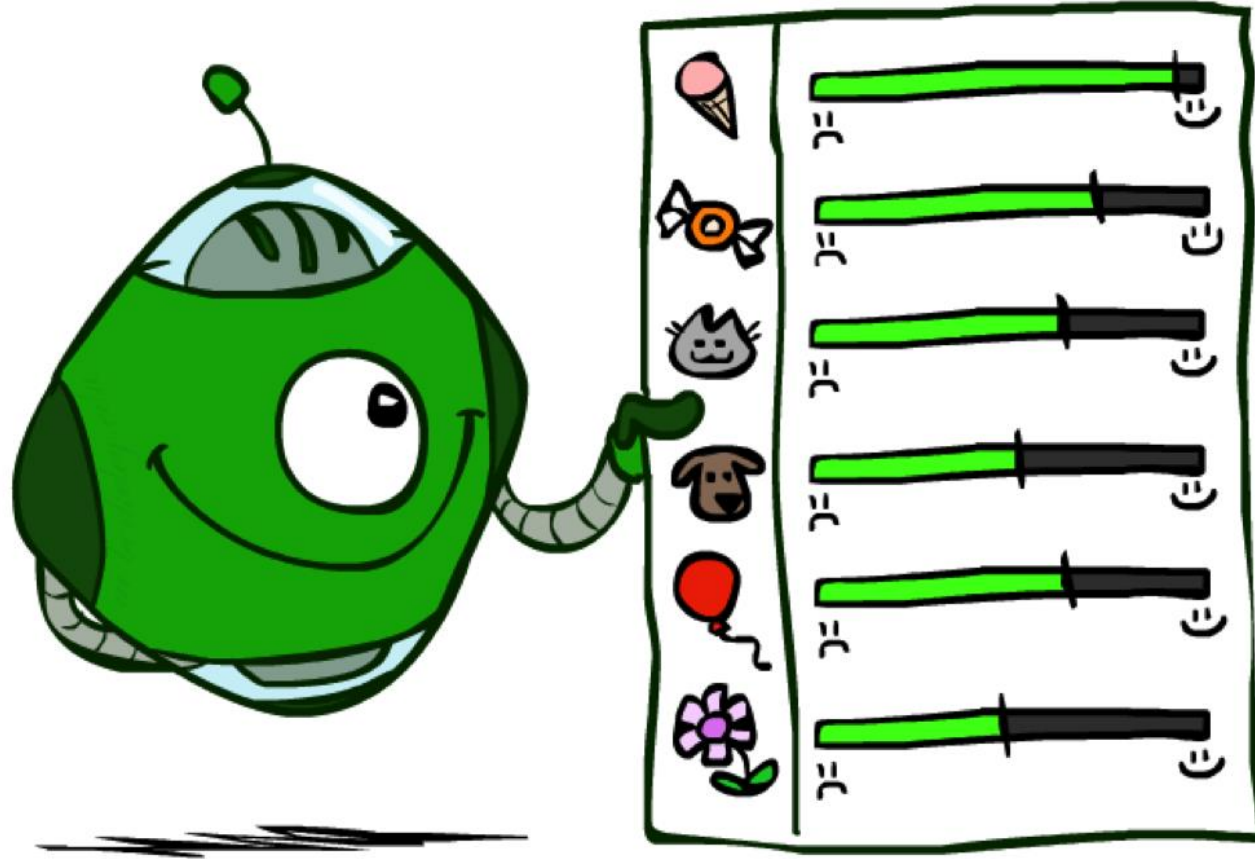


# Multi-Agent Utilities

- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically...



