Q1

May 9, 2024

Mahdi Anvari 610700002 Homework2 of ML Question1

[187]:
```
# importing libraries
import numpy as np
import matplotlib.pyplot as plt
from sklearn.cluster import KMeans
from sklearn.preprocessing import StandardScaler
from scipy.cluster.hierarchy import linkage, fcluster
from sklearn.cluster import SpectralClustering
from sklearn import metrics
import pandas as pd
from sklearn.metrics import adjusted_rand_score
import seaborn as sbn
```

a. Cluster the data using the following methods. Use the same number of clusters as specified in the dataset. For each method, describe what distance or similarity metric you have used. i. K-means ii. Hierarchical clustering (average linkage) iii. Hierarchical clustering (single linkage) iv. Hierarchical clustering (complete linkage) v. Spectral clustering. Describe how you defined the graph.

[24]:
```
# Pathbased dataset
# N=300, k=3, D=2
```

[25]:
```
PathbasedDataset = np.loadtxt('pathbased.txt')
PathbasedScaler = StandardScaler()
ScaledPathbasedDataset = PathbasedScaler.fit_transform(PathbasedDataset[:,:2])
print(PathbasedDataset[:,2])
```

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[1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
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[26]:

```python
# K-means
PathbasedKmeans = KMeans(n_clusters=3, n_init = 10)
PathbasedKmeans.fit(ScaledPathbasedDataset)
PathbasedClusters = PathbasedKmeans.labels_
print(PathbasedClusters)
```

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[27]:

```python
# Hierarchical clustering (average linkage)
PathbasedAverageLinkage = linkage(ScaledPathbasedDataset, method='average',␣
 ↪metric='euclidean')
PathbasedClusters2 = fcluster(PathbasedAverageLinkage, t=3 ,␣
 ↪criterion='maxclust')
print(PathbasedClusters2)
```

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[2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1
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```

[28]:

```python
# Hierarchical clustering (single linkage)
PathbasedSingleLinkage = linkage(ScaledPathbasedDataset, method='single',␣
 ↪metric='euclidean')
PathbasedClusters3 = fcluster(PathbasedSingleLinkage, t=3 ,␣
 ↪criterion='maxclust')
print(PathbasedClusters3)
```

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[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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[29]:
```
# Hierarchical clustering (complete linkage)
PathbasedCompleteLinkage = linkage(ScaledPathbasedDataset, method='complete',␣
 ↪metric='euclidean')
PathbasedClusters4 = fcluster(PathbasedCompleteLinkage, t=3 ,␣
 ↪criterion='maxclust')
print(PathbasedClusters4)
```

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[3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 1 1 1 1 1 1 1 1 1 1 1
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 3 3 3 3]
```

[30]:
```
# Spectral clustering
PathbasedSpectral = SpectralClustering(n_clusters= 3)
PathbasedClusters5 = PathbasedSpectral.fit_predict(ScaledPathbasedDataset)
print(PathbasedClusters5)
```

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[2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0
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 2 2 2 2]
```

[31]:
```
# Spiral dataset
# N=312, k=3, D=2
```

[32]:
```
SpiralDataset = np.loadtxt('spiral.txt')
SpiralScaler = StandardScaler()
ScaledSpiralDataset = SpiralScaler.fit_transform(SpiralDataset[:,:2])
print(SpiralDataset[:,2])
```

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[3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3.
 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3. 3.
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 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
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```

[33]:
```
# K-means
SpiralKmeans = KMeans(n_clusters=3 , n_init=10)
SpiralKmeans.fit(ScaledSpiralDataset)
SpiralClusters = SpiralKmeans.labels_
print(SpiralClusters)
```

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[2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1
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 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1]
```

[34]:
```
# Hierarchical clustering (average linkage)
SpiralAverageLinkage = linkage(ScaledSpiralDataset, method='average',␣
 ↪metric='euclidean')
SpiralClusters2 = fcluster(SpiralAverageLinkage, t=3 , criterion='maxclust')
print(SpiralClusters2)
```

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[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 3 3 3 3 3 3 3 3 3
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 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3]
```

[35]:
```
# Hierarchical clustering (single linkage)
SpiralSingleLinkage = linkage(ScaledSpiralDataset, method='single',␣
 ↪metric='euclidean')
SpiralClusters3 = fcluster(SpiralSingleLinkage, t=3 , criterion='maxclust')
print(SpiralClusters3)
```

```
[2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
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 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3]
```

[36]:
```
# Hierarchical clustering (complete linkage)
SpiralCompleteLinkage = linkage(ScaledSpiralDataset, method='complete',␣
 ↪metric='euclidean')
SpiralClusters4 = fcluster(SpiralCompleteLinkage, t=3 , criterion='maxclust')
print(SpiralClusters4)
```

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[3 3 3 3 3 3 3 3 3 3 3 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
```

[37]:
```
# Spectral clustering
SpiralSpectral = SpectralClustering(n_clusters= 3)
SpiralClusters5 = SpiralSpectral.fit_predict(ScaledSpiralDataset)
print(SpiralClusters5)
```

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[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2
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```

[38]:
```
# Jain dataset
# N=373, k=2, D=2
```

[39]:
```
JainDataset = np.loadtxt('jain.txt')
JainScaler = StandardScaler()
ScaledJainDataset = JainScaler.fit_transform(JainDataset[:,:2])
print(JainDataset[:,2])
```

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[2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2.
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```

[40]:

```python
# K-means
JainKmeans = KMeans(n_clusters=2 , n_init=10)
JainKmeans.fit(ScaledJainDataset)
JainClusters = JainKmeans.labels_
print(JainClusters)
```

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[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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```

[41]:

```python
# Hierarchical clustering (average linkage)
JainAverageLinkage = linkage(ScaledJainDataset, method='average',␣
 ↪metric='euclidean')
JainClusters2 = fcluster(JainAverageLinkage, t=2 , criterion='maxclust')
print(JainClusters2)
```

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[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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2 2 2]
```

[42]:
```python
# Hierarchical clustering (single linkage)
JainSingleLinkage = linkage(ScaledJainDataset, method='single',␣
 ↪metric='euclidean')
JainClusters3 = fcluster(JainSingleLinkage, t=2 , criterion='maxclust')
print(JainClusters3)
```

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[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1]
```

[44]:
```python
# Hierarchical clustering (complete linkage)
JainCompleteLinkage = linkage(ScaledJainDataset, method='complete',␣
 ↪metric='euclidean')
JainClusters4 = fcluster(JainCompleteLinkage, t=2 , criterion='maxclust')
print(JainClusters4)
```

```
[2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
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 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1]
```

[45]:
```python
# Spectral clustering
JainSpectral = SpectralClustering(n_clusters= 2)
JainClusters5 = JainSpectral.fit_predict(ScaledJainDataset)
print(JainClusters5)
```

```
[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 0 0 0]
```

[46]:
```
# Flame dataset
# N=240, k=2, D=2
```

[47]:
```
FlameDataset = np.loadtxt('flame.txt')
FlameScaler = StandardScaler()
ScaledFlameDataset = FlameScaler.fit_transform(FlameDataset[:,:2])
print(FlameDataset[:,2])
```

```
[1. 1. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2.
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 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

[48]:
```
# K-means
FlameKmeans = KMeans(n_clusters=2 , n_init=10)
FlameKmeans.fit(ScaledFlameDataset)
FlameClusters = FlameKmeans.labels_
print(FlameClusters)
```

```
[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
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 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
```

[49]:
```
# Hierarchical clustering (average linkage)
FlameAverageLinkage = linkage(ScaledFlameDataset, method='average',␣
 ↪metric='euclidean')
FlameClusters2 = fcluster(FlameAverageLinkage, t=2 , criterion='maxclust')
print(FlameClusters2)
```

```
[2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 1 1 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
```

```
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 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2]
```

[50]:
```
# Hierarchical clustering (single linkage)
FlameSingleLinkage = linkage(ScaledFlameDataset, method='single',␣
 ↪metric='euclidean')
FlameClusters3 = fcluster(FlameSingleLinkage, t=2 , criterion='maxclust')
print(FlameClusters3)
```

```
[2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
```

[51]:
```
# Hierarchical clustering (complete linkage)
FlameCompleteLinkage = linkage(ScaledFlameDataset, method='complete',␣
 ↪metric='euclidean')
FlameClusters4 = fcluster(FlameCompleteLinkage, t=2 , criterion='maxclust')
print(FlameClusters4)
```

```
[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
```

[52]:
```
# Spectral clustering
FlameSpectral = SpectralClustering(n_clusters= 2)
FlameClusters5 = FlameSpectral.fit_predict(ScaledFlameDataset)
print(FlameClusters5)
```

```
[1 1 1 1 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
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 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
```

[53]:
```
# S1 dataset
# Synthetic 2-d data with N=5000 vectors and k=15 Gaussian clusters with␣
 ↪different degree of cluster overlap
```

```python
[70]: S1Dataset = np.loadtxt('s1.txt')
      S1Scaler = StandardScaler()
      ScaledS1Dataset = S1Scaler.fit_transform(S1Dataset)
```

```python
[63]: # K-means
      S1Kmeans = KMeans(n_clusters=15 , n_init=10)
      S1Kmeans.fit(ScaledS1Dataset)
      S1Clusters = S1Kmeans.labels_
      print(S1Clusters[:1000])
```

```
[10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
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  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  5  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  5  0  0  6  6
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
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 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11]
```

[64]:

```
# Hierarchical clustering (average linkage)
S1AverageLinkage = linkage(ScaledS1Dataset, method='average',␣
 ↪metric='euclidean')
S1Clusters2 = fcluster(S1AverageLinkage, t=15 , criterion='maxclust')
print(S1Clusters2[:1000])
```

```
[4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
 4 4 4 4 4 4 4 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 3 4 4 4 4 4 4 4 4 4 4 4 4 4
 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
 4 4 4 4 2 2 2 2 2 7 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 7 2 2 2 2 2
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 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6
 6]
```

[65]:

```
# Hierarchical clustering (single linkage)
S1SingleLinkage = linkage(ScaledS1Dataset, method='single', metric='euclidean')
S1Clusters3 = fcluster(S1SingleLinkage, t=15 , criterion='maxclust')
print(S1Clusters3[:1000])
```

```
[10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
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 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
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 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12  4  4  4  4  4  4  4  4
  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4
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  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4
  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4
  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4
  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4
  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12]
```

[66]:

```python
# Hierarchical clustering (complete linkage)
S1CompleteLinkage = linkage(ScaledS1Dataset, method='complete',␣
 ↪metric='euclidean')
S1Clusters4 = fcluster(S1CompleteLinkage, t=15 , criterion='maxclust')
print(S1Clusters4[:1000])
```

```
[9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
```

```
 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
 9 9 9 9 9 9 9 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
 9 9 9 9 5 5 5 5 5 6 5 5 5 5 5 5 6 5 5 5 5 5 5 5 5 5 5 5 5 5 5 7 6 5 5 5 5
 5 5 5 5 6 5 5 5 5 5 6 6 5 5 5 5 5 5 5 5 5 6 5 5 5 5 6 5 5 5 5 5 5 5 5 5 5
 5 5 5 6 6 5 5 5 5 7 5 6 5 5 5 5 5 5 5 5 5 5 6 5 5 6 5 5 5 5 6 5 5 5 5 6 5
 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 6 5 5 5 5 6
 6 6 6 5 5 5 5 5 6 5 5 5 5 5 5 5 5 5 5 6 5 5 5 5 6 5 5 6 5 5 5 5 5 6 5 5 5
 5 6 5 5 5 5 5 5 5 5 5 5 5 5 6 5 6 5 5 5 5 5 5 5 5 6 6 6 5 5 5 5 5 6 5 5 5
 5 5 5 6 5 5 5 5 5 5 6 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 6 5 5 6 5 5 5 5
 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 6 5 5 5 5 5 5 5 5 6 5 5 5 5 5 5 5 5 5 5 5
 5 5 5 5 5 5 5 6 5 5 5 5 5 5 5 5 5 5 5 5 6 5 5 5 3 3 3 3 3 3 3 3 3 3 3 3 3
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 3 3 3 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2]
```

[68]:
```python
# Spectral clustering
S1Spectral = SpectralClustering(n_clusters= 15)
S1Clusters5 = S1Spectral.fit_predict(ScaledS1Dataset)
print(S1Clusters5[:1000])
```

```
[12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
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 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
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 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 13 12 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 13 12 12 12 12 12 12 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
 12 12 12 12 12 12 12 12 12 12 12 12  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  8  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  8  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
```

```
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6 10 10 10 10 10 10 10 10
 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
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 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10  1 10
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 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
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 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
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 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14
 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14
 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14]
```

[69]:
```
# S4 dataset
# Synthetic 2-d data with N=5000 vectors and k=15 Gaussian clusters with different degree of cluster overlap
```

[71]:
```
S4Dataset = np.loadtxt('s4.txt')
S4Scaler = StandardScaler()
ScaledS4Dataset = S4Scaler.fit_transform(S4Dataset)
```

[74]:
```
# K-means
S4Kmeans = KMeans(n_clusters=15 , n_init=10)
S4Kmeans.fit(ScaledS4Dataset)
S4Clusters = S4Kmeans.labels_
print(S4Clusters[:1000])
```

```
[ 4  4  7  9  4  4  4  9  4  4  4  4  7  4  4  4  7  4  7  9  6 10  4  4
  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4  7  4 10  4
  4  4  7  6  7  4  4  4  9  4  7  4  4  4  4  4  4  4  4  7  4  6  4  4
  4  4  4  4  4  4  4  4  4  7  6  4  4  9  4 10  4  9  4  4  4  4  4  4
  9  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4 10  4  4  4  4  4  4  4
  9  4  4  4  4  4  4  4  4  4  9  7  4  4  4  7  4  7  9  4  9  4  4  4
  4  4  4  4  4  4  9  4  4 10  4  4 10  4  4  4  7  4 10  4  4  4  4  9
  4  4  4  4  4  4  4  4  4  4  7  4  4  4  4  4  4  4  4  4  4  4  4  9
```

```
 7  4  4 10  9  4  4  4  4 10  4  9  4  7  4  4  4  4  4  4  4  4  4  4
 4  4  4  4  4  4  4  4  4  4  7  4  4  4  4  4  4  4  4  4  4  4  4  4
 4  4  4  7  4  4  4  4  4  4  4  4  4  4  4  4  7  4  4 10  7  4  4  4
 4  7  4  7  4  4  4  4  4  4  4  4  9  6  4  4  4  4  4  4  4  7  4  4
 4  7  7  4  4  4  4  4  6  4  4  4 13 13 13 13 13 13 13  9 11 13 13 13
 6 13  3  0 13 13 13 13 13 13 13 13 13 13 13 13 13  6 13  6 13 13 13  9
13  6 13 13 13 13 13  0 13 13 13  9  6 13  0 13 13 13  6 13 13 13  9 13
13 13  3  9 13 13  3 13 13 13 13 13 13 13  6 13  3 13  3  3 13 13 13 13
 9 13  0 13 13 13 13 13 13 13 13 13 13 13 13  0 13 13 13 13  6 13  0 13
 3  3 13 13 13  6 13 13 13 13 13  3 13 13  6 13 13 13 13 13 13 13 13 13
13 13  9 13 13 13 13 13 13 13  9  6  3 13  3 13 13  3 13 13  3 13 13 13
 9 13  3 13 13 13 13 13 13 13 13  3 13 13 13 13 13 13 13 13 13 13 11 13
13 13 13  0 13 13 13 13  3 13 13 13 13 13 13 13 13 13 13 13 13 13 13  9
13  3 13 13  6 13 13  6 13  6 13 13  6 13 13 13 13  3  9 13  3 13 13 13
13 13 13 13 13 13 13  6  9 13  3 13 13 13  0  6 13 13 13  3 13 13 13 13
13 13 13 13 13 13 13 13  9  3 13 13 13  3 13 13  6 13 13 13  0  0 13 13
13 11 13 13  6 13  6 13 13 13 13 13  6 13 13 13 13 13 13  0 13 13  6 13
13 13 13 13 13 13 13 13 13 13  6  0  3 13 13 13  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2 11  2  2  2  2 11  2  2  2  2  2  2  2
 2  2  2  2 11  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2 11  2 11  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2 11  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2 11  2 11  2  2  2  2  2  2  2  2  2  2
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 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
 2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2 11  2  2  2
 2  2  2  2  2  2  2 10 10 10 10 10  7 10 10 10  2 10 10 10 10 10 10 10
10 10 10  4 10 10 10 10 10 10  2 10  4 10 10 10 10 10 10 10 10 10 10 10
10 10 10 10 10 10 10 10 10 10 10 10 10  2 10  2]
```

