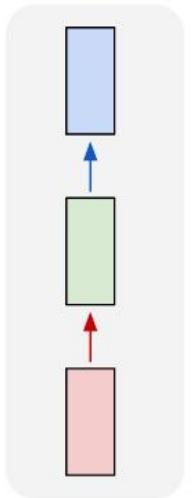


Lecture 10: Recurrent Neural Networks

“Vanilla” Neural Network

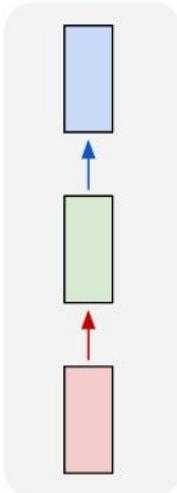
one to one



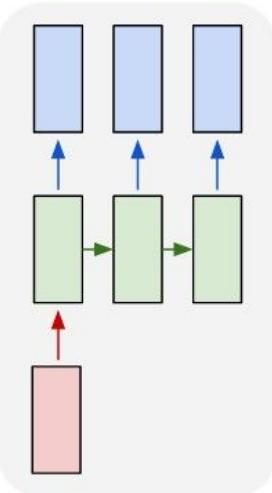
Vanilla Neural Networks

Recurrent Neural Networks: Process Sequences

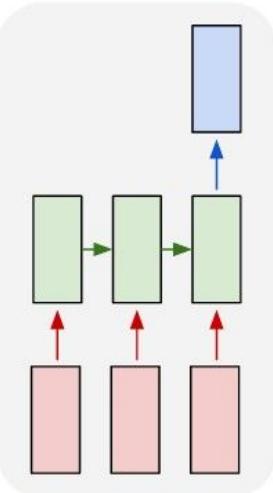
one to one



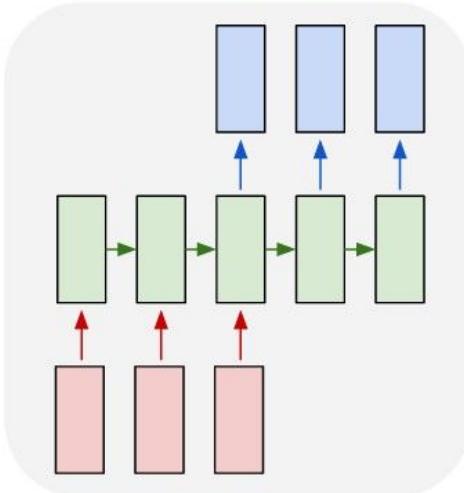
one to many



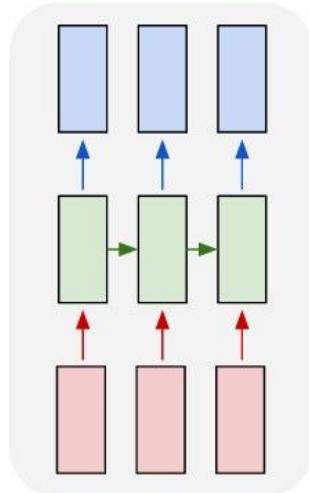
many to one



many to many



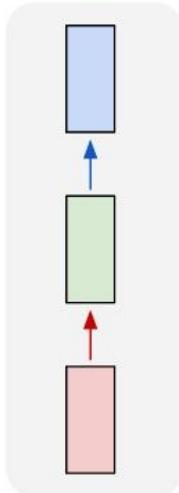
many to many



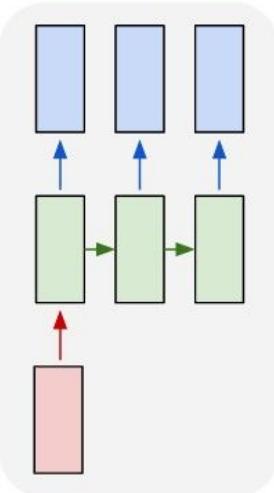
→
e.g. **Image Captioning**
image -> sequence of words

Recurrent Neural Networks: Process Sequences

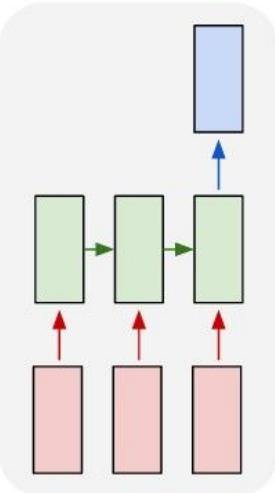
one to one



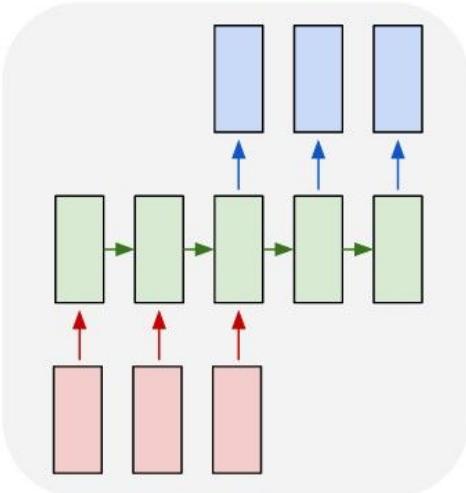
one to many



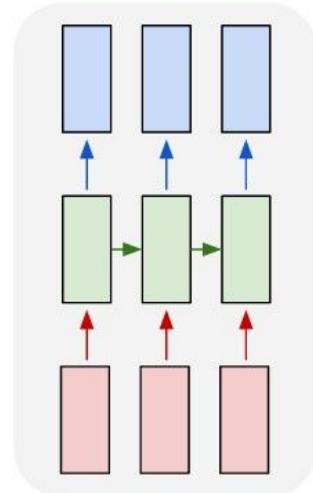
many to one



many to many



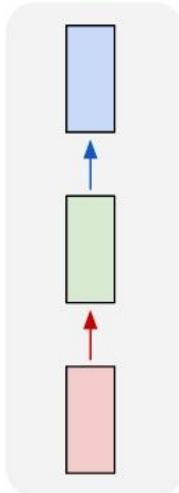
many to many



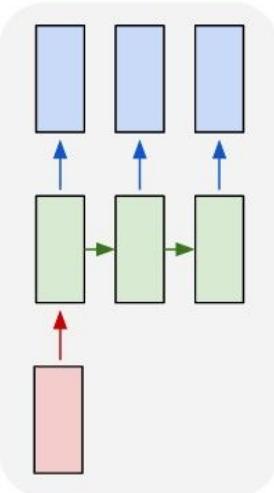
e.g. **Sentiment Classification**
sequence of words -> sentiment

Recurrent Neural Networks: Process Sequences

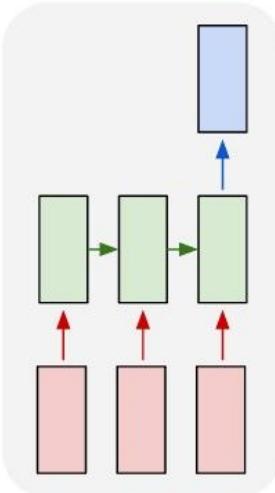
one to one



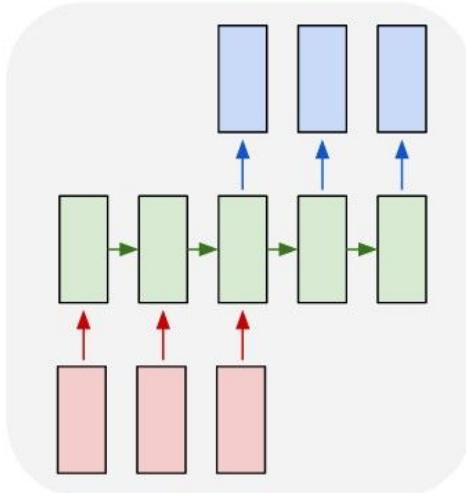
one to many



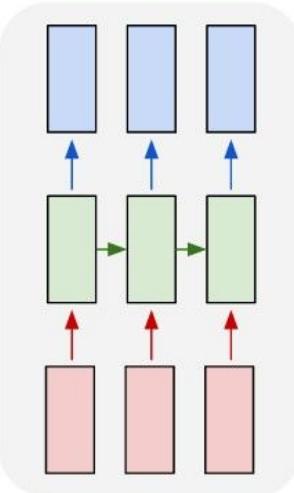
many to one



many to many



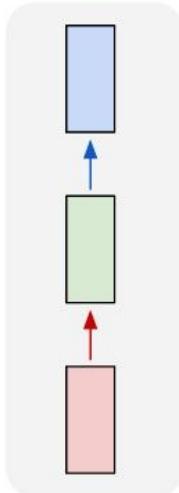
many to many



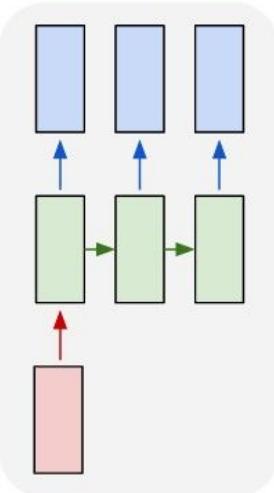
↑
e.g. **Machine Translation**
seq of words -> seq of words

