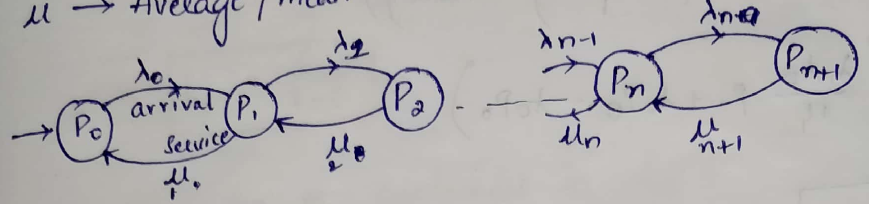
13/9/19

Queuing Theory :-

$\lambda \rightarrow$ Average rate of arrival.

$\mu \rightarrow$ Average / mean Service rate.



Steady-state

* → Expected Rate of arrival into state $1 = \lambda_0 P_0 + \mu_2 P_2$

* → " " " state $n = \lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1}$ --- ①

* Expected rate of flow out at level $1 = \lambda_1 P_1 + \mu_1 P_1$

* " " " at level $n = \lambda_n P_n + \mu_n P_n$ --- ②

Imp → meaning of ① = ② is ? ✓ This is "Steady state of System"

→ Is this Morkovian?

Random Variable ? $X : 0 \; 1 \; 2 \; \cdots \; n \cdots$

Random Process ? $P(x) : P_0 \; P_1 \; P_2 \cdots P_n \cdots$

Expectation ?

$$\boxed{E[x] = \sum x P(x = x)}$$

⇒ Expectation of Service ⇒ $\boxed{E[x] = \sum_{k=0}^{\infty} k P_k}$

$$\lambda_0 P_0 = \mu_1 P_1 \Rightarrow \boxed{P_1 = \frac{\lambda_0}{\mu_1} P_0}$$

$$\lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1} = \lambda_n P_n + \mu_n P_n$$

$$\lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1} = (\lambda_n + \mu_n) P_n$$

$$\boxed{P_{n+1} = \frac{1}{\mu_{n+1}} \left((\lambda_n + \mu_n) P_n - \lambda_{n-1} P_{n-1} \right)}$$

Is it possible to represent this in terms of P_0?

$$\Rightarrow \boxed{P_1 = \frac{\lambda_0}{\mu_1} P_0}$$

$$P_2 = \frac{1}{\mu_2}\left((\lambda_1+\mu_1)\cancel{P_1} - \lambda_0 P_0\right)$$

$$= \frac{1}{\mu_2}\left((\lambda_1+\mu_1)\left(\frac{\lambda_0}{\mu_1}P_0\right) - \lambda_0 P_0\right)$$

$$= \frac{1}{\mu_2}\left(\frac{\lambda_1\lambda_0}{\mu_1}P_0 + \lambda_0 P_0 - \lambda_0 P_0\right)$$

$$\boxed{P_2 = \frac{\lambda_1\lambda_0}{\mu_2\mu_1} P_0}$$ //

$$\Rightarrow \boxed{P_n = \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_1} P_0}$$ //.

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★ when $\lambda_i = \lambda$ → arrival rate

$\mu_i = \mu$ → Service rate.

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$$

let $\rho = \frac{\lambda}{\mu}$ $\quad \rho < 1 \Rightarrow \boxed{\lambda < \mu}$

$$\boxed{P_n = (\rho)^n P_0}$$

Sub this in $E(X)$

$$\Rightarrow \sum_{k=0}^{\infty} k P_k = \sum k \cdot \rho^k P_0$$

$$= P_0 \sum_{k=0}^{\infty} k \rho^k$$

$$= P_0 \frac{\rho}{(1-\rho)^2}$$

$\left[\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots \quad |x| < 1 \right.$

$\frac{d}{dx}\left(\frac{1}{1-x}\right) = 1 + 2x + 3x^2 + \cdots + nx^{n-1} + \cdots$

