«فصل دوم»

آنالیز زمانی سیستم‌های خطی و تغییر ناپذیر با زمان:

کانولوشن سیستم‌های پیوسته

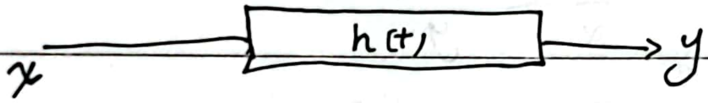
$$y(t)=x(t)*h(t)=h(t)*x(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau$$

فرمولی

$$=\int_{-\infty}^{\infty} x(t-\tau)\,h(\tau)\,d\tau$$

تصویری

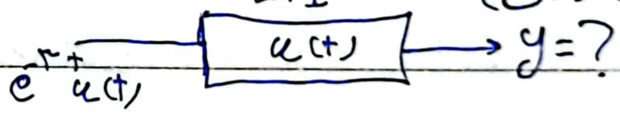
(تغییر ناپذیر با زمان خطی) LTI (Linear time Invaraint)



تمرین) خروجی سیستم LTI زیر را بیابید.

$x(t)=e^{-3t}u(t)$, $h(t)=u(t)$

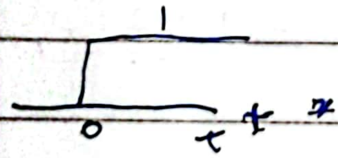
حل فرمولی)



$$y=x(t)*h(t)=\int_{-\infty}^{\infty} e^{-3\tau}\,\underline{u(\tau)}\,\underline{u(t-\tau)}\,d(\tau)$$

$u(\tau)$

$u(t)$

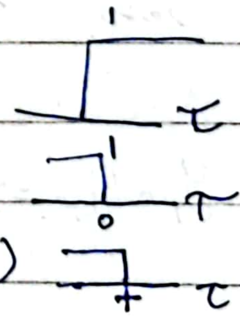


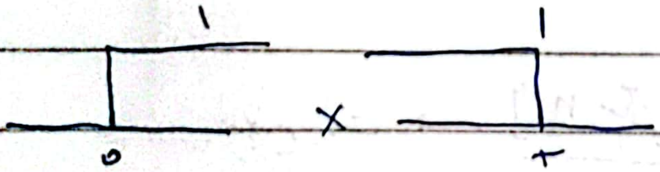
$u(t-\tau)$

① $u(\tau)$

② $u(-\tau)$

③ $u(t-\tau)$





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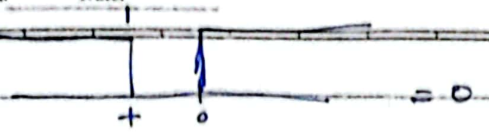
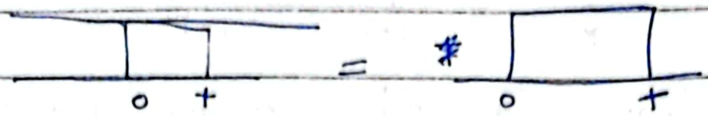
$t<0$



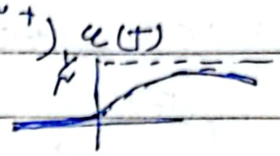
$= 0$

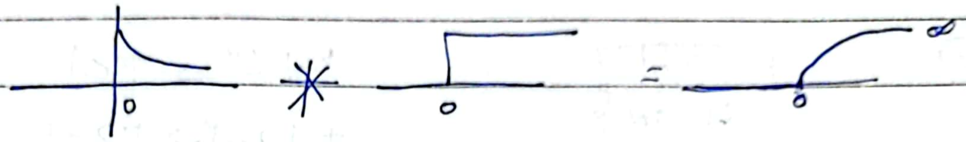
$t>0$



$$\int_0^t e^{-3\tau}\,d\tau = -\frac{1}{3}e^{-3\tau}\Big|_0^t = -\frac{1}{3}(e^{-3t}-1) \rightarrow t>0$$

$$y(t)=\begin{cases}0 & t<0\\ \frac{1-e^{-3t}}{3} & t>0\end{cases} = \frac{1}{3}(1-e^{-3t})\,u(t)$$





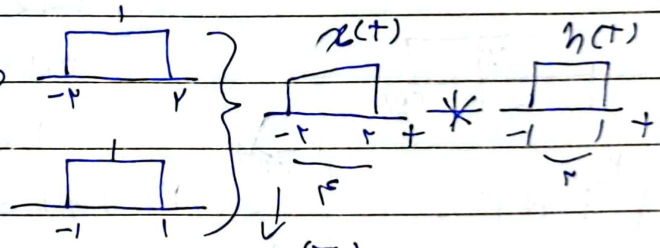
فرمول تستی : $e^{-at}u(t) * u(t) = \frac{1}{a}(1-e^{-at})\,u(t)$

