Assignment: 02

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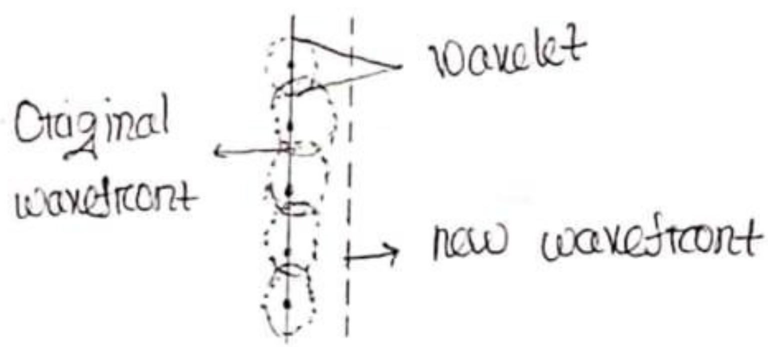
Chapter 2: Diffraction

Wavefront: A wavefront is the locus of all particles in the medium which are vibrating in the same phase.

→ An imaginary surface.

Two Types – ① Plane wavefront
② Spherical wavefront

Wavelet: A wavelet is a point on any given point of wavefront that acts a new source of disturbance to move in all possible directions with the velocity of light.



plane wavefront

Diffraction:

Diffraction is the bending or spreading of waves when they encounter an slit or obstacle.

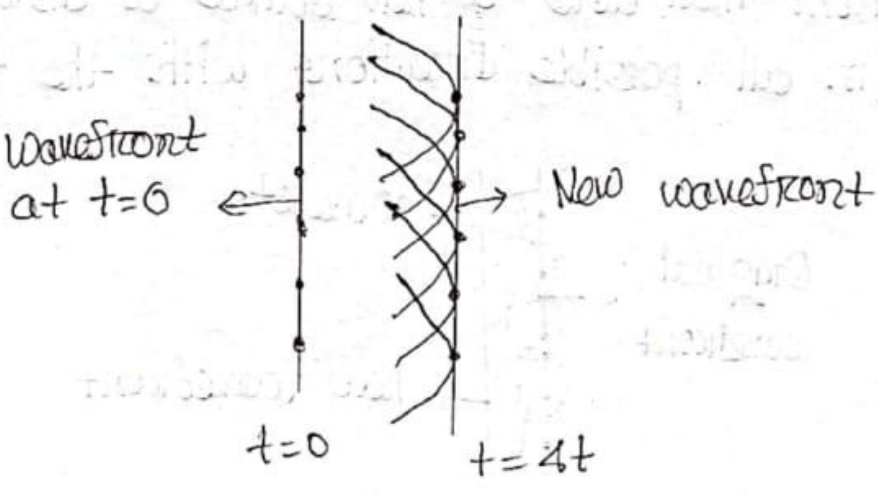
→ It is the fundamental wave phenomenon seen with light, sound and water.

→ It cannot be explained with geometric optics.
→ Requires wave optics.

There are two types of diffraction
1) Fresnel 2) Fraunhofer Diffraction

Hygen's Principle

All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.



Why there are no 'true maxima' in single slit diffraction. (except central one)
= Single slit diffraction does have a central maximum. but the other "bright fringes" on either

side of the central maximum are not true maxima like interference.

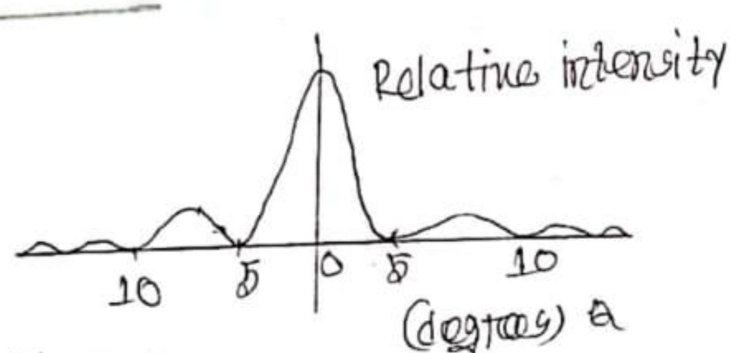
They are weaker, uneven and not equally spaced and their intensity is much lower.