$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + \cdots + nx^n + \cdots$

$\left. \boxed{\frac{x}{(1-x)^2} = \sum_{n=0}^{\infty} n \cdot x^n} \right]$

$$\sum_{i=0}^{\infty} P_i = 1$$

$$P_0 + P_1 + \cdots + P_n + \cdots = 1$$

$$\cancel{P_0 + \rho P_0 + \cdots + \rho^n P_0 +} \; P_0 + \rho P_0 + \cdots + \rho^n P_0 + \cdots = 1$$

$$P_0(1 + \rho + \rho^2 + \cdots + \rho^n + \cdots) = 1$$

$P_0 \cdot \frac{1}{1-\rho} \Rightarrow \boxed{P_0 = 1-\rho}$

$$\Rightarrow \boxed{\text{Expectation} = (1-\rho)\frac{\rho}{(1-\rho)^2} = \frac{\rho}{1-\rho}}$$ //.

$$\Rightarrow \boxed{L_s = \frac{\rho}{1-\rho}}$$

L_s: Mean no. of customers in system.

L_q : mean no. of customers in queue.

W_s: mean wait in the system.

W_q : mean wait in the queue.

C : No. of servers

$\bar{C}$: Expected no. of busy servers.

(not compliment of c)

Q.

No. of cust in store	No. of counters in op
1 to 3	1
4 to 6	2
>6	3

Customer arrive in the counters area acc to a Poission distr with a mean rate of 10 cust/hr. The avg check-out time per cust is exponential with mean 12 min. Determine the steady-state probability P_n of n cust in check-out-area.

Sol:. $\left.\begin{array}{l}\lambda = 10 \text{ cust/hr} \\ \mu = 5 \text{ cust/hr.}\end{array}\right\}$ should be in same units.

$$\mu_n \begin{cases} 5 \text{ cust/hr} & n = 0,1,2,3 \\ 2\times5 = 10 \text{ cust/hr} & n = 4,5,6 \\ 3\times5 = 15 \text{ cust/hr} & n = 7,8,\dots \end{cases}$$

$\Rightarrow P_1 = \left(\frac{10}{5}\right)P_0 = 2P_0$

$$P_2 = \left(\frac{10}{5}\right)^2 P_0 = 4P_0$$

$$P_3 = \left(\frac{10}{5}\right)^3 P_0 = 8P_0$$

$$P_4 = \left(\frac{10}{5}\right)^3 \left(\frac{10}{10}\right) P_0 = 8P_0$$

$$P_5 = \left(\frac{10}{5}\right)^3 \left(\frac{10}{10}\right)^2 P_0 = 8P_0$$

$$P_6 = \left(\frac{10}{5}\right)^3 \left(\frac{10}{10}\right)^3 P_0 = 8P_0$$

$$\Rightarrow P_{n \geq 7} = \left(\frac{10}{5}\right)^3 \left(\frac{10}{10}\right)^3 \left(\frac{10}{15}\right)^{n-6} P_0 = 8\left(\frac{2}{3}\right)^{n-6} P_0$$

$$* \; P_0 \left\{ 31 + 8\left(1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots\right) \right\} = 1$$

$$\Rightarrow \boxed{P_0 = \frac{1}{55}}$$

//.

$$P_n = \begin{cases} \frac{1}{55} & n = 0 \\ \frac{2}{55} & n = 1 \\ \frac{4}{55} & n = 2 \\ \frac{8}{55} & n = 3, 4, 5, 6 \\ \frac{8}{55}\left(\frac{2}{3}\right)^{n-6}, & n \geq 7. \end{cases}$$

//.

n	0	1	2	3	4	5	6	7	---
λ_n	10	10	10	10	10	10	10	10	---
μ_n	5	5	5	5	10	10	10	15	---

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** Queueing Model Classification :-

→ Arrival Process / Service Time / Servers / Maxoccupancy

Arrival Process	Service Time	Servers	Max occupancy
Interarrival times τ	Service times X	1 Server	K customers
M = exponential	M = exponential	C Servers	unspecified if unlimited.
D = deterministic	D = deterministic	∞	
G = general	G = general		
Arrival rate	Service Rate:		
$\lambda = 1/E[\tau]$	$\mu = 1/E[X]$		