$h(t)=rect(\frac{t}{2})$

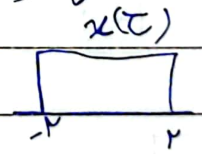
شکل ترکیبی ⟸ $x(t)=rect(\frac{t}{4}) \longrightarrow \boxed{LTI} \longrightarrow y(t)=?$

$x(t) = rect(\frac{t}{4}) \rightarrow$

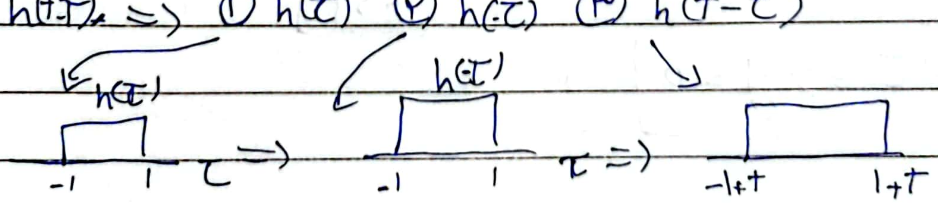
$h(t) = rect(\frac{t}{2}) \rightarrow$



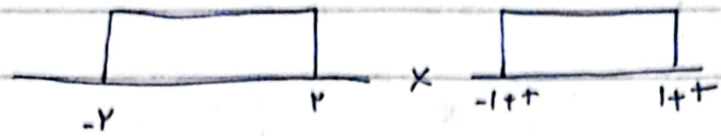
$$y=\int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau =$$



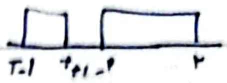
$h(t-\tau) \Rightarrow$ 1 $h(\tau)$ 2 $h(-\tau)$ 3 $h(t-\tau)$



$x(\tau)$



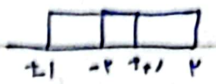
1 $t+1<-۲ \Rightarrow \boxed{t<-۳} \to y=0$



۲ $t+1>-۲ \Rightarrow t>-۳$

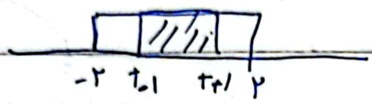
$t-1<-۲ \to t<-1$

$\boxed{-۳<t<-1}$



$$y=\int_{-۲}^{t+1} 1\,d\tau = \tau\Big|_{-۲}^{t+1} = t+1+۲ = \underline{t+۳}$$

۳



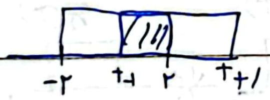
$t+1<۲ \to t<1$

$t-1>-۲ \to t>-1$

$-1<t<1$

$$\int_{t-1}^{t+1} 1\,d\tau = \tau\Big|_{t-1}^{t+1} = t+1-t+1=۲ \Rightarrow \underline{y=۲}$$

۴



$t-1<۲ \Rightarrow t<۳$

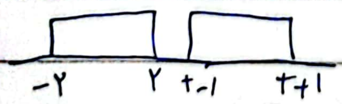
$t+1>۲ \Rightarrow t>1$

$1<t<۳$

$$\int_{t-1}^{۲} 1\,d\tau = \tau\Big|_{t-1}^{۲} = ۲-t+1 = \underline{-t+۳}$$

$$\underline{y=-t+۳}$$

۵



$t-1>۲$

$\underline{t>۳}$

$$\int_{۲}^{+\infty} 0\,d\tau \Rightarrow \underline{y=0}$$

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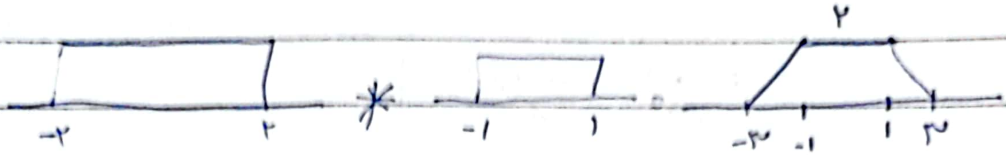
$$y=\begin{cases}0 & t<-3\\ t+3 & -3<t<-1\\ 2 & -1<t<1\\ 3-t & 1<t<3\\ 0 & t>3\end{cases}$$



