

# Manarat International University (MIU)

Department of Computer Science and Engineering

Mid-term Examination (Summer 2019)

Artificial Intelligence (CSE-411)

## Problem Set Solutions

- 4 Adult heights can be considered to be normally distributed,
- Adult women have a mean height of 65 inches and a standard deviation of 3.5 inches. What is the probability that a randomly selected adult woman is over 72 inches?
  - What is the probability that a randomly selected woman is between 63 and 65 inches?

Ans. Given,  $\mu = 65$ ,  $\sigma = 3.5$

$$\begin{aligned} a) P(h > 72) &= 1 - P(h < 72) = 1 - \Phi\left(\frac{72 - 65}{3.5}\right) \\ &= 1 - \Phi(2) = 1 - 0.977 = 0.023 \end{aligned}$$

$$\begin{aligned} b) P(63 < h < 65) &= P(h < 65) - P(h < 63) = \Phi\left(\frac{65 - 65}{3.5}\right) - \Phi\left(\frac{63 - 65}{3.5}\right) \\ &= \Phi(0) - \Phi(-0.57) = 0.5 \{1 - \Phi(0.57)\} \\ &= 0.5 - (1 - 0.7157) = 0.216 \end{aligned}$$

- 5 Following statistics shows preference (like or not) data of 40 student of CSE-40<sup>th</sup> batch for the tv sitcoms *Friends*, *Big Bang Theory* and *How I Met Your Mother*. Each training example has  $x_1$ ,  $x_2$  and  $y$  where

$x_1$  is whether or not the student liked *Friends*,

$x_2$  is whether or not the student liked *Big Bang Theory* and

$y$  is whether or not the student liked *How I Met Your Mother*.

For the 40 training examples the MLE estimates are as follows:

|                              |   | $X_1$ |    | <i>Friends</i> |   |
|------------------------------|---|-------|----|----------------|---|
|                              |   | Y     |    | 0              | 1 |
| <i>How I Met Your Mother</i> | 0 | 10    | 6  |                |   |
|                              | 1 | 4     | 20 |                |   |

|                              |   | $X_2$ |   | <i>Big Bang Theory</i> |   |
|------------------------------|---|-------|---|------------------------|---|
|                              |   | Y     |   | 0                      | 1 |
| <i>How I Met Your Mother</i> | 0 | 8     | 8 |                        |   |
|                              | 1 | 18    | 6 |                        |   |

Md Ibrahim, a new student, likes *Friends* ( $x_1 = 1$ ) but not *Big Bang Theory* ( $x_2 = 0$ ). What do you predict that he will like *How I Met Your Mother*?

Ans:

| Y | $X_1$ |    | $P(X_1 Y)$      |                 |
|---|-------|----|-----------------|-----------------|
|   | 0     | 1  | 0               | 1               |
| 0 | 10    | 6  | $\frac{10}{16}$ | $\frac{6}{16}$  |
| 1 | 1     | 20 | $\frac{1}{24}$  | $\frac{20}{24}$ |

| Y | $X_2$ |   | $P(X_2 Y)$      |                |
|---|-------|---|-----------------|----------------|
|   | 0     | 1 | 0               | 1              |
| 0 | 8     | 8 | $\frac{8}{16}$  | $\frac{8}{16}$ |
| 1 | 18    | 6 | $\frac{18}{24}$ | $\frac{6}{24}$ |

| Y | P(Y)            |
|---|-----------------|
| 0 | $\frac{16}{40}$ |
| 1 | $\frac{24}{40}$ |

$$\begin{aligned} \text{for } y=0 & \log \hat{P}(Y=0) + \log \hat{P}(X_1=1|Y=0) + \log \hat{P}(X_2=0|Y=0) \\ &= \log\left(\frac{16}{40}\right) + \log\left(\frac{6}{16}\right) + \log\left(\frac{8}{16}\right) = -2.59 \end{aligned}$$

$$\begin{aligned} \text{for } y=1 & \log \hat{P}(Y=1) + \log \hat{P}(X_1=1|Y=1) + \log \hat{P}(X_2=0|Y=1) \\ &= \log\left(\frac{24}{40}\right) + \log\left(\frac{20}{24}\right) + \log\left(\frac{18}{24}\right) = -0.980 \end{aligned}$$

$-0.980$  is larger. so our prediction is  $(\hat{y}=1)$

The new student will likely to like How I Met Your Mother.

- 6 Shazam is an application developed by Apple Inc. which can identify music based on a short sample played on the device. Based on the frequency of requests it's been getting these days, Shazam has found that:

80% of songs are *Hello* by Lionel Richie

20% of songs are *Can't Get Used to Losing You* by Andy Williams

When a request is made, Shazam receives an audio sample that it uses to update its belief. From one particular audio sample (S), Shazam estimates that:

- S would have a 50% chance of appearing if *Hello* were playing.
- S would have a 90% chance of appearing if *Can't Get Used to Losing You* were playing

What is the updated probability that the song is *Hello* given the audio sample heard?

Hint:  $P(X_1) = 0.8$ ,  $P(X_2) = 0.2$ ,  $P(S|X_1) = 0.5$ ,  $P(S|X_2) = 0.9$   
Find  $P(X_1|S)$  using Bayes Theorem and Law of Total Probability

Ans. According to the Bayes Theorem we know

$$P(X_1|S) = \frac{P(S|X_1)P(X_1)}{P(S)}$$

Using Law of Total Probability for  $P(S)$

$$= \frac{P(S|X_1)P(X_1)}{P(S|X_1)P(X_1) + P(S|X_2)P(X_2)} = \frac{(0.5 \times 0.8)}{(0.5 \times 0.8) + (0.9 \times 0.2)}$$

$$= 0.69$$

Updated Probability that the Song is Hello = 0.69