** M/M/1 Queues:-

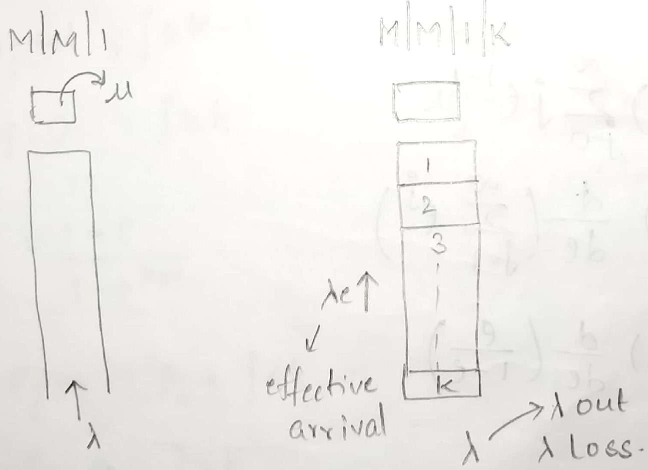
- Customers arrives according to a Poisson Process with rate λ.

$\lambda_j = \lambda \ , \ j \geq 0$

Ex: User activity $M|M|\infty$, $M|G|\infty$

Multiplexer model $M|M|1|K$, $M|M|1$, $M|G|1$, $M|P|1$.

Trunking models $M|M|c|c$, $M|G|c|c$.



⇒ $\lambda_j = \lambda \quad j \geq 0$

$\mu_j = \mu \quad j \geq 1$

$\mu_0 = 0$

1 Server.

customers arrive at poisson process with rate λ.

Service time - exponetial - μ.

$$\boxed{\rho = \lambda/\mu}$$

$$c_j = \rho^j = \frac{\lambda_0 \lambda_1 \cdots \lambda_{j-1}}{\mu_1 \mu_2 \cdots \mu_j} = \left(\frac{\lambda}{\mu}\right)^j.$$

$$P_j = \rho^j P_0$$

$$1 = P_0 + P_1 + \cdots + P_j + \cdots$$

$$1 = P_0 + \rho P_0 + \cdots + \rho^j P_0 + \cdots$$

$$1 = P_0(1 + \rho + \rho^2 + \cdots + \rho^j \cdots)$$

$$1 = P_0 \left(\frac{1}{1-\rho}\right)$$

$$\boxed{P_0 = 1-\rho}$$

$$\boxed{P_j = \rho^j (1-\rho)} \quad , j \geq 1$$

L - no of entities in System - expected.

$$L_s = \sum_{j=0}^{\infty} j \times P_j$$

$$= \sum_{j=0}^{\infty} j\rho^j (1-\rho)$$

$$= (1-\rho)(\rho) \sum_{j=0}^{\infty} j \rho^{j-1}$$

$$= (1-\rho)(\rho) \frac{d}{d\rho}\left(\sum_{j=1}^{\infty} \rho^j\right)$$

$$= (1-\rho)(\rho) \frac{d}{d\rho}\left(\frac{\rho}{1-\rho}\right)$$

$$\boxed{L_s = \frac{\rho}{1-\rho}}$$

μ, λ - both must be of some unit.

$$\boxed{L_s = \frac{\lambda}{\mu - \lambda}}$$

$$L_q = \sum_{n=1}^{\infty} (n-1) P_n$$

because one will be getting served at the customer.

$$= \sum_{j=1}^{\infty} nP_n - \sum_{j=1}^{\infty} P_n$$

$$= L - (1 - P_0)$$

$$= L - e.$$

$$\boxed{L_q = \frac{e^2}{1-e} = \frac{\lambda^2}{\mu(\mu-\lambda)}}$$

W_s - expected waiting time in system = time in line + time in service

W_q - waiting time in queue.

$$\boxed{\begin{array}{l} L = \lambda W \\ L_q = \lambda W_q \end{array}} \qquad \boxed{W_s = \frac{L_s}{\lambda} = \frac{1}{\mu - \lambda}}$$

↓

Little's Formula

$$\Rightarrow \boxed{W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu-\lambda)}}$$