Slits ↓ \ Screen →	Maxima	Minima
Double	$d\sin\theta = m\lambda$	$d\sin\theta = (2m-1)\frac{\lambda}{2}$
Single	×	$a\sin\theta = m\lambda$

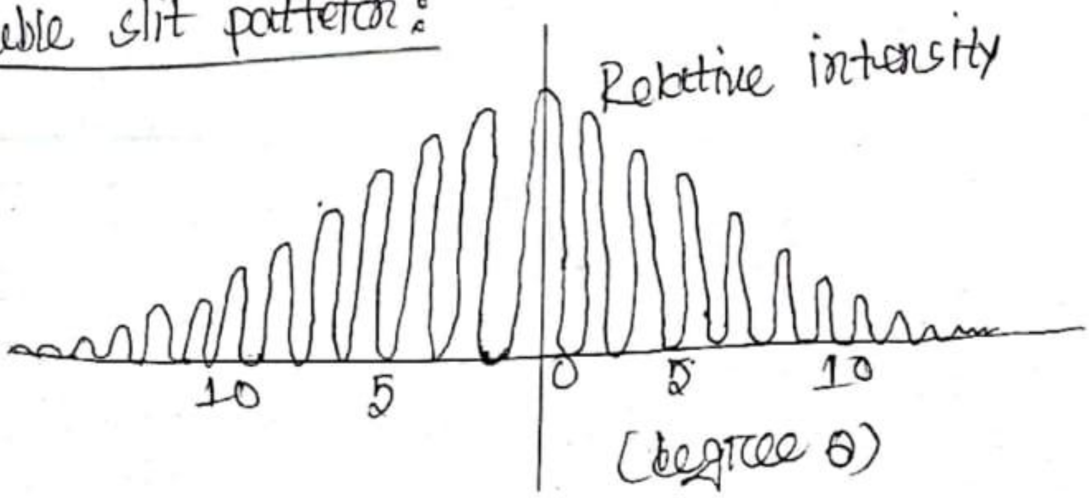
* Path length difference of single slit minima a{

$$\Delta l = \frac{a}{2}\sin\theta.$$

Single slit pattern:



Double slit pattern:



Diffraction Grating

A diffraction grating consists of surface with many equally spaced parallel slits or lines. When light passes through or reflects off the grating, it diffracts and forms an interference pattern.

It can be two types,

i) Transmission ii) Reflection

Intensity pattern in a diffraction grating:

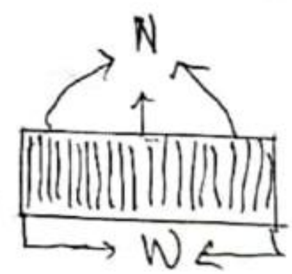
i) Monochromatic light
ii) Sharp / narrow intense peaks ⇒ maxima
iii) Broader dark spots ⇒ minima

Formula

for maxima,

$$d\sin\theta = m\lambda$$

$$\Rightarrow \frac{W}{N}\sin\theta = m\lambda$$



W = Width of the diffraction grating
N = Number of slits / rulings

Prism Vs Diffraction Grating

Feature	Prism	Diff. Grating
Working principle	Reflection	Diffraction and Interference
Wavelength Dependence	Shorter wavelength bend more than longer ones	each wavelength has its own diffraction angle
Spectrum output	Produces continuous spectrum	Produces line or discreate spectrum
Accuracy	less precise	Very precise and better resolution
Order	Only one spectrum per beam	multiple order produces ($m = 1, 2, 3 \ldots$)

Electromagnatic Spectrum table

wave	wavelength (m)	Frequency (Hz)
Radio waves	10^{3}	10^{4}
Micro waves	10^{-2}	10^{8}
IR	10^{-5}	10^{12}
visible	(400 – 700 nm)	10^{15}
UV	10^{-8}	10^{16}
X-Ray	10^{-10}	10^{18}
Gamma-Ray	10^{-12}	10^{18}

