

# Introduction to **Information Retrieval**

CS276: Information Retrieval and Web Search

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Lecture 12: Clustering

# Today's Topic: Clustering

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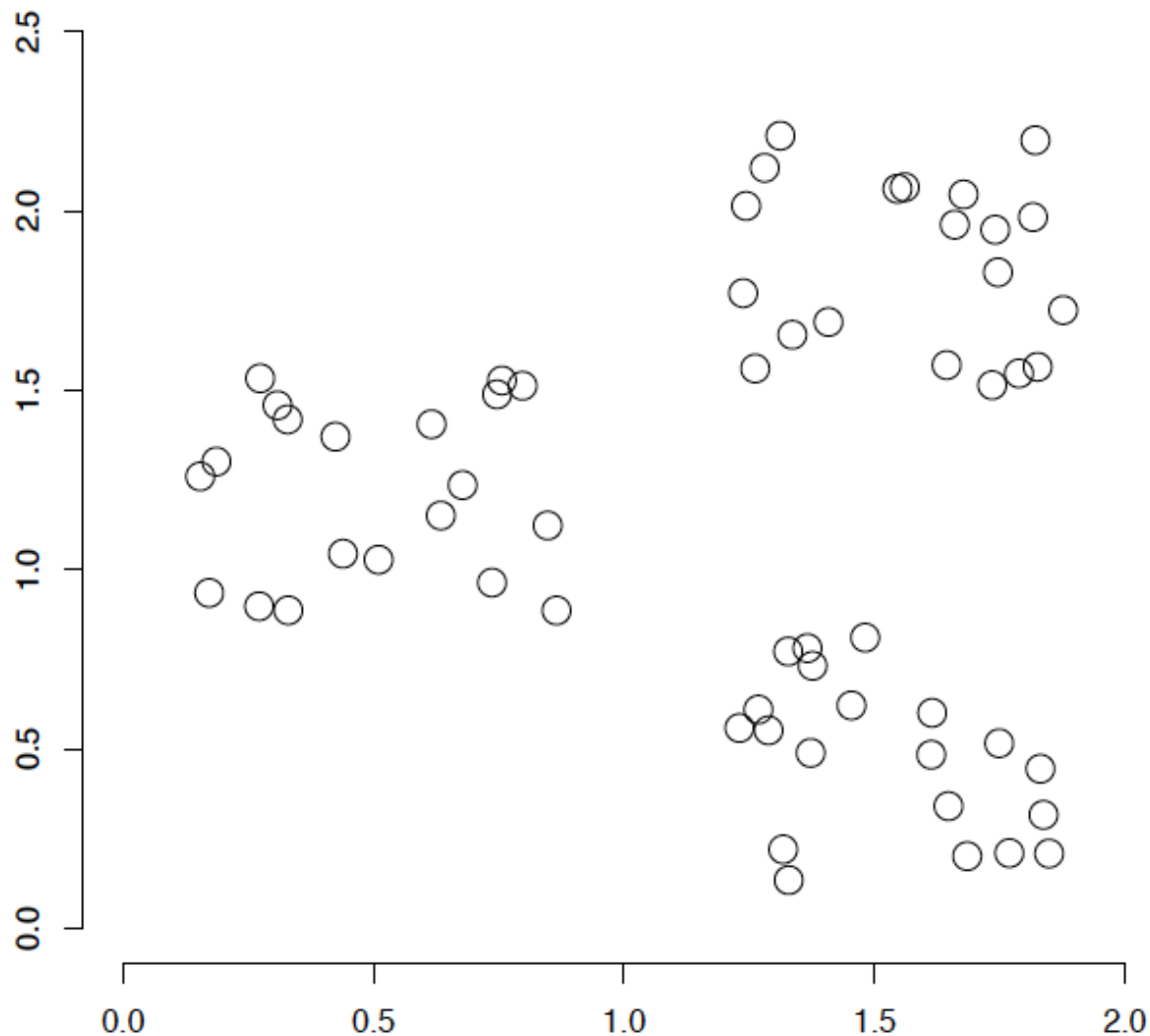
- Document clustering
  - Motivations
  - Document representations
  - Success criteria
- Clustering algorithms
  - Partitional
  - Hierarchical

# What is clustering?

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- **Clustering**: the process of grouping a set of objects into classes of similar objects
  - Documents within a cluster should be similar.
  - Documents from different clusters should be dissimilar.
- The commonest form of *unsupervised learning*
  - Unsupervised learning = learning from raw data, as opposed to supervised data where a classification of examples is given
  - A common and important task that finds many applications in IR and other places

# A data set with clear cluster structure



- How would you design an algorithm for finding the three clusters in this case?

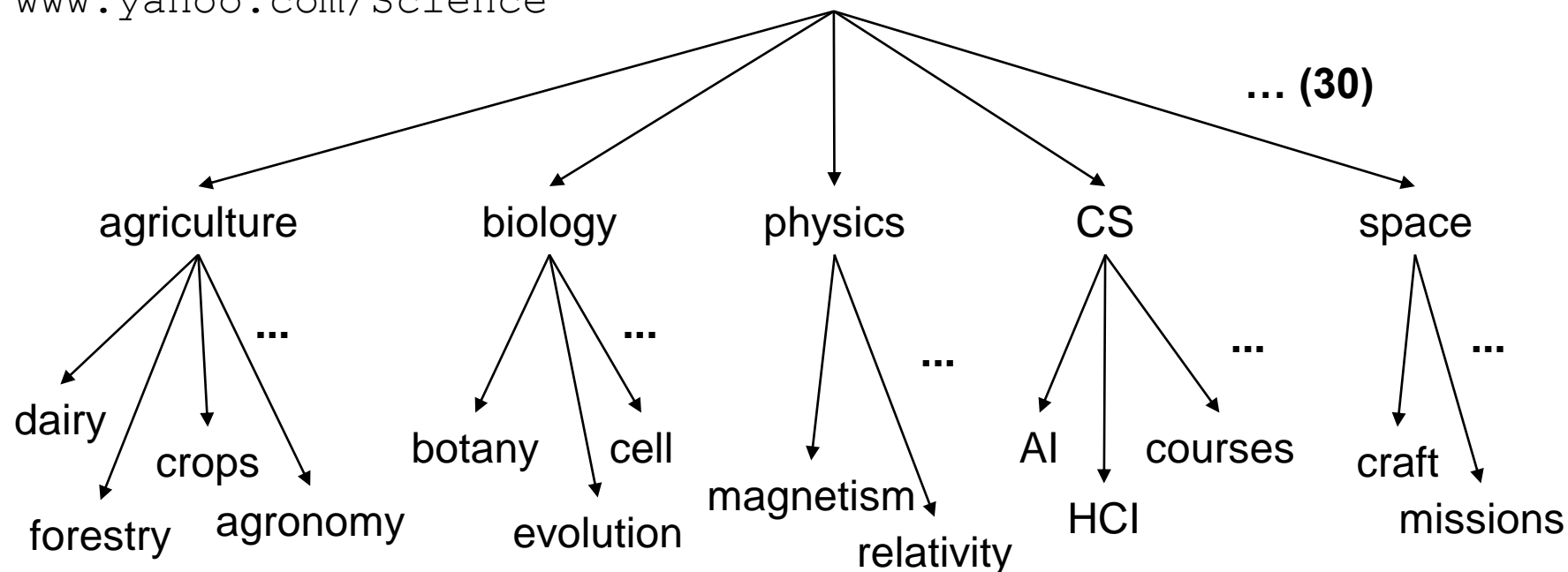
# Applications of clustering in IR

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- Whole corpus analysis/navigation
  - Better user interface: search without typing
- For improving recall in search applications
  - Better search results (like pseudo RF)
- For better navigation of search results
  - Effective “user recall” will be higher
- For speeding up vector space retrieval
  - Cluster-based retrieval gives faster search

