Modelling Corona-Virus using SIR model

December 2, 2020

The SIR model (S $\equiv$Susceptible, $I \equiv$ Infected, R $\equiv$Recovered) is:

$$\frac{dS}{dt} = -\frac{\beta}{N}SI \tag{0.1}$$

$$\frac{dI}{dt} = \frac{\beta}{N}SI - \gamma I \tag{0.2}$$

$$\frac{dR}{dt} = \gamma I \tag{0.3}$$

Here, $N \equiv S + I + R$ is independent of time t denotes total population size.

1 Estimating β and γ:

1.1 Estimating $\beta - \gamma$

At the onset of infection, almost entire population is susceptible: $S \approx N$,

(0.2) $\Rightarrow \frac{dI}{dt} = \frac{\beta}{N}NI - \gamma I \Rightarrow \frac{dI}{dt} \sim (\beta - \gamma)I$.

So $I(t)$ first grows exponentially $I = I_0 e^{(\beta-\gamma)t}$.

(But later maybe power low?!)

$$\frac{dI}{dt} \sim mI \ \textit{where}\ m = \beta - \gamma \Rightarrow I(t) \sim I_0 e^{mt}.$$

We can estimate m by looking at the data on a log-plot and fit the best line.

$\ln I = mt + \ln I_0$ *where* $m = \beta - \gamma$ *is slope of the line.*

1.2 Estimating γ

- Suppose $I(t) \approx I_0$ constant.

  Then $\frac{dR}{dt} = \gamma I_0 \Rightarrow R(t) = \gamma t I_0$.

  Q: When everybody get recovered?

  A: When $R = I_0$.

  If it takes T days to recover, Then $R(T) = I_0 \Rightarrow \gamma t I_0 = I_0 \Rightarrow \gamma T = 1$.

  So $\gamma \approx \frac{1}{T}$ where T is recovery period.

  EX: Estimates for $T \approx 2 \, to \, 5$ weeks recovery period $\Rightarrow \gamma \approx \frac{1}{35} \, to \, \frac{1}{14}$.

- Estimate γ directly from the data

  $(0.3) \Rightarrow \frac{dR}{dt} = \gamma I \Rightarrow \frac{R(t+a)-R(t)}{a} = \gamma I$.

  for $a = 1 \Rightarrow \gamma = \frac{R(t+1)-R(t)}{I(t)}$.

  Theoretically, this should be almost constant, but in the real world it changes for various reasons, such as the number of tests performed, quarantine measures, and so on.

1.3 Estimating β

Then use $\beta = m + \gamma_0$.