[75]:
```
# Hierarchical clustering (average linkage)
S4AverageLinkage = linkage(ScaledS4Dataset, method='average',␣
 ↪metric='euclidean')
S4Clusters2 = fcluster(S4AverageLinkage, t=15 , criterion='maxclust')
print(S4Clusters2[:1000])
```

```
[12 13 13 12 13 13 13 12 13 12 13 13 13 13 13 13 13 13 12 13 10 13 13
 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 10 13
 13 12 13 15 13 13 13 13 12 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13
 13 13 13 12 13 13 13 13 13 13 13 13 13 12 13 10 13 12 13 13 13 12 13 13
 12 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 10 13 13 13 13 13 13 13
 12 13 13 13 13 13 13 13 13 13 12 13 13 13 13 13 13 13 13 13 12 13 13 13
 13 13 13 13 13 13 12 13 13 10 13 13 10 13 13 13 13 13 10 13 13 13 13 12
```

```
13 13 13 13 13 13 13 13 13 13 13 12 13 13 13 13 13 13 13 13 13 12
13 13 13 10 13 13 13 13 13 10 13 13 13 13 13 13 12 13 13 13 13 13 13 13
12 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13
13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 10 13 13 13 13
13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13
13 13 13 13 13 13 13 13 15 13 13 13  4  4  4  4  4  4  4 12 15  4  4  4
15  4  2 15  4  4  4  4  4  4  4  4  4  4  4  4  4 15  4 15  4  4  4 12
 4 15  4  4  4  4  4 15  4  5  4 12 15  4 15  4 15 15 15  4  4  4  4  4
15  4  2 12  4  2  2  4  4  4  4  4  4  4 15 15  5  4  5  2  4  4  4  4
12  4 15  4  4 15  4  4 15  4  4  4  4  4  4 15  4  4  4  4 15  4 15  4
 2  5  4  4  4 13  4  4 15  4  4  2  4  5 15  4  4  4  4  4  4  4  4  4
 4 15 12  4  4  4  4  4  4  5 12 15  2  4  2  4  4  2  4  4  2  4  4  4
 4  4  2  4  6  4  4  4  4  4  4  2  4  4 15  4  4  4  4  4 15  4 14  4
 4  4  4 15 15  5  4  4  2  4  4 15  4  4  4  2  4  4  4 15  5  4  4  4
 4  2  4  4 15  4  4 15  4 15  4  4 15  4  4  4 15  2 15  4  2  4  4  4
 4  4  4  4  4  4  4 15 12  4  2  4  4  4  2 15  5  4  4  5  4 15 15  4
 4  4  5  4  4  5  4 15 12  5  4  4  4  5 15  4 15  4  4  4 15 15  4  5
 4 14  4  4 15 15 15  4  4  4  4  4 15  4  5  4  4  4  4 15  4  4 13  4
 4  4  4  4  4  4  4  4  4  4 15  2  5  4  4  4 11 11 11 11 11 11 11 11
11 11 11 11 11 11 11 11 11 11 11 14 11 11 11 11 14 11 11 11 11 11 11 11
11 11 11 14 14 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11
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11 11 11 11 11 11 14 11 11 11 14 14 11 14 11 11 11 11 11 11 11 11 11 11
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11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11
11 11 14 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 14 11 11 11
11 11 11 11 11 11 11 10 10 10 10 10 13 10 10 10 10 10 10 10 10 10 11 10
10 10 10 13 10 10 10 10 10 10 10 10 13 10 10 10 10 10 10 10 11 10 10 10
10 10 11 10 10 10 10 10 10 10 10 10 10 11 10 11]
```

[76]:
```
# Hierarchical clustering (single linkage)
S4SingleLinkage = linkage(ScaledS4Dataset, method='single', metric='euclidean')
S4Clusters3 = fcluster(S4SingleLinkage, t=15 , criterion='maxclust')
print(S4Clusters3[:1000])
```

```
[ 5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
```

```
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5 11  5  5  5  5  5  3  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  4  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5 15  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  4  5  5  5  5  5  5  5  5  5  5  5  5  5  5  4  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  3  5  5  5  5
  5  5  4  5  5  5  5  5  5  3  5  5  5  5  5  5  5  5  5  5  5  5  5  4
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  4  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  3  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5]
```

[77]:

```
# Hierarchical clustering (complete linkage)
S4CompleteLinkage = linkage(ScaledS4Dataset, method='complete',␣
↪metric='euclidean')
S4Clusters4 = fcluster(S4CompleteLinkage, t=15 , criterion='maxclust')
print(S4Clusters4[:1000])
```

```
[ 8  8  9  1  8  8  8  1  8  8  8  9  9  9  8  8  4  4  4  1  4  9  9  8
  8  8  8  8  8  8  8  8  8  4  8  8  8  8  9  8  8  8  8  8  9  8  9  8
  8  8  4  4  9  8  4  8  1  8  4  8  8  8  8  8  4  8  9  9  8  4  8  8
  4  8  9  1  8  4  8  8  8  4  4  4  8  1  8  9  8  1  1  8  8  1  8  8
  1  8  8  8  8  9  8  8  8  8  8  8  8  8  8  9  9  8  8  8  1  8  8  8
  1  8  8  8  8  8  8  8  8  8  1  9  8  8  8  9  8  9  1  8  1  8  8  8
```

```
 8  8  8  9  8  8  1  8  9  9  8  8  9  8  9  8  4  8  9  8  8  8  8  1
 8  8  8  8  8  8  4  8  8  8  9  9  8  8  8  8  8  8  8  9  8  8  8  4  1
 4  8  1  9  4  8  8  8  8  9  4  1  8  4  8  8  8  8  9  8  9  8  8  1
 8  4  8  8  8  8  9  8  8  8  9  8  8  8  8  1  4  4  1  8  8  8  8  8
 8  8  8  4  9  8  8  9  8  8  8  8  8  8  8  8  9  4  8  9  4  8  8  8
 8  4  8  9  8  8  8  8  8  8  8  8  1  4  8  1  8  1  8  4  8  4  8  8
 8  9  4  8  8  4  8  8  4  1  8  8  2  2  2  2  2  2  2  1  5  2  2  2
 4  2 10  4  2  4  2  3  2  2  3  2  2  2  2  4  2  4  2  4  2  2  2  1
 2  2  2  3  2  2  2  4  2  3  3  1  4  2  5  2  4  2  4  2  3  2  2  2
 4  2 10  1  2 10 10  2  2  2  2  2  2  2  4  4  3  2 10 10  2  2  2  2
 1  2  5  2  2  4  2  2  2  2  2  2  3  2  2  5  2  2  2  2  4  3  4  2
10 10  3  2  4  4  2  2  2  2  2 10  2  3  4  2  2  2  2  2  2  2  3  2
 2  4  1  2  2  3  2  2  2  3  1  4 10  2 10  2  2 10  2  2 10  2  2  2
 2  2 10  2  3  2  2  2  2  2  2 10  2  2  2  2  3  2  2  2  4  2  4  2
 2  2  2  4  2  3  2  2 10  4  2  4  2  2  2 10  2  2  2  4  3  2  2  2
 2 10  2  2  4  2  2  4  2  4  2  2  4  2  3  2  2 10  1  2 10  2  2  2
 2  2  4  2  2  2  3  4  1  2 10  2  2  2 10  4  3  2  2  3  2  2  2  2
 2  2  3  3  2  3  2  4  1  3  4  2  2 10  4  2  4  2  4  2  5  5  2  3
 2  6  3  4  4  2  4  2  2  2  2  2  4  2  3  2  2  3  2  4  2  2  4  2
 3  2  2  3  2  2  2  4  2  2  4 10  3  2  2  2  7  7  7  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  6  7  7  7  7  6  7  7  7  7  7  7  7
 7  7  7  6  6  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  6  7  6  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  6  7
 7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7
 7  7  7  7  7  7  7  6  7  6  7  7  7  7  7  7  7  6  7  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  6  7  6  7  7  7  7  7  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7
 7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7
 7  7  6  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7  6  7  7  7
 7  7  7  7  7  7  7  9  9  9  9  9  9  9  9  9  9  9  9  9  9  7  9
 9  9  9  9  9  9  9  9  9  9  9  9  8  9  9  9  9  9  9  9  7  9  9  9
 9  9  7  7  9  9  9  9  9  9  9  9  9  7  9  7]
```

[78]:
```
# Spectral clustering
S4Spectral = SpectralClustering(n_clusters= 15)
S4Clusters5 = S4Spectral.fit_predict(ScaledS4Dataset)
print(S4Clusters5[1:1000])
```

```
[14  3  5 14 14 14  5 14 14  3  3  3  3 14 14  3  3  3  5  5 12  3 14 14
 14 14 14 14 14 14 14 14  3 14 14 14 14  3 14 14 14 14 14  3 14 12  3 14
 14  5  5  3 14  3 14  5 14  5  3 14 14 14 14  3 14  3  3 14  5 14 14  3
 14  3 14 14  5 14 14 14  3  5  3  3 13 14  3 14 14  5 14 14 14 14  3  5
 14 14 14 14 14 14 14 14 14 14 14 14 14 14  3  3 14 14 14 14 14 14 14  5
 14 14 14 14 14 14 14  3 14 13  3 14 14 14  3 14  3  5 14 14 14 14 14 14
```

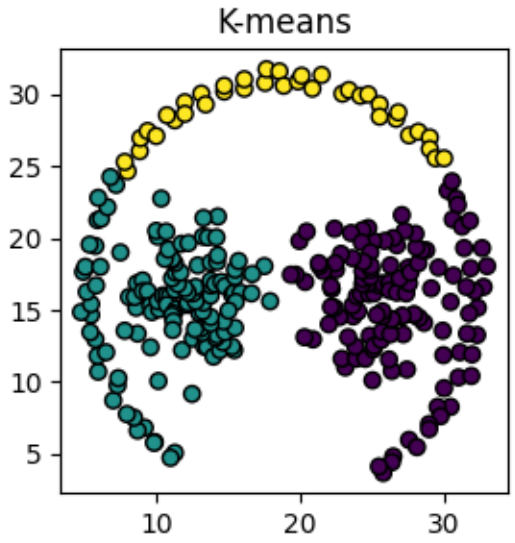
```
14 14  3 14 14  5 14  3 12 14 14  3 14  3 14  3 14  3 14 14 14 14 13 14
14  3 14 14 14  3 14 14 14  3  3 14 14 14 14 14 14  3 14 14 14  3  5  3
14  5  3  5 14 14 14 14 12  5  5 14  5 14 14 14 14 14 14  3 14 14 14 14
 5 14 14 14 14  3 14 14 14  3 14 14 14 14 14  5  3 14 14 14 14 14 14 14
14 14  3 14 14 14  3 14 14 14 14 14 14 14 14  3  3 14  3  5 14 14 14 14
 3 14  3 14  3 14 14 14 14 14  3  5  5 14  5 14  5 14  5 14  3 14 14 14
 3  3 14 14  3 14 14  5 14 14 14  9  9  9  9  9  9  9  5  2  9  9  9  5
13  7  2  9  9  9  9  9  9  9  9  9  9  9  9 11  5  9  5  9  9  9  5 11
 5  9 11 11  9  9  2  9  4  9  5  5  9  2  9  9  5  5  9  9  9 13  9  9
 9  7  5  9  9  7  9  9  9 11 13  9  9  5  5  7  9  7  9  9  9  9  9  5
 9  2  9  9  5  9  9  9  9  9  9 11  9  9  2  9  9  9 11  5 11  2  9  2
 7 11  9  9  5  9  9  5  9  9  2  9 11  5  9  9  9  9  9  9 11  9  9 13
 9  5  9  9  9  9  9  9  4 13  5  2 13  7  9  9  2  9  9  7  9 13  9 13
 9  7 11  4  9  9  9  9  9  9  7  9  9  5  9 11  9 11  9  9 13  2 13  9
 9  9  5  9  4  9  9  7  9  9  5  9  9  9  2  9 11  9  9  4  9  9 13  9
 7  9  9  5 11  9  5  9  5  9 11  5  9  9 13  9  7 13  9  7  9  9  9  9
 9  9  9  9  9 11  5 13  9  2  9  9  9  2  5  4  9 13  7  9  5  5  9  9
 9  4  9  9  9  9  5  5  9  9  9  9  7  5  9  5  9  9  9  2  2  9  4  9
 6 11  9  5  9  5  9  9  9  9  9  5  9  4  9  9 11  9  2  9  9  5  9 11
 9  9  9  9  9  9  9  9  9  5  2  4  9  9  9  1  1  1  1  1  1  1  1  1
 1  1  1  1  1  1  1  1  1  1  6  1  1  1  6  6  6  1  1  1  1  1  1  1
 1  1  6  6  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  6  1  6  1  1  1  1  1  1
 1  1  1  1  1  1  1  6  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 1  1  1  1  6  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  6  1  1
 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 1  1  1  1  1  1  6  1  6  1  1  1  1  1  1  1  6  1  1  1  1  1  1  1
 1  1  1  1  1  6  1  1  1  6  6  1  6  1  1  1  1  1  1  1  1  1  1  1
 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  6  1
 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 1  6  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  6  1  1  1  1
 1  1  1  1  1  1 12 12 12 12 12  3 12 12  3  1 12  1 12 12 12 12 12 12
12  3  3 12 12 12 12 12 12  1 12 14 12  3 12 12 12 12 12 12 12 12 12 12
 3 12 12 12  3 12 12 12 12 12 12 12 12 12  1]
```

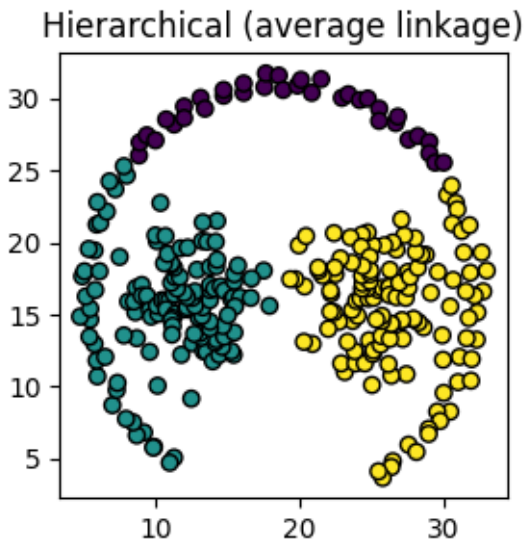
b. Visualize the resulting clustering (use the scatter plot of the data points and color the points by cluster assignment).