Recurrent Neural Networks: Process Sequences

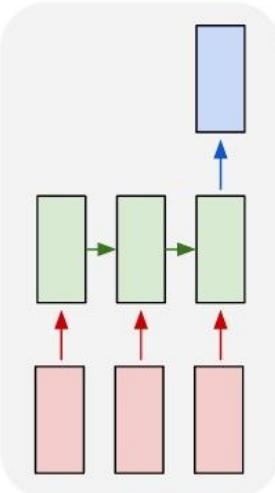
one to one



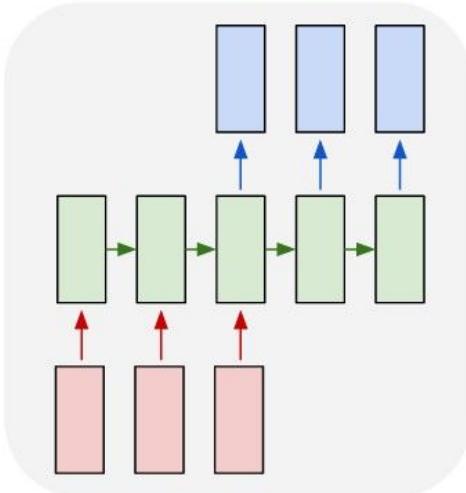
one to many



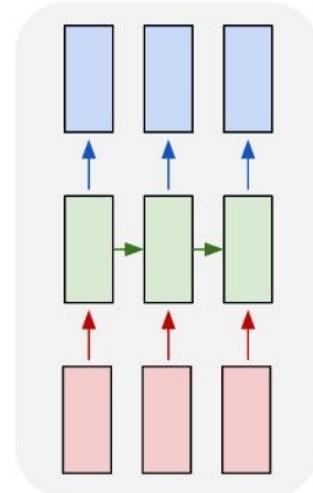
many to one



many to many



many to many



e.g. Video classification on frame level

Sequential Processing of Non-Sequence Data

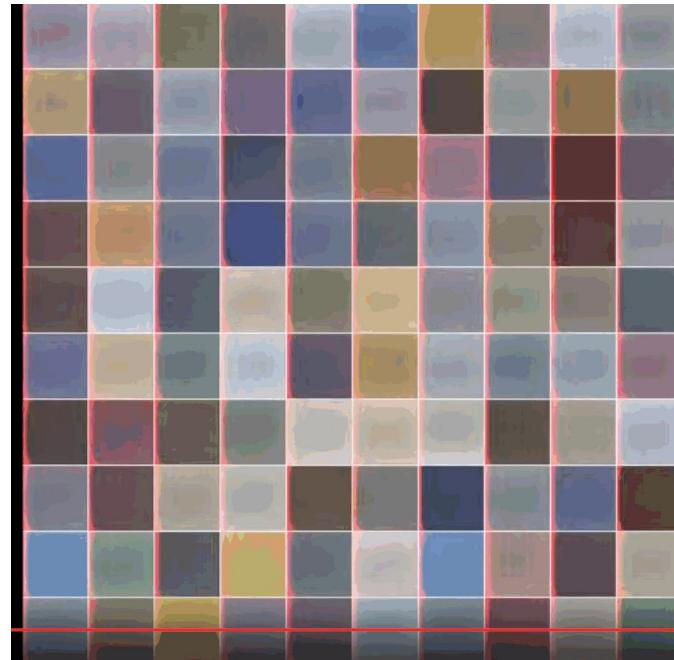
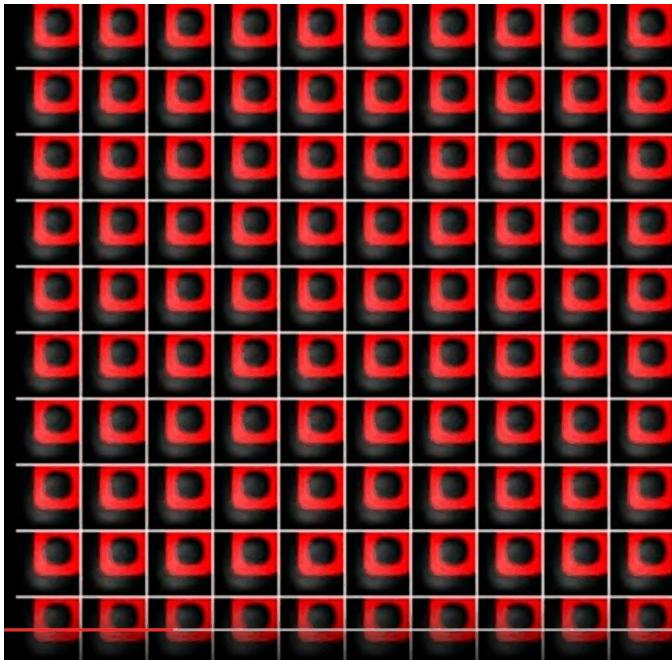
Classify images by taking a series of “glimpses”



Ba, Mnih, and Kavukcuoglu, “Multiple Object Recognition with Visual Attention”, ICLR 2015.
Gregor et al, “DRAW: A Recurrent Neural Network For Image Generation”, ICML 2015
Figure copyright Karol Gregor, Ivo Danihelka, Alex Graves, Danilo Jimenez Rezende, and Daan Wierstra, 2015. Reproduced with permission.

Sequential Processing of Non-Sequence Data

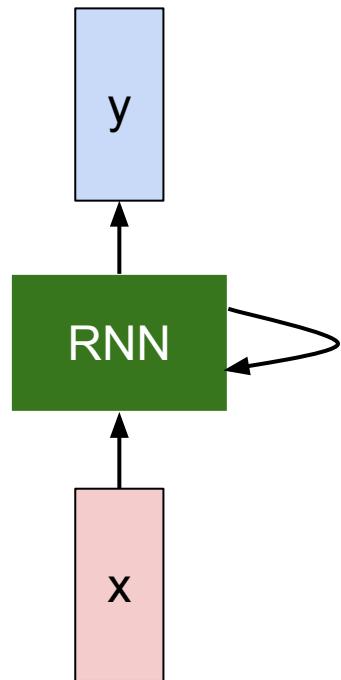
Generate images one piece at a time!



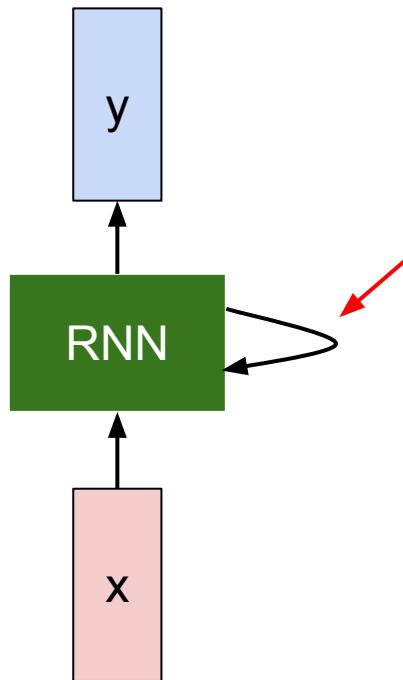
Gregor et al, "DRAW: A Recurrent Neural Network For Image Generation , ICML 2015

Figure copyright Karol Gregor, Ivo Danihelka, Alex Graves, Danilo Jimenez Rezende, and Daan Wierstra, 2015. Reproduced with permission.