*** M/M/1/K Queues :-

$\lambda_j = \lambda \; , \; j \le k-1$ $\qquad \mu_0 = 0$

$\lambda_j = 0 \; , \; j \ge k$ $\qquad \mu_j = \mu \; , \; j \ge 1$

$$P_0 = \frac{1-e}{1-e^{k+1}}$$

$$P_j = e^j P_0 \; , \; j \le k$$

$$P_j = 0 \; , \; j > k$$

$$L_s = \frac{e(1-(k+1)e^k + ke^{k+1})}{(1-e^{k+1})(1-e)}$$

$$l_q = l - (1 - P_0)$$

$$= \frac{\rho\left(1-(k+1)\rho^k + k\rho^{k+1}\right)}{(1-\rho^{k+1})(1-\rho)} - \frac{\rho(1-\rho)}{1-\rho^{k+1}}$$

$$W_s = \frac{l_s}{\lambda(1-P_k)} = \frac{l_s}{\lambda e}$$

$$W_q = \frac{l_q}{\lambda(1-P_k)} = \frac{l_q}{\lambda e}$$

$$\boxed{\lambda e = \lambda(1-P_k)}$$

$$\lambda = \lambda e + \lambda_{loss}$$

$$\lambda_{loss} = \lambda - \lambda e$$

$$\lambda_{loss} = \lambda - \lambda(1-P_k)$$

$$\boxed{\lambda_{loss} = \lambda P_k}$$ //.

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Steady State Distribution:-

Solution of Balance eq^{n}s :

$$Eq① \Rightarrow \lambda P_0 = \mu P_1 \Rightarrow \rho_1 = (\lambda/\mu) P_0$$

$$\lambda P_1 = \mu P_2 \Rightarrow P_2 = \frac{\lambda}{\mu} P_1 = \left(\frac{\lambda}{\mu}\right)^2 P_0$$

$$\vdots$$

$$\lambda P_{k-1} = \mu P_k$$

$$P_k = \left(\frac{\lambda}{\mu}\right)^k P_0$$

$$\sum_{k=0}^{k} P_k = 1$$

$$P_0 + P_1 + \dots + P_k = 1$$

$$P_0 + \left(\frac{\lambda}{\mu}\right) P_0 + \dots + \left(\frac{\lambda}{\mu}\right)^k P_0 = 1$$

$$P_0\left(1+\frac{\lambda}{\mu}+\left(\frac{\lambda}{\mu}\right)^2 - - - +\left(\frac{\lambda}{\mu}\right)^k\right)=1$$

$$P_0(1)\left(\frac{1-(\lambda/\mu)^{k+1}}{1-(\lambda/\mu)}\right)=1$$

$$\boxed{P_0 = \frac{1-\rho}{1-\rho^{k+1}}}$$

★ $\boxed{P_n = \rho^n\left(\frac{1-\rho}{1-\rho^{k+1}}\right)}$ $n=1,2,3,--k.$

ρ can be any value

★ **$M/M/1/c$ Queues** :-

$$W = \frac{L}{\lambda(1-P_c)}$$

$$W_q = \frac{L_q}{\lambda(1-P_c)}$$

$P_k \rightarrow$ Probability of the system that has k customers (queues is full)

$(1-P_k) \rightarrow$ Prob of the queues is not full i.e it can accomodate customers

$$\boxed{\lambda_e = \lambda(1-P_k)}$$

$$\boxed{W_s = \frac{L_s}{\lambda_e}} \quad \boxed{W_q = \frac{L_q}{\lambda_e}}$$

$$\boxed{L_s = \frac{\lambda_e}{\mu-\lambda_e}} \quad \boxed{L_q = L_s - \frac{\lambda_e}{\mu}}$$

① **Effective Arrival Rate λ_e:**

$\lambda_e = \lambda \cdot P_r(\{\text{an arrival enters the System}\})$

$= \lambda \cdot P_r\{\text{System is not full}\}$

$= \lambda \cdot [P_0 + P_1 + P_2 + --- + P_{k-1}]$

$= \lambda \cdot [1-P_k]$ = Through Put Rate

$\lambda_{Lost}/\lambda_b = \lambda \cdot P_r[\{\text{an arrival can't enter the System}\}$

$= \lambda \cdot P_r\{\text{System is free}\}$

$= \lambda \cdot P_k$

(X) Since we are dealing with finite space q, Even if λ is more, when the queue is filled λ_{loss} occurs $\Rightarrow$ If Syst is full $\lambda=0$; $\boxed{P_n=0}$

② Average Customers in System L_s:

$$\star \quad L_s = \sum_{n=0}^{k} n \cdot P_n \qquad \text{Finite Sum.}$$

③ Average Busy Server :-

$$L_B = E[\text{busy Servers}] = E[\# \text{Cust in Service}]$$

$$L_B = 0 \cdot P_0 + 1(P_1 + P_2 + \ldots) = 1 - P_0$$

④ Utilization of the System U :-

$$U = P_r\{n > u\} = P_1 + P_2 + P_3 + \ldots + P_n = 1 - P_0$$

⑤ Average Customers in Queue L_q!

$$L_q = L_s - L_B \qquad \rho = \frac{\lambda}{\mu}$$

Example 1 :- Consider the following scenario: the inter-arrival time is exponentially distributed with a mean of 10 min and the Service this is also exponentially distributed with a mean of 8 means, find the (i) mean wait in the queue, (ii) mean numbers in the queue.

(iii) The mean wait in the System.

(iv) mean number in the System

(v) proportion of time ~~in the~~ Server is idle.

Sol.. $\lambda = 1/10(\text{min}) = 6/\text{hr}$

$\mu = 1/8^{\text{min}} = \frac{60}{8} = 7.5\text{hr}$

$\rho = \lambda/\mu = 6/7.5 = 4/5 = 0.8$

$$L_q = \frac{\rho^2}{1-\rho} = 3.2$$

$$W_q = \frac{L_q}{\lambda} = \frac{3.2}{6} \text{ hr} = 0.5333 \text{ hr} = 32 \text{ min}$$

$$W = W_q + 1/\mu$$

$$= 0.5333 + \frac{1}{7.5}$$

$$= 0.6667 \text{ hr} = 2/3 \text{ hr}$$

$$= 40 \text{ min.}$$

$$L = W\lambda$$

$$= \left(\frac{2}{3}\right)(6)$$

$$= 4.$$

(or) $L_s = \frac{\rho}{1-\rho} = \frac{0.8}{1-0.8} = 4.$

⇒ No of customers = Prop of time the Seever is idle.
utilization factor = P_0

$$= 1-\rho$$

$$= 1-0.8$$

$$= 0.2.$$

2. poison distribution.

$\lambda = 2/5 \text{ min} = 24 \text{ hr}$

10 cars → including the one being serve ~~cars~~.

Still an infinite queue becoz car can wait outside if necessary.

$\mu = 40/\text{hr}$

$\rho = \frac{\lambda}{\mu} = 0.6$

$P_0 = 1-\rho = 0.4$

} Probability that facility is idle.

expected no. of cust waiting time until a cust reaches window – W_q

Prob that waiting line will exceed capacity – $p(n>10)$

18/9/19

① ~~Arrivals~~ Arrivals at the telephone booth are considered to be in Poisson with an avg time of 12 min. The length of a phone calls is assummed to be distributed exponentially with mean 4 min.

(i) Find the avg no. of persons waiting in the system (w_s)

(ii) What is the prob that it will take more than 10 min altogether to wait for the phone & complete a person's call.

(iii) What is prob that a person arriving at the booth will have to wait in the queue

(iv) Estimate the fraction of the day when the phone will be in use

(v) Telephone department install a second booth when convinced that the arrival has to wait on avg for atleast 3 min for phone. By how much the flow of arrival will ↑se in order to justify a second booth.

(vi) what is the avg length of queues that forms from time to time.

Sol:.

$\lambda = 5$ per hr

$\mu = 15$ per hr

$\rho = \frac{5}{13} = \frac{1}{3}$, $P_0 = 1 - \rho = \frac{2}{3}$

(i) $W_s = \frac{L_s}{\lambda} = \frac{1/2}{5} = \frac{1}{10} = 0.1$

(ii) $\boxed{P(W_s) > 10}$ Poisson process

(iii) $1 - P_0 = 1 - \frac{2}{3} = \frac{1}{3}$ [The probability ~~that~~ that a person has to wait in the queue].

(iv) $P_0 \rightarrow$ The system is idle.

$1 - P_0 = 1 - \frac{2}{3} = \frac{1}{3}$ [The fraction of the day when the phone will be in use].

for day $\frac{1}{3} \times 24 = 8$ hrs.

(v) $W_q = \frac{L_q}{\lambda}$, $L_q = \rho^2/1-\rho = \frac{(1/3)^2}{1-1/3} = \frac{1}{6}$

$W_q = \frac{1/6}{5} = 1/30$

when $W_q > 3$ min what should be the λ value?

i.e) $W_q > \frac{3}{60}$ hrs

↓

$$W_q = \frac{(\lambda/\mu)^2}{(1-\lambda/\mu)} \times \frac{1}{\lambda} = \frac{\lambda^2/\mu^2}{\left(\frac{\mu-\lambda}{\mu}\right)} \cdot \frac{1}{\lambda} = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$\rightarrow W_q = \frac{\lambda_N}{15(15-\lambda_N)} > \frac{3}{60} \Rightarrow \lambda_N = ?$$

$$\lambda_N > \frac{3}{60}\left(15(15-\lambda_N)\right)$$

$$\lambda_N + \frac{3}{4}\lambda_N > \frac{45}{4}$$

$$\frac{7\lambda_N}{4} > \frac{45}{4} \Rightarrow \lambda_N > \frac{45}{7} \approx 6.42..$$

when the average arrival rate is around 6.42 persons per hr.

the second booth will be installed.

(vi) Average length of the queue / from time to time.

$$\frac{L_q}{P(n>1)} = \frac{L_q}{1-P_0-P_1} \qquad P_1 = \rho P_0 = \frac{1}{3}\left(\frac{2}{3}\right) = 2/9$$

$$= \frac{1/6}{1-\frac{2}{3}-\frac{2}{9}} = \frac{3}{2} \text{ persons.}$$

2Q) If ppl arrive to purchase cinema tickets at the average rate of 6 per min. It takes an avg of 7.5 sec to purchase a ticket. If a person arrives 2 mins before the picture starts and if it takes exactly 1.5 min to reach the correct seat after purchasing the ticket (i) Can he expect to be seated for the start of the picture (ii) What is the prob that he will be seated for the start of the picture.

(iii) How early must he arrive inorder to be 99% sure of being seated for start of picture.

3Q) A duplicating maintained for office use is operated by an office assistant who earns Rs: 5/job. The time to complete each job varies acc to an expo dist with mean 6 min. assume a poisson input with an avg arrival rate of 5 jobs per hr if an 8hr day is used as a base. determine

(i) The % idle time of machine

(ii) The avg time a job is in the system.

(iii) The avg earning per day of the assistant.

Solutions:-

2Ans:- $\lambda = \frac{1}{6}$ cust per min

$\mu = 8$ cust/hr

$\rho = \frac{\lambda}{\mu} = \frac{1}{48}$.

(i) $L_s = \cancel{\frac{1}{47}} \cdot \frac{\rho}{1-\rho} = \frac{1}{47}$

$W_s = \frac{L_s}{\lambda} = \frac{6}{47} \simeq 0.12.$

(ii)

3Ans:- $\lambda = 5$ jobs/hr

$\mu = 10$ jobs/hr

$\rho = \frac{5}{10} = 0.5$ //.

(i) idle time :- $P_0 = 1 - \rho = 0.5$

$\Rightarrow$ 50% of the time the machine will be idle.

(ii) $L_s = \frac{\rho}{1-\rho} = 1$

(iii) Avg earning is $L_s \times 5 \times 8 = 40$ Rs per day.

(4Q) A parking lot is limited to 5 spaces. Cars making use of this space arrive acc to a poisson distribution at the rate of 6 cars/hr. parking time is exponentially distributed with a mean of 30 min. Visiters who can't find an empty space on arrival may temp wait inside the lot. until a parked car leaves. the temp space can hold only 3 cars. other cars that can't park or if the temp space must go else where. Determine the foll.

(i) The probability of n cars in the system. The effective arrival rate for cars that actually use the lot. and all other queing factors.

Sol:- $\lambda = 6$ per hr.

$\mu = 2$ car/hr

5
3

Total is 8

$M/M/5/k$

$\boxed{k = 8}$

n	0	1	2	3	4	5	6	7	8

$$\lambda e = \lambda(1 - P_k)$$

$$P_0 = \frac{1-\rho^{k+1}}{1-\rho} \Rightarrow \rho = \frac{\lambda}{\mu} = 3$$

$$P_1 = \left(\frac{\lambda_0}{\mu_1}\right) P_0 = \frac{6}{2} P_0 = 3P_0$$

$$P_2 = \left(\frac{\lambda_0 \lambda_1}{\mu_2 \mu_4}\right) P_0 = \frac{6^2}{4 \times 2} P_0$$