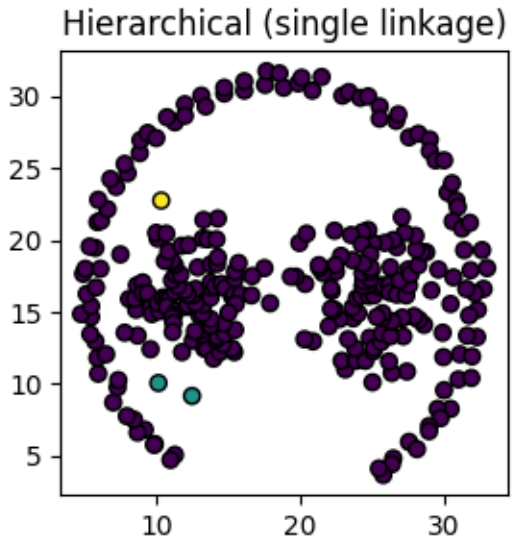
[99]:
```
# Pathbased dataset
# N=300, k=3, D=2
```

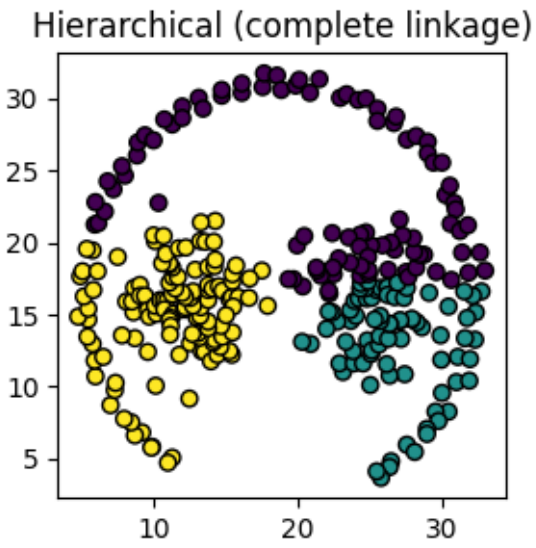
[122]:
```
plt.figure(figsize=(3, 3))
plt.scatter(PathbasedDataset[:,0], PathbasedDataset[:,1], c=PathbasedClusters,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("K-means")
plt.show()
```

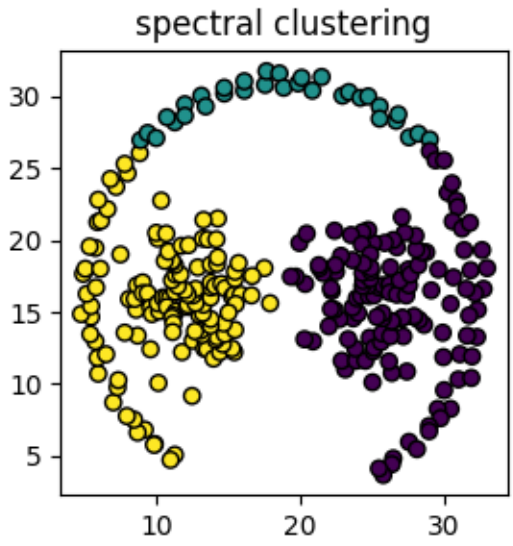
```
plt.figure(figsize=(3, 3))
plt.scatter(PathbasedDataset[:,0], PathbasedDataset[:,1], c=PathbasedClusters2,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (average linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(PathbasedDataset[:,0], PathbasedDataset[:,1], c=PathbasedClusters3,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (single linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(PathbasedDataset[:,0], PathbasedDataset[:,1], c=PathbasedClusters4,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (complete linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(PathbasedDataset[:,0], PathbasedDataset[:,1], c=PathbasedClusters5,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("spectral clustering")
plt.show()
```




Hierarchical (average linkage)
30
25
20
15
10
5
10
20
30


Hierarchical (single linkage)
30
25
20
15
10
5
10
20
30

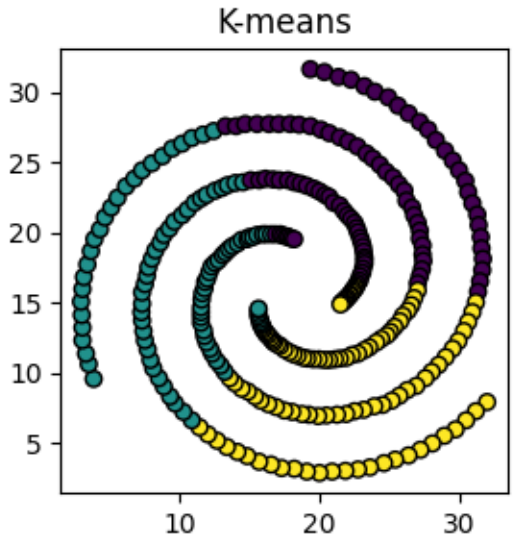


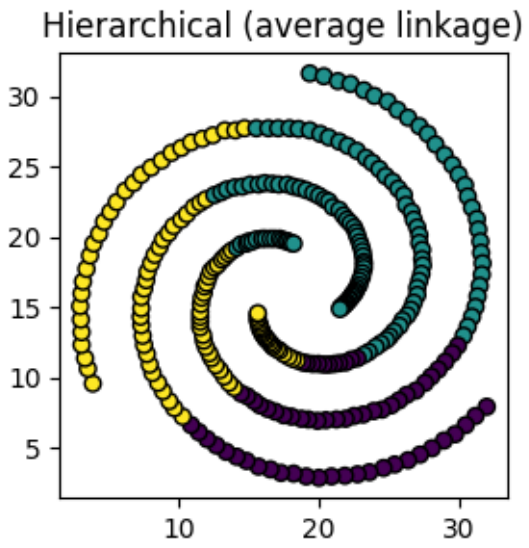


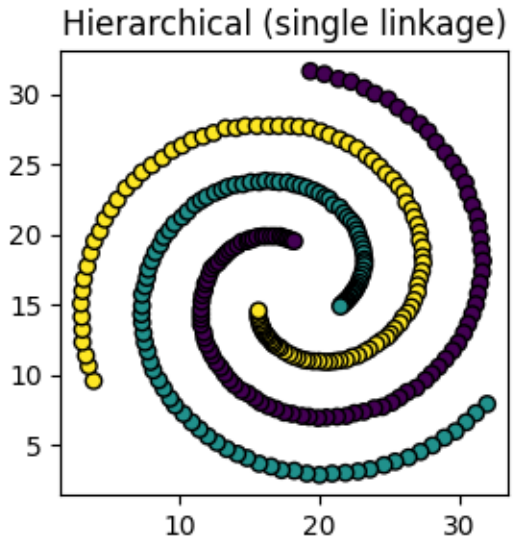
[103]:
```
# Spiral dataset
# N=312, k=3, D=2
```

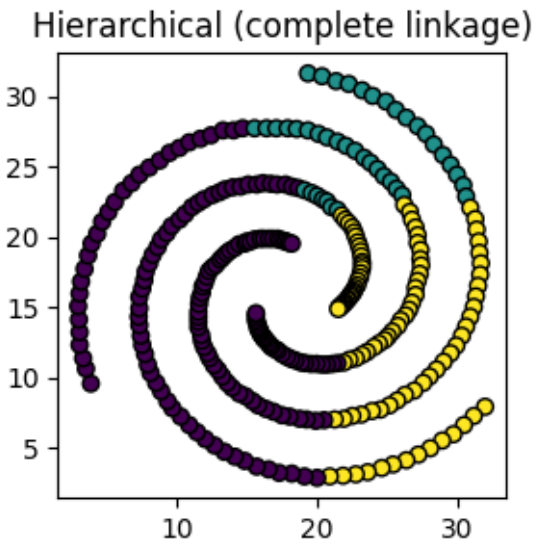
[123]:
```
plt.figure(figsize=(3, 3))
plt.scatter(SpiralDataset[:,0], SpiralDataset[:,1], c=SpiralClusters,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("K-means")
plt.show()
```

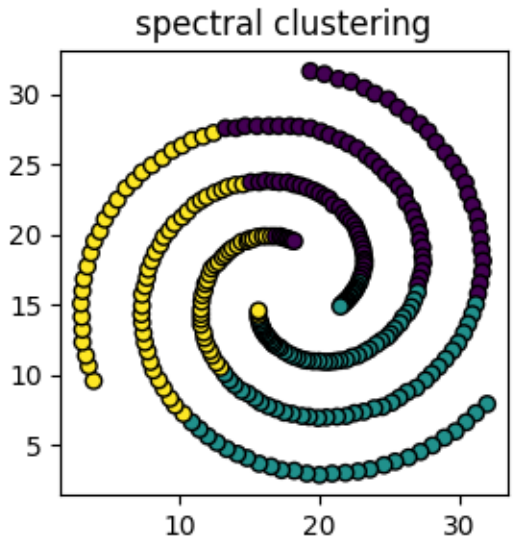
```
plt.figure(figsize=(3, 3))
plt.scatter(SpiralDataset[:,0], SpiralDataset[:,1], c=SpiralClusters2,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (average linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(SpiralDataset[:,0], SpiralDataset[:,1], c=SpiralClusters3,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (single linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(SpiralDataset[:,0], SpiralDataset[:,1], c=SpiralClusters4,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (complete linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(SpiralDataset[:,0], SpiralDataset[:,1], c=SpiralClusters5,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("spectral clustering")
plt.show()
```




Hierarchical (average linkage)
30
25
20
15
10
5
10
20
30


Hierarchical (single linkage)
30
25
20
15
10
5
10
20
30

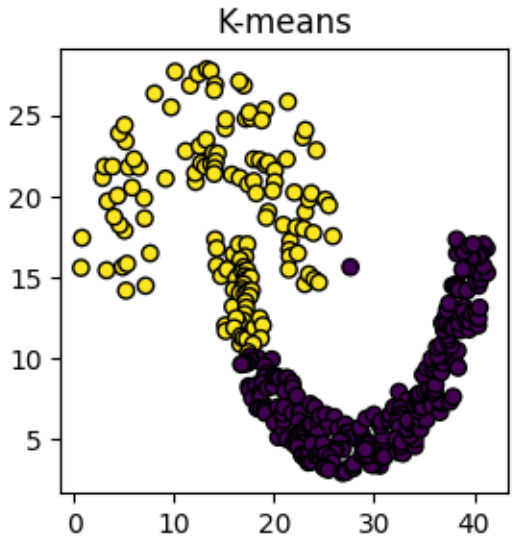


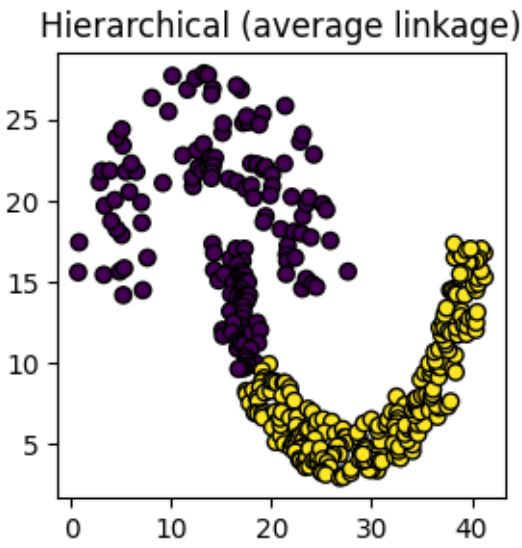


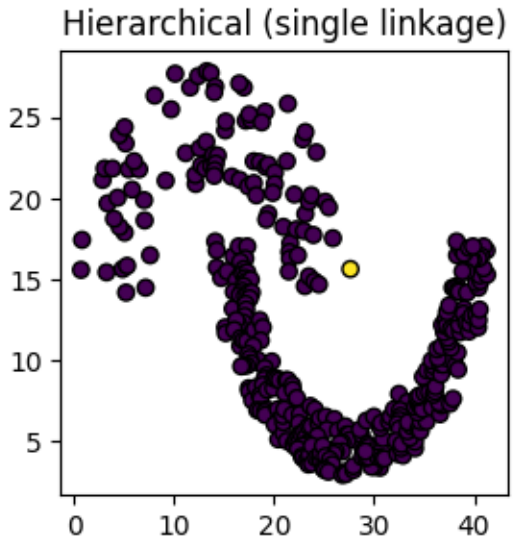
[108]:
```
# Jain dataset
# N=373, k=2, D=2
```

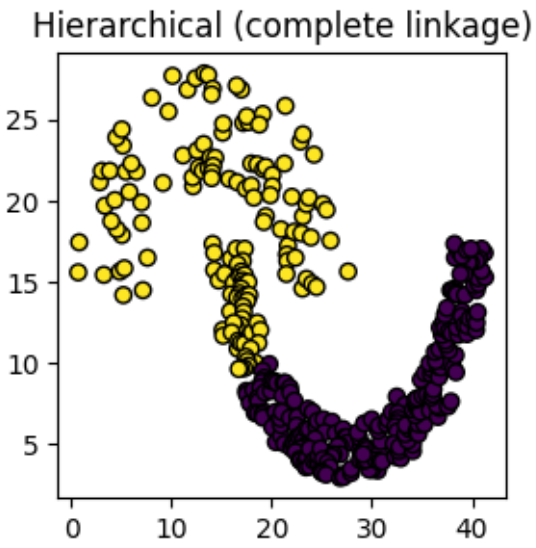
[126]:
```
plt.figure(figsize=(3, 3))
plt.scatter(JainDataset[:,0], JainDataset[:,1], c=JainClusters, cmap='viridis',␣
↪edgecolor='k')
plt.title("K-means")
plt.show()
```