Recurrent Neural Network



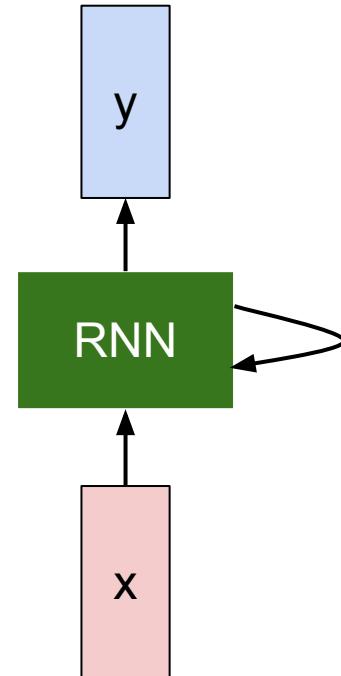
Recurrent Neural Network



Key idea: RNNs have an “internal state” that is updated as a sequence is processed

Recurrent Neural Network

We can process a sequence of vectors \mathbf{x} by applying a **recurrence formula** at every time step:

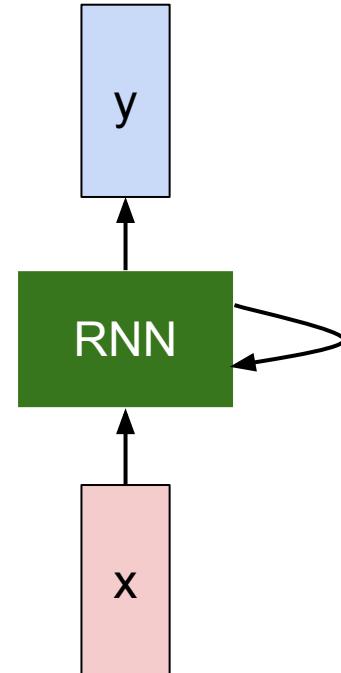


Recurrent Neural Network

We can process a sequence of vectors \mathbf{x} by applying a **recurrence formula** at every time step:

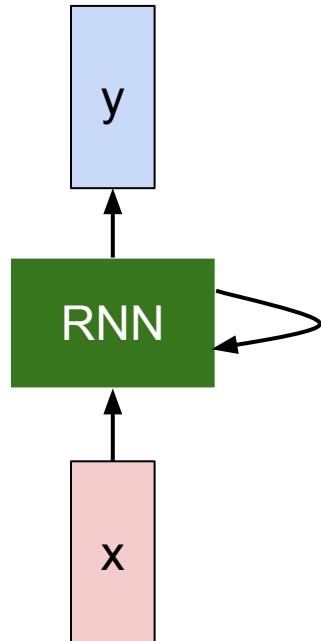
$$h_t = f_W(h_{t-1}, x_t)$$

Notice: the same function and the same set of parameters are used at every time step.



(Simple) Recurrent Neural Network

The state consists of a single “*hidden*” vector \mathbf{h} :



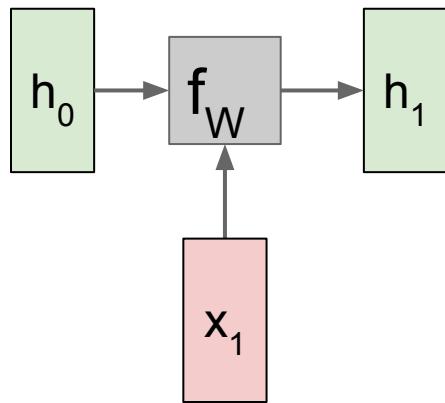
$$\mathbf{h}_t = f_W(\mathbf{h}_{t-1}, \mathbf{x}_t)$$