$$P_3 = \frac{6^3}{6 \times 4 \times 2} P_0$$

$$P_4 = \frac{6^4}{8 \times 6 \times 4 \times 2} P_0$$

$$P_5 = \frac{6^5}{10 \times 8 \times 6 \times 4 \times 2} P_0$$

$$P_6 = \frac{6^6}{10 \times 10 \times 8 \times 6 \times 4 \times 2} P_0$$

$$P_7 = \frac{6^7}{10 \times 10 \times 10 \times 8 \times 6 \times 4 \times 2} P_0$$

$$P_8 = \frac{6^8}{10 \times 10 \times 10 \times 10 \times 8 \times 6 \times 4 \times 2} P_0$$

$$\boxed{P_9 = 0}$$

$$\boxed{\sum_{n=0}^{\infty} P_n = 1} \rightarrow P_0$$

$$\mu = \begin{cases} n(60/30) = 2n & n = 1 \text{ to } 5 \\ 10 & n = 6 \text{ to } 8 \\ - & n > 8. \end{cases}$$

23/9/19

1Q. Cafiteria can accomodate max 50 persons. cust arrive in a poisson String of rate 10 per hr and are Served at the rate of 12/hr. What is the prob that an arriving cust will not eat in the cafeteria becoz it is full.

2Q. Passion arrive at 1 doctor clinic acc to poisson dist at a rate of 20 patients per hour. A waiting time can't accomodate more than 14 patients. Examination time for patient is exponential with a mean of 8min.

i. What is the prob than an arriving rate of patient will not wait

ii) What is the prob that an arriving patient will sit in the room?

(iii) what is the expected ~~arrival~~ total time that a patient Spends in the clinic?

Solution:- M/M/1/50

① P_{50} ; $\lambda = 10/hr$; $\mu = 12/hr$

$\boxed{k=50}$

$$\rho = \frac{\lambda}{\mu} = \frac{10}{12} = \frac{5}{6}$$

$$P_1 = \rho \cdot P_0 \,;\; P_2 = \rho^2 P_0 \,, \ldots, P_n = \rho^n P_0$$

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}}$$

$$= \frac{1-(5/6)}{1-(5/6)^{51}}$$

$$\boxed{P_{50} \approx 0.00002}$$

2Ans :- $\boxed{\lambda = 20}$, $\boxed{k = 15}$ M/M/1/15

$\Rightarrow \mu = 7.5/hr.$

(i) $P_0 = \dfrac{1-\rho}{1-\rho^{k+1}} = \dfrac{1-\left(\frac{20}{7.5}\right)}{1-\left(\frac{20}{7.5}\right)^{15+1}} \approx 0.0000003$

(ii) $\sum_{n=1}^{14} P_n = 1 - P_0 - P_{15}$ // $\left[\sum_{n=0}^{15} P_n = 1\right]$.

$\Rightarrow P_{15} = \dfrac{1-\rho}{1-(\rho)^{15+1}}$

$\Rightarrow \sum_{n=1}^{14} P_n = 1 - 0.0000003 -$

$= 0.37.$

(iii) $W_s = \dfrac{L_s}{\lambda_e}$

Try

$\lambda_e = \lambda(1 - P_K)$

$= \lambda(1 - P_{15})$

$\Rightarrow L_s = \sum_{n=0}^{15} n \cdot P_n.$

$\Rightarrow P_n = \rho^n \cdot P_0$, $P_0 = \dfrac{1-\rho}{1-\rho^{k+1}}$

(or)

$L_s = \rho\left(\dfrac{1-(k+1)\rho^k + k\rho^{k+1}}{(1-\rho^{k+1})(1-\rho)}\right) \approx 14.4$

0	1	2	3	4	5	6	7	8	9	10	11	12	13
3×10^{-7}	8×10^{-7}	2.15×10^{-6}	5.689×10^{-6}	1.517×10^{-5}	4.04×10^{-5}	0.00011	0.00029	0.00077	0.00205	0.00546	0.01435	0.03879	

$W_s = \dfrac{L_s}{\lambda_e} \approx 1.92$ hrs.

Exponential Distribution :-

$$f(t) = \lambda e^{-\lambda t}, \; t \geq 0$$

$$\boxed{E(t) = 1/\lambda}$$

$$p(t < T) = \int_0^T f(t)\,dt = 1 - e^{-\lambda T}.$$