Digital Image Processing

Morphological Image Processing

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Distance/online Course: Session 05 Episode 01

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Aim and Goal

- Used to extract image components that are useful in the representation, description and modification of region shape, such as:
  - Boundaries extraction
  - Skeletons
  - Convex hull
  - Morphological filtering
  - Thinning
  - Pruning

Mathematical Background

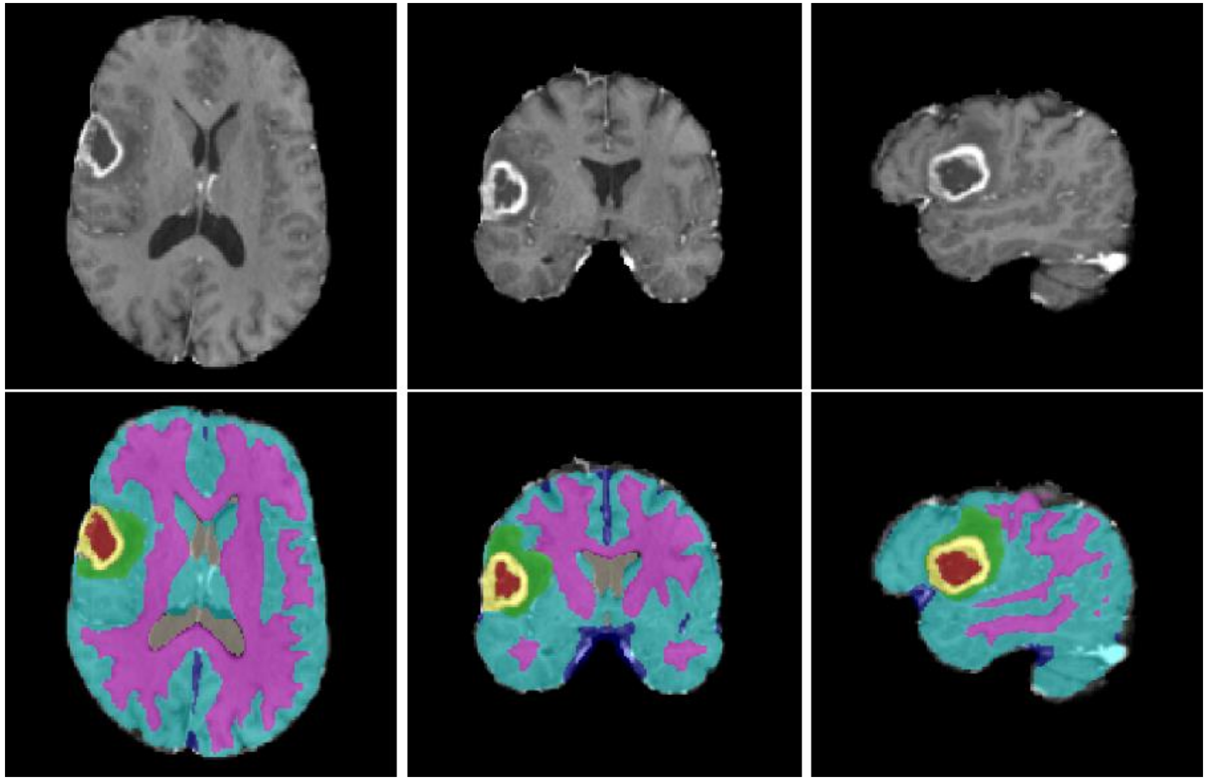
- The language of mathematical morphology is set theory

- In binary images, the sets in question are members of the 2-D integer space Z^2 , where each element of a set is a tuple (2-D vector) whose coordinates are the coordinates of an object (typically foreground) pixel in the image.

- Grayscale digital images can be represented as sets whose components are in Z^3 . In this case, two components of each element of the set refer to the coordinates of a pixel, and the third corresponds to its discrete intensity value

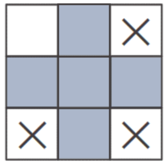
Why Binary Image

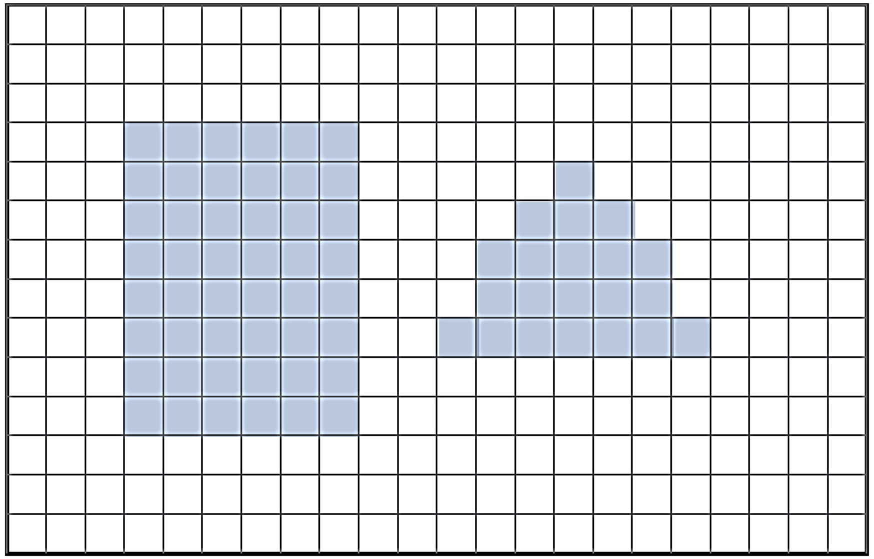
- After segmentation step, we have:



Structuring Elements (SE's)

- Morphological operations are defined in terms of sets. In image processing, we use morphology with two types of sets of pixels: objects and structuring elements (SE's).
- Its rule is like as impulse response in signal/image processing
- SE's member:
  - Background (0)
  - Foreground (1)
  - Don't Care (×)

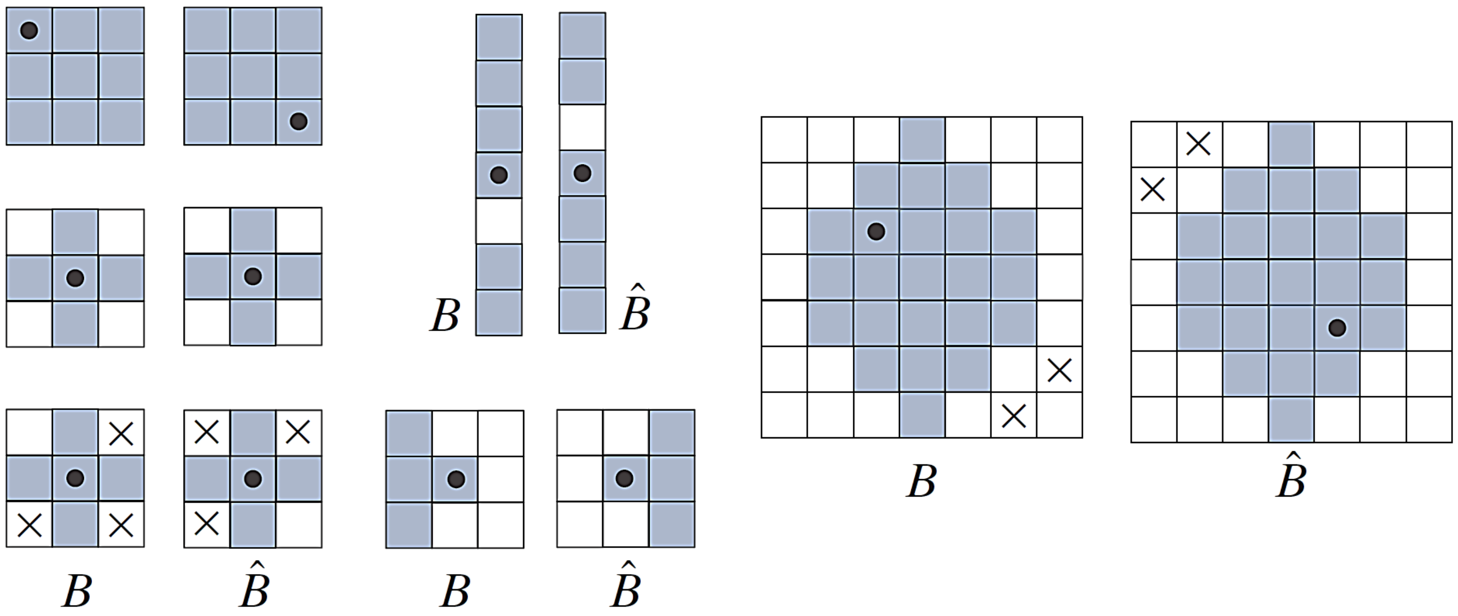




Set Reflection

- Set Reflection (about its origin):

$$\hat{B} = \{ w \mid w = -b, \text{ for } b \in B \}$$



Set Translation

- Set Translation by point $z = (z_1, z_2)$:

$$(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$$

An Example

› An (I) image and a SE’s (B):