Visible light

(nm)

Violet $\longrightarrow$ 380 - 450

Blue $\longrightarrow$ 450 - 485

Cyan $\longrightarrow$ 485 - 500

Green $\longrightarrow$ 500 - 565

Yellow $\longrightarrow$ 565 - 590

Orange $\longrightarrow$ 590 - 620

Red $\longrightarrow$ 620 - 750

Two conditions, when diffraction doesn't happen.

i) When, $\lambda > d$ or When no sharp edge/obstacle/slits.

ii) When, $\lambda <<< d$

Because: ① When the opening is very wide compared to the wave length of light, the wave passes straight with little bending why? Theres not enough interaction between the edge of the wave and the aperture to cause noticeable interference or spreading and:

$$\lambda > d$$

$$\Rightarrow \frac{\lambda}{d} > 1$$

$$\Rightarrow \sin\theta > 1$$ it does not exist.

Why we can not detect diffraction of X-Ray using diffraction gratings?

⇒ While using diffraction grating to detect diffraction of x-ray, the relation of wavelength and slits becomes $\lambda <<<< d$ which does not follow the condition of diffraction.

X- Ray diffraction:

The diffraction x-ray created by crystaline solide, for X-ray maxima,

$$2d \sin\theta = m\lambda .$$

Mathematical Problems

Single and Double Slit Diffraction

1. In a double-slit arrangement the slits are separated by a distance equal to 100 times the wavelength of the light passing through the slits. What is the angular separation in radians between the central maximum and an adjacent maximum?
2. Light that has a 600-nm wavelength is incident on a long narrow slit. Which of the following slits will bend the light most?
   i. 1.0 mm
   ii. 0.1 mm, and
   iii. 0.01 mm.
3. Red light of wavelength of 700 nm falls on a double slit separated by 400 nm.
   a. At what angle is the first-order maximum in the diffraction pattern?
   b. What is unreasonable about this result?
   c. Which assumptions are unreasonable or inconsistent?You are conducting a single-slit diffraction experiment with light of wavelength .
4. What appears on a distant viewing screen, at a point at which the top and bottom rays through the slits have a path length difference equal to
   a. 5 λ and
   b. 4.5 λ?
5. What must be the ratio of the slit width to the wavelength for a single slit to have the first diffraction minimum at $\theta = 45^o$?

6. What is the wavelength of the light whose first side diffraction maximum is at $\theta = 15^0$, thus coinciding with the first minimum for the red light?
7. A single slit is illuminated by light of wavelengths a and b chosen so that the first diffraction minimum of the a component coincides with the second minimum of the b component.
   a. If $\lambda_b = 350\, nm$, what is λ_a?
   b. For what order number m_b (if any) does a minimum of the λ_b component coincide with the minimum of the λ_a component in the order number
      i. ma = 2, and
      ii. ma = 3?

Math: Single slit Grating:

1). Solution:

We know, for double slit maxima,

$$d\sin\theta = m\lambda$$

$$\Rightarrow \sin\theta = \frac{m\lambda}{d}$$

$$\Rightarrow \sin\theta = \frac{1 \times \lambda}{100\lambda}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{100}\right)$$

$$= 0.573^{\circ}$$

In Radians, $\theta = 0.01$ radians. (Ans)

Here,
$m=1$ [adjacent]
$d=100\lambda$
$\frac{d}{\lambda} = 100$

2. Solution:

for single slit,

$$a\sin\theta = m\lambda$$

$$\sin\theta = m\lambda \frac{1}{a}$$

Therefore, $\sin\theta$ is proportional to the slit separation.

so, smallest slit, 0.01 mm bends light the most.

Answer = (iii) 0.01 mm

3. Solution:

ⓐ for double slit maxima,

$$d\sin\theta = m\lambda$$
$$\Rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$
$$\Rightarrow \theta = \sin^{-1}\left(\frac{700\times10^{-9}}{400\times10^{-9}}\right)$$
$$\Rightarrow \theta = \text{doesn't exist}$$

$m = 1$,
$\lambda = 700\times10^{-9}$ m
$d = 400\times10^{-9}$ m

ⓑ
since, $\lambda > d$ but the condition is $d > \lambda$
thats why, diffraction doesnot happen here,
there would be central maxima.