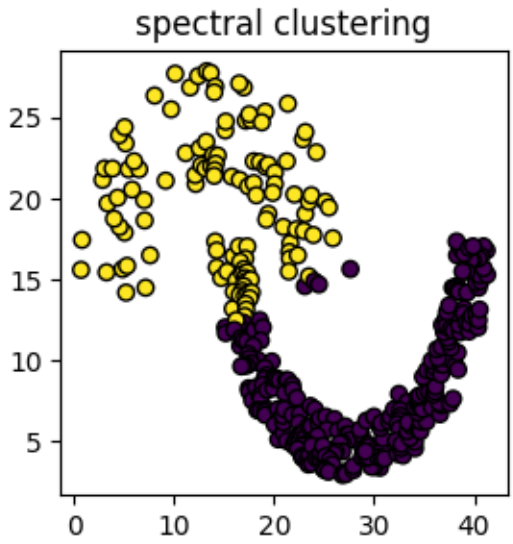
```
plt.figure(figsize=(3, 3))
plt.scatter(JainDataset[:,0], JainDataset[:,1], c=JainClusters2,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (average linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(JainDataset[:,0], JainDataset[:,1], c=JainClusters3,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (single linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(JainDataset[:,0], JainDataset[:,1], c=JainClusters4,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (complete linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(JainDataset[:,0], JainDataset[:,1], c=JainClusters5,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("spectral clustering")
plt.show()
```




Hierarchical (average linkage)
25
20
15
10
5
0
10
20
30
40


Hierarchical (single linkage)
25
20
15
10
5
0
10
20
30
40

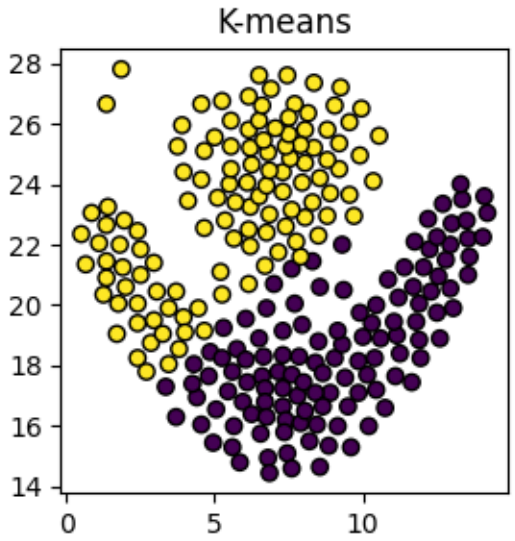


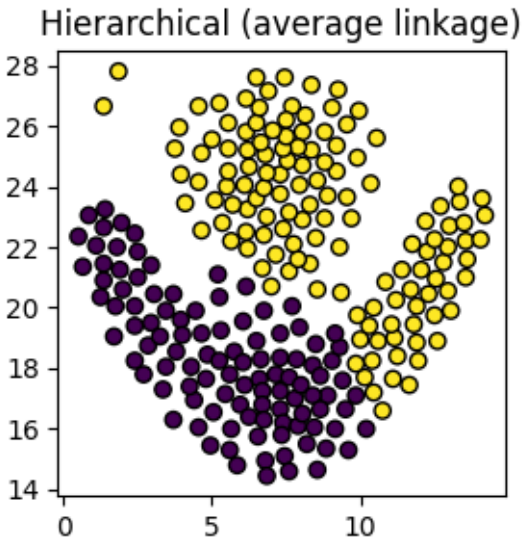


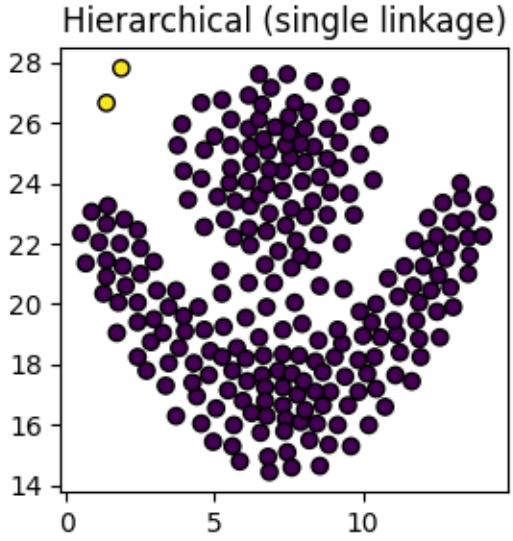
[110]:
```
# Flame dataset
# N=240, k=2, D=2
```

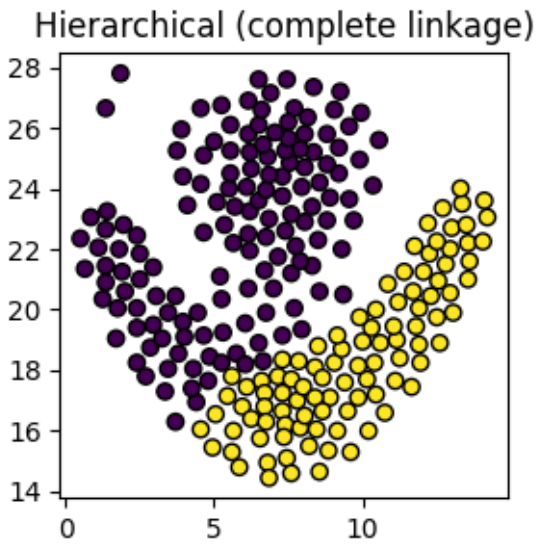
[127]:
```
plt.figure(figsize=(3, 3))
plt.scatter(FlameDataset[:,0], FlameDataset[:,1], c=FlameClusters,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("K-means")
plt.show()
```

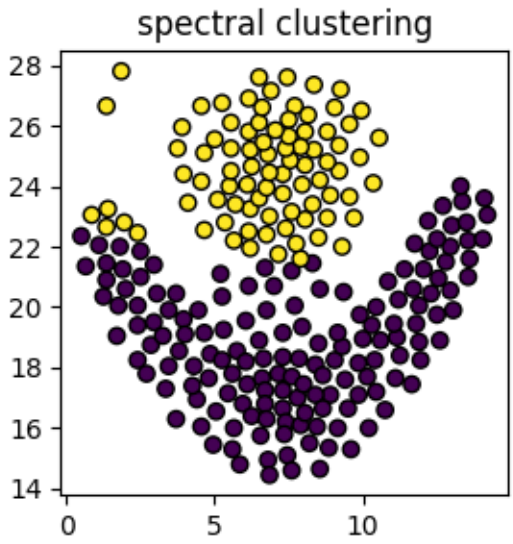
```
plt.figure(figsize=(3, 3))
plt.scatter(FlameDataset[:,0], FlameDataset[:,1], c=FlameClusters2,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (average linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(FlameDataset[:,0], FlameDataset[:,1], c=FlameClusters3,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (single linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(FlameDataset[:,0], FlameDataset[:,1], c=FlameClusters4,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("Hierarchical (complete linkage)")
plt.show()

plt.figure(figsize=(3, 3))
plt.scatter(FlameDataset[:,0], FlameDataset[:,1], c=FlameClusters5,␣
 ↪cmap='viridis', edgecolor='k')
plt.title("spectral clustering")
plt.show()
```




Hierarchical (average linkage)
28
26
24
22
20
18
16
14
0
5
10

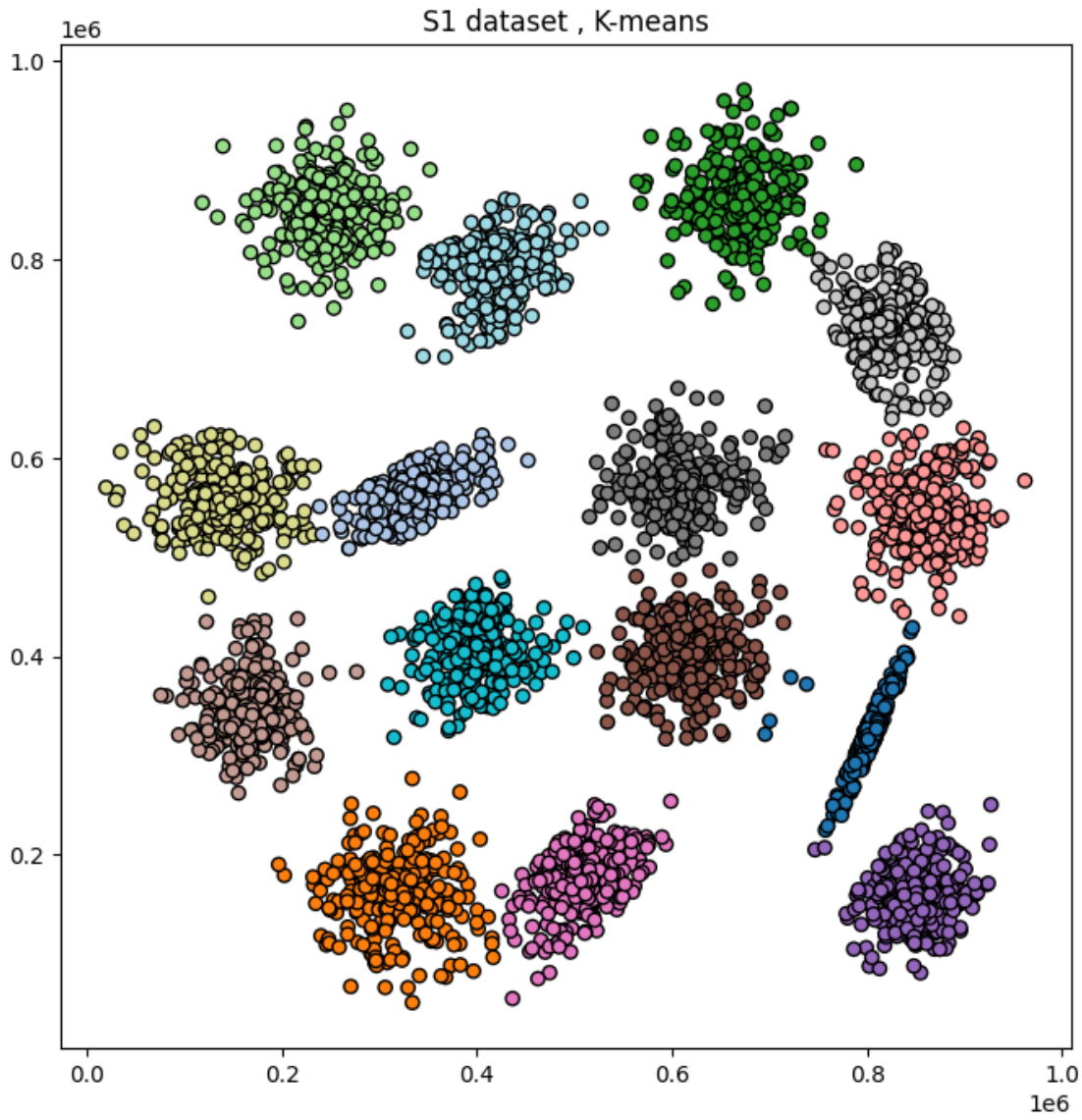

Hierarchical (single linkage)
28
26
24
22
20
18
16
14
0
5
10



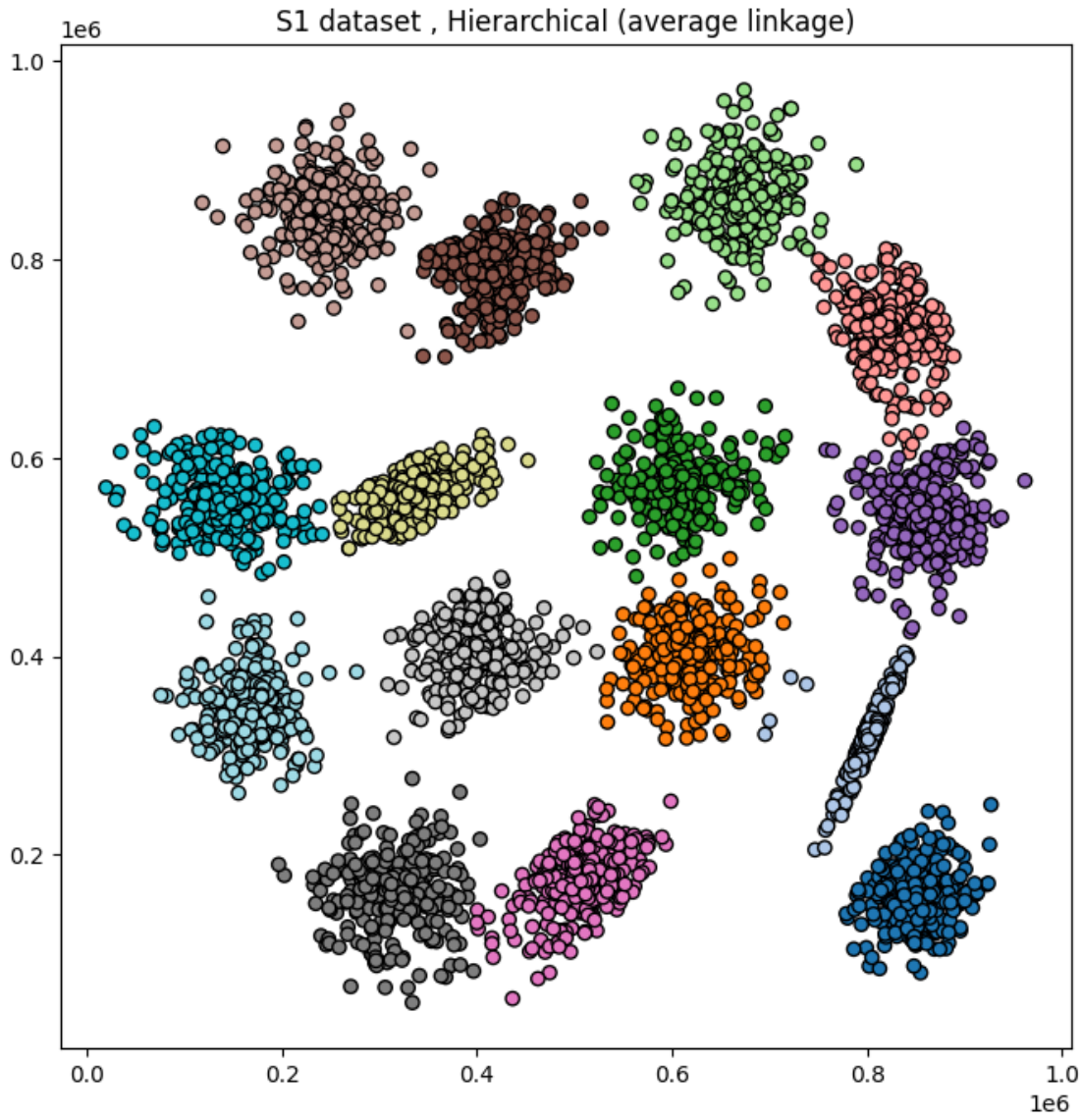


[120]:
```
# S1 dataset
# Synthetic 2-d data with N=5000 vectors and k=15 Gaussian clusters with␣
 ↪different degree of cluster overlap
```