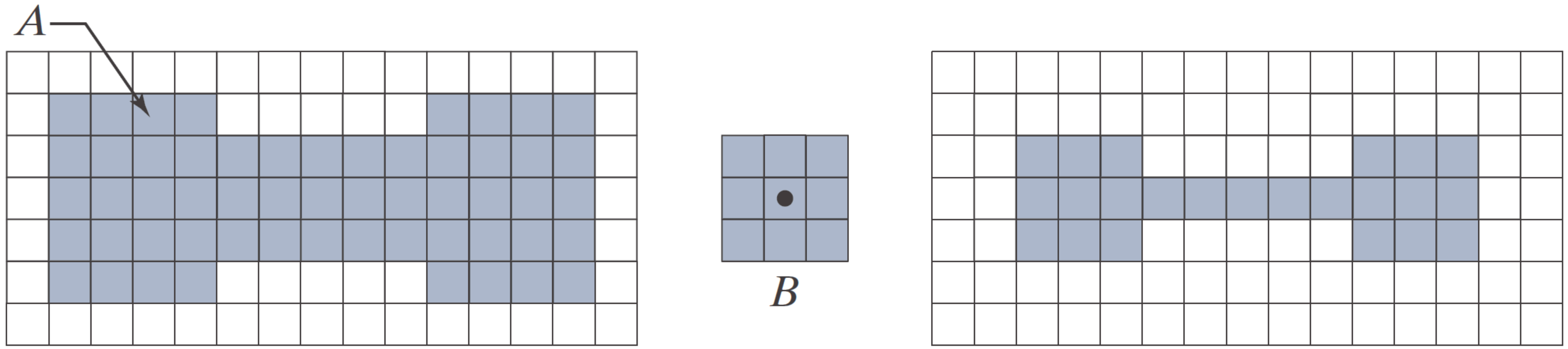


Image I

Image after morphological operation

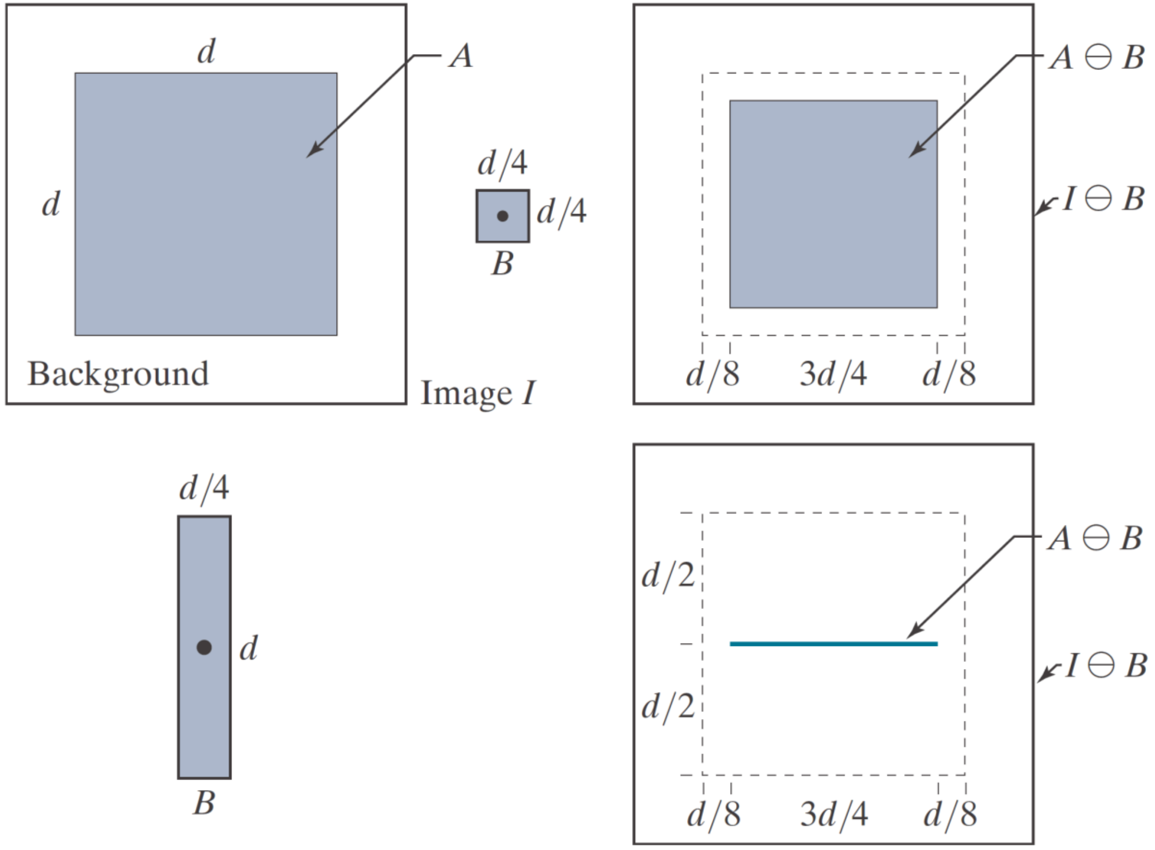
Two Fundamental Operator

- There are two fundamental operation:
  - Dilation
  - Erosion

Erosion

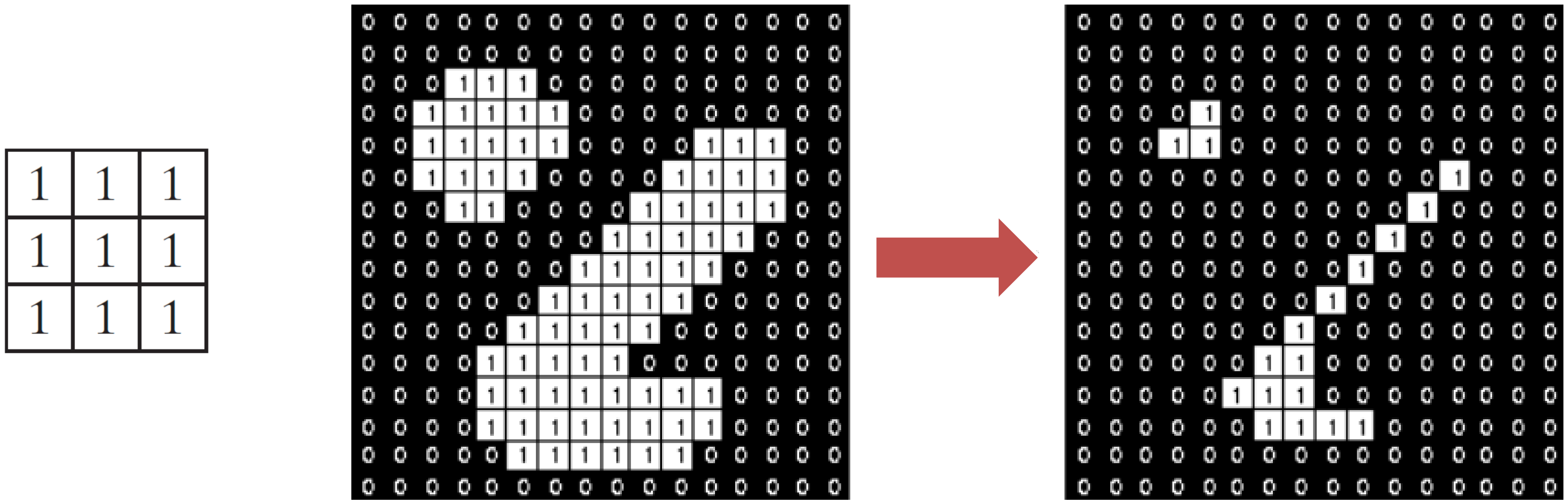
- Erosion Definition:

$$A \ominus B = \left\{ z \,\middle|\, (B)_z \subseteq A \right\}$$



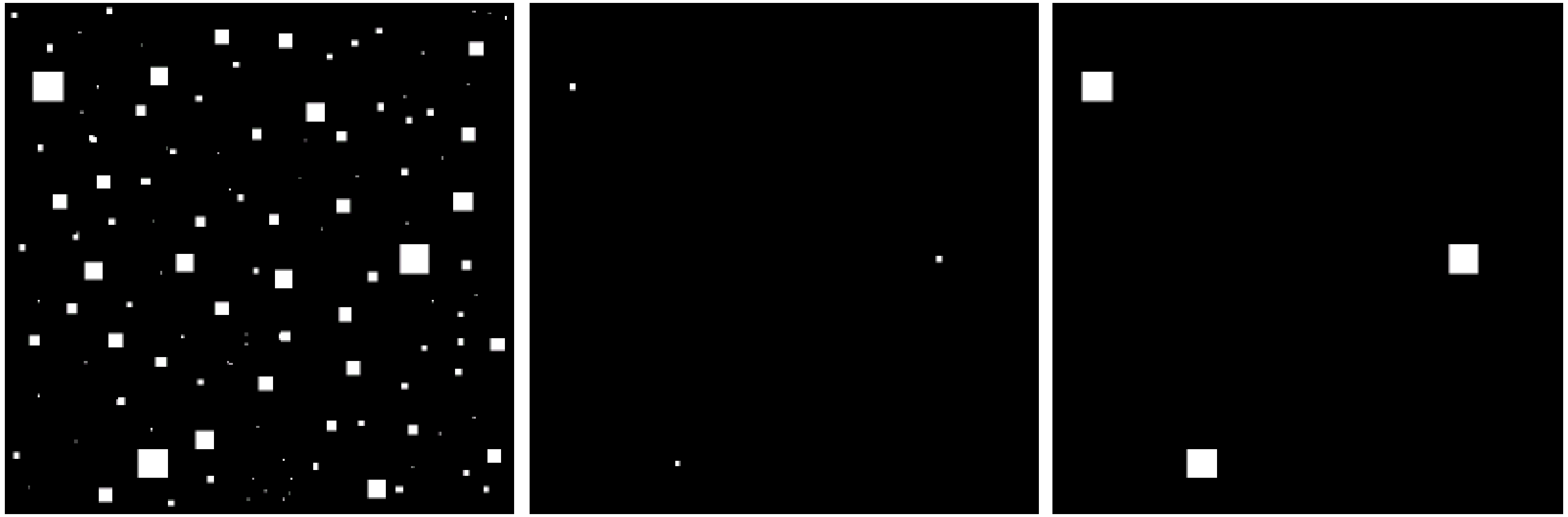
Erosion - Example

› Image erosion with a full 3×3 SE:



Erosion - Example

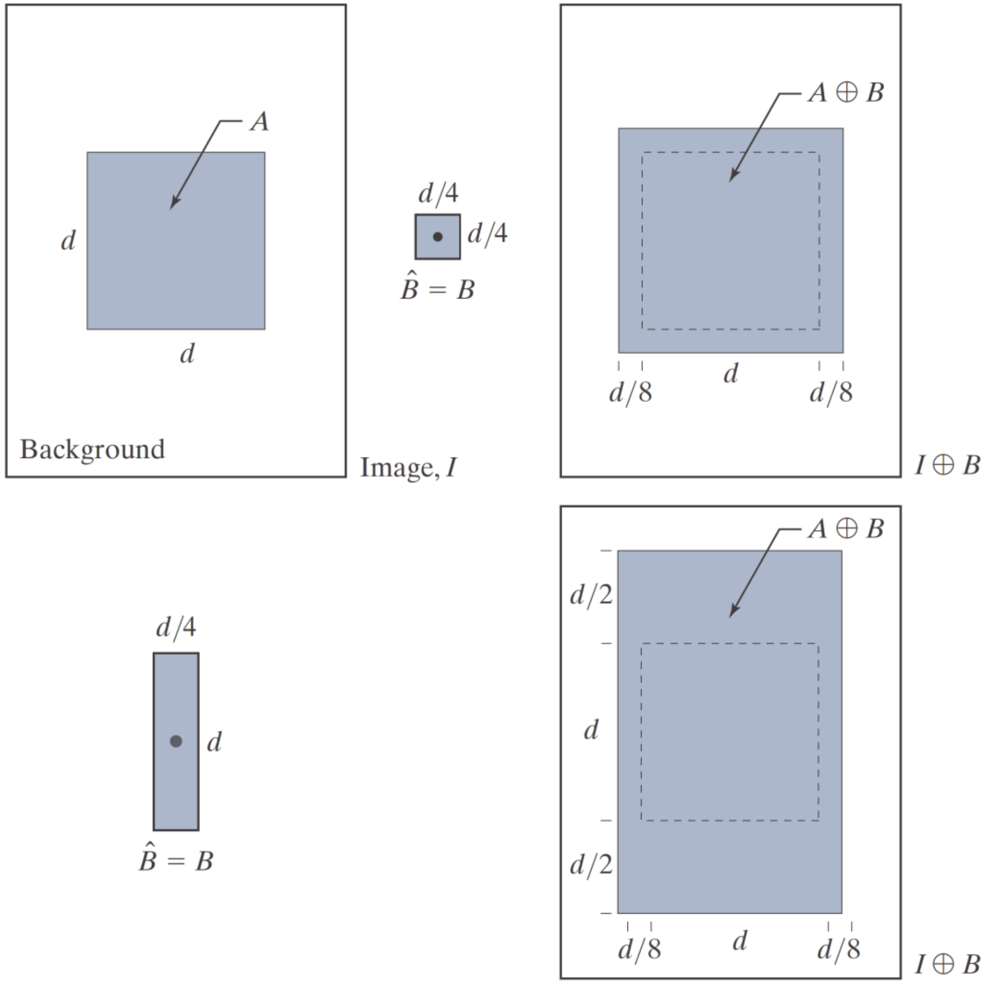
- Remove small objects:
- Object size in input image: $1 \times 1, 3 \times 3, 5 \times 5, 7 \times 7, 9 \times 9, 15 \times 15$
- Erosion with a full 13×13 SE then dilation with same SE



Dilation

› Dilation Definition:

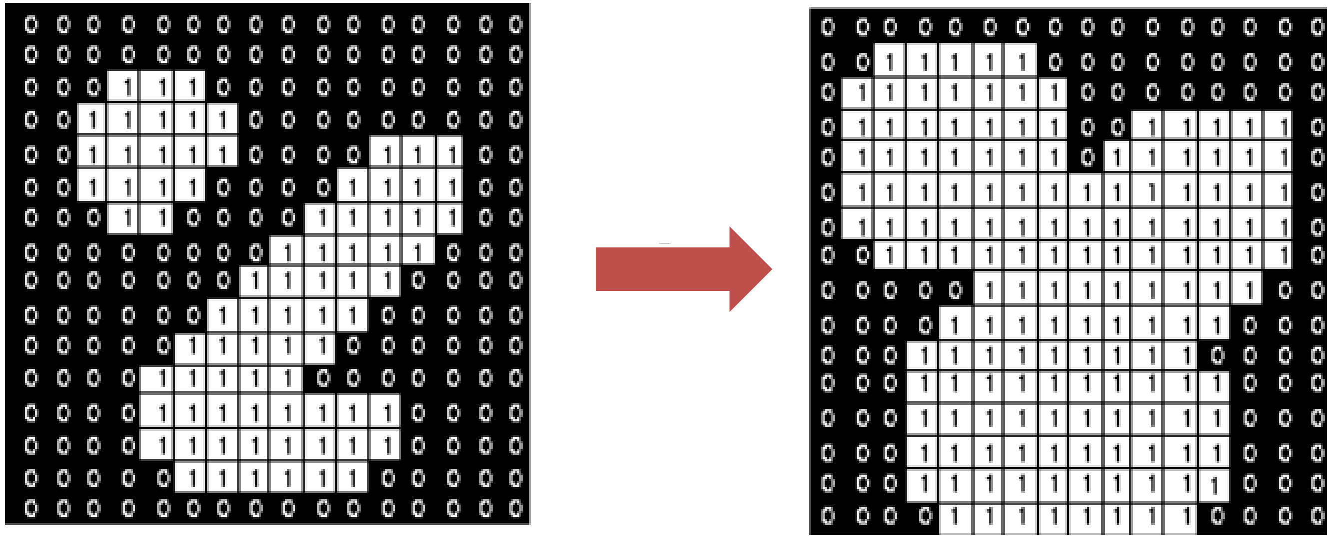
$$A \oplus B = \left\{ z \,\middle|\, [(\hat{B})_z \cap A] \subseteq A \right\}$$