$$\mathbf{h}_t = \tanh(W_{hh}\mathbf{h}_{t-1} + W_{xh}\mathbf{x}_t)$$

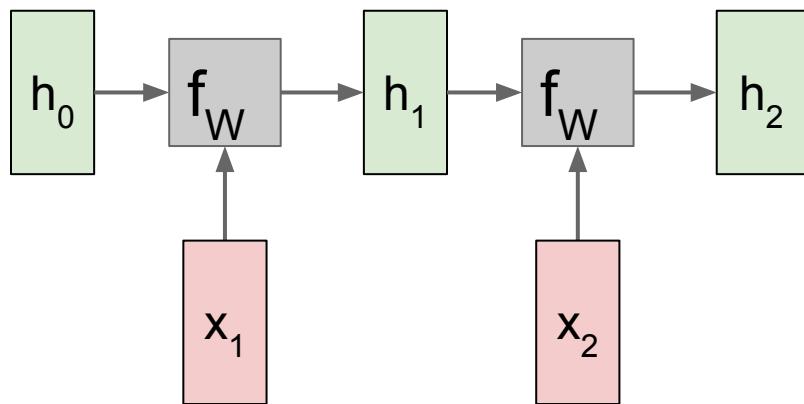
$$y_t = W_{hy}\mathbf{h}_t$$

Sometimes called a “Vanilla RNN” or an
“Elman RNN” after Prof. Jeffrey Elman

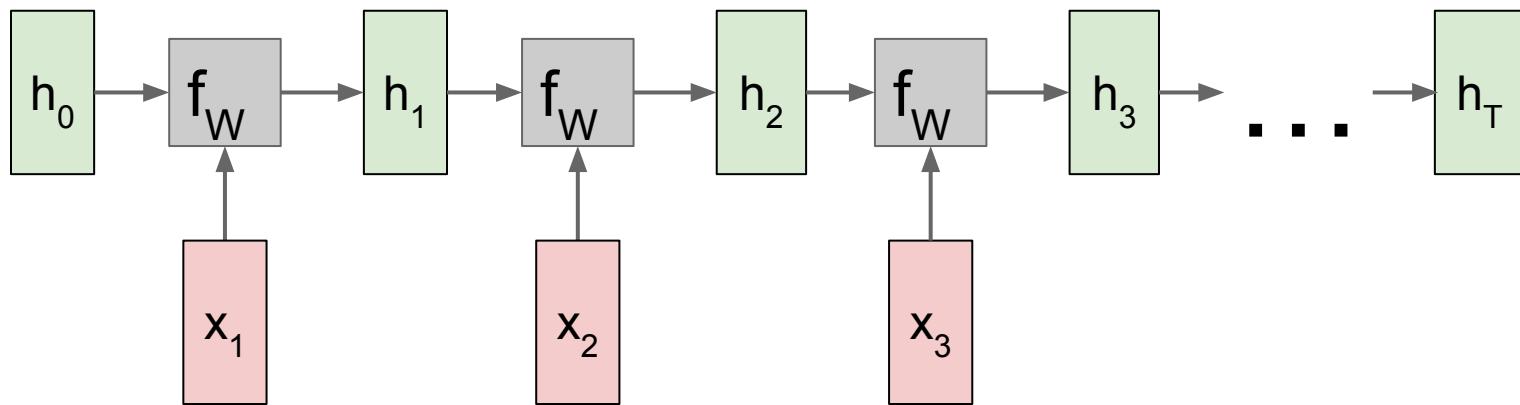
RNN: Computational Graph



RNN: Computational Graph

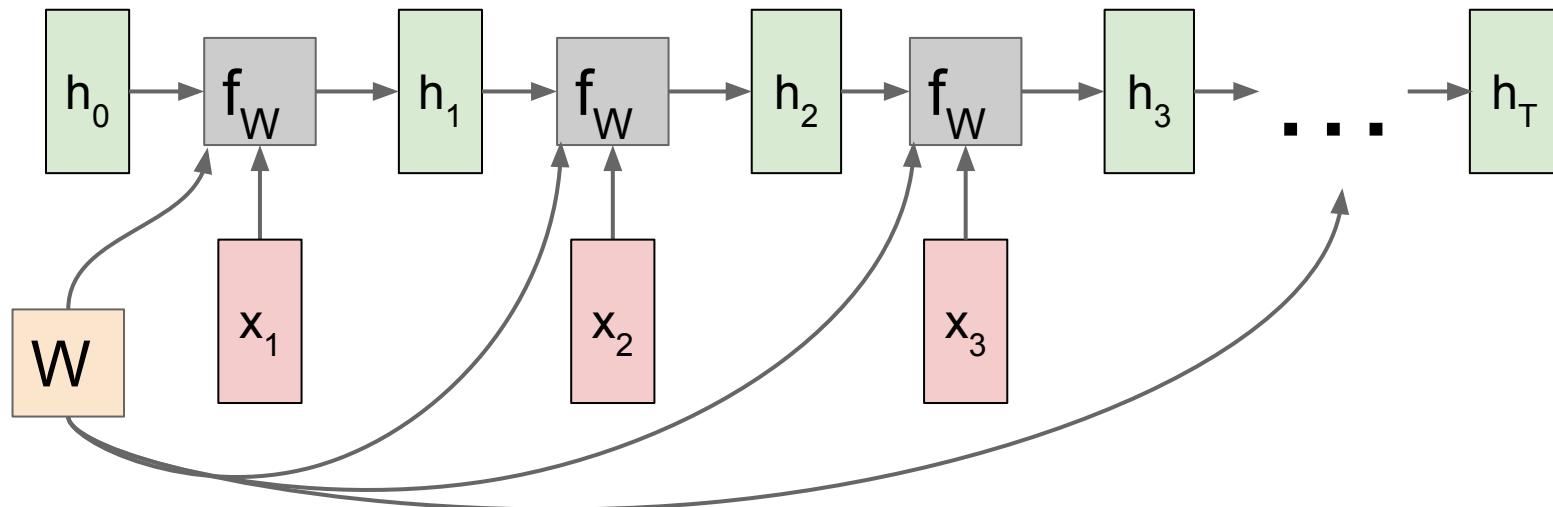


RNN: Computational Graph

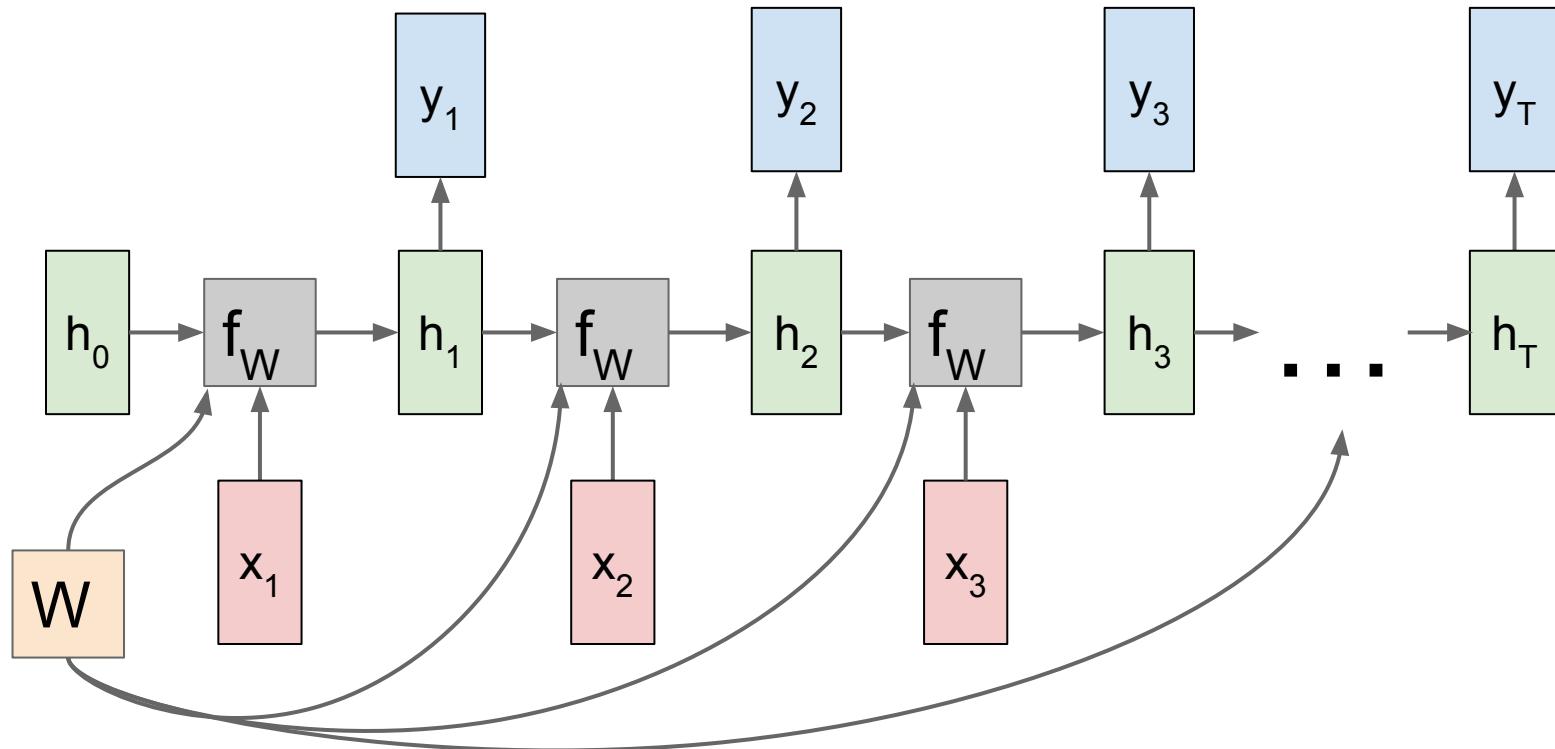


RNN: Computational Graph

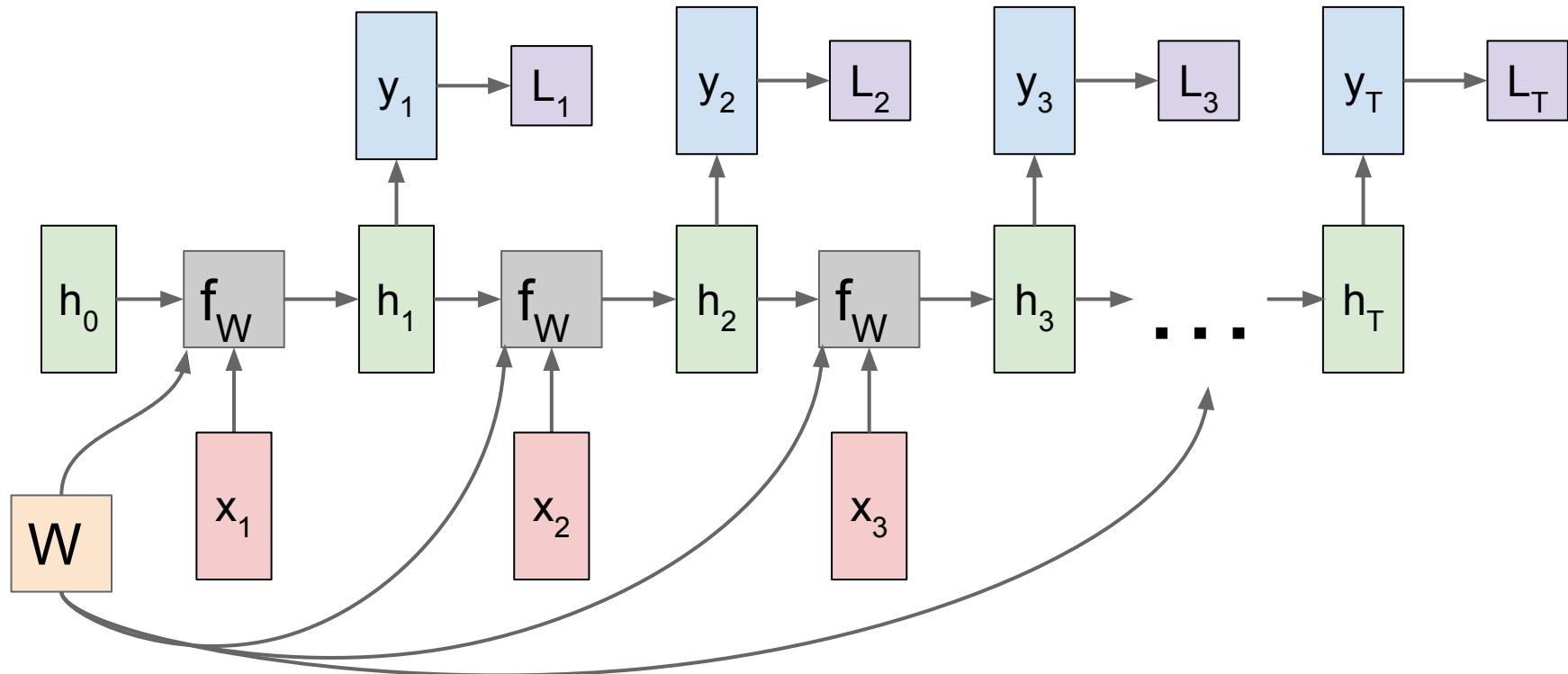
Re-use the same weight matrix at every time-step