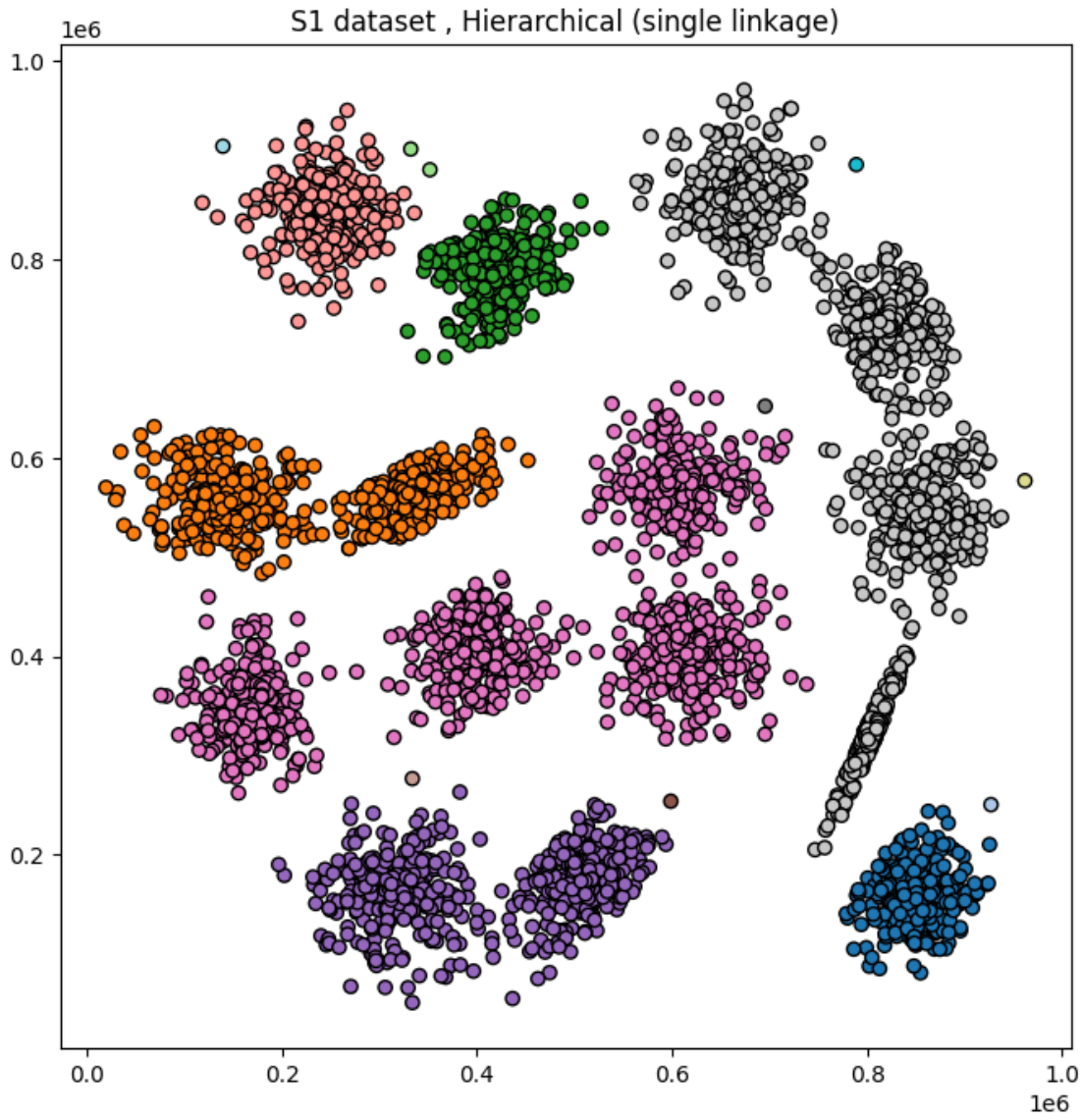
[144]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S1Dataset[:,0], S1Dataset[:,1], c=S1Clusters, cmap='tab20',␣
 ↪edgecolor='k')
plt.title("S1 dataset , K-means")
plt.show()
```



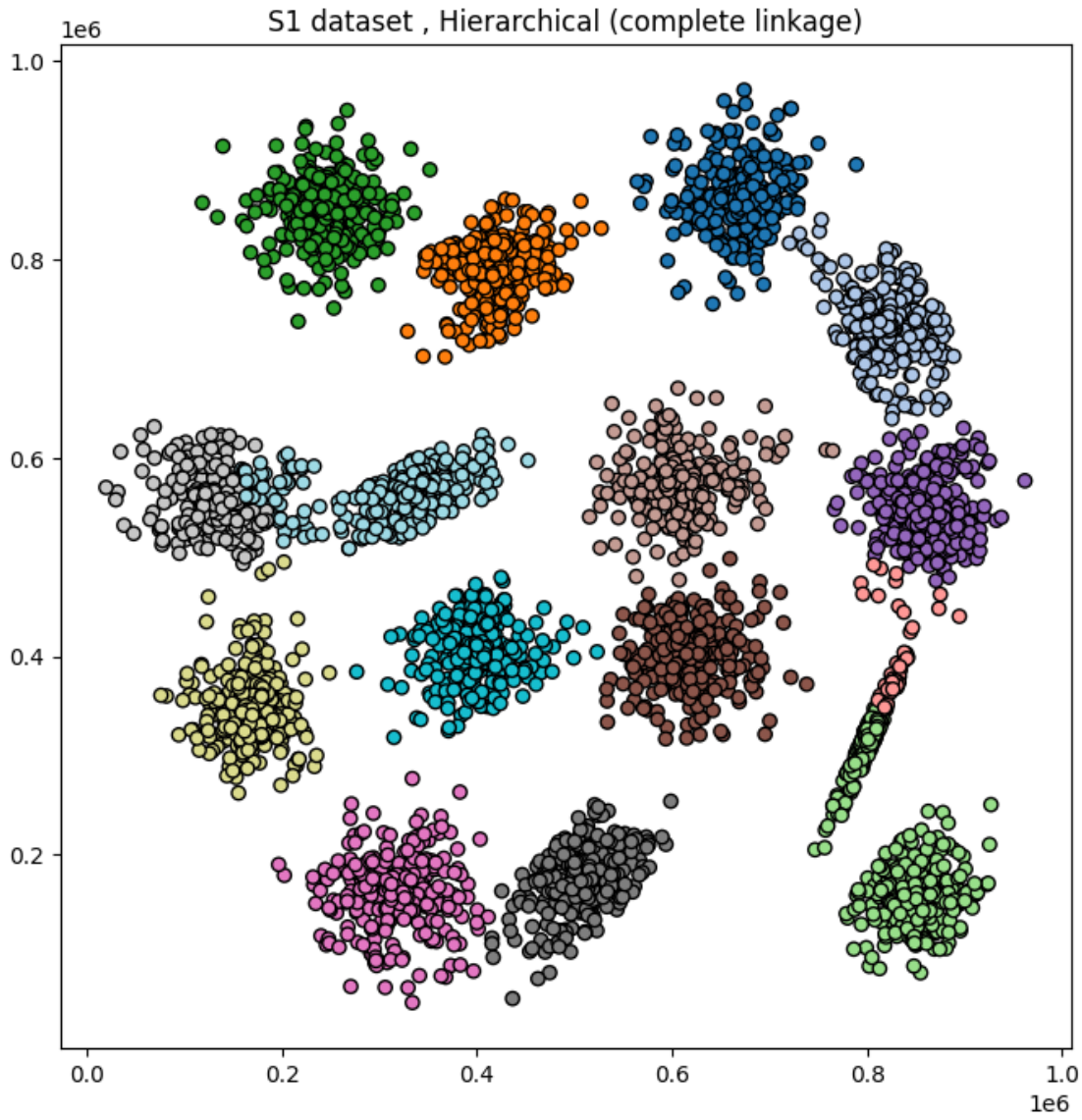
[145]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S1Dataset[:,0], S1Dataset[:,1], c=S1Clusters2, cmap='tab20',␣
↪edgecolor='k')
plt.title("S1 dataset , Hierarchical (average linkage)")
plt.show()
```



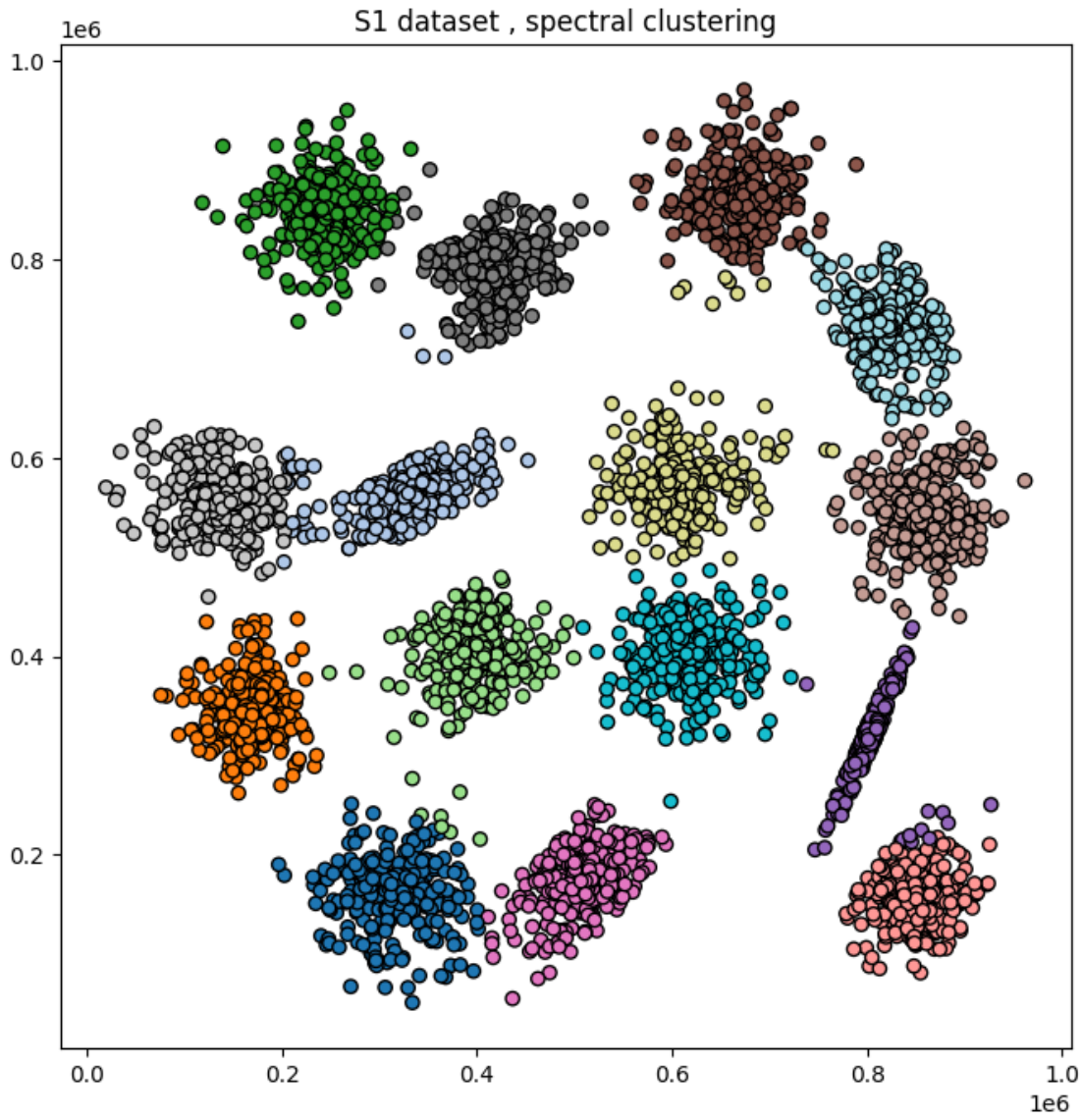
[146]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S1Dataset[:,0], S1Dataset[:,1], c=S1Clusters3, cmap='tab20',␣
↪edgecolor='k')
plt.title("S1 dataset , Hierarchical (single linkage)")
plt.show()
```



[147]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S1Dataset[:,0], S1Dataset[:,1], c=S1Clusters4, cmap='tab20',␣
↪edgecolor='k')
plt.title("S1 dataset , Hierarchical (complete linkage)")
plt.show()
```

