$$y = x * h = 4x(t+2) - 3x(t+2/5) - 6x(t) + 9x(t+\tfrac{1}{2})$$
$$8x(t-\tfrac{1}{2})$$

فصل سوم سری فوریه:

سری فوریه پیوسته:

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}\,dt \qquad \omega_0 = \frac{2\pi}{T}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

سری فوریه گسسته:

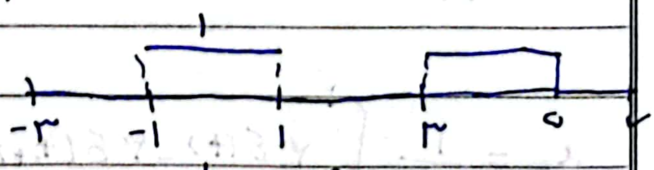
$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\omega_0 n} \qquad \omega_0 = \frac{2\pi}{N}$$

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\omega_0 n}$$

$$a_k = a_{k+N} \qquad N = 4 \quad a_0 = a_4 \;,\; a_1 = a_5$$

تمرین) نمایش سری فوریه سیگنال زیر را بیابید؟

$$f(t) = \begin{cases} 1 & -1 < t < 1 \\ 0 & 1 < t < 3 \end{cases} \qquad T = 4$$



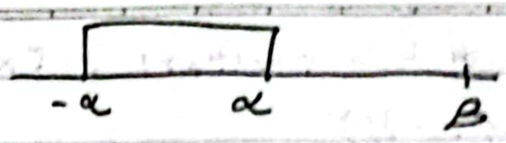
$$a_k = \frac{1}{4}\int_{-1}^{3} x(t)e^{-jk\frac{\pi}{2}t}\,dt = \frac{1}{4}\int_{-1}^{1} e^{-jk\frac{\pi}{2}t}\,dt$$

$$\boxed{\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}}$$

$$= \frac{1}{4}\,\frac{e^{-jk\frac{\pi}{2}t}}{-jk\frac{\pi}{2}}\Bigg|_{-1}^{1} = \frac{1}{4}\,\frac{e^{-jk\frac{\pi}{2}} - e^{+jk\frac{\pi}{2}}}{-jk\frac{\pi}{2}}$$

$$= \frac{1}{4}\,\frac{+2\cancel{j}\sin\frac{k\pi}{2}}{+jk\frac{\pi}{2}} = \frac{\sin k\frac{\pi}{2}}{k\pi} = \frac{1}{2}\mathrm{sinc}(k/2)$$

$a_k = d \operatorname{sinc}(kd)$



$d = \dfrac{\text{[illegible]}}{\text{[illegible]}} = \dfrac{2\alpha}{\beta-\alpha}$

$a_0 = \frac{1}{2}$ $\qquad a_2 = 0$

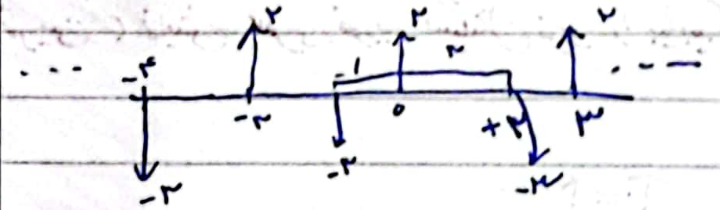
$a_1 = \frac{1}{\pi}$ $\qquad a_{-2} = 0$

$a_{-1} = \frac{1}{\pi}$

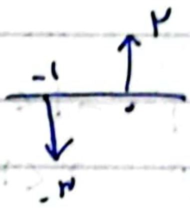
$$x(t) = \sum_{k=-\infty}^{k=\infty} a_k e^{jk\frac{\pi}{2}t} = \dots + \frac{1}{\pi} e^{-j\frac{\pi}{2}t} + \frac{1}{2} + \frac{1}{\pi} e^{j\frac{\pi}{2}t} + \dots$$

$$f(t) = \sum_{k=-\infty}^{\infty} 2\delta(t+3k) - 3\delta(t+3k+1)$$

$$= \dots + 2\delta(t-3) - 3(t-2) + 2\delta(t) - 3\delta(t+1) + 2\delta(t+3) - 3\delta(t+4) + \dots$$



$T = 3$



$$a_k = \frac{1}{3}\int_{-1}^{2} 2\delta(t) - 3\delta(t+1)\, e^{-jk\frac{2\pi}{3}t}\, dt = \frac{1}{3}\int 2\delta(t) e^{-jk\frac{2\pi}{3}t}\, dt$$

$$-\frac{1}{3}\int 3\delta(t+1) e^{-jk\frac{2\pi}{3}t}\, dt = \frac{2}{3} - e^{jk\frac{2\pi}{3}}$$

$$f_s\{k\,\delta(t)\} = \frac{k}{T}$$

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$f(t) = 4\sin^2(2t) - \cos(2t) + 1$