Dilation - Example

› Image dilation with a full 3 × 3 SE:

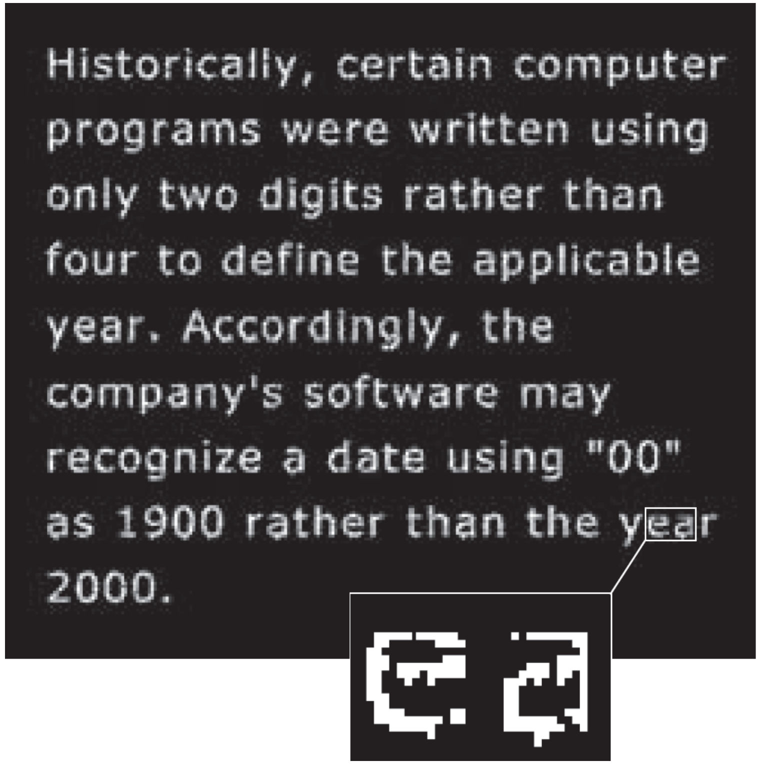
1	1	1
1	1	1
1	1	1

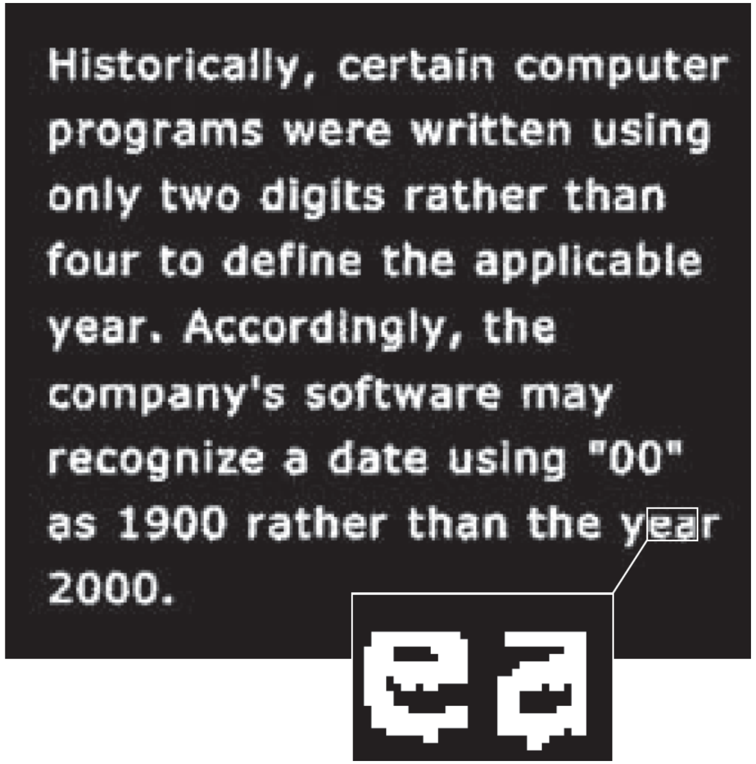


Dilation - Example

› Image dilation with a full 3 × 3 SE:

1	1	1
1	1	1
1	1	1





Opening and Closing

- Dilation expands and Erosion shrinks.
  - Opening:
    - Smooth contour
    - Break narrow isthmuses
    - Remove thin protrusion
  - Closing:
    - Smooth contour
    - Fuse narrow breaks,
    - and long thin gulfs.
    - Remove small holes, and fill gaps.

Opening and Closing

- Opening:
  - An erosion followed by a dilation using the same SE for both:

$$A \circ B = (A \ominus B) \oplus B$$

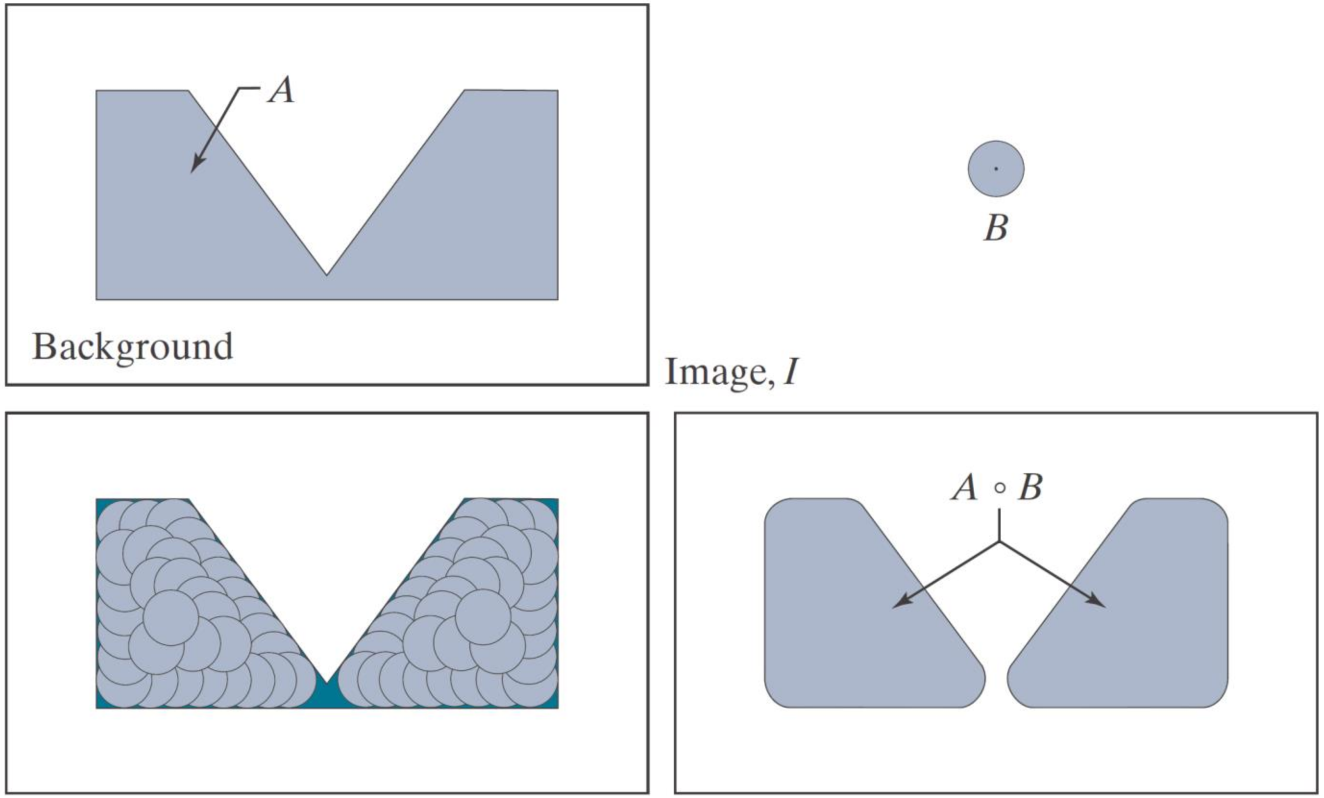
- Closing:
  - An dilation followed by a erosion using the same SE for both:

$$A \bullet B = (A \oplus B) \ominus B$$

Opening Example

› Opening Example:

$$A \circ B = (A \ominus B) \oplus B$$



Opening - Example

› Image opening with a full 3×3 SE:

1	1	1
1	1	1
1	1	1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	1	0	0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
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0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
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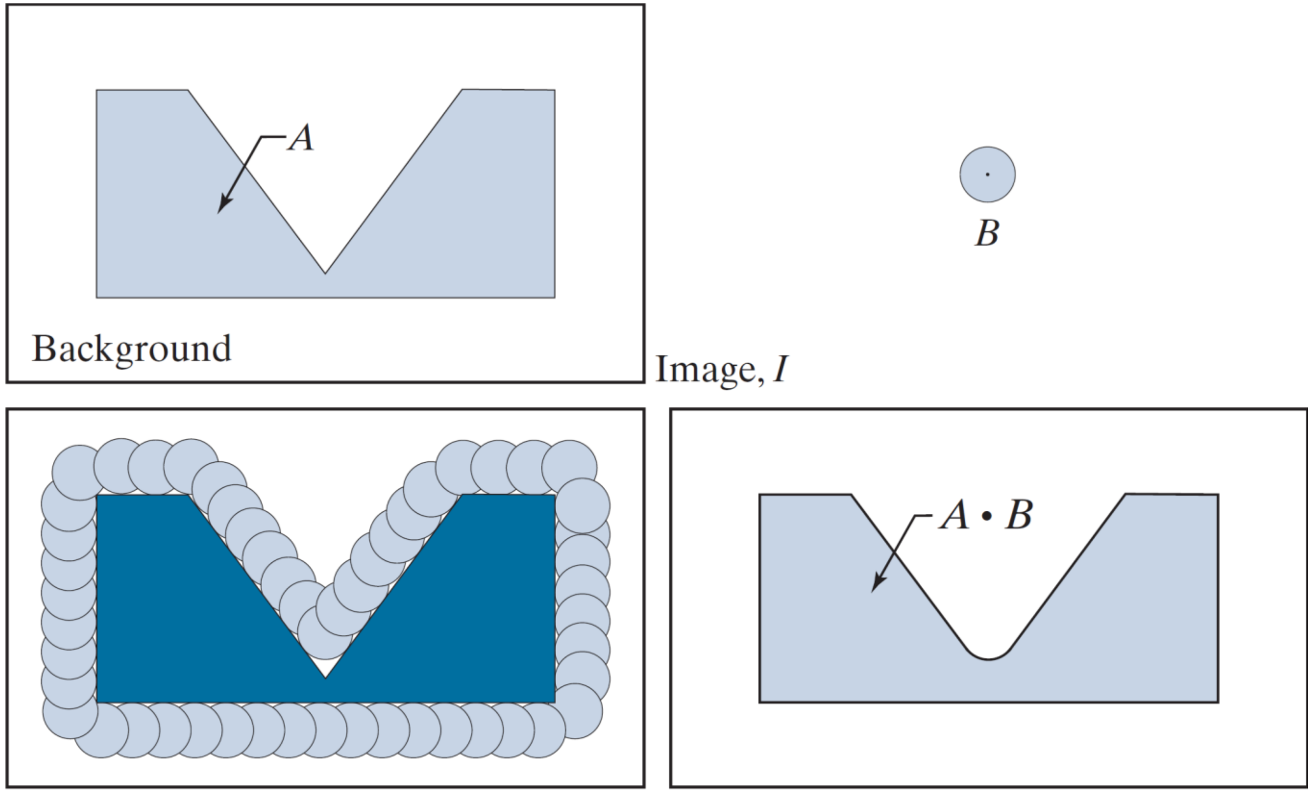
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Closing Example

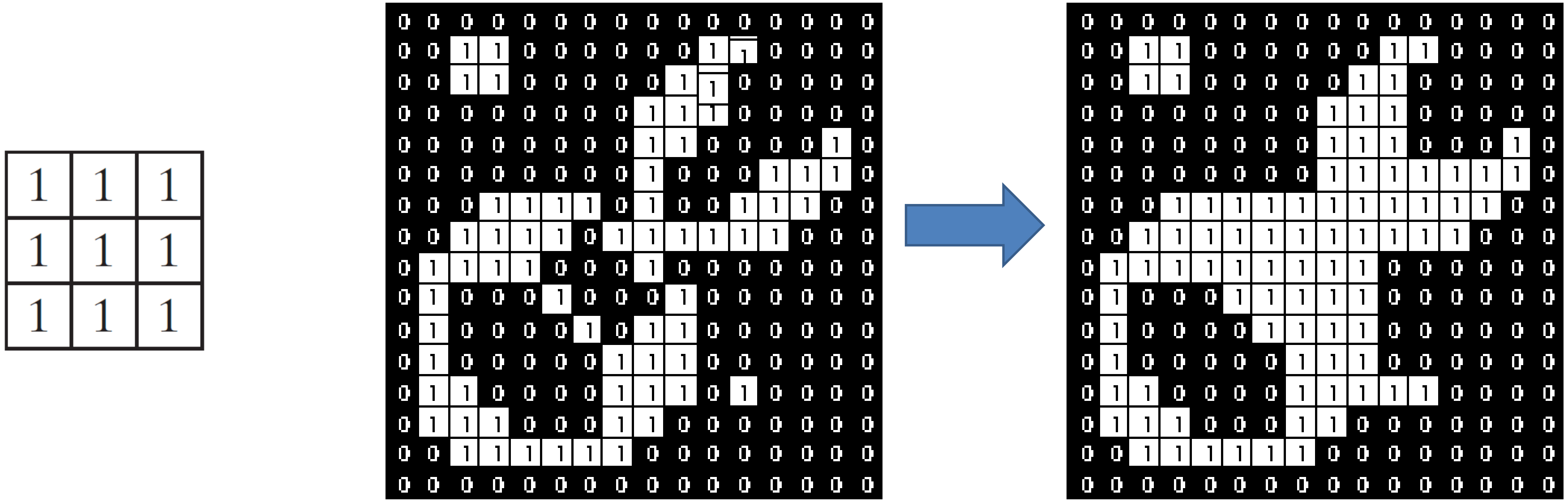
- Closing Example:

$$A \bullet B = (A \oplus B) \ominus B$$



Closing - Example

› Image closing with a full 3 × 3 SE:



Opening and Closing Properties

› Morphological opening has the following properties:

(a) $A \circ B$ is a subset of A.

(b) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.

(c) $(A \circ B) \circ B = A \circ B$.

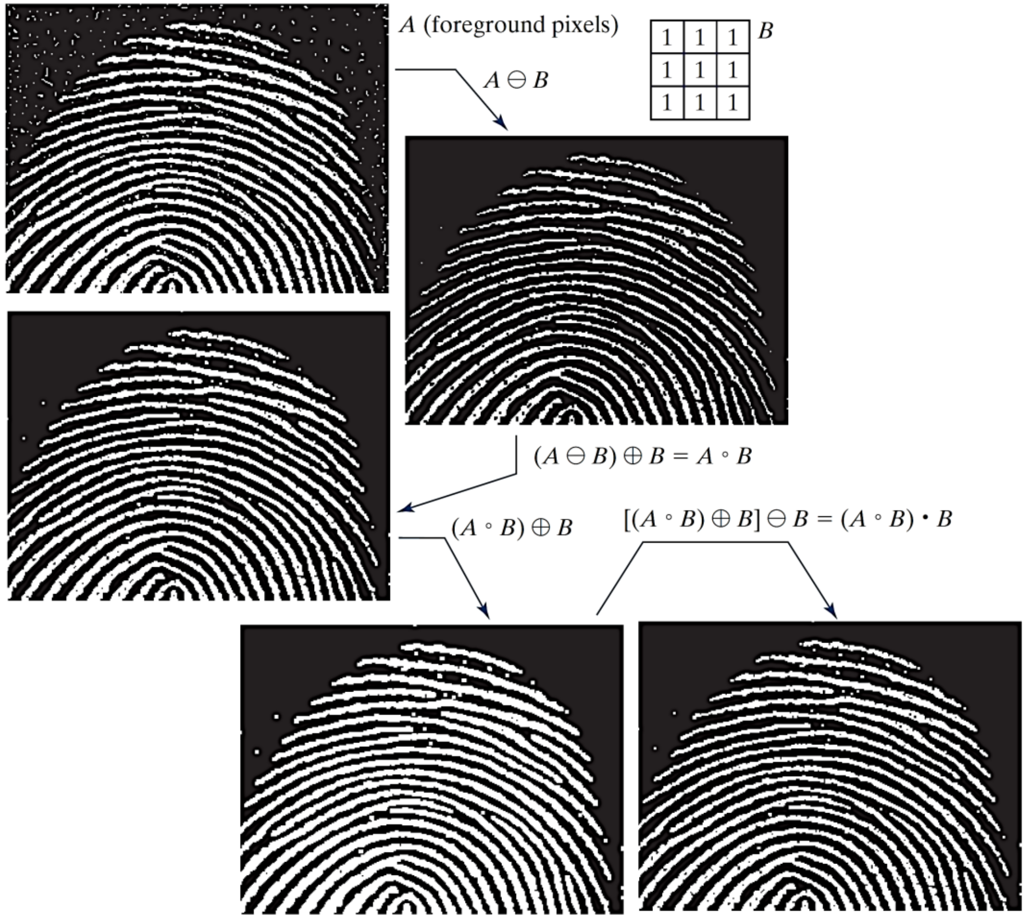
(a) A is a subset of $A \bullet B$.

(b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.

(c) $(A \bullet B) \bullet B = A \bullet B$.

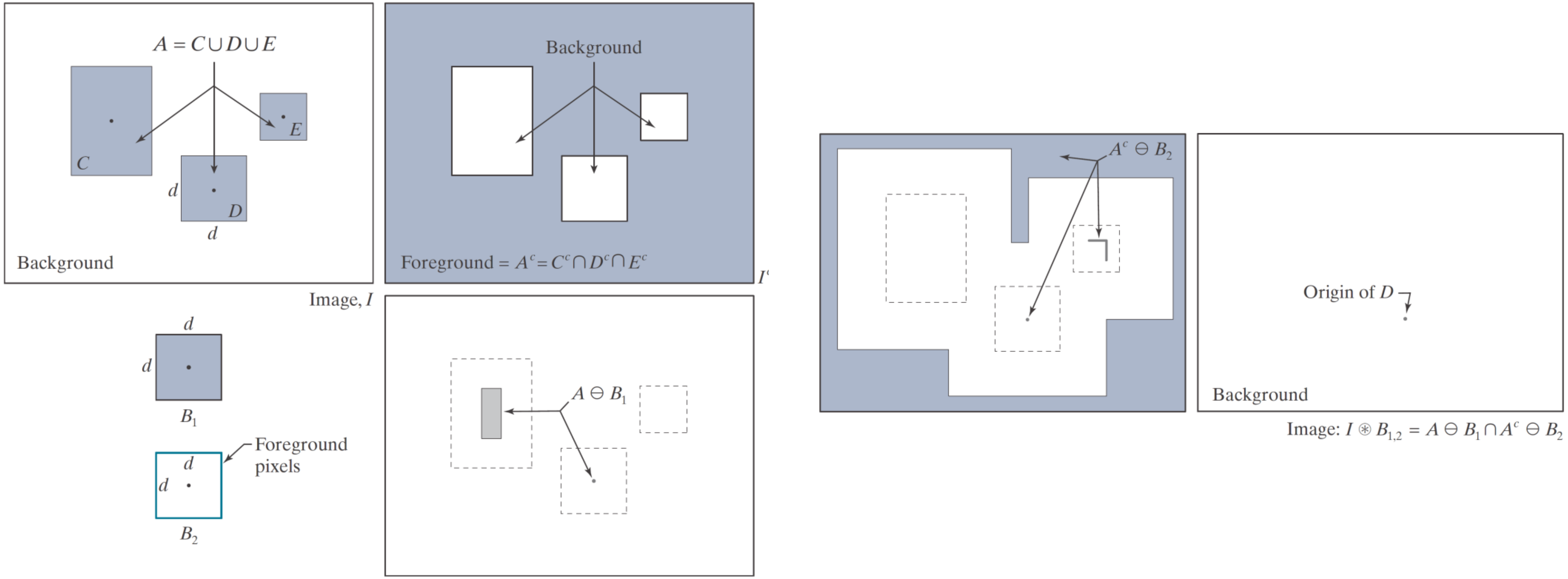
Morphological Filtering

- Noise Removal:



Hit-or-Miss

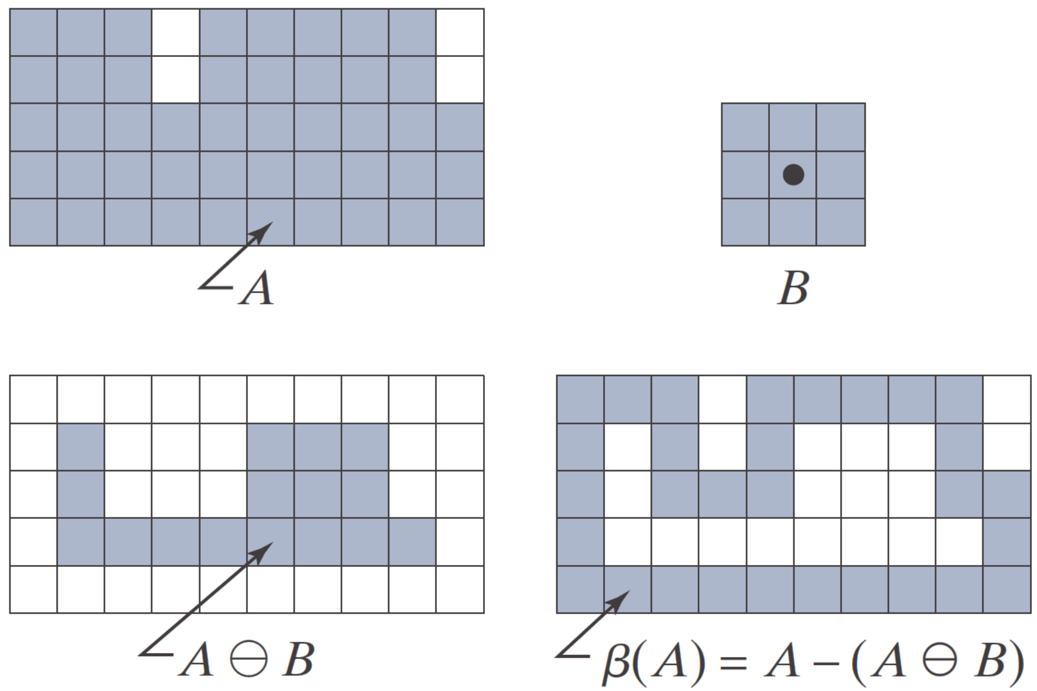
- The morphological hit-or-miss transform (HMT) is a basic tool for shape detection using template.



Application (1)

› Boundary Extraction Formulation:

$$\beta(A) = A - (A \ominus B)$$



Application (1)

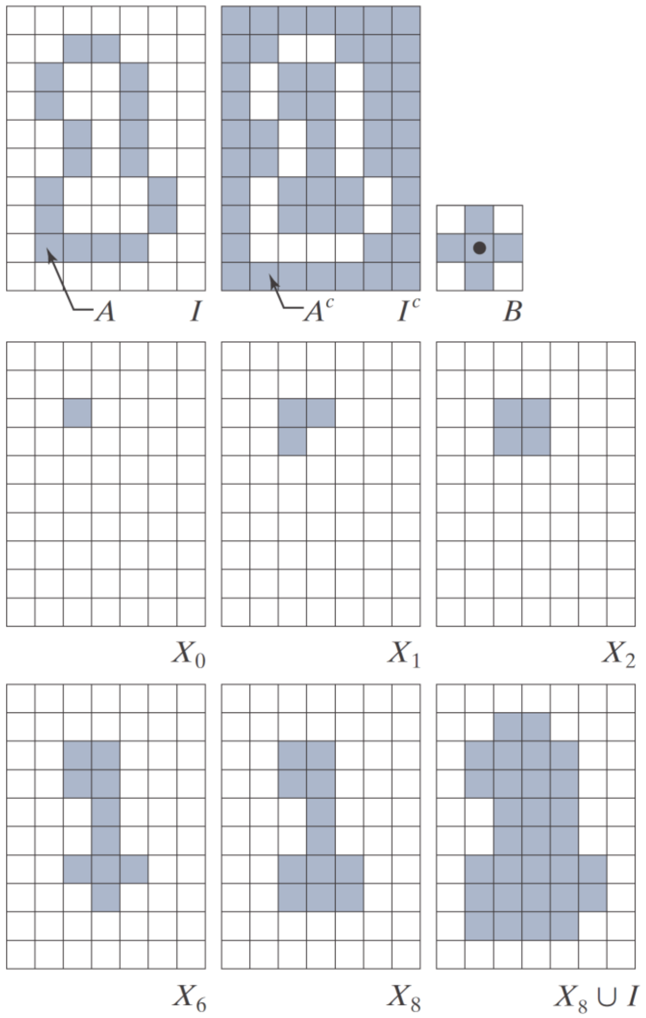
› Boundary Extraction Example:

Application (2)

- Hole Filling Formulation:

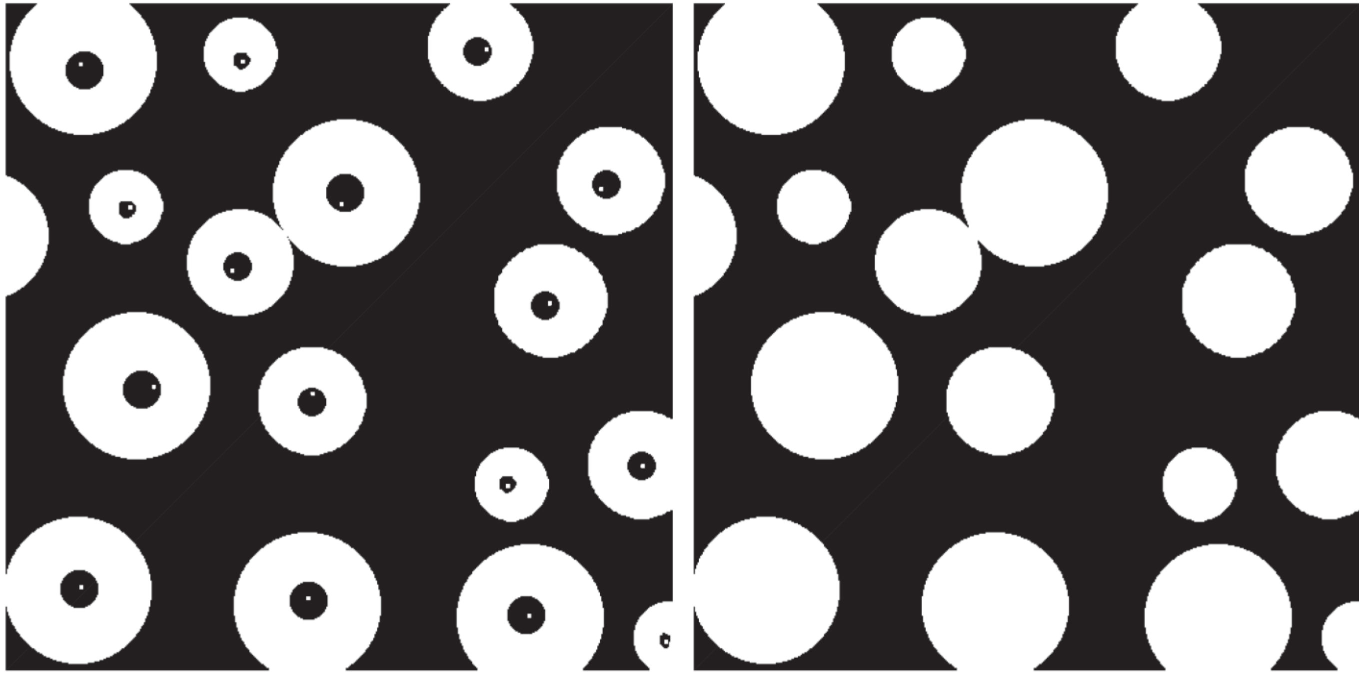
$$X_k = \left(X_{k-1} \oplus B\right) \cap I^c$$

- Start inside the hole
- Repeat until convergence



Application (2)

› Hole Filling Example:

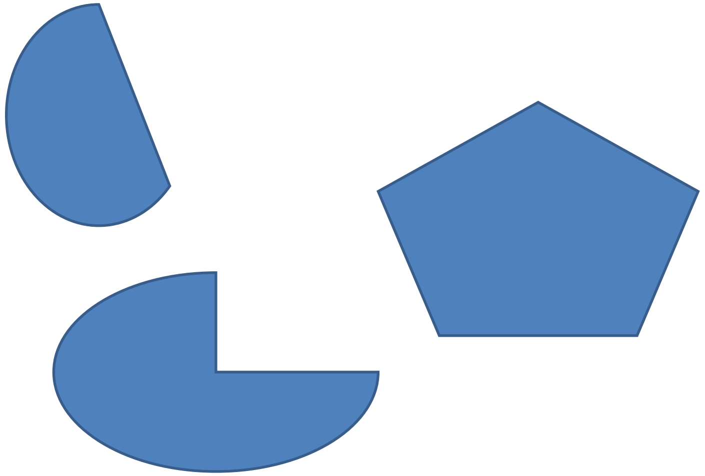


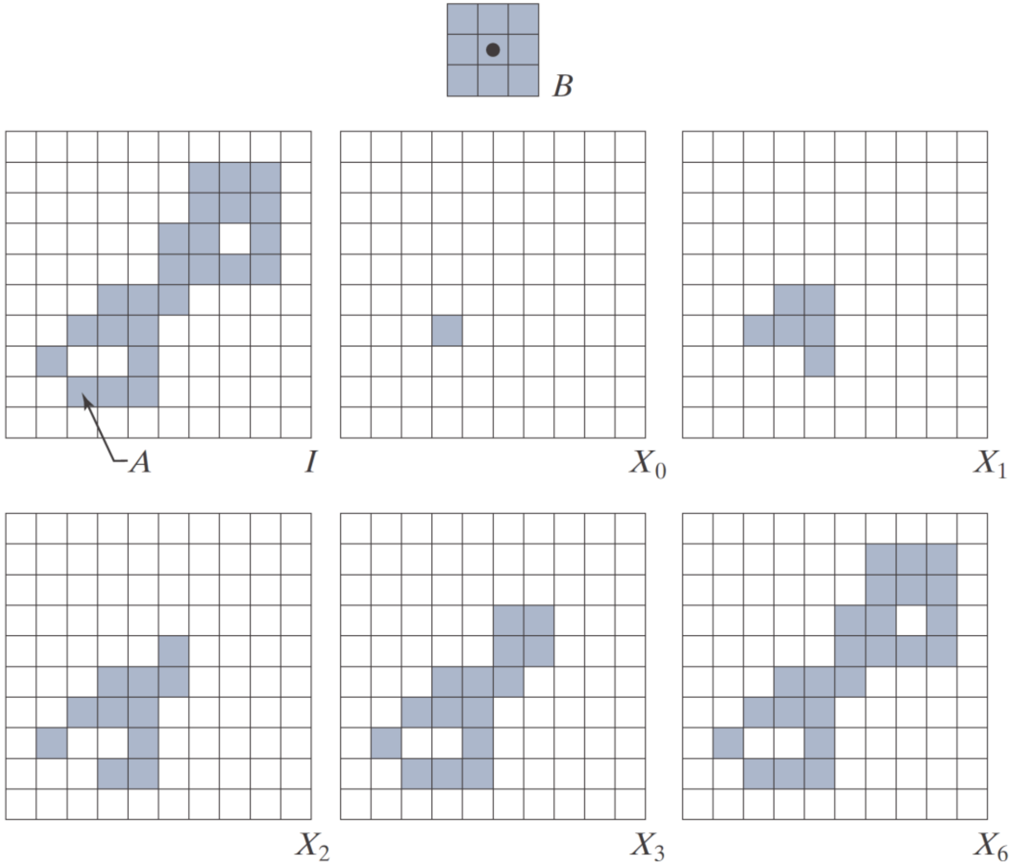
Application (3)

- Connected Component Extraction Formulation:

$$X_k = \left(X_{k-1} \oplus B\right) \cap I$$

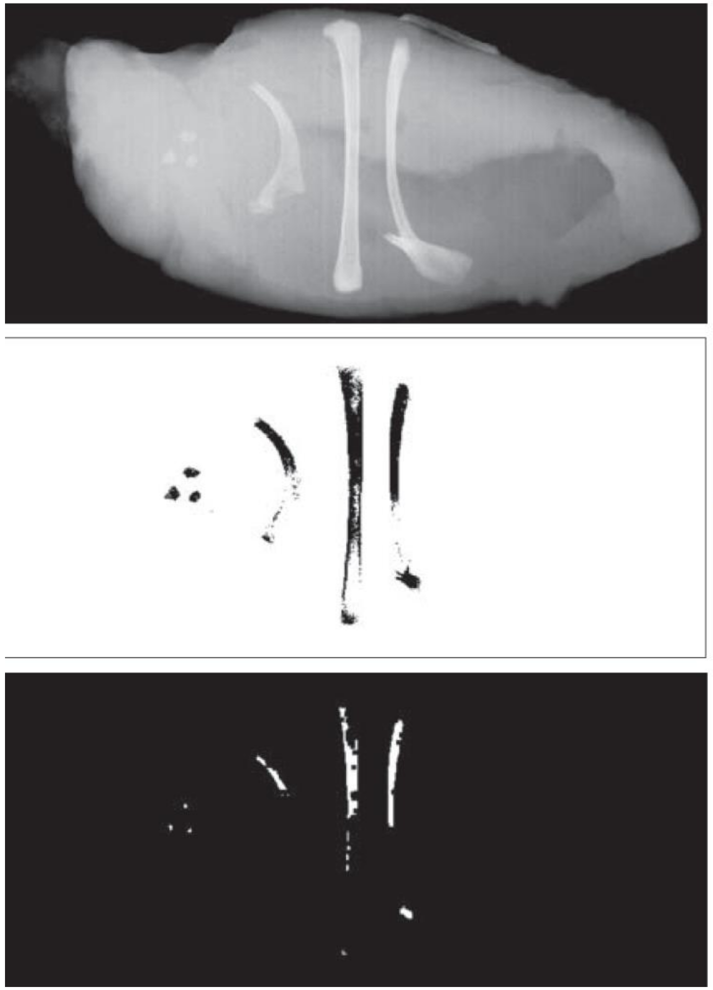
- Start inside the region
- Repeat until convergence





Application (3)

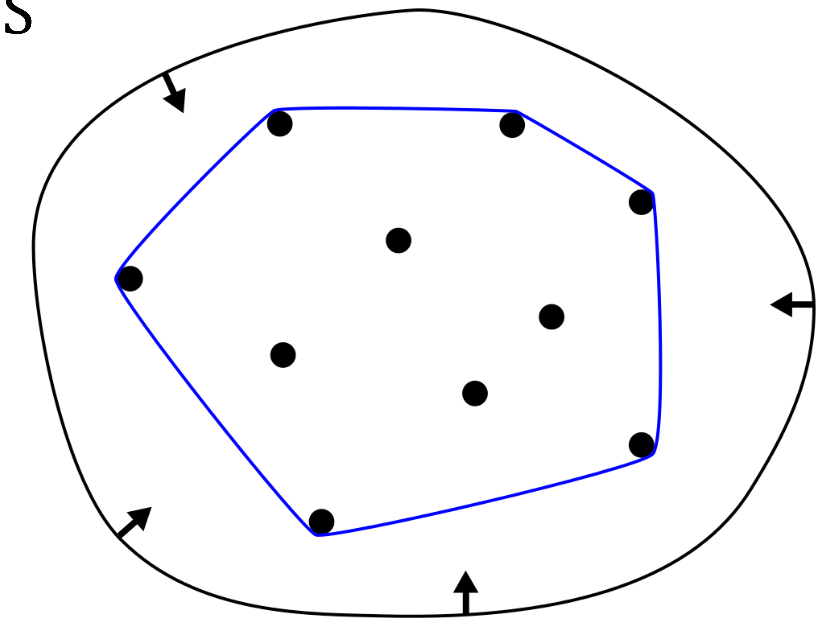
- Connected Component Extraction Formulation:



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

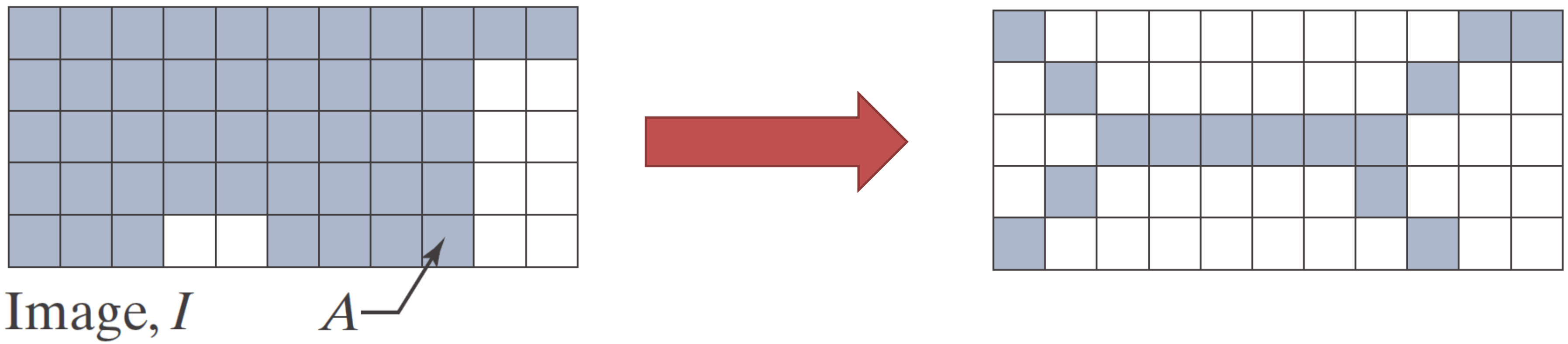
Application (4)

- Convex Hull Extraction:
  - Smallest Convex set H, containing S



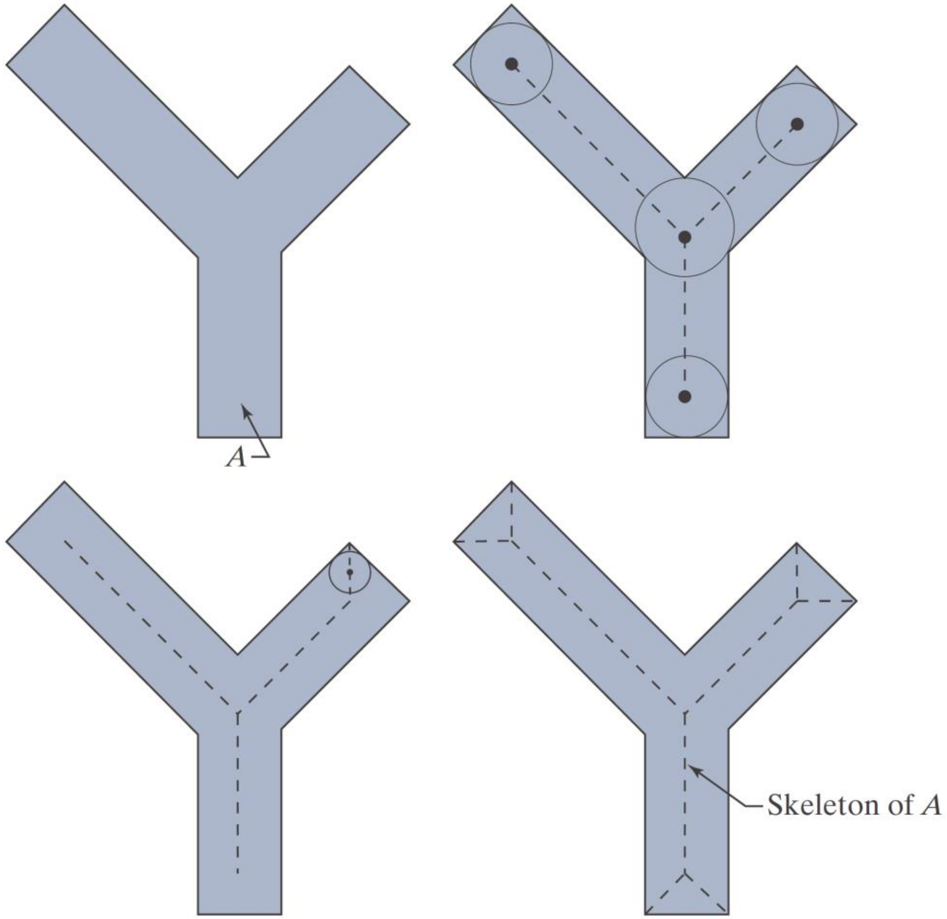
Application (5)

› *Thinning* and Skeletonization



Application (5)

› Thinning and *Skeletonization*



Matlab Command

- *strel*: Create morphological structuring element
- imerode, imdilate
- imclose, imopen
- bwhitmiss, imtophat
- imfill: Fill image regions and holes
- conndef: Create connectivity array

The End

› AnY QuEsTiOn?