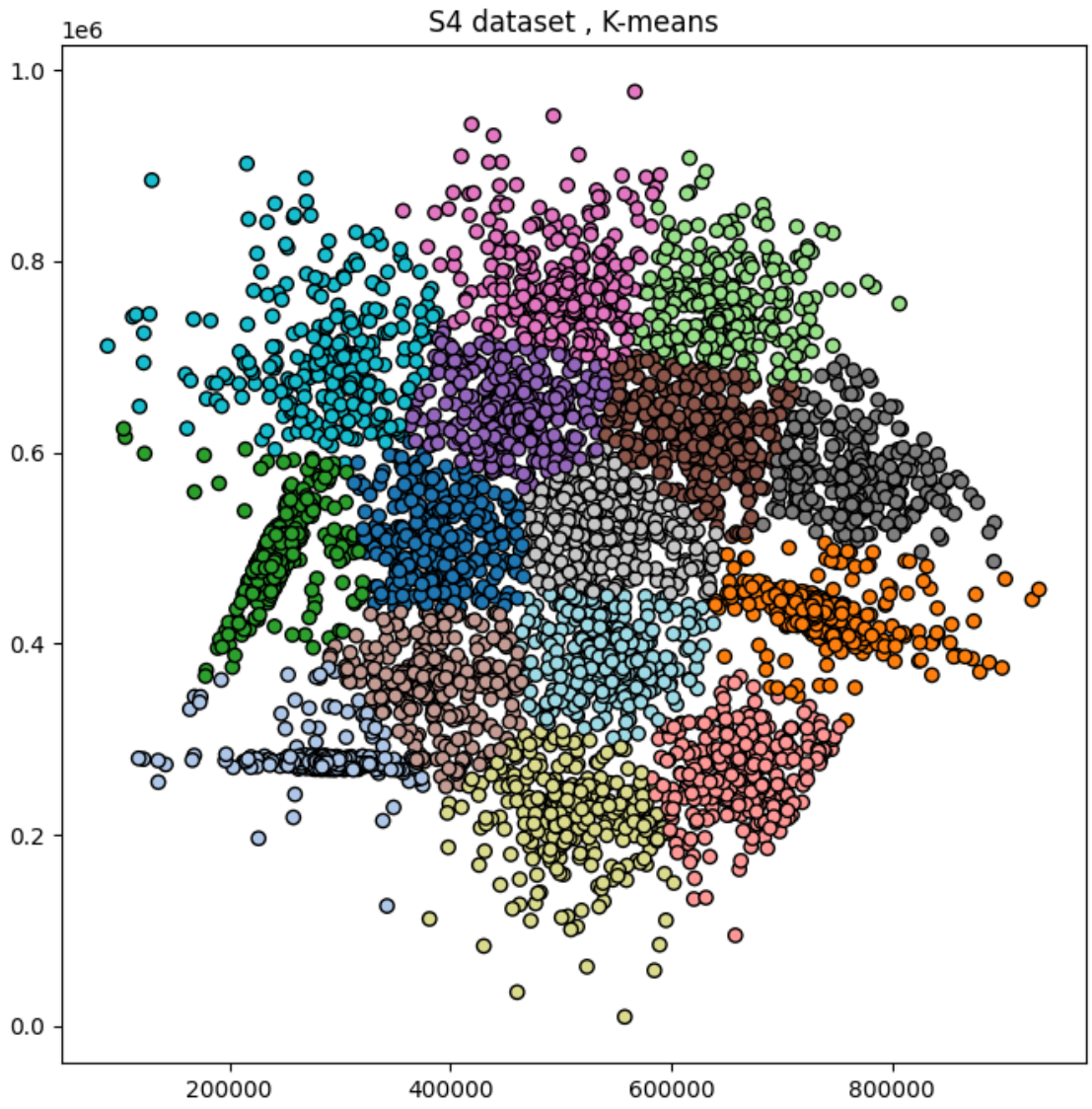


[148]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S1Dataset[:,0], S1Dataset[:,1], c=S1Clusters5, cmap='tab20',␣
↪edgecolor='k')
plt.title("S1 dataset , spectral clustering")
plt.show()
```

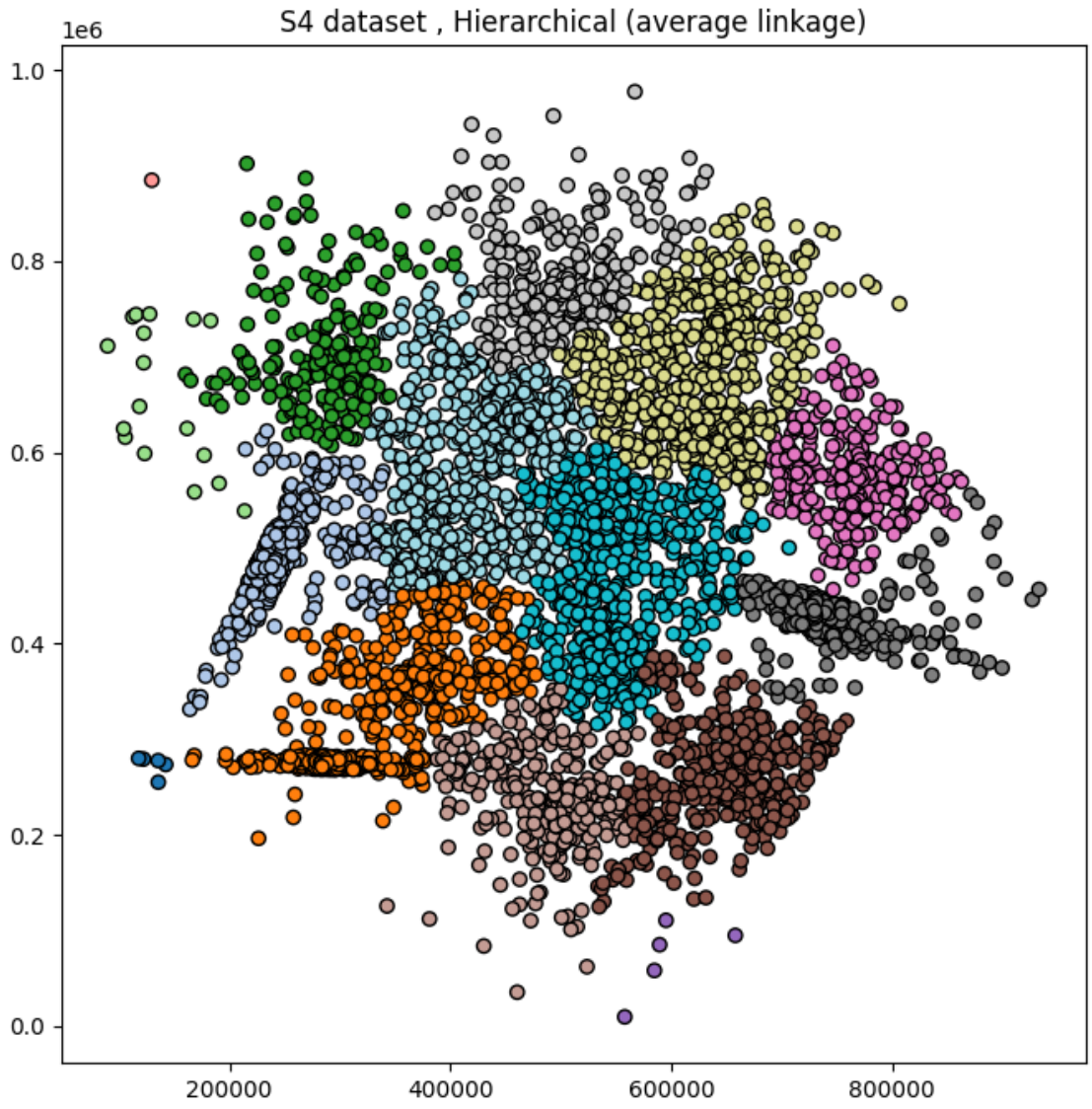


[124]:
```
# S4 dataset
# Synthetic 2-d data with N=5000 vectors and k=15 Gaussian clusters with␣
 ↪different degree of cluster overlap
```

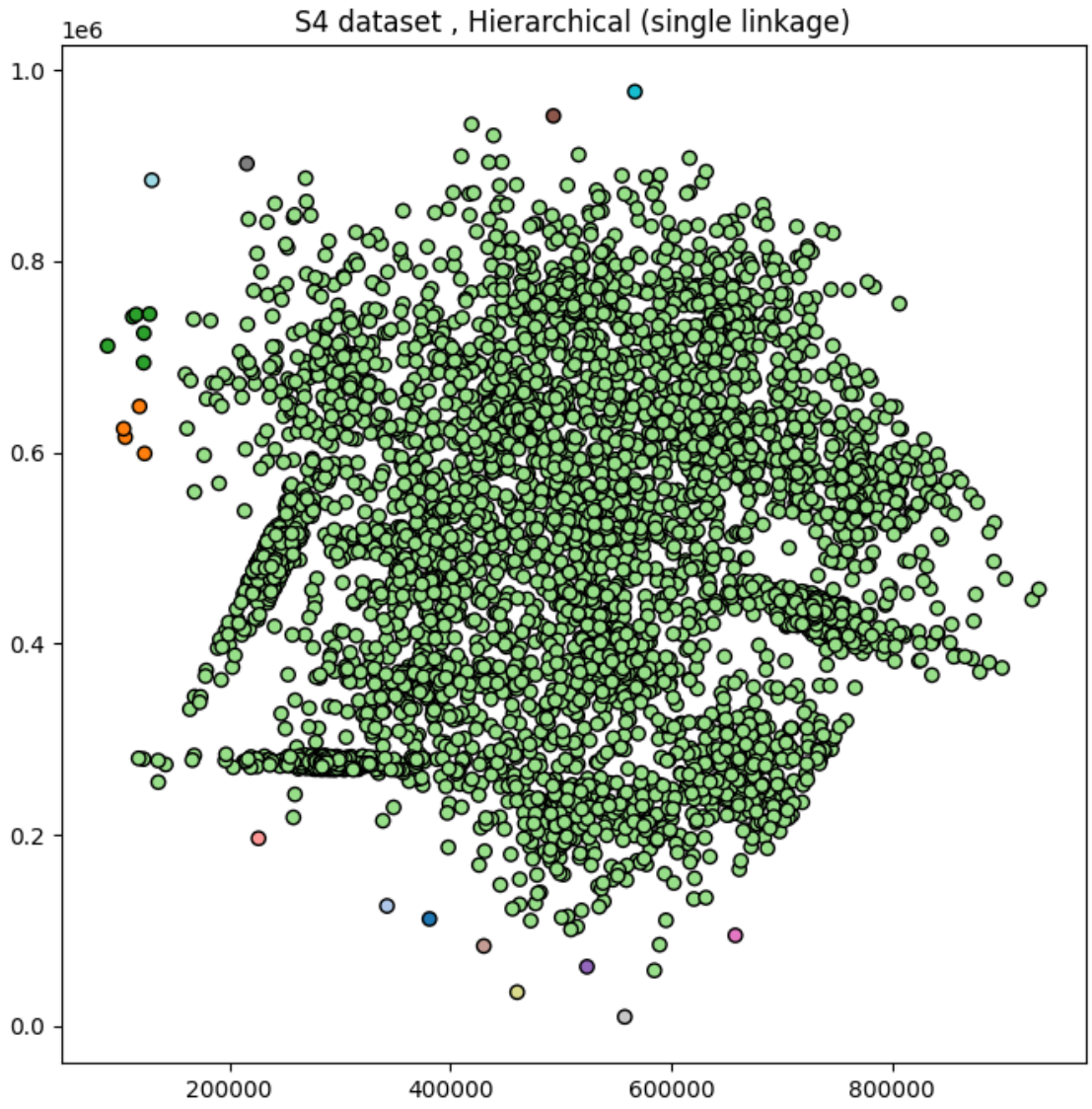
[149]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S4Dataset[:,0], S4Dataset[:,1], c=S4Clusters, cmap='tab20',␣
 ↪edgecolor='k')
plt.title("S4 dataset , K-means")
plt.show()
```



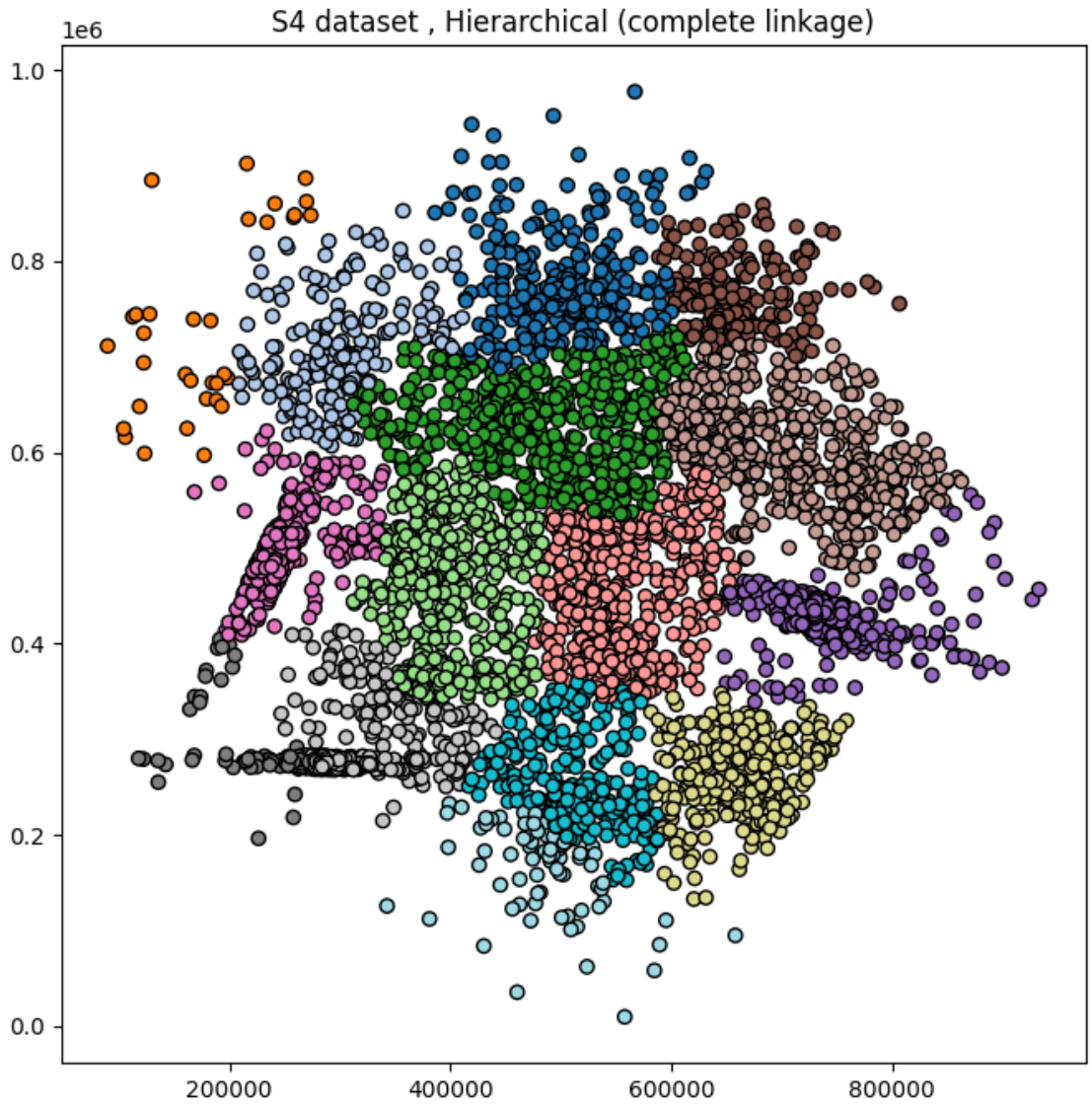
[150]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S4Dataset[:,0], S4Dataset[:,1], c=S4Clusters2, cmap='tab20',␣
↪edgecolor='k')
plt.title("S4 dataset , Hierarchical (average linkage)")
plt.show()
```



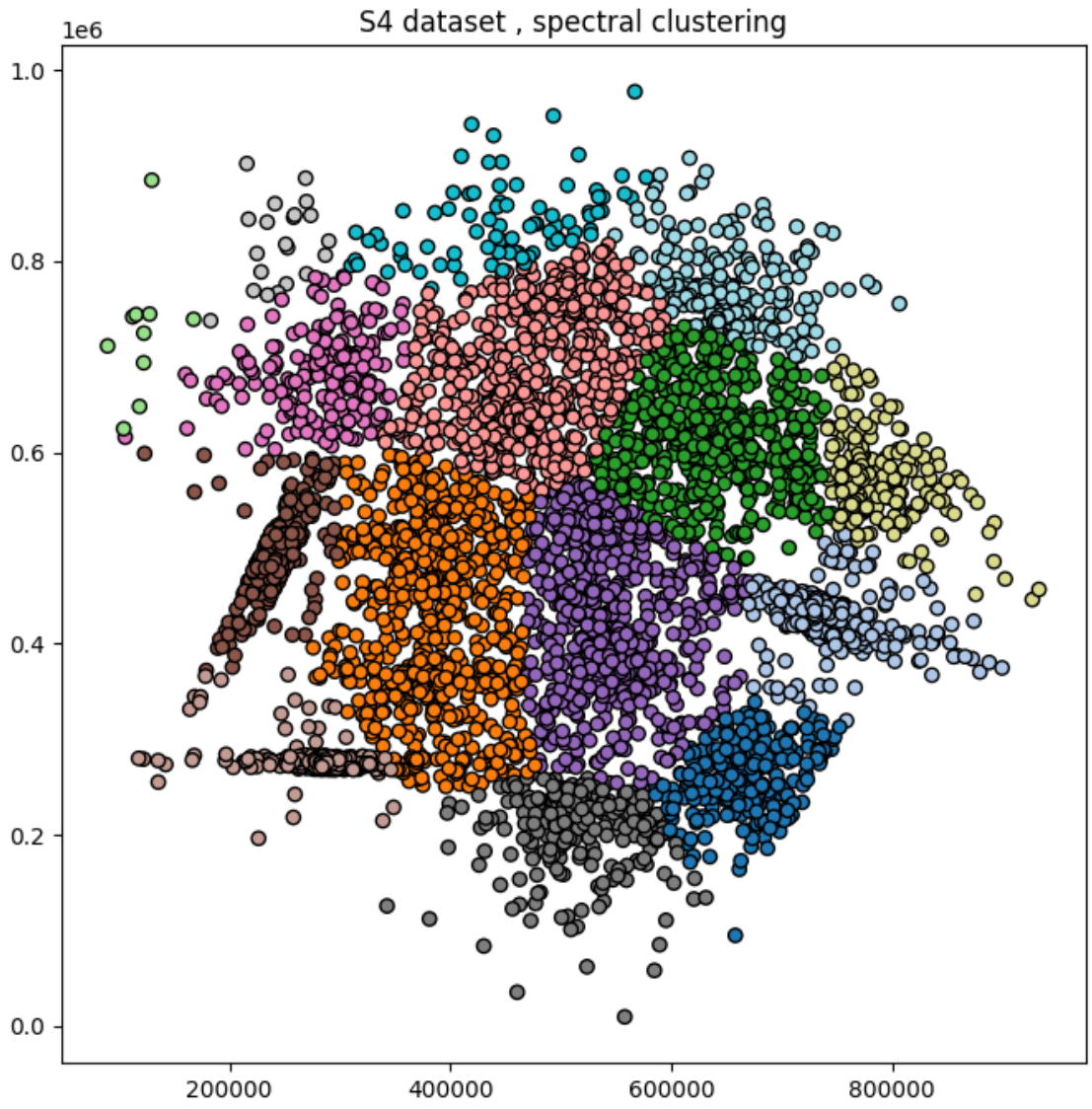
[151]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S4Dataset[:,0], S4Dataset[:,1], c=S4Clusters3, cmap='tab20',␣
↪edgecolor='k')
plt.title("S4 dataset , Hierarchical (single linkage)")
plt.show()
```