# Yahoo! Hierarchy *isn't* clustering but *is* the kind of output you want from clustering

[www.yahoo.com/Science](http://www.yahoo.com/Science)



# Google News: automatic clustering gives an effective news presentation metaphor

The screenshot shows the Google News interface in a web browser. The address bar displays <http://news.google.com/>. The page is organized into two main columns: 'World' on the left and 'U.S.' on the right. Each column contains several news stories with headlines, sources, and timestamps. Small thumbnail images are placed next to some stories. At the bottom of each column, there are buttons to 'Show more stories' or 'Show fewer stories'. The browser's taskbar at the top shows several open tabs, including 'Goog...', 'Ed 602 ...', 'Class ja...', 'Java De...', 'Free Wi...', 'DeNova...', 'LDC Syntacti...', 'Digg - ...', 'Free Co...', and 'Iwuit:'. The status bar at the bottom of the browser window shows the URL: <http://www.google.com/hostednews/ap/article/ALeqM5hGjNbXI6O23C8QzqZMY0pGPAik-AD94INTG1>.

**World »** [edit](#) [X](#)

**Pirates Demand \$25 Million Ransom for Hijacked Tanker (Update1)**  
 Bloomberg - 36 minutes ago  
 By Caroline Alexander and Hamsa Omar Nov. 20 (Bloomberg) -- Somali pirates are demanding \$25 million in ransom to release an oil-laden Saudi supertanker seized off the East African coast, and called on the ship's owners to pay up "soon."  
[Somali pirates demand \\$25M for Saudi ship](#) United Press International  
[African Union says Somali politicians fuel piracy](#) Washington Post  
[BBC News](#) - [guardian.co.uk](#) - [Aljazeera.net](#) - [RIA Novosti](#)  
[all 4,015 news articles »](#)

**Pakistan protests over US missile strikes**  
 Reuters - 2 hours ago  
 By Simon Cameron-Moore ISLAMABAD (Reuters) - Pakistan summoned US ambassador Anne Patterson on Thursday to protest over missile strikes launched by pilotless drone aircraft against militant targets in Pakistan.  
[Pakistan protests US drone attacks, Taliban warns of reprisals](#) AFP  
[Pakistan warns US over missile strike](#) CNN International  
[Telegraph.co.uk](#) - [China Daily](#) - [Xinhua](#) - [PRESS TV](#)  
[all 560 news articles »](#)

**Nighttime attack on Thai antigovernment protesters wounds at least 20**  
 Christian Science Monitor - 30 minutes ago  
 The government denied attacking demonstrators, who have called for the ouster of the prime minister. By Huma Yusuf One person has been killed and 23 others wounded in a grenade attack Thursday against antigovernment protesters occupying the Thai prime ...  
[Blast Kills 1, Wounds 23 at Thai Prime Minister's Office](#) Washington Post  
[Anti-government protestor in Thailand dies in grenade attack](#) International Herald Tribune  
[Xinhua](#) - [United Press International](#) - [The Associated Press](#) - [AsiaOne](#)  
[all 688 news articles »](#)

[Show more stories](#) [Show fewer stories](#)

**U.S. »** [edit](#) [X](#)

**Top Court in California Will Review Proposition 8**  
 New York Times - 1 hour ago  
 By JESSE MCKINLEY SAN FRANCISCO - Responding to pleas for legal clarity from those on both sides of the issue, the California Supreme Court said Wednesday that it would take up the case of whether a voter-approved ban on same-sex unions was ...  
[California Supreme Court to decide fate of Prop. 8 same-sex ...](#)  
[San Jose Mercury News](#)  
[Prop. 8 gay marriage ban goes to Supreme Court](#) Los Angeles Times  
[The Miami Herald](#) - [San Diego Union Tribune](#) - [Indiana Daily Student](#) - [San Francisco Chronicle](#)  
[all 1,241 news articles »](#)

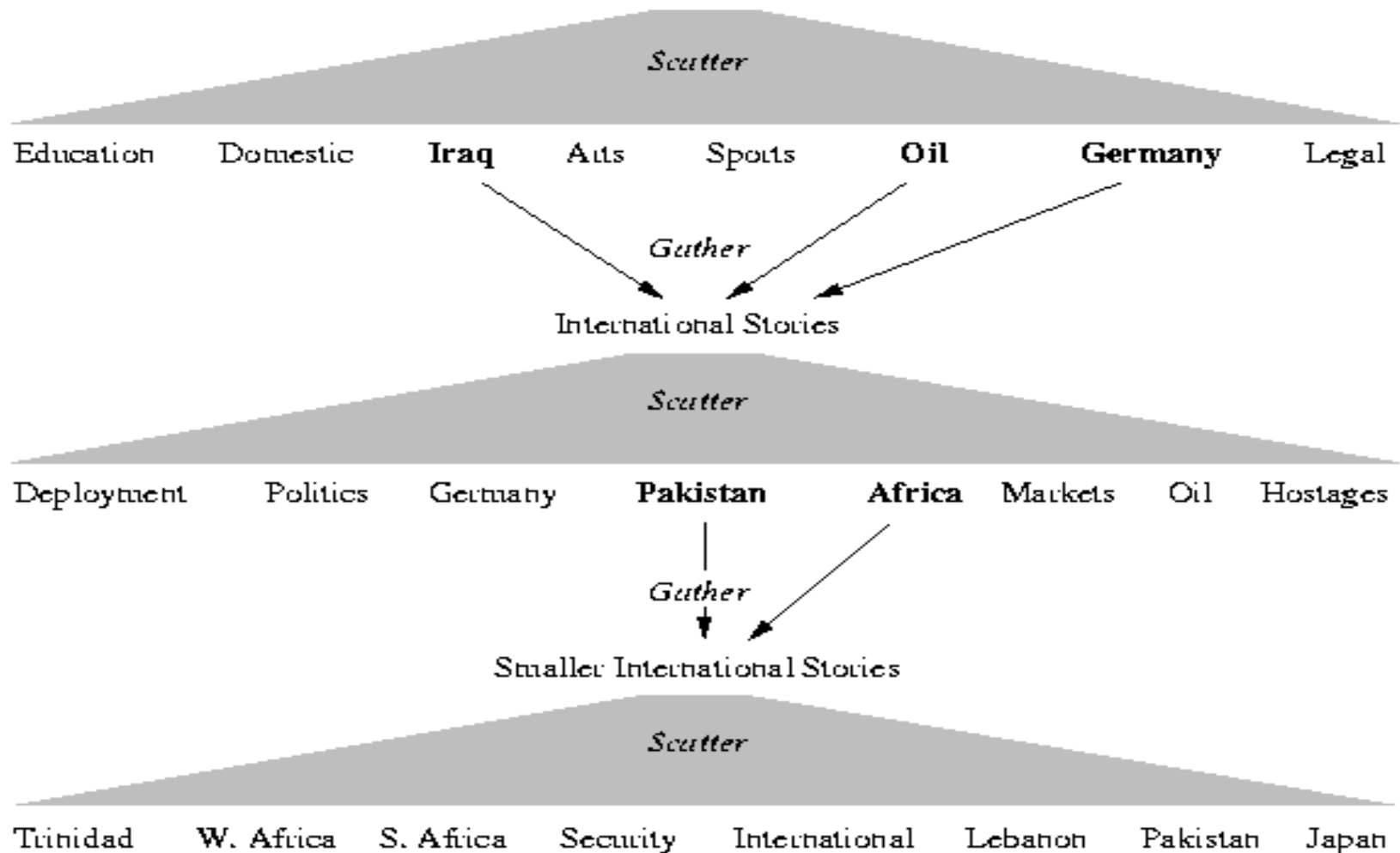
**Drop That Cigarette, Today Is The Great American Smokeout**  
 dBTechno - 1 hour ago  
 Washington (dBTechno) - Today marks the annual Great American Smokeout hosted by the American Cancer Society, and is trying to get people all across the US to drop their cigarettes for just one day.  
[Great American Smokeout: Time to kick the habit](#) Capital Times  
[National Smoke Out Day is Thursday; be a quitter](#) Las Cruces Sun-News  
[MPNnow.com](#) - [eMaxHealth.com](#) - [Times Tribune of Corbin](#) - [ABC15.com \(KNXV-TV\)](#)  
[all 338 news articles »](#)