برای مثلثاتی از انتگرال نمی رویم:

$f(t) = 4\,\dfrac{1-\cos 4t}{2} - \cos 2t + 1 \qquad \sin^2 a = \dfrac{1-\cos 2a}{2}$

$2 - 2\cos 4t - \cos 2t + 1 = 3 - 2\cos 4t - \cos 2t$

$\omega_0 = 4 \Rightarrow \dfrac{2\pi}{T} = 4 \to T = \dfrac{\pi}{2}$

$\to$ ک.م.م $(\pi, \frac{\pi}{2}) = \pi = T$

$\omega_0 = 2 \to \dfrac{2\pi}{T} = 2 \to T = \pi$

$\omega_0 = \dfrac{2\pi}{T} = 2$

$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

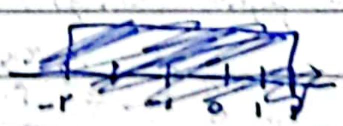
$f(t) = 3 - e^{j4t} - e^{-j4t} - \frac{1}{2}e^{j2t} - \frac{1}{2}e^{-j2t} = \sum a_k e^{jk2t}$

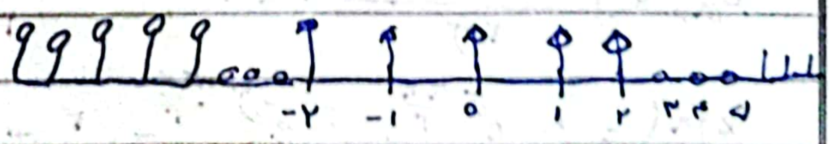
$\boxed{a_0 = 3}$ $\boxed{a_2 = -1}$ $\boxed{a_{-2} = -1}$ $\boxed{a_1 = -\frac{1}{2}}$ $\boxed{a_{-1} = -\frac{1}{2}}$

$a_k = 0 \qquad k \neq \pm 1, \pm 2, 0$

$x[n] = \text{rect}(\frac{n}{5}) \qquad N = 8$; $\qquad \text{rect}(\frac{n}{2N+1})$

$2N+1 = 5 \Rightarrow N = 2$





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$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\omega_0 k n}$$

$$a_k = \frac{1}{8}\sum_{-2}^{2} e^{-jk\frac{\pi}{4}n} = \frac{1}{8}\sum_{n=-2}^{n=2}\left(e^{-jk\frac{\pi}{4}}\right)^n$$

$$\sum_{m=\alpha}^{\beta} q^m = \frac{q^{\alpha}\left(1-q^{\beta-\alpha+1}\right)}{1-q}$$

$$= \frac{1}{8}\,\frac{e^{-jk\frac{\pi}{4}(-2)}\left(1-e^{-jk\frac{\pi}{4}(5)}\right)}{1-e^{-jk\frac{\pi}{4}}}$$

$$= \frac{1}{8}\,\frac{e^{jk\frac{\pi}{2}}\left(1-e^{-jk\frac{5\pi}{4}}\right)}{1-e^{-jk\frac{\pi}{4}}} = \frac{1}{8}\,\frac{e^{jk\frac{\pi}{2}}\, e^{-jk\frac{3\pi}{4}}}{1-e^{-jk\frac{\pi}{4}}}$$

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{\pi}{4}n}$$

$$x[n] = j\sin\left(\frac{\pi n}{4}\right) - 4\cos^2\left(\frac{\pi n}{3}\right) \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\cos\frac{\pi n}{3} = \frac{1+\cos\frac{2\pi n}{3}}{2}$$

$$x[n] = j\sin\left(\frac{\pi n}{4}\right) - 2\left(1+\cos\frac{2\pi n}{3}\right)$$

$$\frac{2\pi}{\frac{\pi}{4}} = 8 \qquad \frac{2\pi}{\frac{2\pi}{3}} = 3$$

$$N = \text{ک.م.م}(8,3) = 24 \qquad \omega_0 = \frac{2\pi}{24} = \frac{\pi}{12}$$

$$x[n] = j\,\frac{e^{j\frac{\pi n}{4}} - e^{-j\frac{\pi n}{4}}}{2j} - 2 - 2\,\frac{e^{j\frac{2\pi n}{3}} + e^{-j\frac{2\pi n}{3}}}{2}$$

$$x[n] = \frac{1}{2}e^{j\frac{\pi n}{4}} - \frac{1}{2}e^{-j\frac{\pi n}{4}} - 2 - e^{j\frac{2\pi n}{3}} - e^{-j\frac{2\pi n}{3}}$$