[152]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S4Dataset[:,0], S4Dataset[:,1], c=S4Clusters4, cmap='tab20',␣
↪edgecolor='k')
plt.title("S4 dataset , Hierarchical (complete linkage)")
plt.show()
```



[143]:
```
plt.figure(figsize=(8, 8))
plt.scatter(S4Dataset[:,0], S4Dataset[:,1], c=S4Clusters5, cmap='tab20',␣
↪edgecolor='k')
plt.title("S4 dataset , spectral clustering")
plt.show()
```



c. For datasets from the "Shape Set" where the true cluster is known, evaluate each clustering result using the purity index.

[168]:
```
def PurityScore(y_true, y_pred):
    contingency_matrix = metrics.cluster.contingency_matrix(y_true, y_pred)
    return np.sum(np.amax(contingency_matrix, axis=0)) / np.
 ↪sum(contingency_matrix)

rows = ['K-means','Hierarchical(average linkage)','Hierarchical(single␣
 ↪linkage)','Hierarchical(complete linkage)','Spectral']
columns = ['Pathbased','Spiral','Jain','Flame']
PurityIndex = pd.DataFrame(index=rows , columns=columns)

for i in range(len(rows)):
```

```
    for j in range(len(columns)):
        if j==0:
            TrueLabels = PathbasedDataset[:,2]
            if i==0:
                PredictedLabels = PathbasedClusters
            if i==1:
                PredictedLabels = PathbasedClusters2
            if i==2:
                PredictedLabels = PathbasedClusters3
            if i==3:
                PredictedLabels = PathbasedClusters4
            if i==4:
                PredictedLabels = PathbasedClusters5

        if j==1:
            TrueLabels = SpiralDataset[:,2]
            if i==0:
                PredictedLabels = SpiralClusters
            if i==1:
                PredictedLabels = SpiralClusters2
            if i==2:
                PredictedLabels = SpiralClusters3
            if i==3:
                PredictedLabels = SpiralClusters4
            if i==4:
                PredictedLabels = SpiralClusters5

        if j==2:
            TrueLabels = JainDataset[:,2]
            if i==0:
                PredictedLabels = JainClusters
            if i==1:
                PredictedLabels = JainClusters2
            if i==2:
                PredictedLabels = JainClusters3
            if i==3:
                PredictedLabels = JainClusters4
            if i==4:
                PredictedLabels = JainClusters5

        if j==3:
            TrueLabels = FlameDataset[:,2]
            if i==0:
                PredictedLabels = FlameClusters
            if i==1:
                PredictedLabels = FlameClusters2
            if i==2:
```

```
                PredictedLabels = FlameClusters3
            if i==3:
                PredictedLabels = FlameClusters4
            if i==4:
                PredictedLabels = FlameClusters5

        Purity = PurityScore(TrueLabels, PredictedLabels)
        PurityIndex.iloc[i,j] = Purity

print(PurityIndex)
```

	Pathbased	Spiral	Jain	Flame
K-means	0.76	0.339744	0.873995	0.820833
Hierarchical(average linkage)	0.753333	0.416667	0.86059	0.8
Hierarchical(single linkage)	0.376667	1.0	0.742627	0.645833
Hierarchical(complete linkage)	0.693333	0.391026	0.86059	0.770833
Spectral	0.74	0.349359	0.906166	0.95

d. For each dataset, compare the results of each pair of clustering methods using the Rand index. Visualize the results in a 5x5 heatmap.

[ ]:
```
# Pathbased dataset
# N=300, k=3, D=2
```

[219]:
```
methods = ['K-means','Average','Single','Complete','Spectral']
PathbasedRandIndex = pd.DataFrame(index=methods , columns=methods)
PathbasedRandIndexNP = np.zeros((5,5))

for i in range(len(methods)):
    for j in range(len(methods)):

        if i==0:
            PredictedLabels1 = PathbasedClusters
        if i==1:
            PredictedLabels1 = PathbasedClusters2
        if i==2:
            PredictedLabels1 = PathbasedClusters3
        if i==3:
            PredictedLabels1 = PathbasedClusters4
        if i==4:
            PredictedLabels1 = PathbasedClusters5

        if j==0:
            PredictedLabels2 = PathbasedClusters
        if j==1:
            PredictedLabels2 = PathbasedClusters2
        if j==2:
            PredictedLabels2 = PathbasedClusters3
```

```
        if j==3:
            PredictedLabels2 = PathbasedClusters4
        if j==4:
            PredictedLabels2 = PathbasedClusters5

        PathbasedRand = adjusted_rand_score(PredictedLabels1,PredictedLabels2)
        PathbasedRandIndex.iloc[i,j] = PathbasedRand
        PathbasedRandIndexNP[i,j] = PathbasedRand

print(PathbasedRandIndex)

plt.figure(figsize=(6,5))
sbn.heatmap(PathbasedRandIndexNP, annot=True, cmap="viridis")
plt.title("Rand Index Comparison for Pathbased dataset")
plt.show()
```

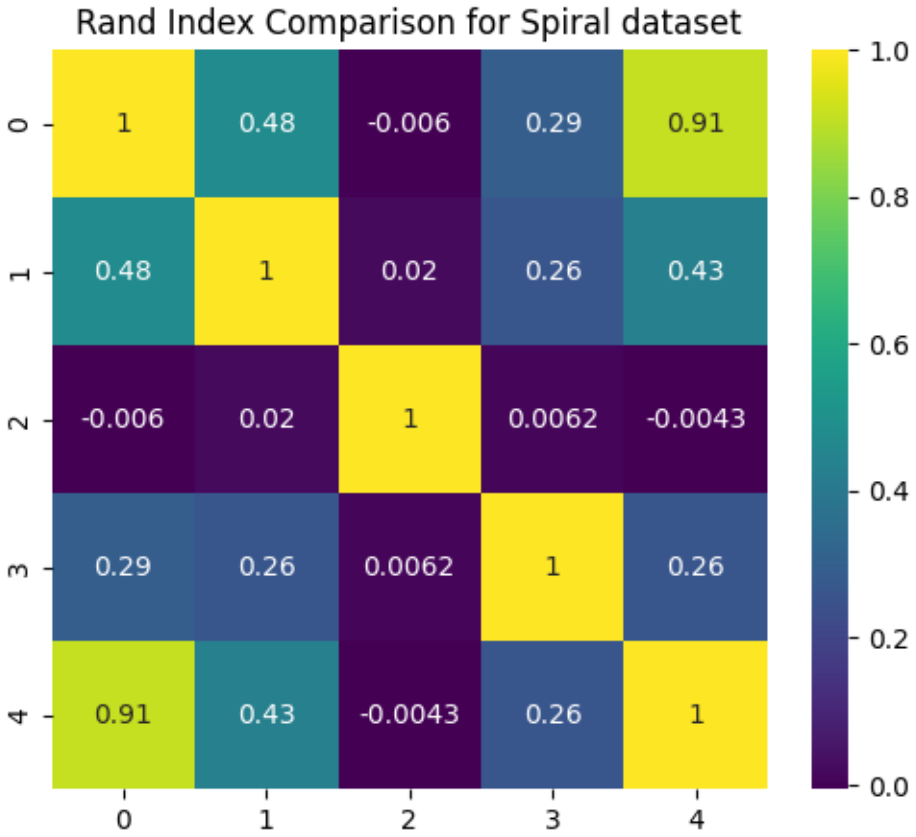
	K-means	Average	Single	Complete	Spectral
K-means	1.0	0.984546	-0.002221	0.63922	0.954234
Average	0.984546	1.0	-0.002404	0.625766	0.969491
Single	-0.002221	-0.002404	1.0	-0.002208	-0.002067
Complete	0.63922	0.625766	-0.002208	1.0	0.612139
Spectral	0.954234	0.969491	-0.002067	0.612139	1.0

[176]:
```
# Spiral dataset
# N=312, k=3, D=2
```

[220]:
```
SpiralRandIndex = pd.DataFrame(index=methods , columns=methods)
SpiralRandIndexNP = np.zeros((5,5))

for i in range(len(methods)):
    for j in range(len(methods)):

        if i==0:
            PredictedLabels1 = SpiralClusters
        if i==1:
            PredictedLabels1 = SpiralClusters2
        if i==2:
            PredictedLabels1 = SpiralClusters3
        if i==3:
            PredictedLabels1 = SpiralClusters4
        if i==4:
            PredictedLabels1 = SpiralClusters5

        if j==0:
            PredictedLabels2 = SpiralClusters
        if j==1:
            PredictedLabels2 = SpiralClusters2
        if j==2:
            PredictedLabels2 = SpiralClusters3
        if j==3:
            PredictedLabels2 = SpiralClusters4
        if j==4:
            PredictedLabels2 = SpiralClusters5

        SpiralRand = adjusted_rand_score(PredictedLabels1,PredictedLabels2)
        SpiralRandIndex.iloc[i,j] = SpiralRand
        SpiralRandIndexNP[i,j] = SpiralRand

print(SpiralRandIndex)

plt.figure(figsize=(6,5))
sbn.heatmap(SpiralRandIndexNP, annot=True, cmap="viridis")
plt.title("Rand Index Comparison for Spiral dataset")
plt.show()
```

```
           K-means   Average    Single  Complete  Spectral
K-means        1.0  0.477081 -0.006025  0.291922  0.913216
Average   0.477081       1.0  0.020269  0.262671   0.42995
```

```
Single   -0.006025  0.020269       1.0  0.006224 -0.004251
Complete  0.291922  0.262671  0.006224       1.0   0.26179
Spectral  0.913216   0.42995 -0.004251   0.26179       1.0
```

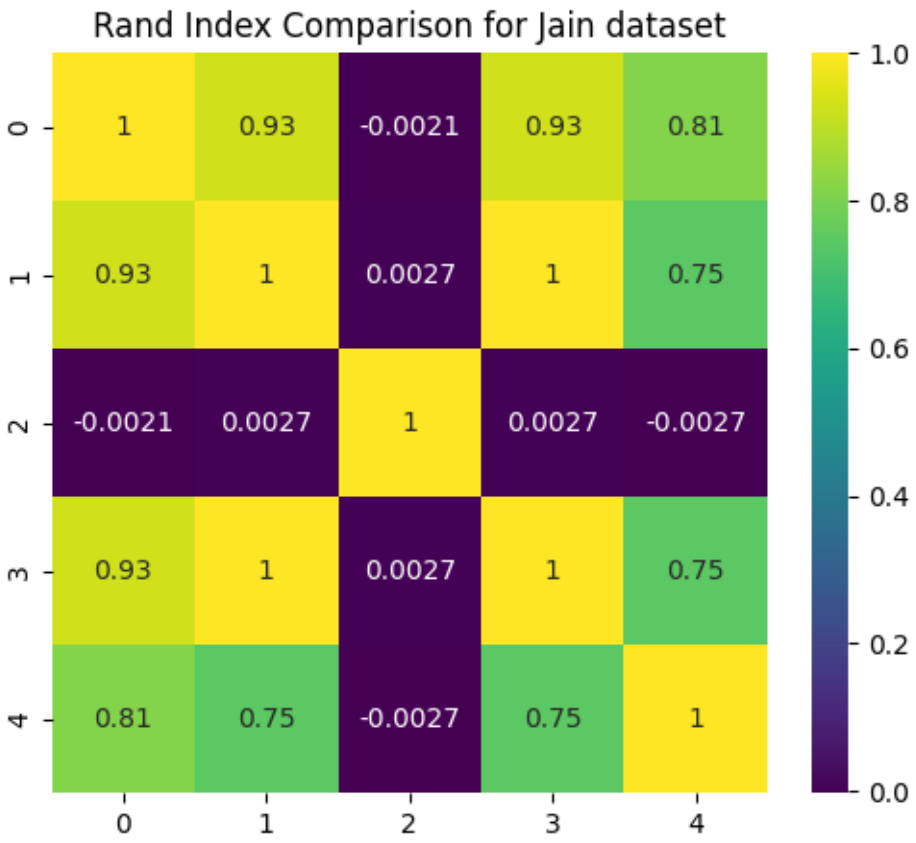


[178]:
```
# Jain dataset
# N=373, k=2, D=2
```

[221]:
```
JainRandIndex = pd.DataFrame(index=methods , columns=methods)
JainRandIndexNP = np.zeros((5,5))

for i in range(len(methods)):
    for j in range(len(methods)):

        if i==0:
            PredictedLabels1 = JainClusters
        if i==1:
            PredictedLabels1 = JainClusters2
        if i==2:
            PredictedLabels1 = JainClusters3
        if i==3:
```

