CPSC 2150 – Algorithms and Data Structures II

Assignment 3: Graphs

Total - 90 Marks

"First, solve the problem. Then, write the code."
- John Johnson

Learning Outcomes

- Design and implement an appropriate data structure for a given graph.
- Design and implement the efficient graph-based algorithm in terms of time and space complexity.
- Develop C++ code based on the existing constraints.

Exercise 1: Suppose that graph G has the following adjacency lists:

```
1 – (2, 3, 4)
2 – (1, 3, 4)
3 – (1, 2, 4)
4 – (1, 2, 3, 6)
5 – (6, 7, 8)
6 – (4, 5, 7)
7 – (5, 6, 8)
8 – (5, 7)
```

1. **[5 marks]** Draw G.
2. **[5 marks]** Give the sequence of vertices visited using depth-first search starting at vertex 6. Draw the possibility tree to justify your answer.
3. **[5 marks]** Give the sequence of vertices visited using breadth-first search starting at vertex 6. Draw the possibility tree to justify your answer.

Exercise 2 [15 marks]: In the lecture we learned algorithms such as Kruskal, Prim and Dijkstra to find the MST and shortest path of a given weighted graph with only positive labels. Do you think these algorithms can be used with a weighted graph with negative and positive labels? Discuss your answer (**answers.pdf**).

Exercise 3 [15 marks]: There are eight small islands in a lake and we want to build seven bridges to connect them so that each island can be reached by the others. The distances in feet between pairs of islands are given in the following table.

	1	2	3	4	5	6	7	8
1	-	240	210	340	300	200	345	120
2	-	-	265	175	215	180	190	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	270	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-

Find which bridges to build so that their total length is minimum. Use the minimum spanning tree algorithm taught in class, starting from island 1 Show the execution of the algorithm using the following tabular format, where each row of the table corresponds to a step of the algorithm. The first row has been filled for your convenience. Complete the table with the remaining 7 rows.

Step	Selected islands	Unselected islands	Selected Bridge (Edge)
0	{1}	{2,3,4,5,6,7,8}	none

Exercise 4 [15 marks]: Let undirected graph G = (V, E) and vertices start, goal ∈ V be given. Assuming all edges in E are of non-negative weight, describe an efficient algorithm for finding the longest acyclic path from start to goal.
Discuss your algorithm for a directed graph with non-negative weight.

Exercise 5 [15 marks]: Consider the following greedy strategy for finding a shortest path from vertex start to vertex goal in a given connected graph.

1. Initialize path to start.
2. Initialize visitedVertices to {start}.
3. If start=goal, return path and exit. Otherwise, continue.
4. Find the edge (start,v) of minimum weight such that v is adjacent to start and v is not in visitedVertices.
5. Add v to path.
6. Add v to visitedVertices.
7. Set start equal to v and go to step 3.

Does this greedy strategy always find a shortest path from start to goal? Either explain intuitively why it works, or give a counter-example.

Exercise 6 [20 marks]: Let G be an undirected graph with vertex set {0, 1,...,n − 1} and with m edges. We denote with (i, j) the edge connecting vertex i and vertex j, and with d(i) the degree of vertex i. We define the following two operations:

- deleteEdge(i, j): delete from G a given edge (i, j).
- deleteIncidentEdges(i): delete from G all the d(i) edges incident on a given vertex i.

Provide a precise analysis of the time complexity of operations deleteEdge and deleteIncidentEdges for the following two representations of graph G:

1. Graph G is represented by an n × n boolean matrix A such that A[i, j] is true if and only if G contains edge (i, j).
2. Graph G is represented by n sequences S1,...,Sn, where sequence Si contains all the d(i) vertices adjacent to vertex i and is realized by means of a doubly-linked list.

SUBMIT to D2L

Submit a zip file named **StudentNumber-Asgn3.zip** including **answers.pdf.** For example, if your student number is 10023449, the submitted file must be named as **10023449-Asgn3.zip**.