**Perino: Bush would sign jobless benefits extension**  
 The Associated Press - 47 minutes ago  
 WASHINGTON (AP) - With weekly jobless claims benefits at a 16-year high, the White House said Thursday that President George W. Bush would quickly sign legislation pending in Congress to provide further unemployment benefits.  
[Bush would sign measure to extend jobless benefits](#) Houston Chronicle  
[Jobless claims show need for benefits extension: White House](#) AFP  
[Washington Times](#) - [Wall Street Journal Blogs](#) - [WOI](#) - [Tampabay.com](#)  
[all 599 news articles »](#)

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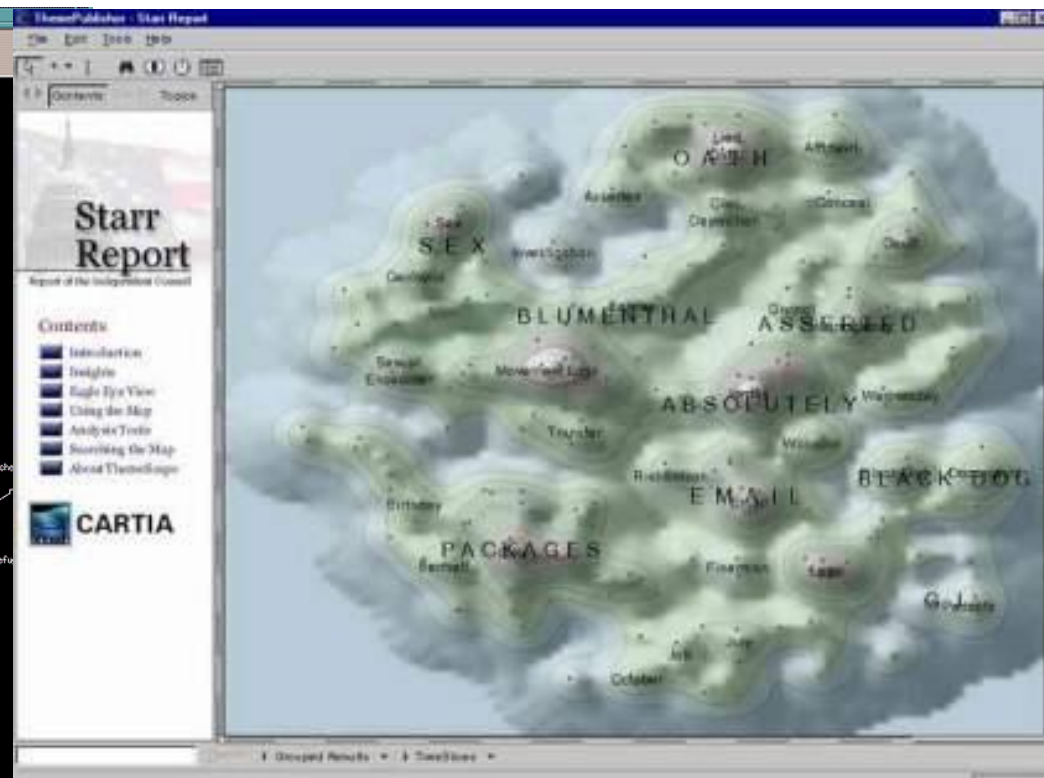
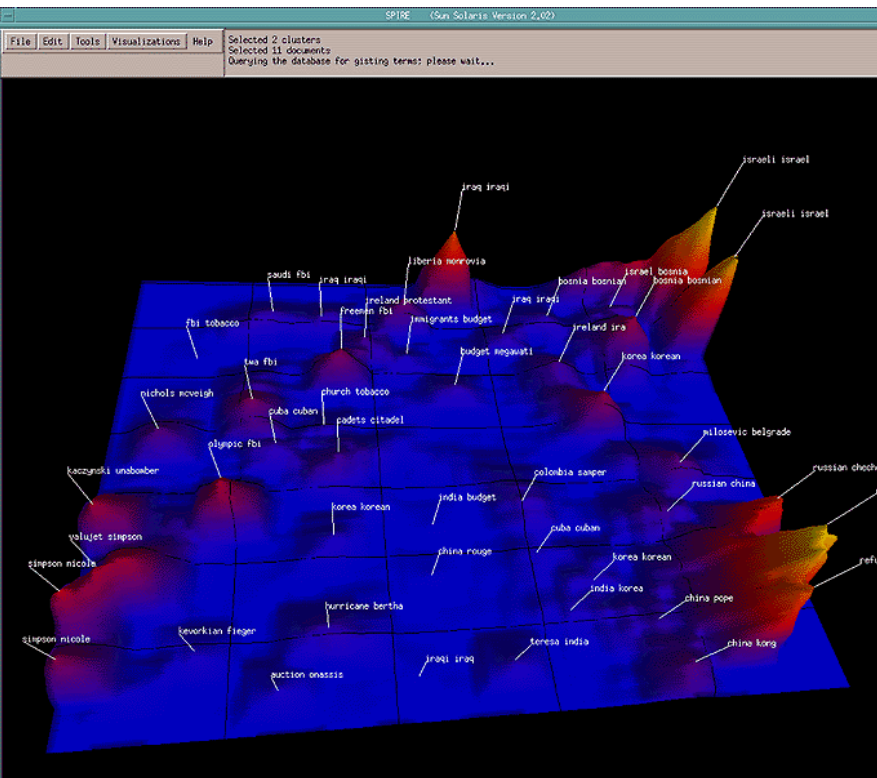
# Scatter/Gather: Cutting, Karger, and Pedersen

New York Times News Service, August 1990





- [Mountain height = cluster size]



# For improving search recall

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- *Cluster hypothesis* - Documents in the same cluster behave similarly with respect to relevance to information needs
- Therefore, to improve search recall:
  - Cluster docs in corpus a priori
  - When a query matches a doc  $D$ , also return other docs in the cluster containing  $D$
- Hope if we do this: The query “car” will also return docs containing *automobile*
  - Because clustering grouped together docs containing *car* with those containing *automobile*.



Why might this happen?

The screenshot shows the Yippy search engine interface. At the top, the address bar displays the URL: [http://search.yippy.com/search?v%3aproject=clusty&v%3afile=viv\\_Xpo6AV&v%3arecluster=&](http://search.yippy.com/search?v%3aproject=clusty&v%3afile=viv_Xpo6AV&v%3arecluster=&). Below the address bar, there are navigation links: "Most Visited", "Getting Started", "Latest Headlines", and "Fridge Filters". The main search bar contains the text "clustering" and a "Search" button. To the right of the search bar, there are links for "advanced" and "preferences".

On the left side, there is a sidebar with a "clouds" tab selected. Under "clouds", there are several categories with counts: "All Results (185)", "Analysis (23)", "Method (22)", "Computing (15)", "Search, Engine (13)", "Hierarchical (16)", "Definition (11)", "High availability (13)", "Linux (11)", "Windows, Microsoft (9)", and "Papers (8)". There is also a "remix" button. At the bottom of the sidebar, there is a "find in clouds:" search bar and a "Find" button.

The main content area displays the search results for the query "clustering". It starts with "Top 179 results retrieved for the query **clustering** ([definition](#)) ([details](#))". The first result is "Clustering" with a description: "Lower Latency In Your Data Center w/ Intel's **Cluster** Ready Solutions!" and the URL [www.intel.com](http://www.intel.com). The second result is "Load Balancing 101" with a description: "Learn the 'Nuts & Bolts' of Load Balancing with F5's White Paper" and the URL [www.f5.com/load\\_balancing](http://www.f5.com/load_balancing). The third result is "Affordable Load Balancers" with a description: "High Performance Load Balancing Solutions From KEMP- See Demo Today" and the URL [kemptechnologies.com](http://kemptechnologies.com).

Below these results, there are three more entries, each with a title, a description, and a URL:

- [Computer cluster - Wikipedia, the free encyclopedia](#) : Middleware such as MPI (Message Passing Interface) or PVM (Parallel Virtual Machine) permits compute **clustering** programs to be portable to a /Computer\_ **cluster**  
[en.wikipedia.org/wiki/Computer\\_cluster](http://en.wikipedia.org/wiki/Computer_cluster) - [cache] - Bing, Yahoo!
- [Writer's Web: Prewriting: Clustering](#) : Prewriting: **Clustering** Melanie Dawson & Joe Essid (printable version here) **Clustering** is a type of prewriting that allows you to explore many ideas  
[writing2.richmond.edu/writing/wwweb/cluster.html](http://writing2.richmond.edu/writing/wwweb/cluster.html)  
[writing2.richmond.edu/writing/wwweb/cluster.html](http://writing2.richmond.edu/writing/wwweb/cluster.html) - [cache] - Bing, Yahoo!
- [Getting Started: Clustering Ideas - CT Community Colleges](#) : **Clustering**. **Clustering** is similar to another process called Brainstorming. **Clustering** is something that you can do on your own or with friends or  
[grammar.ccc.commnet.edu/grammar/composition/brainstorm\\_cluster.htm](http://grammar.ccc.commnet.edu/grammar/composition/brainstorm_cluster.htm)  
[grammar.ccc.commnet.edu/grammar/composition/brainstorm\\_clustering.htm](http://grammar.ccc.commnet.edu/grammar/composition/brainstorm_clustering.htm) - [cache] - Bing, Yahoo!

At the bottom of the main content area, there is a link: [Advanced Clustering | Home](#) .

At the bottom left, there is a "Yippy Approved Shakespeare Searched" logo.

At the bottom right, there is a footer: "yippy.com – grouping search results 11".

# Issues for clustering

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- Representation for clustering
  - Document representation
    - Vector space? Normalization?
      - Centroids aren't length normalized
  - Need a notion of similarity/distance
- How many clusters?
  - Fixed a priori?
  - Completely data driven?
    - Avoid “trivial” clusters - too large or small
      - If a cluster's too large, then for navigation purposes you've wasted an extra user click without whittling down the set of documents much.

# Notion of similarity/distance

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- Ideal: semantic similarity.
- Practical: term-statistical similarity
  - We will use cosine similarity.
  - Docs as vectors.
  - For many algorithms, easier to think in terms of a *distance* (rather than similarity) between docs.
  - We will mostly speak of Euclidean distance
    - But real implementations use cosine similarity

# Clustering Algorithms

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- Flat algorithms
  - Usually start with a random (partial) partitioning
  - Refine it iteratively
    - $K$  means clustering
    - (Model based clustering)
- Hierarchical algorithms
  - Bottom-up, agglomerative
  - (Top-down, divisive)

# Hard vs. soft clustering

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- Hard clustering: Each document belongs to exactly one cluster
  - More common and easier to do
- Soft clustering: A document can belong to more than one cluster.
  - Makes more sense for applications like creating browsable hierarchies
  - You may want to put a pair of sneakers in two clusters: (i) sports apparel and (ii) shoes
  - You can only do that with a soft clustering approach.
- We won't do soft clustering today. See IIR 16.5, 18

# Partitioning Algorithms

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- Partitioning method: Construct a partition of  $n$  documents into a set of  $K$  clusters
- Given: a set of documents and the number  $K$
- Find: a partition of  $K$  clusters that optimizes the chosen partitioning criterion
  - Globally optimal
    - Intractable for many objective functions
    - Ergo, exhaustively enumerate all partitions
  - Effective heuristic methods:  $K$ -means and  $K$ -medoids algorithms

See also Kleinberg NIPS 2002 – impossibility for natural clustering



# K-Means

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- Assumes documents are real-valued vectors.
- Clusters based on *centroids* (aka the *center of gravity* or mean) of points in a cluster,  $c$ :

$$\vec{\mu}(c) = \frac{1}{|c|} \sum_{\vec{x} \in c} \vec{x}$$

- Reassignment of instances to clusters is based on distance to the current cluster centroids.
  - (Or one can equivalently phrase it in terms of similarities)

# K-Means Algorithm

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Select  $K$  random docs  $\{s_1, s_2, \dots, s_K\}$  as seeds.

Until clustering *converges* (or other stopping criterion):

For each doc  $d_i$ :

Assign  $d_i$  to the cluster  $c_j$  such that  $\text{dist}(x_i, s_j)$  is minimal.