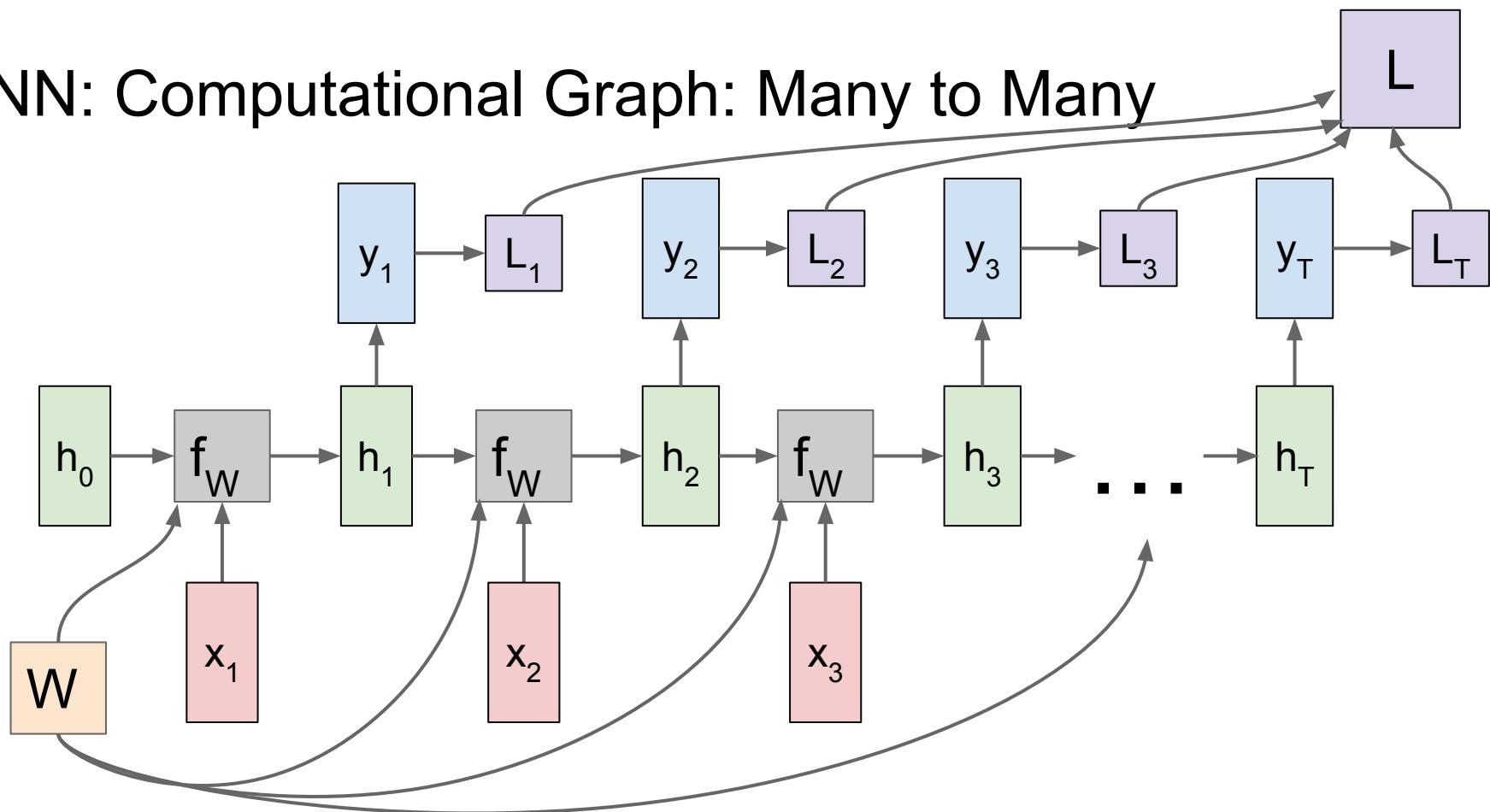
RNN: Computational Graph: Many to Many



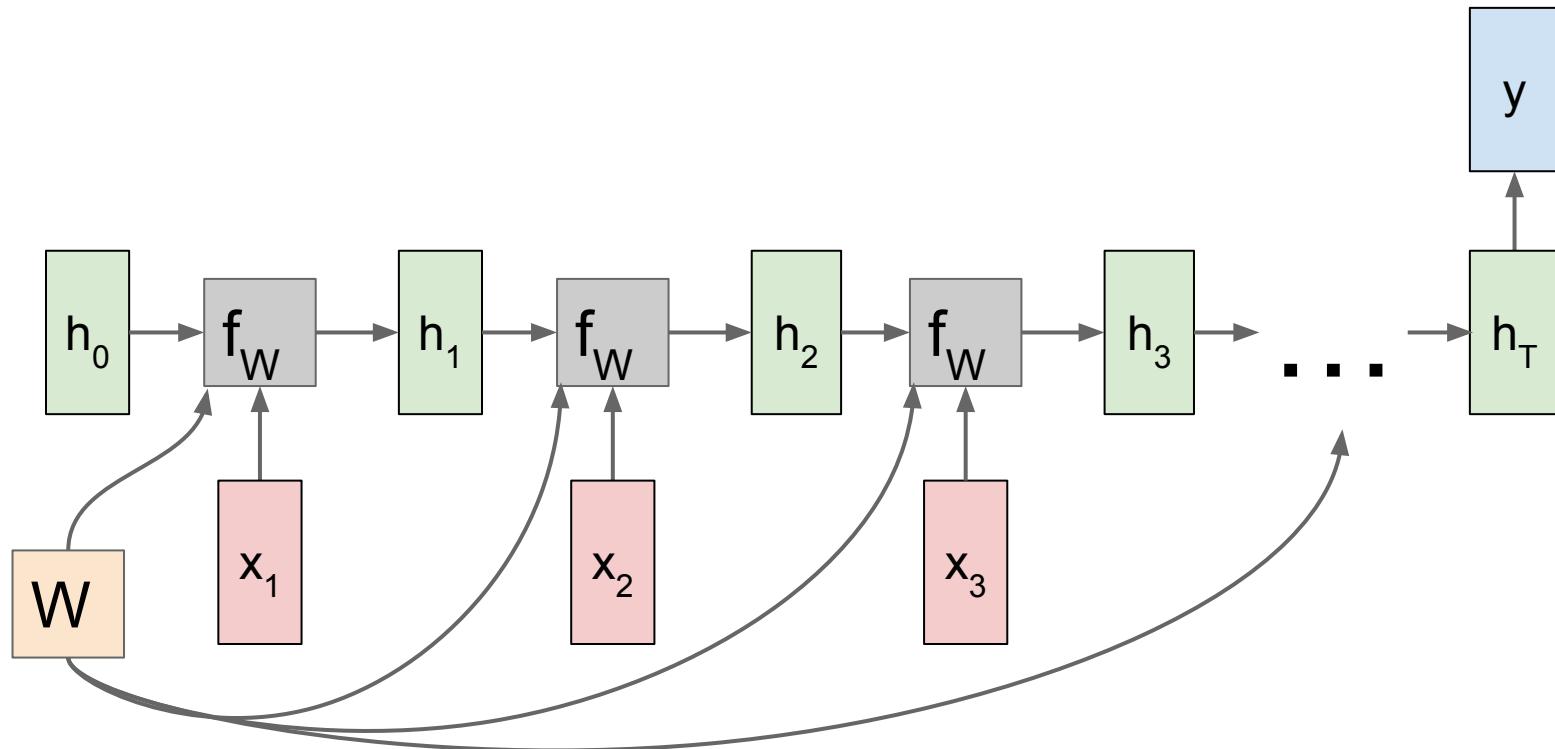
RNN: Computational Graph: Many to Many



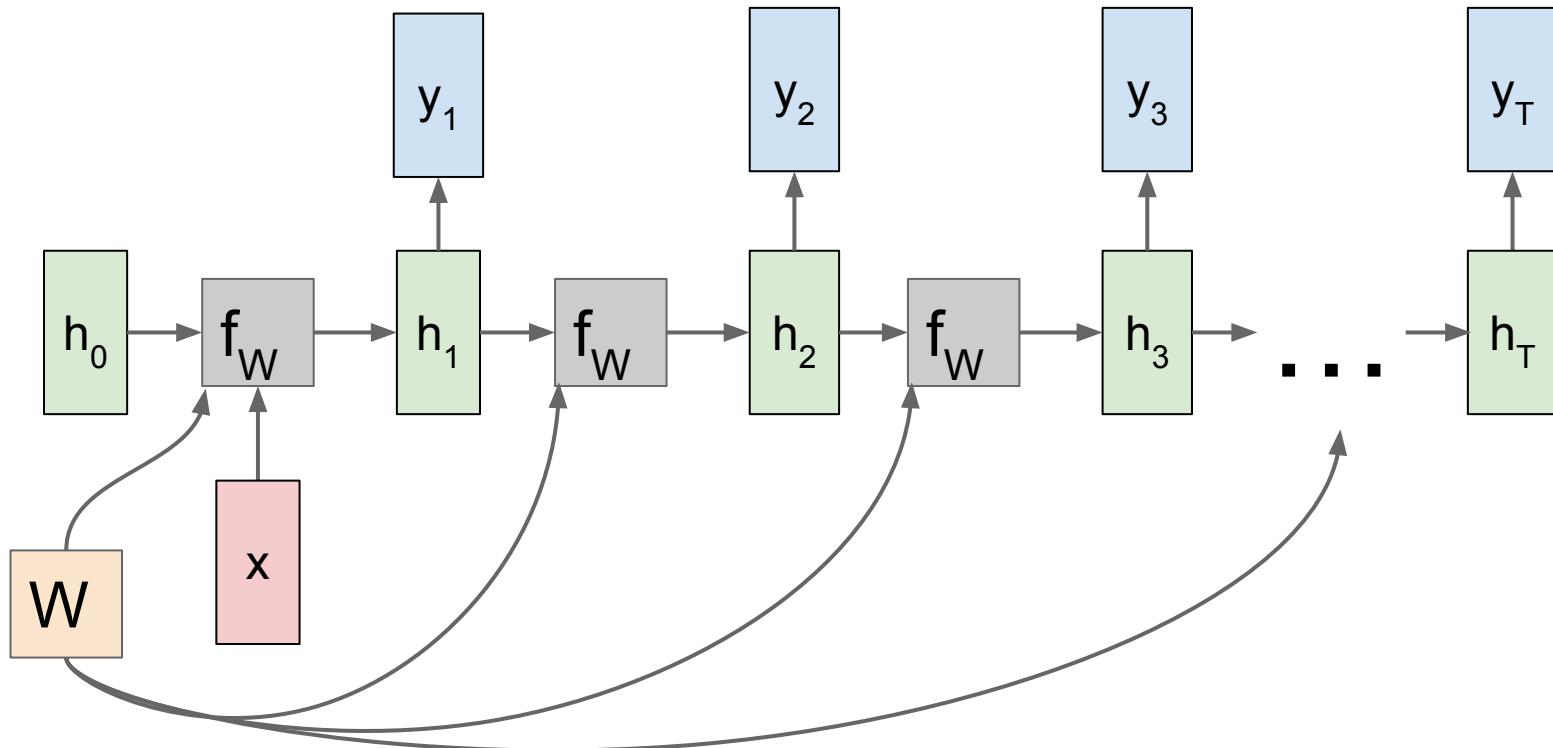
RNN: Computational Graph: Many to Many



RNN: Computational Graph: Many to One

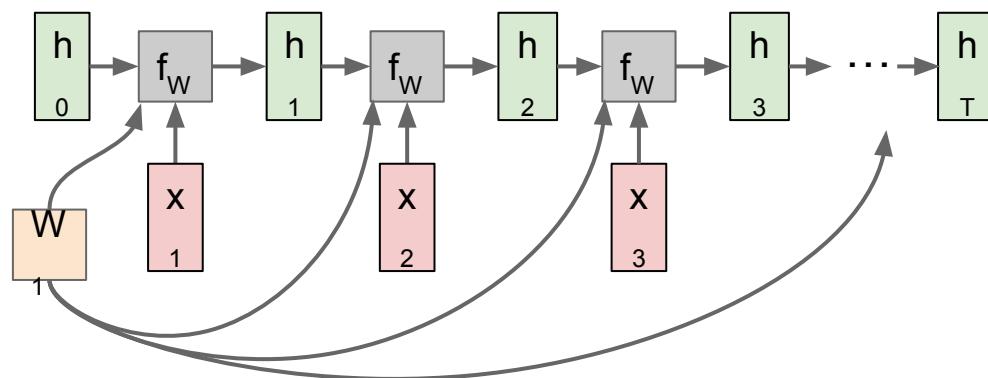


RNN: Computational Graph: One to Many



Sequence to Sequence: Many-to-one + one-to-many

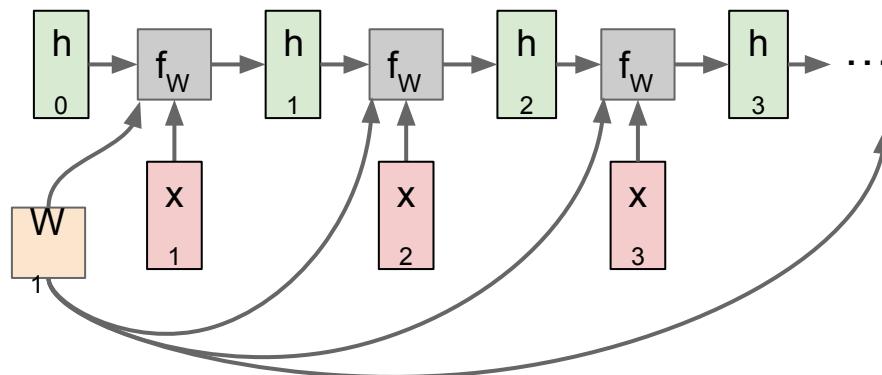
Many to one: Encode input sequence in a single vector



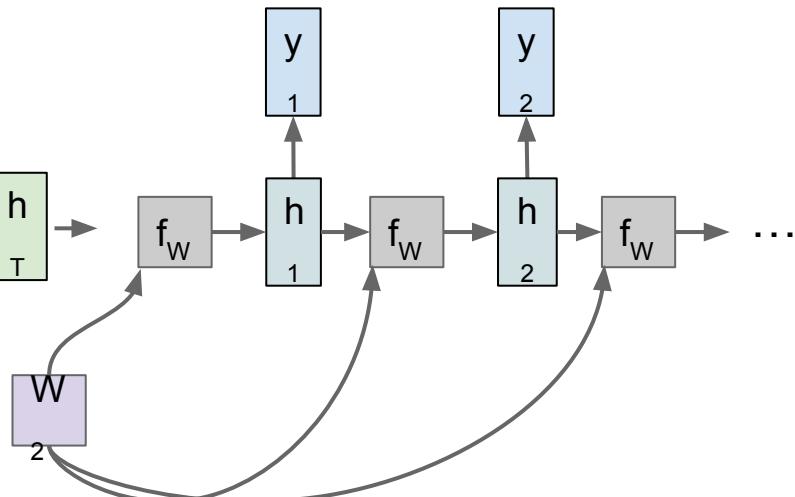