$x[n] = \sum a_k e^{jk\frac{\pi}{۱۲}n}$

$\boxed{a_{۳} = \frac{1}{۲}}$ $\boxed{a_{-۳} = -\frac{1}{۲}}$ $\boxed{a_0 = -۲}$ $\boxed{a_{۸} = -1}$

$a_{-۸} = -1$

$a_k = 0 \quad k \neq 0, \pm ۳, \pm ۸$

ویژگی های سری فوریه:

$x(t) \rightarrow a_k$ $\qquad x(-t) \rightarrow a_{-k}$

$x(t - T_0) \rightarrow a_k e^{-jk\omega_0 T_0}$

$x^*(t) \rightarrow a^*_{-k}$ $\qquad \frac{dx}{dt} \rightarrow jk\omega_0 a_k$

$x(t)$: Real $\quad a_k = a^*_{-k}$ $\qquad \int_{-\infty}^{t} x(\tau)\, d\tau = \frac{a_k}{jk\omega_0}$

$x(t)$ = Real & even $\quad a = a^*_k$ (Real & even)

$x(t)$ = Real & odd $\quad -a_k = a^*_k$ (Im & odd)

$\frac{1}{T}\int_{-\infty}^{\infty} |x(t)|^۲ dt = \sum_{k=-\infty}^{\infty} |a_k|^۲$

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تمرین: فرض کنید ضرایب سری فوریه سیگنال $x(t)$ به صورت زیر باشد.
در مورد ضرایب سری فوریه توابع خواسته شده را به دست آورید؟

$a_k = (\frac{1}{2})^{|k|}$, $T=4$ $\quad a_1 = \frac{1}{2} \quad a_{-1} = \frac{1}{2}$

$a_k =$ زوج و حقیقی

a) $\int_{-\infty}^{t} x(\beta)d\beta = \frac{a_k}{jk\omega_0} = \frac{(\frac{1}{2})^{|k|}}{jk\frac{2\pi}{4}} = \frac{2}{jk\pi}(\frac{1}{2})^{|k|}$

b) $x''(t) = y$ $\quad x(t) — a_k$

$x' \to jk\omega_0 a_k$

$b_k = -k^2(\frac{\pi}{2})^2(\frac{1}{2})^{|k|}$ $\quad x'' \to (jk\omega_0)^2 a_k$

c) $odd\{x(t)\} = \frac{x(t)-x(-t)}{2} = \frac{a_k - a_{-k}}{2} = \frac{(\frac{1}{2})^{k} - (\frac{1}{2})^{-k}}{2}$

$= \frac{(\frac{1}{2})^{k} - (\frac{1}{2})^{k}}{2} = 0$

d) $Im\{x(t)\} = \frac{x(t) - x^*(t)}{2} = \frac{a_k - a^*_{-k}}{2} = \frac{(\frac{1}{2})^{|k|} - (\frac{1}{2})^{|-k|}}{2} = 0$

e) $x(t-2) =$

$b_k = a_k e^{-jk\frac{2\pi}{4}\times 2} = \cancel{a_k e}\ a_k e^{-jk\pi} = a_k(-1)^k$

$e^{-jk\pi} = \cos k\pi - j\sin k\pi = (-1)^k$

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اگر سیگنال x_1 و ضرایب سری فوریه سیگنال x_2 به صورت زیر باشند درمورد حقیقی بودن، زوج و یا فرد بودن سیگنال ها بحث کنید.

$$x_1 = \sum k \cos\left(\frac{k\pi}{2}\right) e^{jk2t}$$

$$b_k = j\frac{k}{5}$$

$$x(t) = \sum a_k e^{jk\omega_0 t}$$

$$a_k = k \cos \frac{k\pi}{2}$$

$$b_k = j\frac{k}{5}$$

$x_1 : a \qquad a_k = k\cos\frac{k\pi}{2}$

$a_k = a_{-k} \rightarrow k\cos\frac{k\pi}{2} = (-k)\cos\left(-\frac{k\pi}{2}\right)$