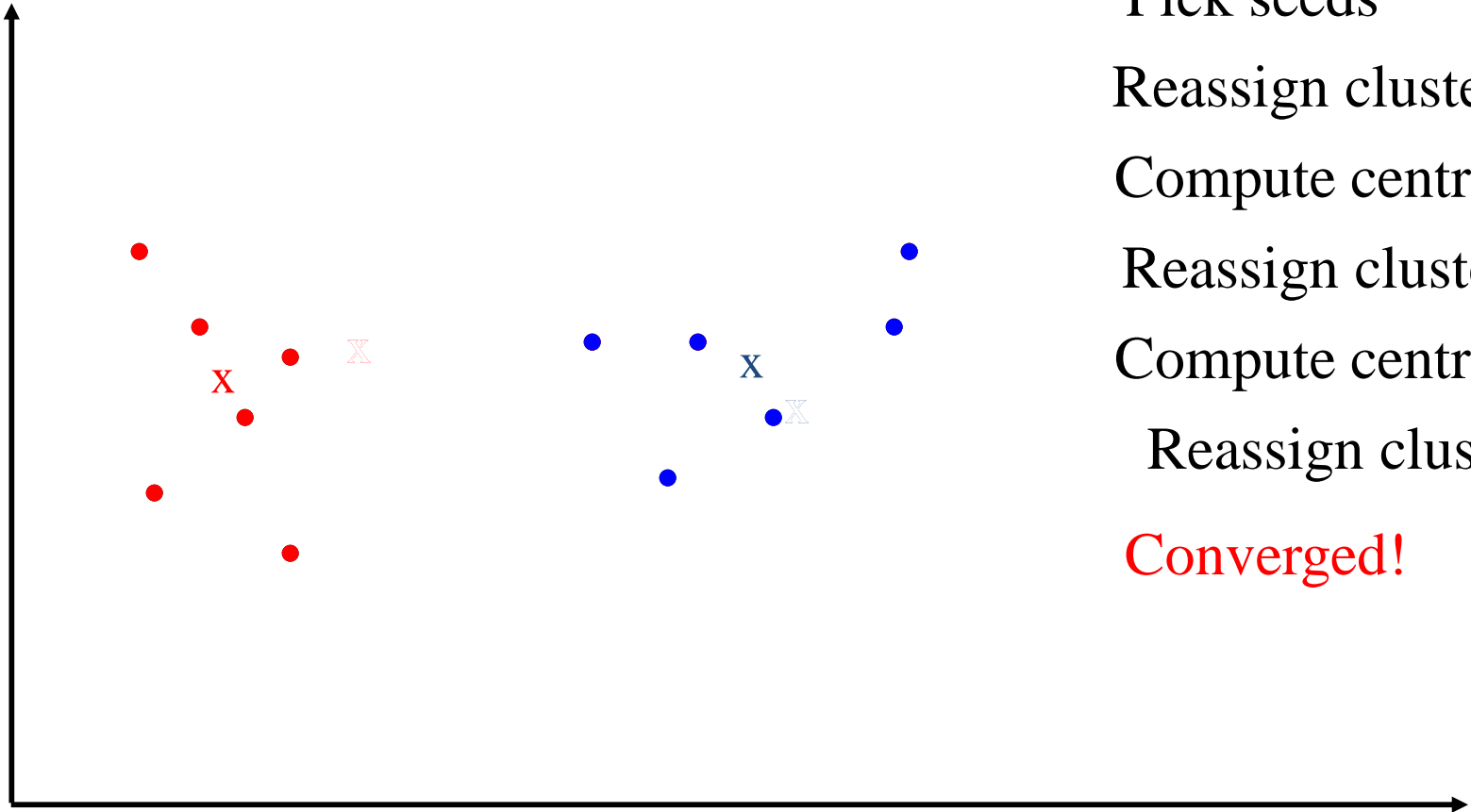
*(Next, update the seeds to the centroid of each cluster)*

For each cluster  $c_j$

$$s_j = \mu(c_j)$$

# K Means Example

## ( $K=2$ )



Pick seeds

Reassign clusters

Compute centroids

Reassign clusters

Compute centroids

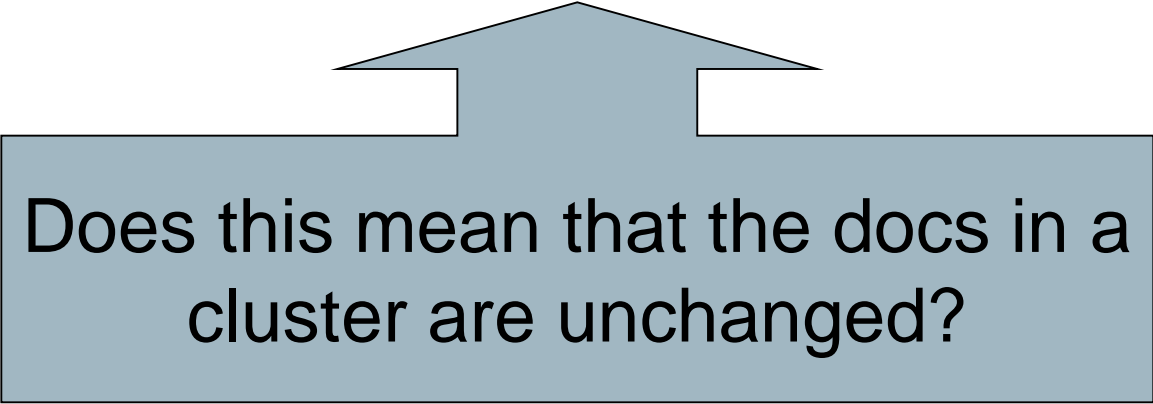
Reassign clusters

**Converged!**

# Termination conditions

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- Several possibilities, e.g.,
  - A fixed number of iterations.
  - Doc partition unchanged.
  - Centroid positions don't change.



Does this mean that the docs in a cluster are unchanged?

# Convergence

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- Why should the  $K$ -means algorithm ever reach a *fixed point*?
  - A state in which clusters don't change.
- $K$ -means is a special case of a general procedure known as the *Expectation Maximization (EM) algorithm*.
  - EM is known to converge.
  - Number of iterations could be large.
    - But in practice usually isn't

Lower case!

# Convergence of $K$ -Means

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- Define goodness measure of cluster  $k$  as sum of squared distances from cluster centroid:
  - $G_k = \sum_i (d_i - c_k)^2$  (sum over all  $d_i$  in cluster  $k$ )
- $G = \sum_k G_k$
- Reassignment monotonically decreases  $G$  since each vector is assigned to the closest centroid.

# Convergence of $K$ -Means

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- Recomputation monotonically decreases each  $G_k$  since ( $m_k$  is number of members in cluster  $k$ ):
  - $\sum (d_i - a)^2$  reaches minimum for:
    - $\sum -2(d_i - a) = 0$
    - $\sum d_i = \sum a$
    - $m_k a = \sum d_i$
    - $a = (1/m_k) \sum d_i = c_k$
- $K$ -means typically converges quickly

# Time Complexity

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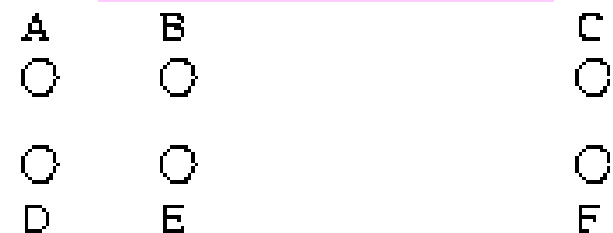
- Computing distance between two docs is  $O(M)$  where  $M$  is the dimensionality of the vectors.
- Reassigning clusters:  $O(KN)$  distance computations, or  $O(KNM)$ .
- Computing centroids: Each doc gets added once to some centroid:  $O(NM)$ .
- Assume these two steps are each done once for  $I$  iterations:  $O(IKNM)$ .



# Seed Choice

- Results can vary based on random seed selection.
- Some seeds can result in poor convergence rate, or convergence to sub-optimal clusterings.
  - Select good seeds using a heuristic (e.g., doc least similar to any existing mean)
  - Try out multiple starting points
  - Initialize with the results of another method.