ⓒ for single slit,
the condition is $d > \lambda$
since here $d \not> \lambda$
Having a diffraction pattern or having more the one minima are unreasonable, as there will not happen any kind of diffraction.

4. Solution:

a) 5λ is equavalent to $m\lambda$, which is the condition of a constractive interference.

So, there will appear a maxima.

b) 4.5λ is equavalent to $(2m-1)\frac{\lambda}{2}$, which is the condition of destructive interference.

So, there will appear a minima.

5. Solution:

We know that

$m = 1$
$\theta = 45^\circ$

$$a \sin\theta = m\lambda$$

$$\Rightarrow \frac{a}{\lambda} = \frac{m}{\sin\theta} = \frac{m}{\sin(45^\circ)} = \sqrt{2} \quad (Ans)$$

6. Solution:

For red light diffraction,

$\lambda = 700\ nm$
$= 700 \times 10^{-9}\ m$
$\theta = 15^\circ$
$m = 1$

$$d \sin\theta = m\lambda$$

$$d = \frac{m\lambda}{\sin\theta}$$

$$= \frac{1 \times 700 \times 10^{-9}}{\sin(15^\circ)}$$

$$= 2.70 \times 10^{-6}\ m$$

For first side diffraction,

we know that,

$d \sin\theta = 1.5\lambda$

$d = 2.7 \times 10^{-6} m$

$m = 1.5$

$\theta = 15^\circ$

$\Rightarrow \lambda = \frac{d \sin\theta}{1.5}$

$= \frac{2.7 \times 10^{-6} \times \sin(15)}{1.5}$

$= 4.65 \times 10^{-7} m$

$= 465$ nm (Ans)

7. Solution

a) Given,

$\lambda_b = 350 \times 10^{-9} m$

$m_b = 2$

$m_a = 1$

$\lambda_a = ?$

We know, $m_a \lambda_a = m_b \lambda_b$

$\Rightarrow \lambda_b = \frac{2 \times 350 \times 10^{-9}}{1} = 700 \times 10^{-9} m$ (Ans)

b) 1. At $m_a = 2$

$\lambda_a = 700 \times 10^{-9} m$

$\lambda_b = 350 \times 10^{-9} m$

$\therefore m_b \lambda_b = m_a \lambda_a$

$$\Rightarrow m_b = \frac{2 \times 700 \times 10^{-9}}{350 \times 10^{-9}} = 4 \text{ (Ans)}$$

b) 2.

At $m_a = 3$

$\therefore m_a \lambda_b = m_a \lambda_a$

$$\Rightarrow m_b = \frac{3 \times 700 \times 10^{-9}}{350 \times 10^{-9}}$$

$$= 6 \text{ (Ans)}$$

Diffraction Grating

1. A diffraction grating 20.0 mm wide has 6000 rulings. Light of wavelength 589 nm is incident perpendicularly on the grating. What are the
    a. Largest
    b. second largest, and
    c. third largest

    values of θ at which maxima appears on a distant viewing screen?

2. If a diffraction grating produces a third-order bright spot for green light (of wavelength 530 nm) at 65.0° from the central maximum, at what angle will the second-order bright spot be for red light (of wavelength 700 nm)?
3. A grating has 400 lines/mm. How many orders of the entire visible spectrum (400–700 nm) can it produce in a diffraction experiment, in addition to the $m = 0$ order?
4. A diffraction grating is made up of slits of width 300 nm with separation 900 nm. The grating is illuminated by monochromatic plane waves of wavelength 600 nm at normal incidence. How many maxima are there in the full diffraction pattern?
5. What is the angular width of a spectral line observed in the first order if the grating has 1000 slits/cm? ($\lambda = 500$ nm)
6. Monochromatic light is at normal incidence on a plane transmission grating. The first-order maximum in the interference pattern is at an angle of 8.94°. What is the angular position of the fourth-order maximum?
7. If a diffraction grating produces a third-order bright spot for red light (of wavelength 700 nm) at 65.0° from the central maximum, at what angle will the second-order bright spot be for violet light?