Sutskever et al, "Sequence to Sequence Learning with Neural Networks", NIPS 2014

Sequence to Sequence: Many-to-one + one-to-many

Many to one: Encode input sequence in a single vector



One to many: Produce output sequence from single input vector

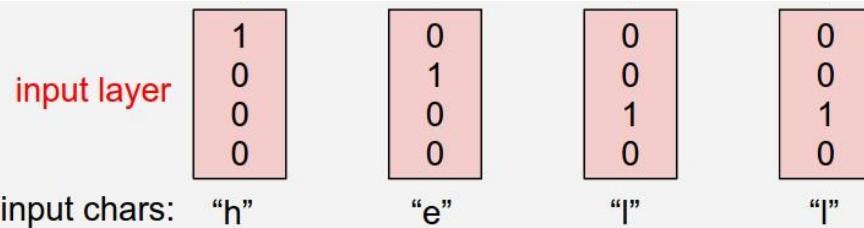


Sutskever et al, "Sequence to Sequence Learning with Neural Networks", NIPS 2014

Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

Example training
sequence:
“hello”

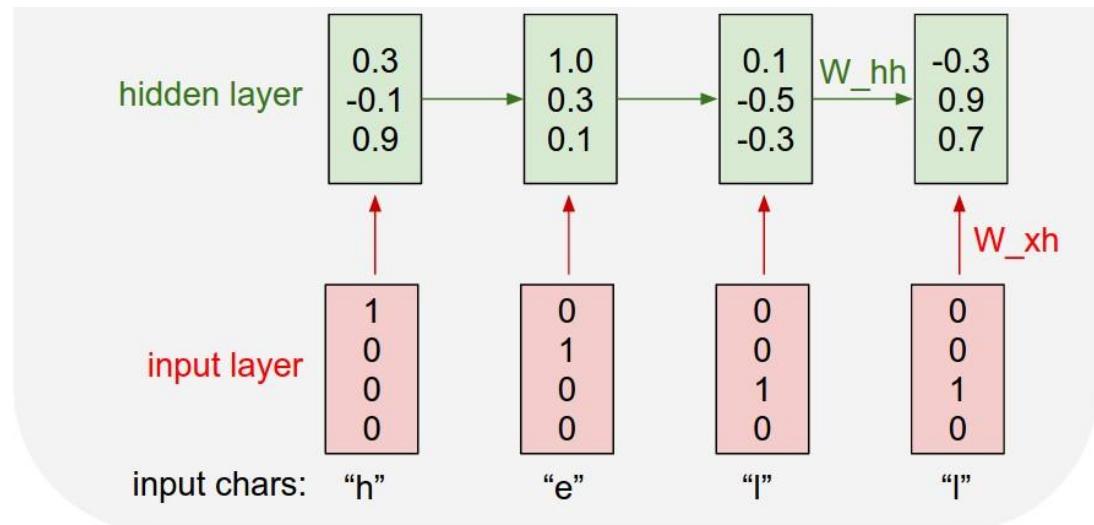


Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

Example training
sequence:
“hello”

