HomeWork Number: 2
Graph Algorithms

Date: 14/12/2018

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Question No: 1

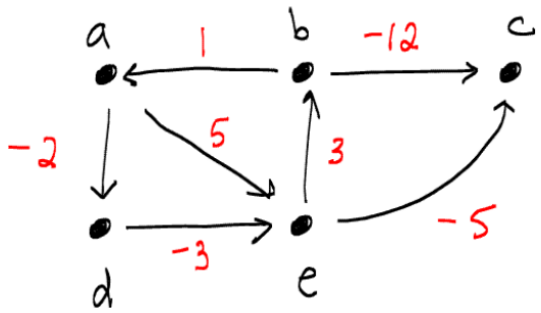
Prove that if the weights on the edges of a connected, undirected graph are distinct, then there is a unique minimum spanning tree.

Question No: 2

Given any connected undirected graph G with positive edge weights w, does there always exist a shortest path tree S such that S is a minimum spanning tree of G? Prove your answer using counter example.

Question No: 3

a. Run the Bellman-Ford shortest path algorithm on the following graph, starting from vertex a. Specifically, fill in the tables below.



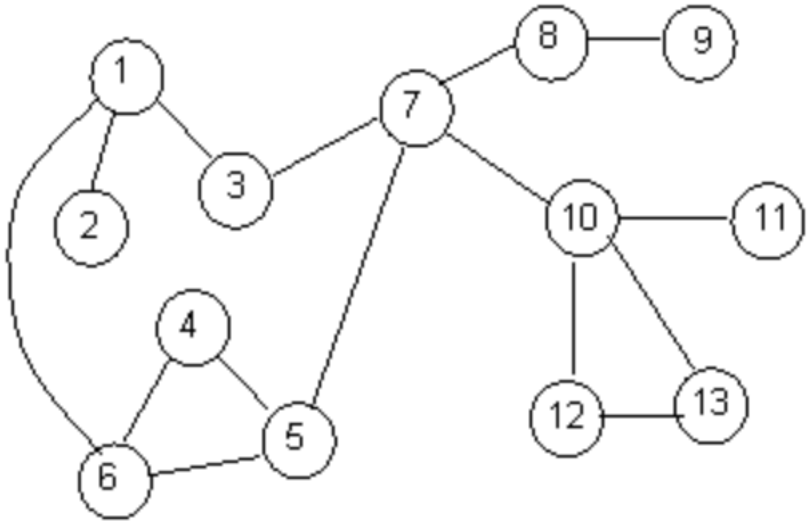
#edges	a	b	c	d	e
0	0	infinity	infinity	infinity	Infinity
1					
2					
3					
4					

vertex	a	b	c	d	e
prev[vertex]					

b. Suppose you are given a directed graph that has negative values on some of the edges. The Bellman Ford algorithm will give the correct solution for shortest paths from some starting vertex s, if there are no negative cycles. But suppose you don't know if the given graph has negative cycles. One way to check for negative cycles is to run Bellman Ford for n iterations, instead of n-1. Explain.

Question No: 4

Find number of bi - connected components of the given graph



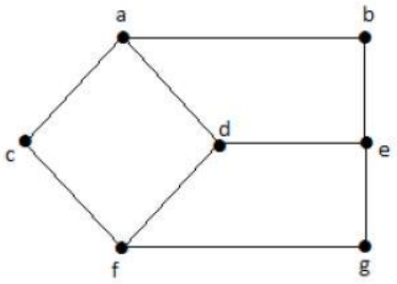
Question No: 5

Given an undirected graph, it can be tested to determine whether or not it is a tree in O(V + E) time. A tree is a connected graph without any cycles.

Question No: 6

The *eccentricity* e(u) of a vertex u in a connected, undirected, *unweighted* graph G is the maximum distance from u to any other vertex in the graph. That is, if δ(u, v) is the shortest path from u to v, then $e(u) = \max_{v \varepsilon V} \delta(u, v)$. Give an efficient algorithm to find the eccentricity of a given vertex s. Analyze its running time.

Example :



In the above graph, the eccentricity of ‘a’ is 3.
The distance from ‘a’ to ‘b’ is 1 (‘ab’),
from ‘a’ to ‘c’ is 1 (‘ac’),
from ‘a’ to ‘d’ is 1 (‘ad’),
from ‘a’ to ‘e’ is 2 (‘ab’-‘be’) or (‘ad’-‘de’),
from ‘a’ to ‘f’ is 2 (‘ac’-‘cf’) or (‘ad’-‘df’),
from ‘a’ to ‘g’ is 3 (‘ac’-‘cf’-‘fg’) or (‘ad’-‘df’-‘fg’).
So the eccentricity is 3, which is a maximum from vertex ‘a’ from the distance between ‘ag’ which is maximum.

Question No: 7

Modify the prims algorithm to check if an undirected, weighted graph is connected. Analyze your algorithm and show the result using order notation.

Question No: 8

Ms.X has joined the graduate program of an open university. She has to take 9 courses to complete the course. All these are computer based and she can decide the order in which these can be taken as long as the pre-requisite condition are met. If a course A has prerequisite courses B and C, then B and C should be completed before A can be taken. The table below lists the courses and their pre-requites

Course	A	B	C	D	E	F	G	H	I
Prerequisite	-	-	B,A	A	B,C	D	D	D	E,F

a)Create a graph capturing the prerequisite relationship.
b)what algorithm can be used to solve this problem? Justify your answer.

Question No: 9

Explain what adjustment if any need to be made in Dijkstra's and/or in an underlying graph to solve the following problems.

a) Solve the Single source shortest paths problem for directed weighted graphs.
b) Find a shortest path between two given vertices of weighted graph or digraph.
c) Find the shortest paths to a given vertex from each other vertex of a weighted graph