Assignment 4 Solution

Find the context free grammars for the following languages

Question 1:
$L=\{a^p b^q c^r \mid p!=q \text{ or } q!=r\}$
(i.e. either there are different number of a's and b's or a different number of b's and c's or both)

Solution:
S -> S_1 | S_2

S_1 -> XC_1 *//S_1 will generate p!=q*
X -> aXb | A_1 | B_1
A_1 -> aA_1 | a *//A_1 will add extra a's to make p>q*
B_1 -> bB_1 | b *//B_1 will add extra b's to make p<q*
C_1 -> cC_1 | lambda *//C_1 will add zero or more c's*

S_2 -> A_2Y *//S_2 will generate q!=r*
Y -> bYc | B_2 | C_2
A_2 -> aA_2 | lambda *//A_2 will add 0 or more*
B_2 -> bB_2 | b *//B_2 will add extra b's to make q>r*
C_2 -> cC_2 | c *//C_2 will add extra c's to make q<r*

Question 2:
$L=\{a^n b^m c^k : k = |n-m|\}$

Solution:
S -> X | Y

X -> aXc | A *//k = n-m => n = k+m*
A -> aAb | lambda

Y -> BC *//k = m-n => m = k+n*
B -> aBb | lambda *//$a^n b^n$*
C -> bCc | lambda *//$b^k c^k$*

Question 3:

L={w : w is in S* and na(w) + nb(w) != nc(w) }
S = {a,b,c}

Solution:

S -> S1 | S2

S1 -> XaX | XbX *//S1 will generate na(w) + nb(w) > nc(w)*
X -> aXC | CXa | bXC | CXb *//X generates*
X -> XX | lambda *//na(w)+nb(w) >= nc(w)*
C -> c | lambda

S2 -> YcY *//S2 will generate na(w) + nb(w) < nc(w)*
Y -> AYc | cYA | BYc | cYB *//Y generates*
Y -> YY | lambda *//na(w)+nb(w) <= nc(w)*
A -> a | lambda
B -> b | lambda

Question 4:

L= {w1cw2 such that w1, w2 are members of {a,b}+ and w1!=w2R}

Solution:

S -> XaYbX | XbYaX

X -> aX | bX | lambda *//X will generate any combination of a's and b's*
Y -> aYa | bYb | c *//Y will generate a palindrome with center symbol = 'c'*