# Utilities

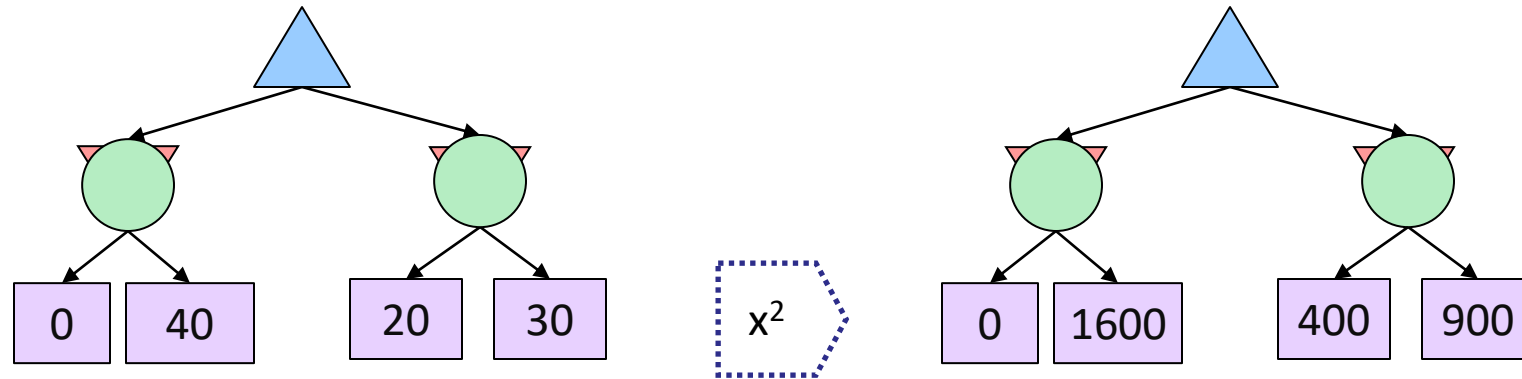


# Maximum Expected Utility

- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility:
  - A rational agent should choose the action that **maximizes its expected utility, given its knowledge**
- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - How do we know that averaging even makes sense?
  - What if our behavior (preferences) can't be described by utilities?



# What Utilities to Use?



- For worst-case minimax reasoning, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this **insensitivity to monotonic transformations**
- For average-case expectimax reasoning, we need *magnitudes* to be meaningful

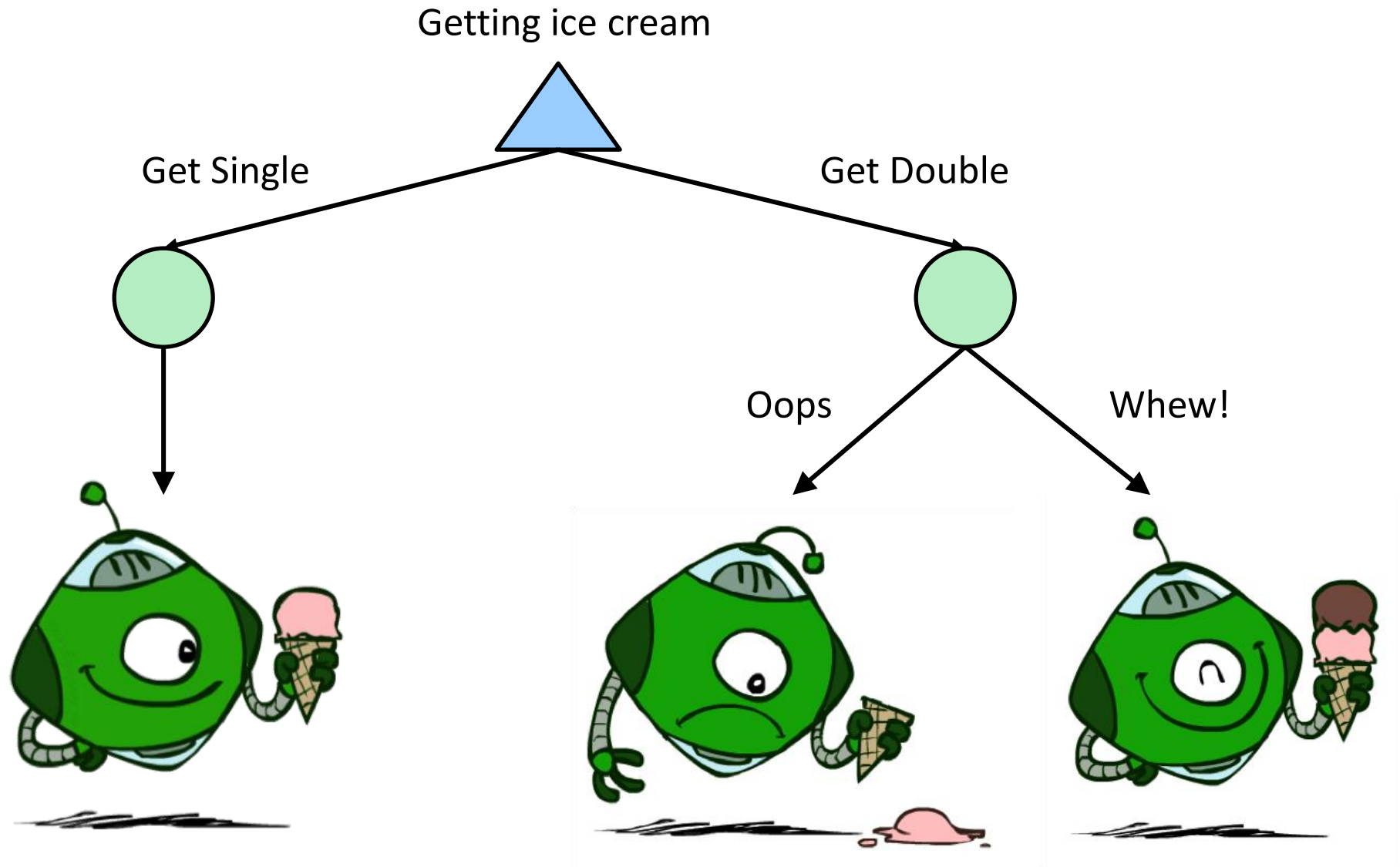
# Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function
- We hard-wire utilities and let behaviors emerge
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?