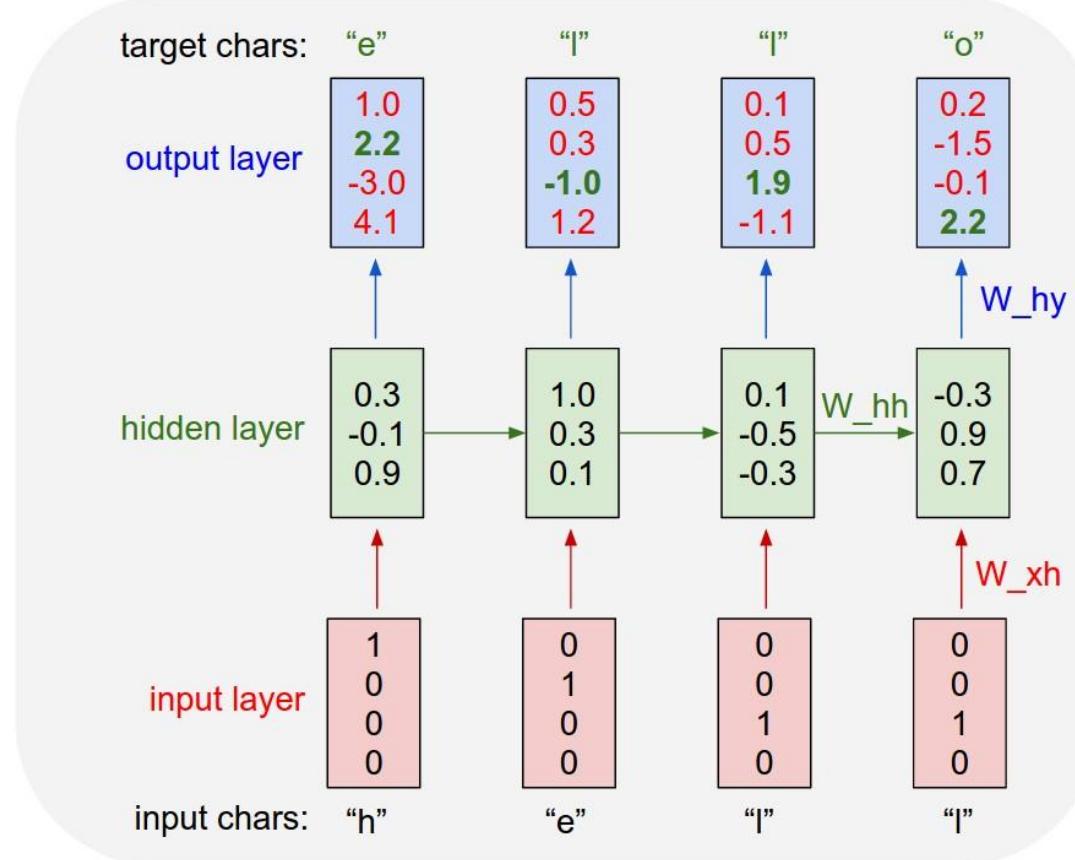
$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$



Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

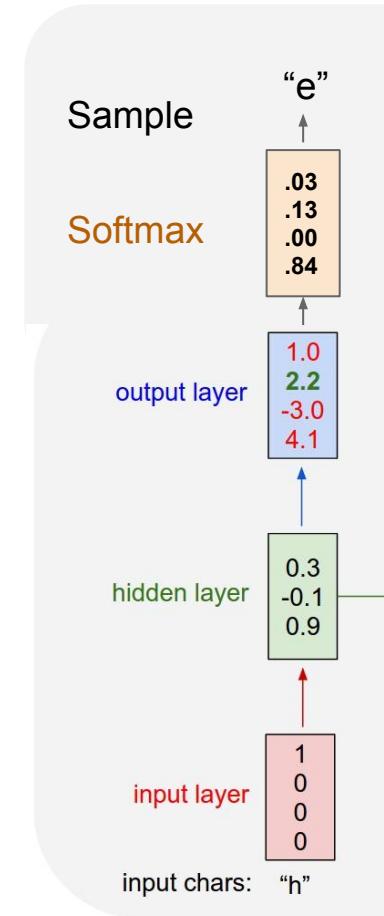
Example training
sequence:
“hello”



Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

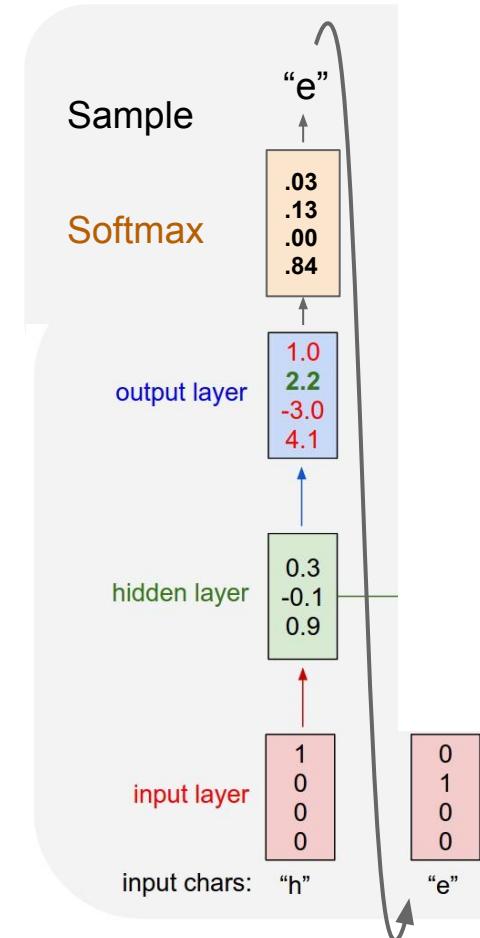
At test-time sample
characters one at a time,
feed back to model



Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

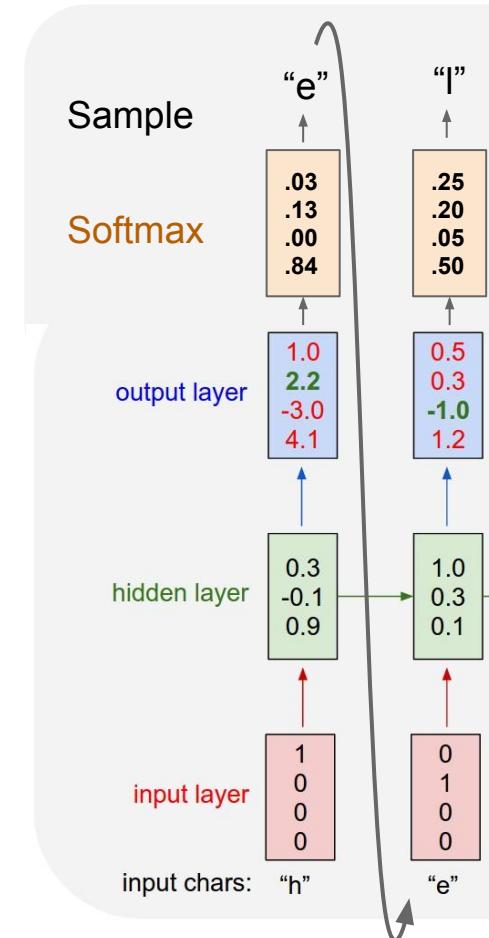
At test-time sample
characters one at a time,
feed back to model



Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

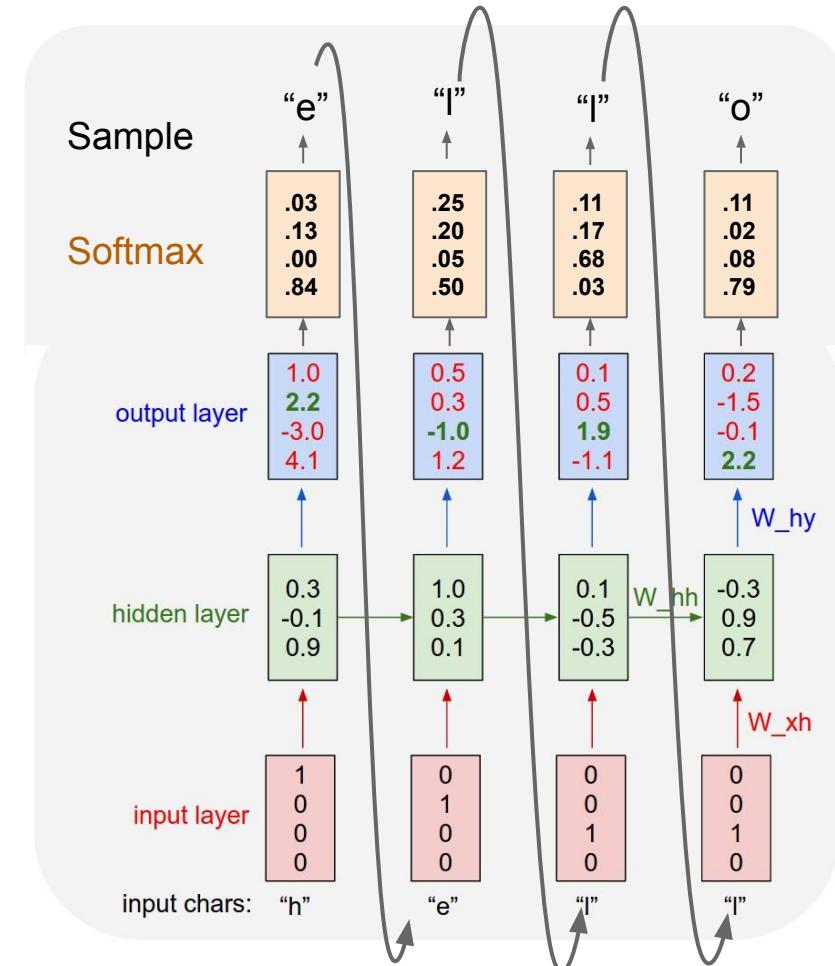
At test-time sample
characters one at a time,
feed back to model