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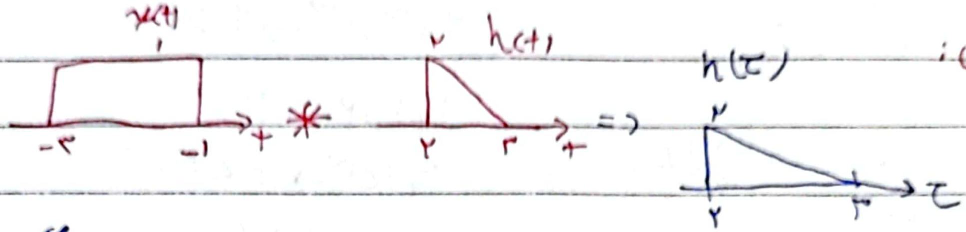
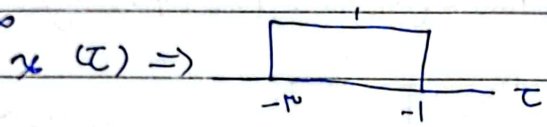


$$\int_{-\infty}^{\infty} x(t-\tau)\,h(\tau)\,d\tau$$

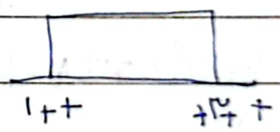
$x(t-\tau)$ = ① $x(\tau)$

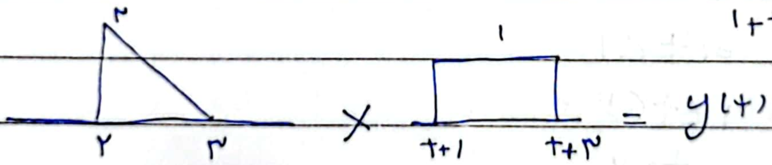
② $x(-\tau)$

③ $x(t-\tau)$



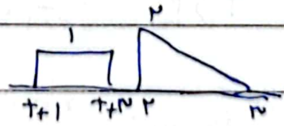
$x(t-\tau)$





① $t+3<2$

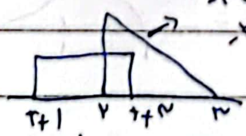
$t<-1$



$y=0$

② $-2\tau+a=y$

$-2\tau+6=y$

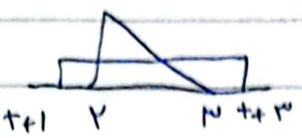


$t+3>2 \to t>-1$

$t+3<3 \to t<0$ $\Big\}\ -1<t<0$

$$\int_{2}^{t+3}(-2\tau+6)\times 1\,d\tau = -\tau^2+6\tau\Big|_{2}^{t+3} = -(t+3)^2+6(t+3)-(-4+12)$$

(۳)



$t+3>3 \rightarrow t>0$

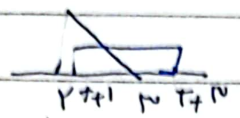
$t+1<2 \rightarrow t<1$

$0<t<1$

$$y=\int_{2}^{3} 1\times(-2\tau+6)\,d\tau = -\tau^{2}+6\tau \Big|_{2}^{3}$$

$$(\underbrace{-9+18}_{9})-(\underbrace{-4+12}_{8})=1$$

(۴)



$t+1>2 \rightarrow t>1$

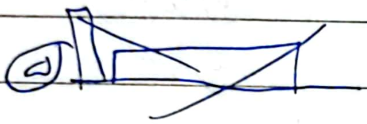
$t+1<3 \rightarrow t<2$

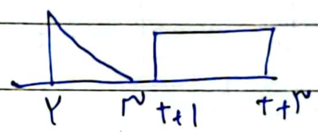
$1<t<2$

$$y=\int_{t+1}^{3} 1\times(-2\tau+6)\,d\tau$$

$$-\tau^{2}+6\tau \Big|_{t+1}^{3} = (-9+18)-(-(t+1)^{2}+6(t+1))=y$$

(۵)

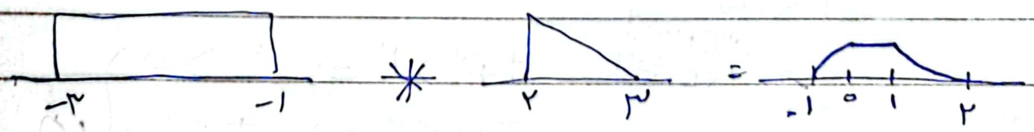




$t+1>3 \rightarrow t>2$

$y=0$

$$y\begin{cases} 0 & t<-1 \\ -(t+3)^{2}+6(t+3)-8 & -1<t<0 \\ 1 & 0<t<1 \\ (t+1)^{2}-6(t+1)+9 & 1<t<2 \\ 0 & t>2 \end{cases}$$



$h(t) \to$ پاسخ ضربه $\qquad \delta = \dfrac{du(t)}{dt}$, $\underbrace{h(t)}_{\text{پاسخ ضربه}} = \dfrac{ds(t)}{dt} \to$ پاسخ پله

$\delta(t) \longrightarrow \boxed{LTI} \longrightarrow h(t)$

$u(t) \longrightarrow \boxed{LTI} \longrightarrow s(t)$

با فرض اینکه پاسخ پله داده شده به شرح زیر باشد.