## Example showing sensitivity to seeds



**In the above, if you start with B and E as centroids you converge to {A,B,C} and {D,E,F}**  
**If you start with D and F you converge to {A,B,D,E} {C,F}**

# *K*-means issues, variations, etc.

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- Recomputing the centroid after every assignment (rather than after all points are re-assigned) can improve speed of convergence of *K*-means
- Assumes clusters are spherical in vector space
  - Sensitive to coordinate changes, weighting etc.
- Disjoint and exhaustive
  - Doesn't have a notion of “outliers” by default
  - But can add outlier filtering

Dhillon et al. ICDM 2002 – variation to fix some issues with small document clusters

# How Many Clusters?

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- Number of clusters  $K$  is given
  - Partition  $n$  docs into predetermined number of clusters
- Finding the “right” number of clusters is part of the problem
  - Given docs, partition into an “appropriate” number of subsets.
  - E.g., for query results - ideal value of  $K$  not known up front
    - though UI may impose limits.
- Can usually take an algorithm for one flavor and convert to the other.

# $K$ not specified in advance

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- Say, the results of a query.
- Solve an optimization problem: penalize having lots of clusters
  - application dependent, e.g., compressed summary of search results list.
- Tradeoff between having more clusters (better focus within each cluster) and having too many clusters

## $K$ not specified in advance

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- Given a clustering, define the Benefit for a doc to be the cosine similarity to its centroid
- Define the Total Benefit to be the sum of the individual doc Benefits.



Why is there always a clustering of Total Benefit  $n$ ?

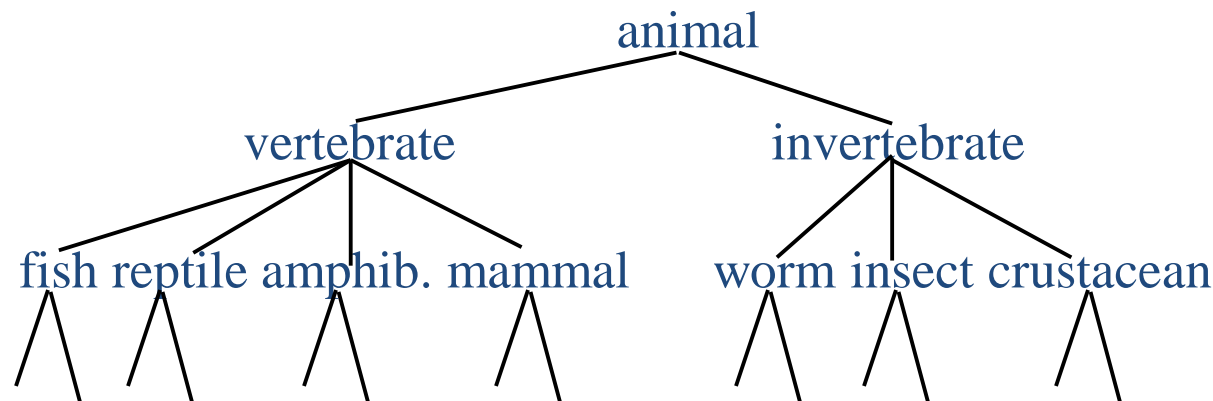
# Penalize lots of clusters

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- For each cluster, we have a Cost  $C$ .
- Thus for a clustering with  $K$  clusters, the Total Cost is  $KC$ .
- Define the Value of a clustering to be =  
 $\text{Total Benefit} - \text{Total Cost}$ .
- Find the clustering of highest value, over all choices of  $K$ .
  - Total benefit increases with increasing  $K$ . But can stop when it doesn't increase by "much". The Cost term enforces this.

# Hierarchical Clustering

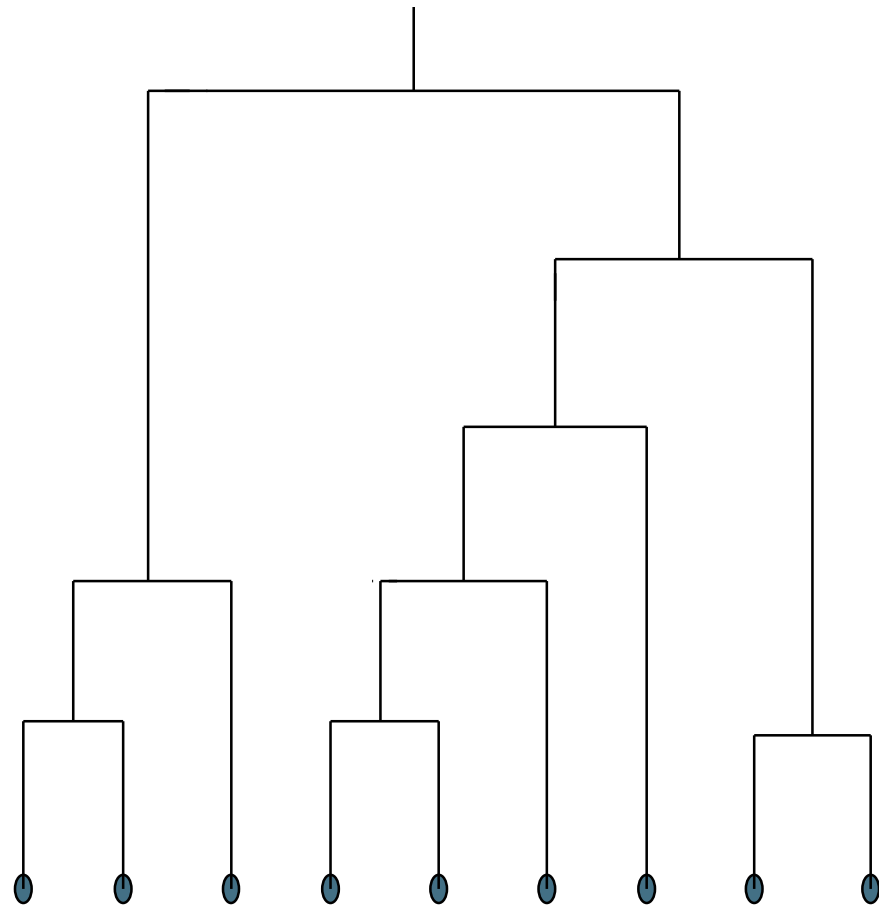
- Build a tree-based hierarchical taxonomy (*dendrogram*) from a set of documents.



- One approach: recursive application of a partitional clustering algorithm.

# Dendrogram: Hierarchical Clustering

- Clustering obtained by cutting the dendrogram at a desired level: each connected component forms a cluster.





# Hierarchical Agglomerative Clustering (HAC)

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- Starts with each doc in a separate cluster
  - then repeatedly joins the closest pair of clusters, until there is only one cluster.
- The history of merging forms a binary tree or hierarchy.