# Utilities: Uncertain Outcomes



# Preferences

- An agent must have preferences among:

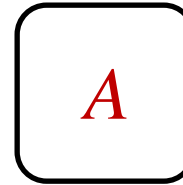
- Prizes:  $A$ ,  $B$ , etc.
- Lotteries: situations with uncertain prizes

$$L = [p, A; (1 - p), B]$$

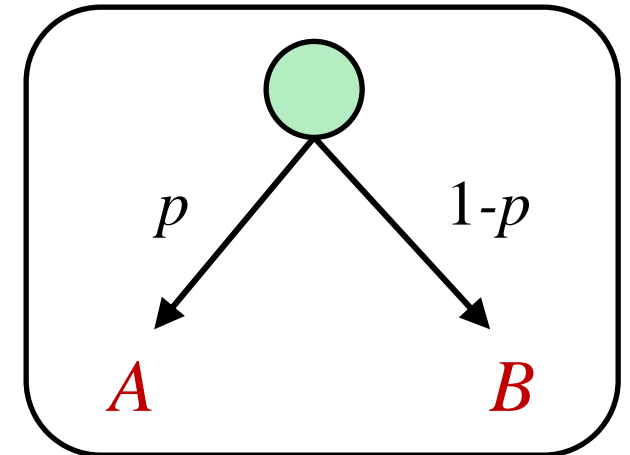
- Notation:

- Preference:  $A \succ B$
- Indifference:  $A \sim B$

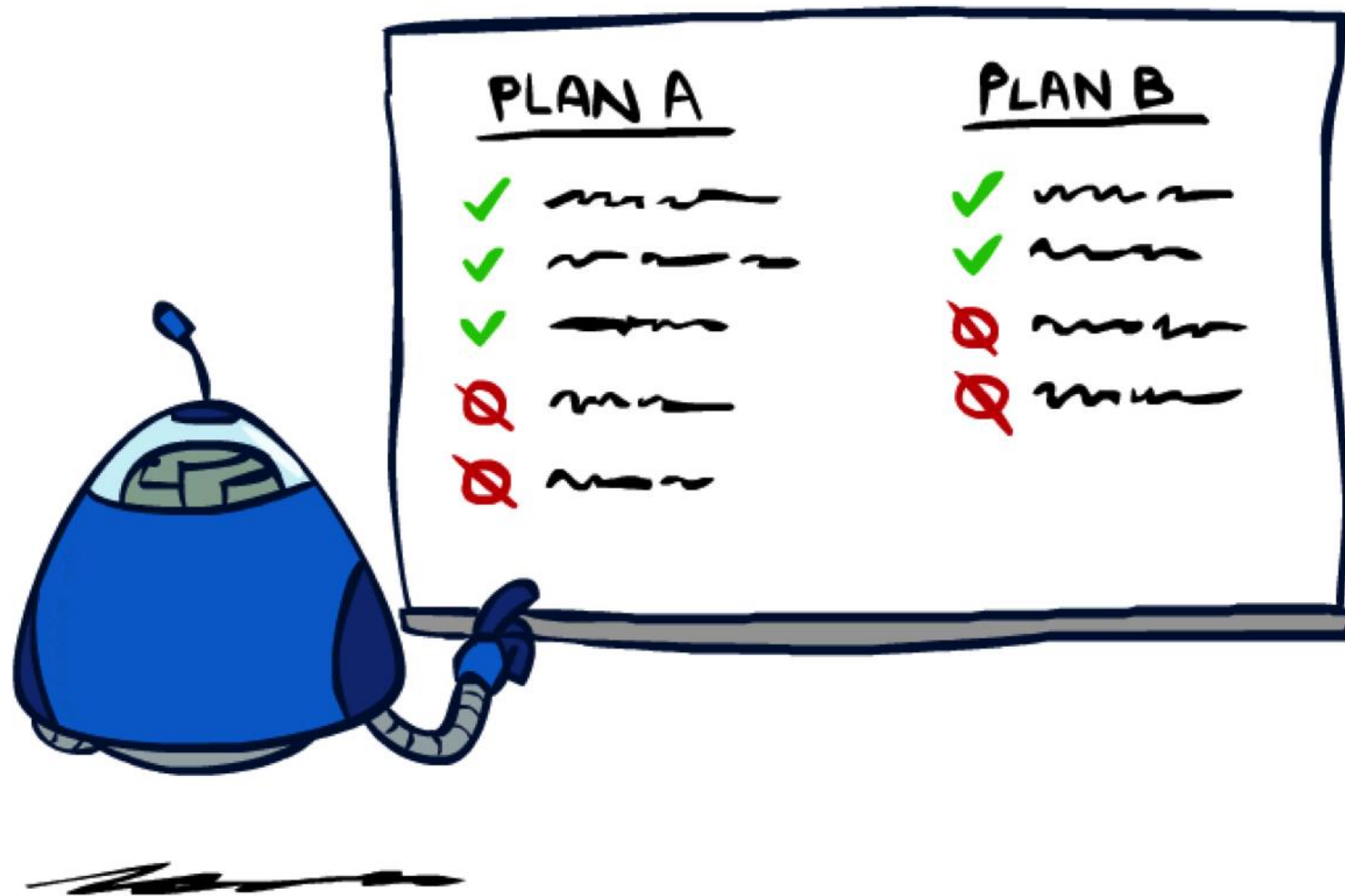
A Prize



A Lottery



# Rationality

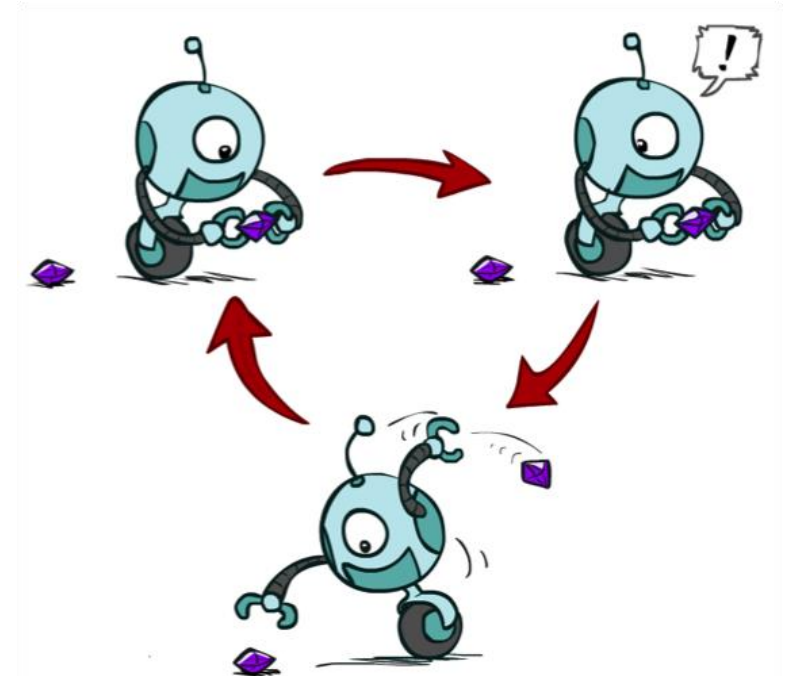


# Rational Preferences

- We want some constraints on preferences before we call them rational, such as:

Axiom of Transitivity:  $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

- For example: an agent with **intransitive preferences** can be induced to give away all of its money
  - If  $B \succ C$ , then an agent with  $C$  would pay (say) 1 cent to get  $B$
  - If  $A \succ B$ , then an agent with  $B$  would pay (say) 1 cent to get  $A$
  - If  $C \succ A$ , then an agent with  $A$  would pay (say) 1 cent to get  $C$



# Rational Preferences

## The Axioms of Rationality

### Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

### Transitivity

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

### Continuity

$$A \succ B \succ C \Rightarrow \exists p [p, A; 1 - p, C] \sim B$$

### Substitutability

$$A \sim B \Rightarrow [p, A; 1 - p, C] \sim [p, B; 1 - p, C]$$

### Monotonicity

$$A \succ B \Rightarrow \\ (p \geq q \Leftrightarrow [p, A; 1 - p, B] \succeq [q, A; 1 - q, B])$$



Theorem: Rational preferences imply behavior describable as maximization of expected utility

# Next Time: MDPs!

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