زوج نیست و حقیقی نیست

$x(t)$: Real $\qquad a_k = a^*_{-k}$

حقیقی نیست

$x(t) = x(-t) \Rightarrow a_k = a_{-k}$ ✗

$x(t) = -x(-t) \Rightarrow \boxed{a_k = -a_{-k}}$ ✓

$x(t)$ فرد است

$b_k = j\frac{k}{5} \qquad \mathrm{Re}$ ✗ $\qquad j\frac{k}{5} = -j\frac{-k}{5}$

x_2 موهومی و فرد است

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اطلاعات زیر در مورد سیگنال در دسترس است. سیگنال هایی که در شرایط زیر صدق می کنند را بیابید؟

$x[n]$ is Real & even $\rightarrow a_k \rightarrow$ حقیقی و زوج

$$\sum_{n=0}^{5} |x[n]|^2 = 54 \qquad N=6 \qquad a_{-4} = 2$$

$$\sum_{n=0}^{5} x[n] e^{j\pi n} = 6 \rightarrow a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi}{6}n}$$

$$x[n] = \sum_k a_k e^{jk\frac{2\pi}{6}n}$$

$$a_3 = \frac{\frac{1}{6}\sum_{n=0}^{5} x[n] e^{3j\pi n}}{6} = 1 \qquad \boxed{a_3 = 1}$$

$$a_{-4} = 2 \qquad a_{-4} = a_{-4+6} = a_2 = 2$$

$$a_2 = a_{-2} = 2 \qquad a_3 = a_{-3} = 1$$

$$\frac{1}{N}\sum |x[n]|^2 = \sum |a_k|^2 \Rightarrow \frac{1}{6}\times 54 = 4+1+4+|a_{-1}|^2+|a_0|^2+|a_1|^2$$

$$9 = 9 + 2|a_{-1}|^2 + |a_0|^2 \Rightarrow 0 = 2|a_{-1}|^2 + |a_0|^2$$

$$\boxed{a_{-1} = a_0 = a_1 = 0}$$

$$x[n] = \sum_{k=-3}^{1} a_k e^{jk\frac{2\pi}{6}n}$$

$$2e^{j(-2\frac{2\pi}{6}n)} + (1)e^{-j3\frac{2\pi}{6}n} + 2e^{j(-2)\frac{2\pi}{6}n}$$

$$x[n] = 2e^{-j\frac{4\pi}{3}n} + e^{-j\pi n} + 2e^{-j\frac{\pi}{3}n}$$

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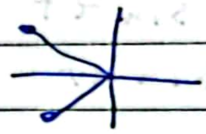
$$x[n] = 2(\cos \frac{4\pi}{3} n - j \cancel{\sin \frac{4\pi}{3} n}) + \cos \frac{2\pi}{3} n - j \cancel{\sin \frac{2\pi}{3} n}) + (-1)^n$$

$$4 \cos \frac{2\pi}{3} n + (-1)^n$$



اطلاعات زیر در مورد سیگنالی در دسترس است . سیگنال های
که در شرایط زیر صدق می کنند را بیابید؟

$x(t)$ is Reel & odd

$T = 2$

$$\int_0^2 |x(t)|^2 dt = 2 \qquad a_k = 0 \quad |k| > 1$$

$$a_2 \quad a_{-2} \quad a_2 = 0$$

$$a_{-1} \quad a_0 \quad a_1$$

$x(t) \rightarrow a_k \rightarrow$ موهومی و فرد $\boxed{a_0 = 0}$

$$\boxed{a_1 = -a_{-1}} \qquad \frac{1}{T}\int_0^T |x(t)|^2 dt = \sum |a_k|^2$$

$$1 = |a_{-1}|^2 + |a_1|^2 = 2|a_1|^2$$

$$|a_1|^2 = \frac{1}{2} \rightarrow a_1 = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{j}{2} \end{cases} \qquad a_1 = \frac{j}{\sqrt{2}} \rightarrow a_{-1} = \frac{-j}{\sqrt{2}}$$

$$a_{-1} = \frac{-j}{\sqrt{2}} \rightarrow a_{-1} = \frac{j}{\sqrt{2}}$$

$$x(t) = \sum_{k=-1}^{k=1} a_k e^{jk\frac{2\pi}{2}} + \frac{j}{2}$$

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$$x(t) = \frac{j}{\sqrt{۲}} e^{j\pi t} - \frac{j}{\sqrt{۲}} e^{-j\pi t} = \frac{j}{\sqrt{۲}} (۲j \sin \pi t)$$

$$= \begin{cases} \sqrt{۲} \sin \pi t \\ \sqrt{۲} \sin \pi t \end{cases} \qquad \frac{۲\pi}{\pi} = ۲$$

$$a_1 = -\frac{1}{\sqrt{۲} j} \qquad a_{-1} = \frac{-1}{\sqrt{۲} j}$$