```
            PredictedLabels1 = JainClusters4
        if i==4:
            PredictedLabels1 = JainClusters5

        if j==0:
            PredictedLabels2 = JainClusters
        if j==1:
            PredictedLabels2 = JainClusters2
        if j==2:
            PredictedLabels2 = JainClusters3
        if j==3:
            PredictedLabels2 = JainClusters4
        if j==4:
            PredictedLabels2 = JainClusters5

        JainRand = adjusted_rand_score(PredictedLabels1,PredictedLabels2)
        JainRandIndex.iloc[i,j] = JainRand
        JainRandIndexNP[i,j] = JainRand

print(JainRandIndex)

plt.figure(figsize=(6,5))
sbn.heatmap(JainRandIndexNP, annot=True, cmap="viridis")
plt.title("Rand Index Comparison for Jain dataset")
plt.show()
```

```
           K-means   Average    Single  Complete  Spectral
K-means        1.0  0.925991 -0.002065  0.925991  0.814684
Average   0.925991       1.0  0.002698       1.0  0.748152
Single   -0.002065  0.002698       1.0  0.002698 -0.002688
Complete  0.925991       1.0  0.002698       1.0  0.748152
Spectral  0.814684  0.748152 -0.002688  0.748152       1.0
```

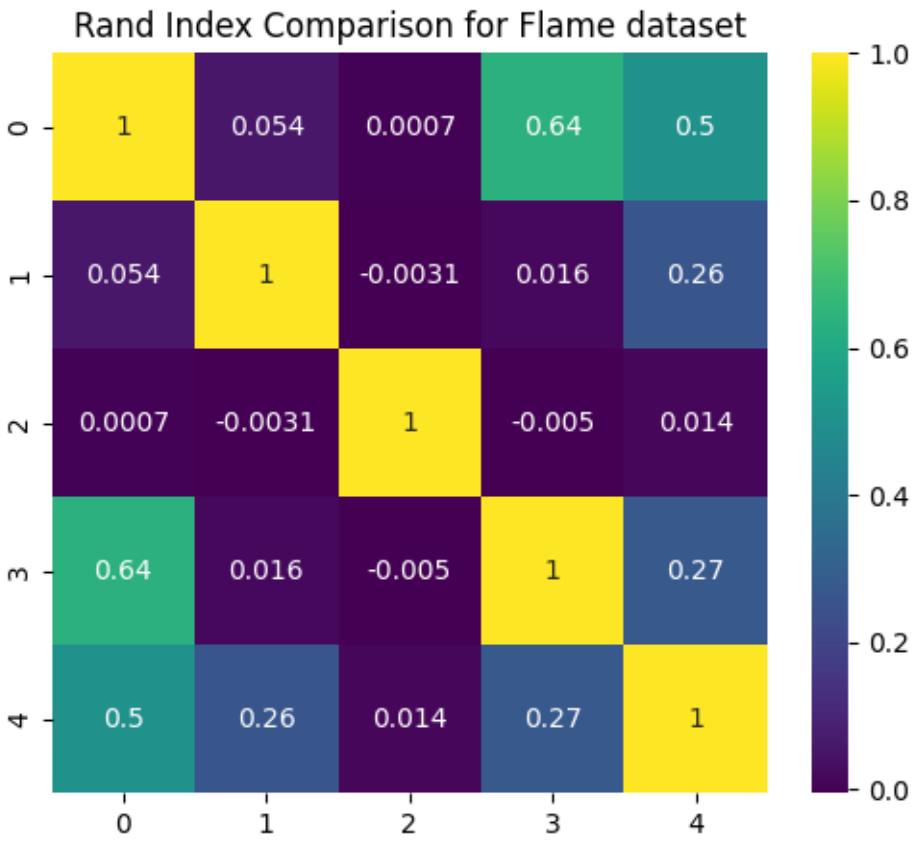


[180]:
```
# Flame dataset
# N=240, k=2, D=2
```

[222]:
```
FlameRandIndex = pd.DataFrame(index=methods , columns=methods)
FlameRandIndexNP = np.zeros((5,5))

for i in range(len(methods)):
    for j in range(len(methods)):

        if i==0:
            PredictedLabels1 = FlameClusters
        if i==1:
            PredictedLabels1 = FlameClusters2
        if i==2:
            PredictedLabels1 = FlameClusters3
        if i==3:
            PredictedLabels1 = FlameClusters4
        if i==4:
            PredictedLabels1 = FlameClusters5
```

```
        if j==0:
            PredictedLabels2 = FlameClusters
        if j==1:
            PredictedLabels2 = FlameClusters2
        if j==2:
            PredictedLabels2 = FlameClusters3
        if j==3:
            PredictedLabels2 = FlameClusters4
        if j==4:
            PredictedLabels2 = FlameClusters5

        FlameRand = adjusted_rand_score(PredictedLabels1,PredictedLabels2)
        FlameRandIndex.iloc[i,j] = FlameRand
        FlameRandIndexNP[i,j] = FlameRand

print(FlameRandIndex)

plt.figure(figsize=(6,5))
sbn.heatmap(FlameRandIndexNP, annot=True, cmap="viridis")
plt.title("Rand Index Comparison for Flame dataset")
plt.show()
```

```
           K-means   Average    Single  Complete  Spectral
K-means        1.0  0.054491  0.000705  0.638535   0.49981
Average   0.054491       1.0 -0.003118  0.015748  0.263422
Single    0.000705 -0.003118       1.0  -0.00502  0.013824
Complete  0.638535  0.015748  -0.00502       1.0  0.270843
Spectral   0.49981  0.263422  0.013824  0.270843       1.0
```

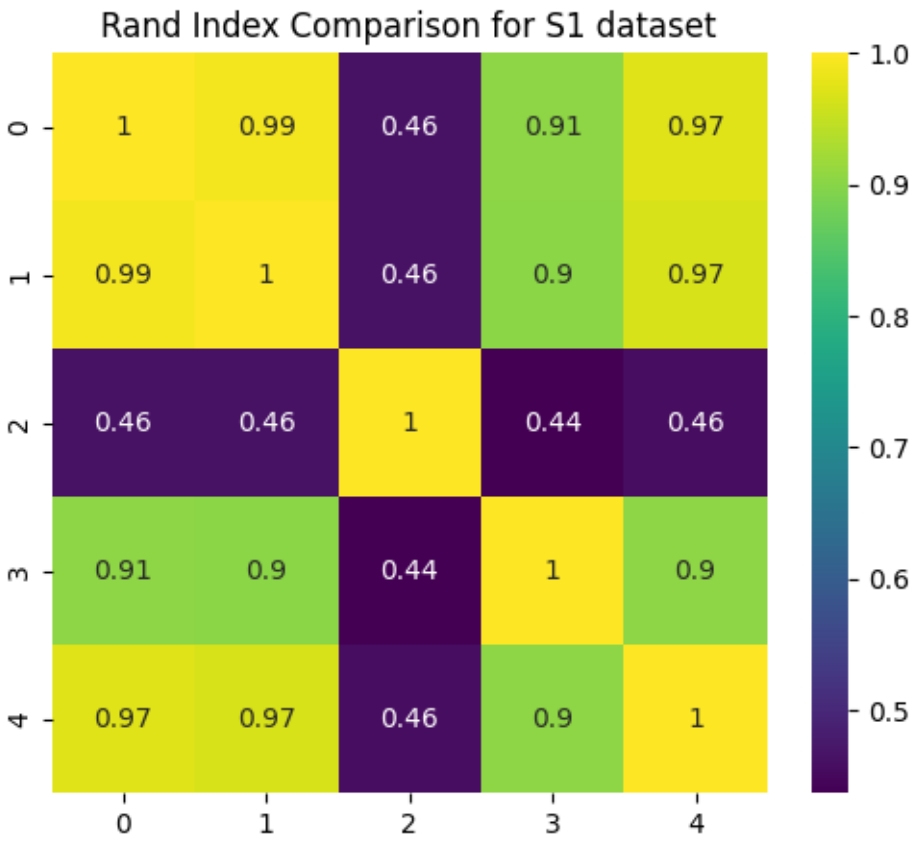


[183]:
```
# S1 dataset
# Synthetic 2-d data with N=5000 vectors and k=15 Gaussian clusters with␣
 ↪different degree of cluster overlap
```

[223]:
```
S1RandIndex = pd.DataFrame(index=methods , columns=methods)
S1RandIndexNP = np.zeros((5,5))

for i in range(len(methods)):
    for j in range(len(methods)):

        if i==0:
            PredictedLabels1 = S1Clusters
        if i==1:
            PredictedLabels1 = S1Clusters2
        if i==2:
            PredictedLabels1 = S1Clusters3
        if i==3:
            PredictedLabels1 = S1Clusters4
        if i==4:
            PredictedLabels1 = S1Clusters5
```

```
        if j==0:
            PredictedLabels2 = S1Clusters
        if j==1:
            PredictedLabels2 = S1Clusters2
        if j==2:
            PredictedLabels2 = S1Clusters3
        if j==3:
            PredictedLabels2 = S1Clusters4
        if j==4:
            PredictedLabels2 = S1Clusters5

        S1Rand = adjusted_rand_score(PredictedLabels1,PredictedLabels2)
        S1RandIndex.iloc[i,j] = S1Rand
        S1RandIndexNP[i,j] = S1Rand

print(S1RandIndex)

plt.figure(figsize=(6,5))
sbn.heatmap(S1RandIndexNP, annot=True, cmap="viridis")
plt.title("Rand Index Comparison for S1 dataset")
plt.show()
```

```
           K-means   Average    Single  Complete  Spectral
K-means        1.0  0.989727  0.462011  0.905319  0.971393
Average   0.989727       1.0  0.462504  0.902849  0.966765
Single    0.462011  0.462504       1.0  0.436653  0.455791
Complete  0.905319  0.902849  0.436653       1.0  0.903843
Spectral  0.971393  0.966765  0.455791  0.903843       1.0
```

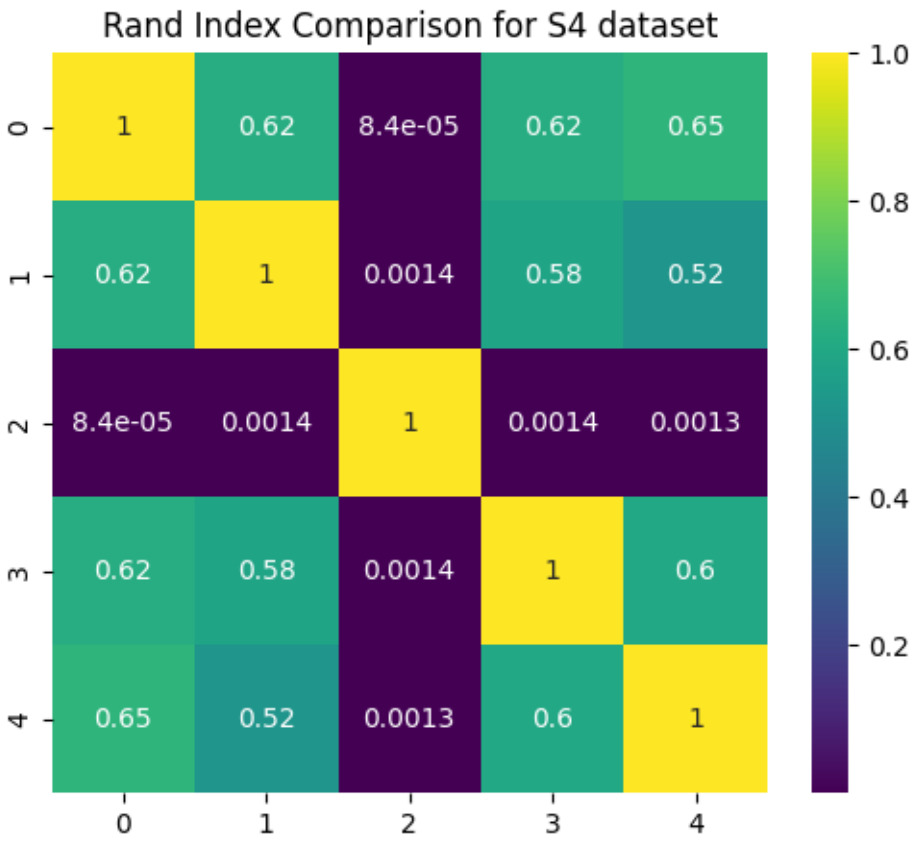


[185]:
```
# S4 dataset
# Synthetic 2-d data with N=5000 vectors and k=15 Gaussian clusters with␣
 ↪different degree of cluster overlap
```

[218]:
```
S4RandIndex = pd.DataFrame(index=methods , columns=methods)
S4RandIndexNP = np.zeros((5,5))

for i in range(len(methods)):
    for j in range(len(methods)):

        if i==0:
            PredictedLabels1 = S4Clusters
        if i==1:
            PredictedLabels1 = S4Clusters2
        if i==2:
            PredictedLabels1 = S4Clusters3
        if i==3:
            PredictedLabels1 = S4Clusters4
        if i==4:
            PredictedLabels1 = S4Clusters5
```

```
        if j==0:
            PredictedLabels2 = S4Clusters
        if j==1:
            PredictedLabels2 = S4Clusters2
        if j==2:
            PredictedLabels2 = S4Clusters3
        if j==3:
            PredictedLabels2 = S4Clusters4
        if j==4:
            PredictedLabels2 = S4Clusters5

        S4Rand = adjusted_rand_score(PredictedLabels1,PredictedLabels2)
        S4RandIndex.iloc[i,j] = S4Rand
        S4RandIndexNP[i,j] = S4Rand

print(S4RandIndex)

plt.figure(figsize=(6,5))
sbn.heatmap(S4RandIndexNP, annot=True, cmap="viridis")
plt.title("Rand Index Comparison for S4 dataset")
plt.show()
```

```
           K-means   Average    Single  Complete  Spectral
K-means        1.0  0.624955  0.000084  0.621982  0.649843
Average   0.624955       1.0  0.001441  0.584414  0.523268
Single    0.000084  0.001441       1.0  0.001359  0.001327
Complete  0.621982  0.584414  0.001359       1.0  0.597986
Spectral  0.649843  0.523268  0.001327  0.597986       1.0
```


Rand Index Comparison for S4 dataset
1
0.62
8.4e-05
0.62
0.65
0.62
1
0.0014
0.58
0.52
8.4e-05
0.0014
1
0.0014
0.0013
0.62
0.58
0.0014
1
0.6
0.65
0.52
0.0013
0.6
1
0
1
2
3
4
1.0
0.8
0.6
0.4
0.2

[ ]: