

**Automata Theory**  
**Assignment #5**  
**Suggested Solutions**  
**Due: June 16, 2008 (before the Exam)**

1. (10 pts) Find a context-free grammar for the language  $L = \{a^n w w^R b^n : w \in \Sigma^*, n \geq 1\}$  where  $\Sigma = \{a, b\}$ .

**Answer:** A suggested answer is

$$\begin{aligned} S &\rightarrow aSb|S_1, \\ S_1 &\rightarrow aS_1a|bS_1b|\lambda. \end{aligned}$$

2. (20 pts) Find context-free grammars for the following languages (with  $n \geq 0, m \geq 0$ ).

- (a)  $L = \{a^n b^m : n \leq m + 3\}$   
(b)  $L = \{a^n b^m : n \neq m - 1\}$   
(c)  $L = \{a^n b^m : n \neq 2m\}$   
(d)  $L = \{a^n b^m : 2 \leq m \leq 3n\}$

**Answer:**

(a)

$$\begin{aligned} S &\rightarrow aSb|A|B, \\ A &\rightarrow a|aa|aaa|\lambda, \\ B &\rightarrow bB|b. \end{aligned}$$

(b)

$$\begin{aligned} S &\rightarrow aSb|A|B, \\ A &\rightarrow aA|\lambda, \\ B &\rightarrow bbC, \\ C &\rightarrow bC|\lambda. \end{aligned}$$

(c)

$$\begin{aligned} S &\rightarrow aaSb|A|B, \\ A &\rightarrow aA|a, \\ B &\rightarrow bB|b. \end{aligned}$$

(d)

$$S \rightarrow aSbb|aSbbb|\lambda.$$

3. (10 pts)

Show that the following grammar is ambiguous.

$$S \rightarrow aSbS|bSaS|\lambda$$

**Answer:**

Consider  $w = abab$ , which has two leftmost derivations

$$S \Rightarrow aSbS \Rightarrow abS \Rightarrow abab$$

and

$$S \Rightarrow aSbS \Rightarrow aSb \Rightarrow abab.$$

4. (20 pts)

Prove the follow result. Let  $G = (V, T, S, P)$  be a context-free grammar in which every  $A \in V$  occurs on the left side of at most one production. Then  $G$  is unambiguous.

**Answer:**

Consider leftmost productions. Since the variable to be expanded occurs on the left side of only one production, there is never a choice.

5. (20 pts) Eliminate useless productions from

$$S \rightarrow a|aA|B|C,$$

$$A \rightarrow aB|\lambda,$$

$$B \rightarrow Aa,$$

$$C \rightarrow cCD,$$

$$D \rightarrow ddd.$$

**Answer:**

$$S \rightarrow a|aA|B,$$

$$A \rightarrow aB|\lambda,$$

$$B \rightarrow Aa.$$

6. (10 pts) Eliminate all  $\lambda$ -productions from

$$S \rightarrow AaB|aaB,$$

$$A \rightarrow \lambda,$$

$$B \rightarrow bbA|\lambda.$$

**Answer:**

$$S \rightarrow a|abb|aa|aabb$$

7. (10 pts) Eliminate all unit-productions from the grammar in Exercise 6 of Section 6.1 in textbook.

**Answer:**

$$S \rightarrow a|aA|cCD,$$

$$A \rightarrow aB|\lambda,$$

$$B \rightarrow Aa,$$

$$C \rightarrow cCD,$$

$$D \rightarrow ddd.$$

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