8. (a) What is the wavelength of light that is deviated in the first order through an angle of 13.5° by a transmission grating having 5000 slits/cm? (b) What is the second-order deviation of this
9. wavelength? Assume normal incidence.

Math Solve: Diffraction Grating:

1. Solution:

$w = 20\ mm = 0.02\ m$

$N = 6000$

$d = \frac{0.02}{6000} = 3.33\times10^{-6} m$

We know that, $d\sin\theta = m\lambda$

$$m_{max} = \frac{d}{\lambda} = \frac{3.33\times10^{-6}}{589\times10^{-9}} = 5.65 \approx 5$$

a) largest $(m=5)$

$$\theta_5 = \sin^{-1}\left(\frac{5\times589\times10^{-9}}{3.33\times10^{-6}}\right) = 61.9^\circ$$

b) Second largest $(m=4)$

$$\theta_4 = \sin^{-1}\left(\frac{4\times589\times10^{-9}}{3.33\times10^{-6}}\right) = 48.1^\circ$$

c) Third largest $(m=3)$.

$$\theta_3 = \sin^{-1}\left(\frac{3\times589\times10^{-9}}{3.33\times10^{-6}}\right) = 34.3^\circ$$

(Ans)

2. Solution:

Given for green,

$m = 3$

$\lambda = 530\ nm = 530\times10^{-9} m$

$\theta = 65^\circ$

$$d = \frac{m\lambda}{\sin\theta} = \frac{3\times530\times10^{-9}}{\sin 65^\circ} = 1.75\times10^{-6}\ m$$

$$\sin\theta_{red} = \frac{2\times700\times10^{-9}}{d}$$

$$\Rightarrow \theta_{red} = \sin^{-1}\left(\frac{2\times700\times10^{-9}}{1.75\times10^{-6}}\right) = 53.13^\circ \text{ (Ans)}$$

3. Solution:

for diffraction,

$$d\sin\theta = m\lambda$$

$$\Rightarrow m = \frac{d\sin\theta}{\lambda}$$

$$\Rightarrow m \propto \frac{1}{\lambda}$$

Therefore $\frac{W}{N}\sin\theta = m\lambda$

$$\Rightarrow m = \frac{W\sin\theta}{N\lambda}$$

$$= \frac{1\times10^{-3}\times\sin(90)}{400\times700\times10^{-9}}$$

$$= 3.57 \approx 3$$

$\theta = 90^\circ$ highest number m

So, 3 orders of the entire visible spectrum could be produced in addition $m=0$.

4. Solution:

Grating maxima condition,

$$d\sin\theta = m\lambda$$

$$\Rightarrow m = \frac{d}{\lambda} = \frac{900}{600} = 1.5 \approx 1$$

Therefore, the total maxima on full diffraction pattern is $(2\times1+1) = 3$

Total maxima = 3 (Ans)

5. solution:

We know,

$$\frac{W}{N}\sin\theta = m\lambda$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{100\times1\times500\times10^{-9}}{0.01}\right)$$

$$= 2.87^\circ \text{ (Ans)}$$

$\lambda = 500\ nm$

$N = 10^5$ slits/m

$\therefore d = \frac{1}{10^5} = 1\times10^{-5}m$

6. Solution:

1st-order maximum at 8.04°

For 1st order:

$$d\sin\theta_1 = \lambda$$

$$\Rightarrow \lambda = d\sin(8.04^\circ)$$

For $m=4$

$$\sin\theta_4 = \frac{4\lambda}{d} = 4\sin(8.04^\circ) = 0.62$$

$$\Rightarrow \theta_4 = \sin^{-1}(0.62) = 38.3^\circ \text{ (Ans)}$$

7. Solution.

Red light ($\lambda_1 = 700\, nm$)
$m = 1,\ \theta = 65°$
violet light ($\lambda_2 = 400\, nm$)
$m = 1,\ \theta = ?$