الف) پاسخ ضربه را بیابید.

ب) به دو روش پاسخ خروجی به ورودی داده شده را مشخص کنید.

$u(t) \longrightarrow \boxed{LTI} \longrightarrow s(t) = ۲\delta(۲-t) - \delta(t+1)$

$x \longrightarrow \boxed{s} \longrightarrow y_1$

$e^{-t}u(t) \longrightarrow \boxed{LTI} \longrightarrow y(t) = ? \qquad x \longrightarrow \boxed{h} \longrightarrow y \qquad \to y = y_1'$

الف) $h = s' = ۲\delta'(t-۲) - \delta'(t+1)$

ب) $y(t) = x(t) * h$

$$y_1 = e^{-t}u(t) * \underbrace{(۲\delta(۲-t) - \delta(t+1))}_{s}$$

$$y_1 = ۲e^{-t}u(t) * \delta(t-۲) - e^{-t}u(t) * \delta(t+1)$$

$$y_1 = ۲e^{-(t-۲)}u(t-۲) - e^{-(t+1)}u(t+1)$$

$$y = y_1' = ۲(-1)e^{-(t-۲)}u(t-۲) + ۲e^{-(t-۲)}\delta(t-۲) + e^{-(t+1)}u(t+1) - \delta(t+1)e^{-(t+1)} = -۲e^{-(t-۲)}u(t-۲) + ۲\delta(t-۲) + e^{-(t+1)}u(t+1) - \delta(t+1) \Rightarrow y$$

② $y = x(t) * h = e^{-t}u(t) * (۲\delta'(t-۲) - \delta'(t+1))$

$= ۲e^{-t}u(t) * \delta'(t-۲) - e^{-t}u(t)\,\delta'(t+1)$

$f(t) * \delta(t-t_0) = f(t-t_0)$

$f(t) * \delta'(t-t_0) = \frac{d}{dt} f \Big|_{t \to t-t_0}$

$= (۲e^{-t}u(t))' \Big|_{t \to t-۲} - (e^{-t}u(t))' \Big|_{t \to t+1}$

$= (-۲e^{-t}u(t) + ۲e^{-t}\delta(t)) - (-e^{-t}u(t) + e^{-t}\delta(t))$

$-۲e^{-(t-۲)}u(t-۲) + ۲\delta(t-۲) + e^{-(t+1)}u(t+1) - \delta(t+1)$

خروجی سیستم LTI زیر را در لحظه داده شده بیابید

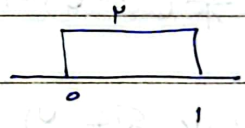
$x(t) = ۲u(t) - ۲u(t-1),\; h(t) = u(t) + e^{-t}\delta(t+1),\; t=۲$

$h(\tau) = u(\tau) + e\,\delta(\tau+1)$

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(۲) = ?$

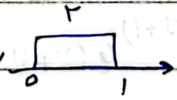
$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\,h(\tau)\,d\tau = y(۲) = \int_{-\infty}^{\infty} x(۲-\tau)\,h(\tau)\,d\tau$

$x(t) = ۲(u(t) - u(t-1)) \Rightarrow$

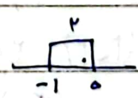


~~x~~ $x(۲-\tau)$,

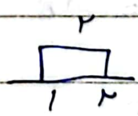
① $x(\tau)$



② $x(-\tau)$



③ $x(۲-\tau)$



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$$y(۲) = \int_{-\infty}^{\infty} \;{}^{۲}_{۱\;\;۲}\; \cdot \left(\;{}^{۱}_{۰}\; + e \;\frac{\delta(\tau+1)}{\tau=-1} \right) = \int_{-\infty}^{\infty} \;{}^{۲}_{۱\;\;۲}\; + e \;{}^{۲}_{۱\;\;۲}\; {}_{-1}$$

$$= ۲(۲-۱) = ۲$$