Note: the resulting clusters are still “hard” and induce a partition

# *Closest pair* of clusters

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- Many variants to defining closest pair of clusters
- **Single-link**
  - Similarity of the *most* cosine-similar (single-link)
- **Complete-link**
  - Similarity of the “furthest” points, the *least* cosine-similar
- **Centroid**
  - Clusters whose centroids (centers of gravity) are the most cosine-similar
- **Average-link**
  - Average cosine between pairs of elements

# Single Link Agglomerative Clustering

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- Use maximum similarity of pairs:

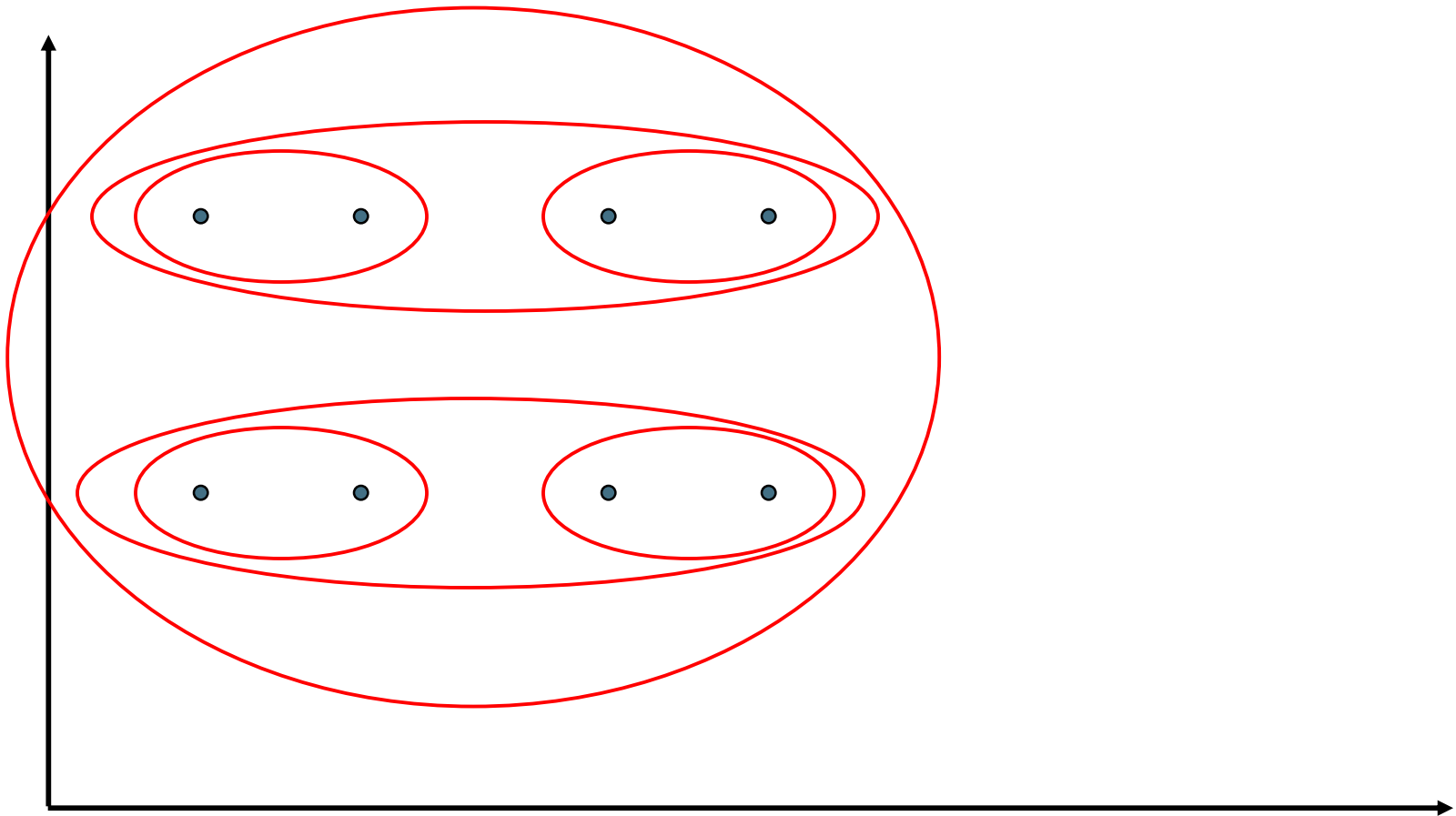
$$\textit{sim}(c_i, c_j) = \max_{x \in c_i, y \in c_j} \textit{sim}(x, y)$$

- Can result in “straggly” (long and thin) clusters due to chaining effect.
- After merging  $c_i$  and  $c_j$ , the similarity of the resulting cluster to another cluster,  $c_k$ , is:

$$\textit{sim}((c_i \cup c_j), c_k) = \max(\textit{sim}(c_i, c_k), \textit{sim}(c_j, c_k))$$

# Single Link Example

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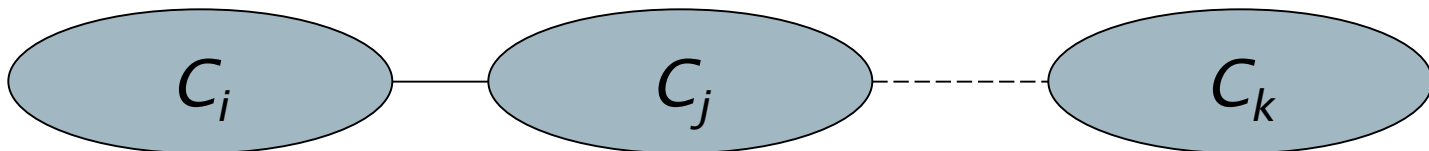
# Complete Link

- Use minimum similarity of pairs:

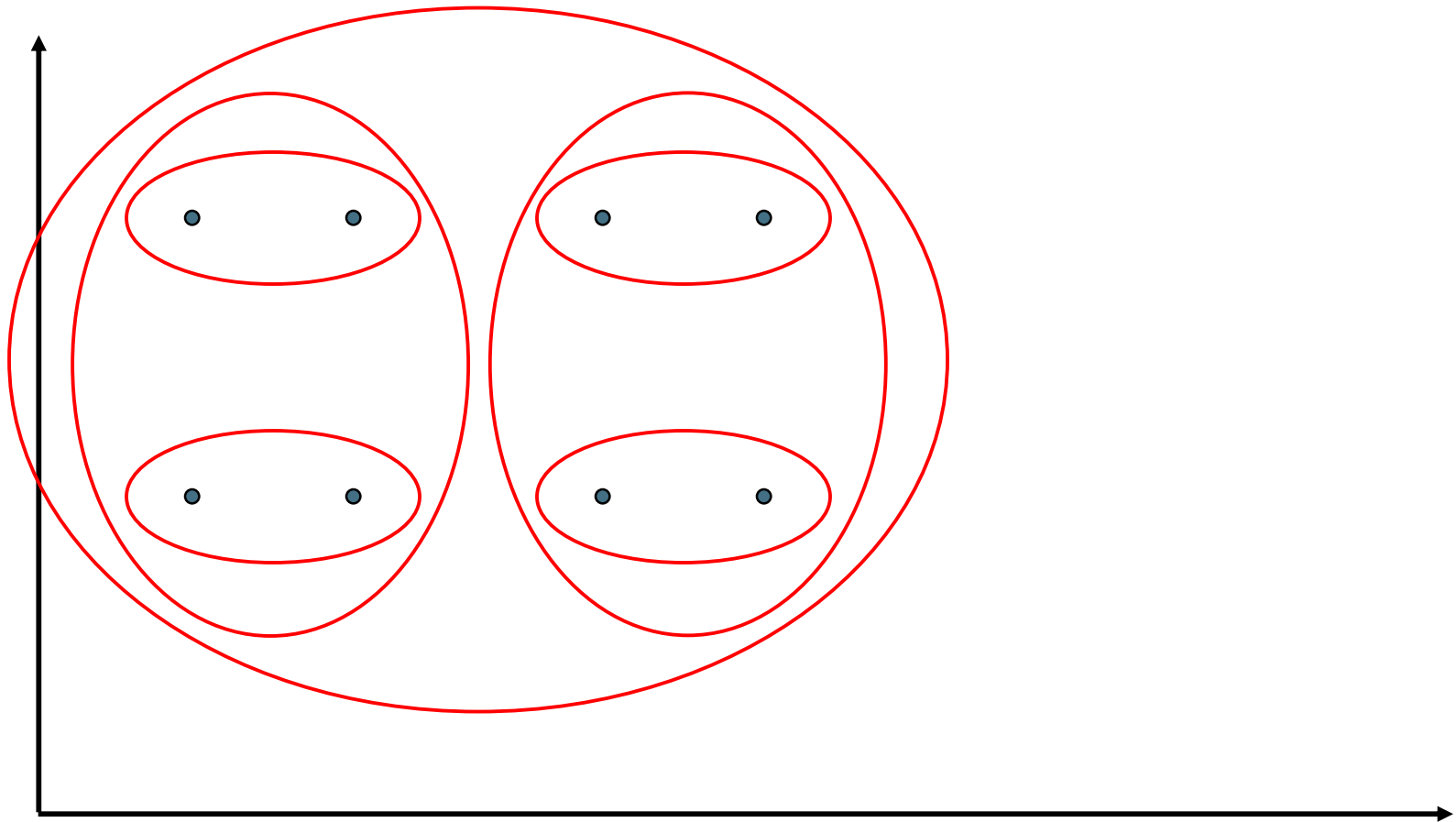
$$\text{sim}(c_i, c_j) = \min_{x \in c_i, y \in c_j} \text{sim}(x, y)$$