From the red light;

$\cancel{d \sin(65°) = 3.70}$

$$\Rightarrow d = \frac{2100}{\sin 65°} = 2317.8\ nm$$

θ for violet light,

$$\sin\theta = \frac{2 \times 400}{2317.8} = 0.345$$

$$\therefore \theta = \sin^{-1}(0.345) = 20.2° \text{ (Ans)}$$

8. Solution:

a) Given,

$N = 5000 \times 100 = 5 \times 10^5$ slits/m.
$m = 1,\ \theta = 13.5°$
$W = 1\ cm = 0.01\ m$
$\lambda = ?$

We know,

$$d \sin\theta = m\lambda$$

$$\lambda = \frac{0.01 \times \sin(13.5°)}{5000} = 4.6 \times 10^{-7}\ m \text{ (Ans)}$$

b. we know,

$d \sin\theta = m\lambda$

$\Rightarrow \sin\theta = \frac{m\lambda}{d}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{2\times 5000 \times 4.6\times 10^{-7}}{0.01}\right)$

$= 27.84$ (Ans)

$m = 2$

$\lambda = 4.6\times 10^{-7}\ m$

$d = \frac{0.01}{5000}$

$\theta = ?$

Bragg's Law (X-ray diffraction)

1. What is the smallest Bragg angle for x rays of wavelength 30 pm to reflect from reflecting planes spaced 0.30 nm apart in a calcite crystal?
2. An x-ray beam of wavelength undergoes first-order reflection (Bragg law diffraction) from a crystal when its angle of incidence to a crystal face is 23°, and an x-ray beam of wavelength 97 pm undergoes third-order reflection when its angle of incidence to that face is 60°. Assuming that the two beams reflect from the same family of reflecting planes, find
   a. the interplanar spacing
   b. the wavelength.
3. X rays of wavelength 0.12 nm are found to undergo second order reflection at a Bragg angle of 28° from a lithium fluoride crystal.What is the interplanar spacing of the reflecting planes in the crystal?

Math: X-Ray Diffraction

1. Solution:

We know,

$$2d \sin\theta = m\lambda$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right)$$

$$= \sin^{-1}\left(\frac{1 \times 30 \times 10^{-12}}{2 \times 0.3 \times 10^{-9}}\right)$$

$$= 2.87^\circ \text{ (Ans)}$$

Given,

$m = 1$.

$\lambda = 30\ pm$

$= 30 \times 10^{-12} m$

$d = 0.30\ nm$

$= 0.3 \times 10^{-9}\ m$

$\theta = ?$

2. Solution:

(a) For second beam,

$$2d \sin\theta_2 = m\lambda_2$$

$$d = \frac{m\lambda_2}{2\sin\theta_2} = \frac{3 \times 97 \times 10^{-12}}{2 \cdot \times \sin 60^\circ}$$

$$= 1.6 \times 10^{-10} m \text{ (Ans)}$$

Given,

$\lambda_2 = 97 \times 10^{-12} m$

$m_2 = 3$

$\theta_2 = 60^\circ$

$d = ?$

b) For first beam,

$2d\sin\theta_1 = m_1\lambda_1$

$\Rightarrow \lambda_1 = \frac{2d\sin\theta_1}{m_1}$

$= \frac{2\times1.68\times10^{-10}\times\sin(23)}{1}$

$= \cancel{1}\ 1.31\times10^{-10}\ m$ (Ans)

$m_1 = 1$
$\theta_1 = 23^\circ$
$d = 1.6\times10^{-10}\ m$

3. Solution:

Bragg's law,

$2d\sin\theta = m\lambda$

$\Rightarrow d = \frac{m\lambda}{2\sin\theta}$

$= \frac{2\times0.12\times10^{-9}}{2\sin(28)}$

$= 2.56\times10^{-10}\ m$

(Ans)

$m = 2$
$\lambda = 0.12\ nm$
$= 0.12\times10^{-9}$
$\theta = 28^\circ$