- Makes “tighter,” spherical clusters that are typically preferable.
- After merging  $c_i$  and  $c_j$ , the similarity of the resulting cluster to another cluster,  $c_k$ , is:

$$\text{sim}((c_i \cup c_j), c_k) = \min(\text{sim}(c_i, c_k), \text{sim}(c_j, c_k))$$



# Complete Link Example



# Computational Complexity

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- In the first iteration, all HAC methods need to compute similarity of all pairs of  $N$  initial instances, which is  $O(N^2)$ .
- In each of the subsequent  $N-2$  merging iterations, compute the distance between the most recently created cluster and all other existing clusters.
- In order to maintain an overall  $O(N^2)$  performance, computing similarity to each other cluster must be done in constant time.
  - Often  $O(N^3)$  if done naively or  $O(N^2 \log N)$  if done more cleverly

# Group Average

- Similarity of two clusters = average similarity of all pairs within merged cluster.

$$\text{sim}(c_i, c_j) = \frac{1}{|c_i \cup c_j|(|c_i \cup c_j| - 1)} \sum_{\vec{x} \in (c_i \cup c_j)} \sum_{\vec{y} \in (c_i \cup c_j): \vec{y} \neq \vec{x}} \text{sim}(\vec{x}, \vec{y})$$

- Compromise between single and complete link.
- Two options:
  - Averaged across all ordered pairs in the merged cluster
  - Averaged over all pairs *between* the two original clusters
- No clear difference in efficacy



# Computing Group Average Similarity

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- Always maintain sum of vectors in each cluster.

$$\vec{s}(c_j) = \sum_{\vec{x} \in c_j} \vec{x}$$

- Compute similarity of clusters in constant time:

$$\text{sim}(c_i, c_j) = \frac{(\vec{s}(c_i) + \vec{s}(c_j)) \bullet (\vec{s}(c_i) + \vec{s}(c_j)) - (|c_i| + |c_j|)}{(|c_i| + |c_j|)(|c_i| + |c_j| - 1)}$$

# What Is A Good Clustering?

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- Internal criterion: A good clustering will produce high quality clusters in which:
  - the intra-class (that is, intra-cluster) similarity is high
  - the inter-class similarity is low
  - The measured quality of a clustering depends on both the document representation and the similarity measure used

# External criteria for clustering quality

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- Quality measured by its ability to discover some or all of the hidden patterns or latent classes in gold standard data
- Assesses a clustering with respect to ground truth ... requires *labeled data*
- Assume documents with  $C$  gold standard classes, while our clustering algorithms produce  $K$  clusters,  $\omega_1, \omega_2, \dots, \omega_K$  with  $n_i$  members.

## External Evaluation of Cluster Quality

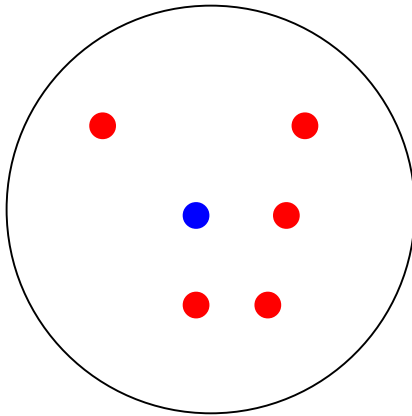
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- Simple measure: purity, the ratio between the dominant class in the cluster  $\pi_i$  and the size of cluster  $\omega_i$

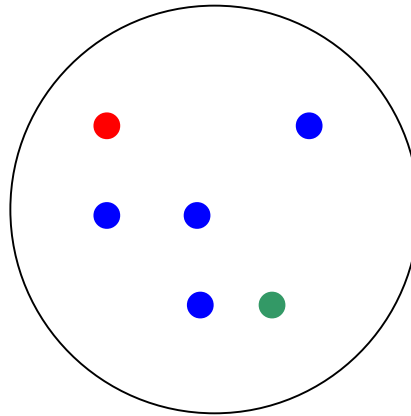
$$Purity(\omega_i) = \frac{1}{n_i} \max_j (n_{ij}) \quad j \in C$$

- Biased because having  $n$  clusters maximizes purity
- Others are entropy of classes in clusters (or mutual information between classes and clusters)

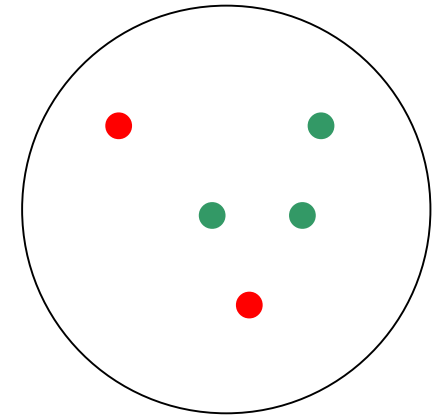
# Purity example



Cluster I



Cluster II



Cluster III

Cluster I: Purity =  $1/6 (\max(5, 1, 0)) = 5/6$

Cluster II: Purity =  $1/6 (\max(1, 4, 1)) = 4/6$

Cluster III: Purity =  $1/5 (\max(2, 0, 3)) = 3/5$

Rand Index measures between pair decisions. Here  $RI = 0.68$

Number of points	Same Cluster in clustering	Different Clusters in clustering
Same class in ground truth	20	24
Different classes in ground truth	20	72

## Rand index and Cluster F-measure

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$$RI = \frac{A + D}{A + B + C + D}$$

Compare with standard Precision and Recall:

$$P = \frac{A}{A + B} \qquad R = \frac{A}{A + C}$$

People also define and use a cluster F-measure, which is probably a better measure.

# Final word and resources

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- In clustering, clusters are inferred from the data without human input (unsupervised learning)
- However, in practice, it's a bit less clear: there are many ways of influencing the outcome of clustering: number of clusters, similarity measure, representation of documents, . . .
- Resources
  - IIR 16 except 16.5
  - IIR 17.1–17.3