Example: Character-level Language Model Sampling

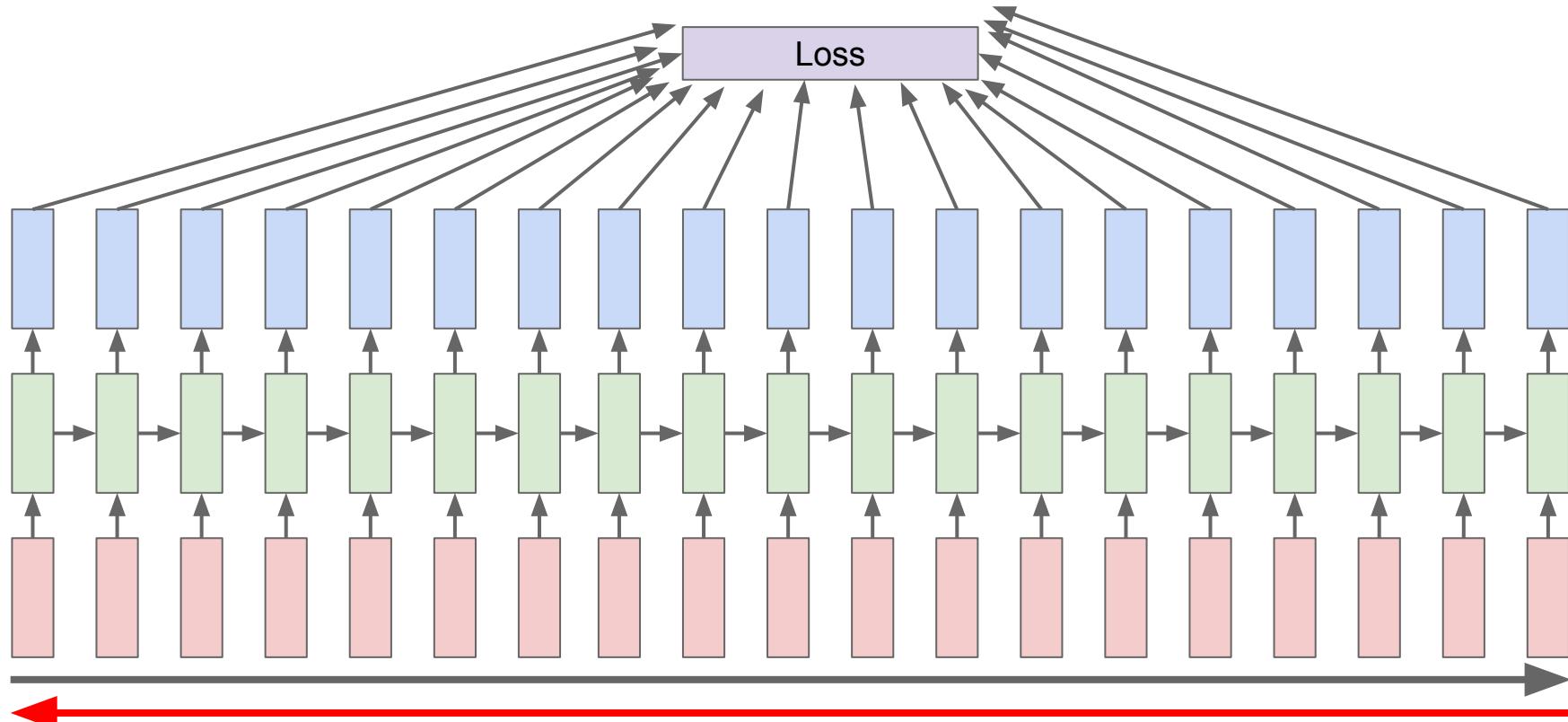
Vocabulary:
[h,e,l,o]

At test-time sample
characters one at a time,
feed back to model

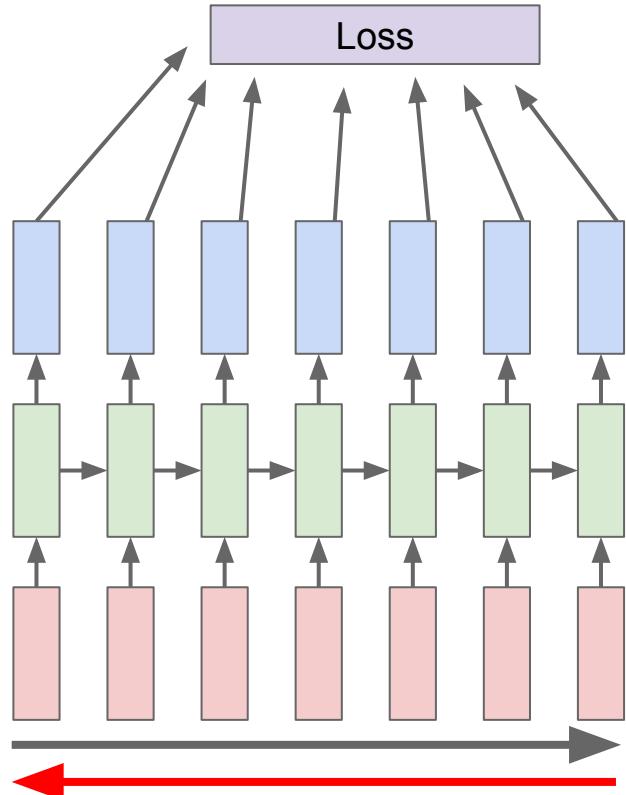


Backpropagation through time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

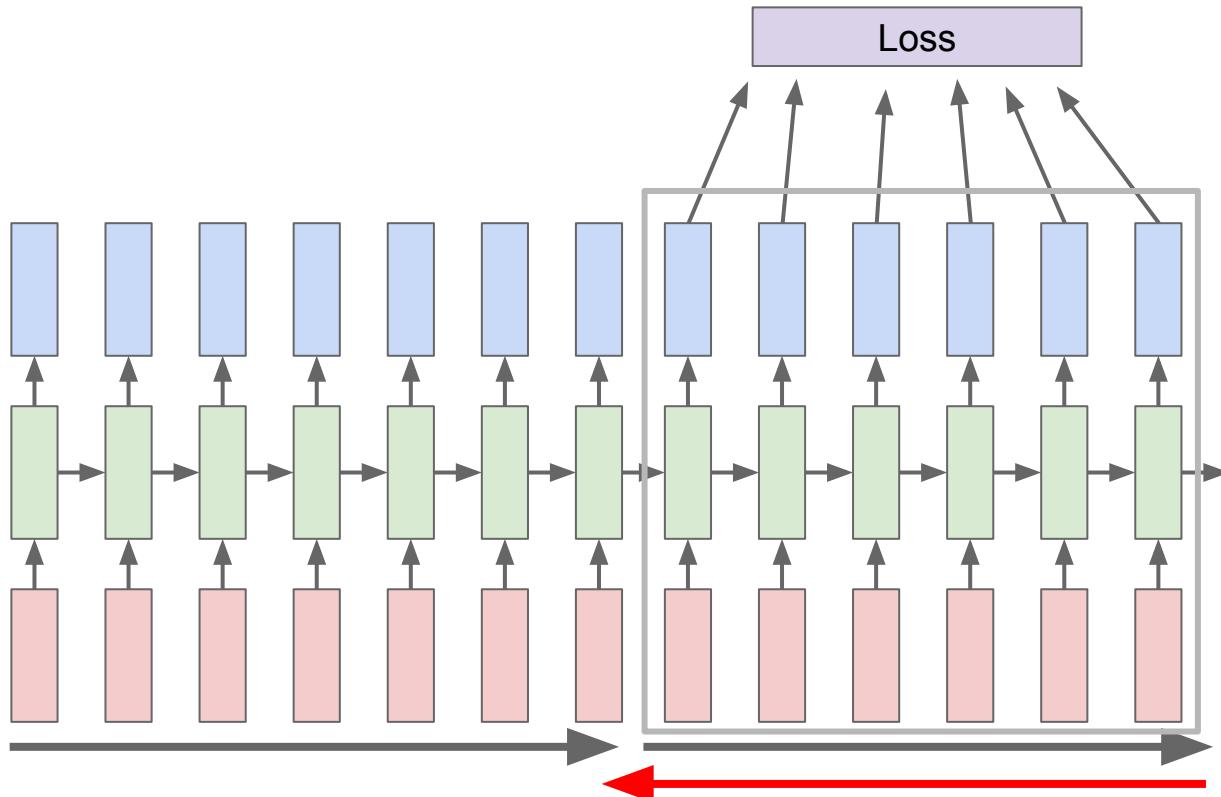


Truncated Backpropagation through time



Run forward and backward
through chunks of the
sequence instead of whole
sequence

Truncated Backpropagation through time



Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

Truncated Backpropagation through time

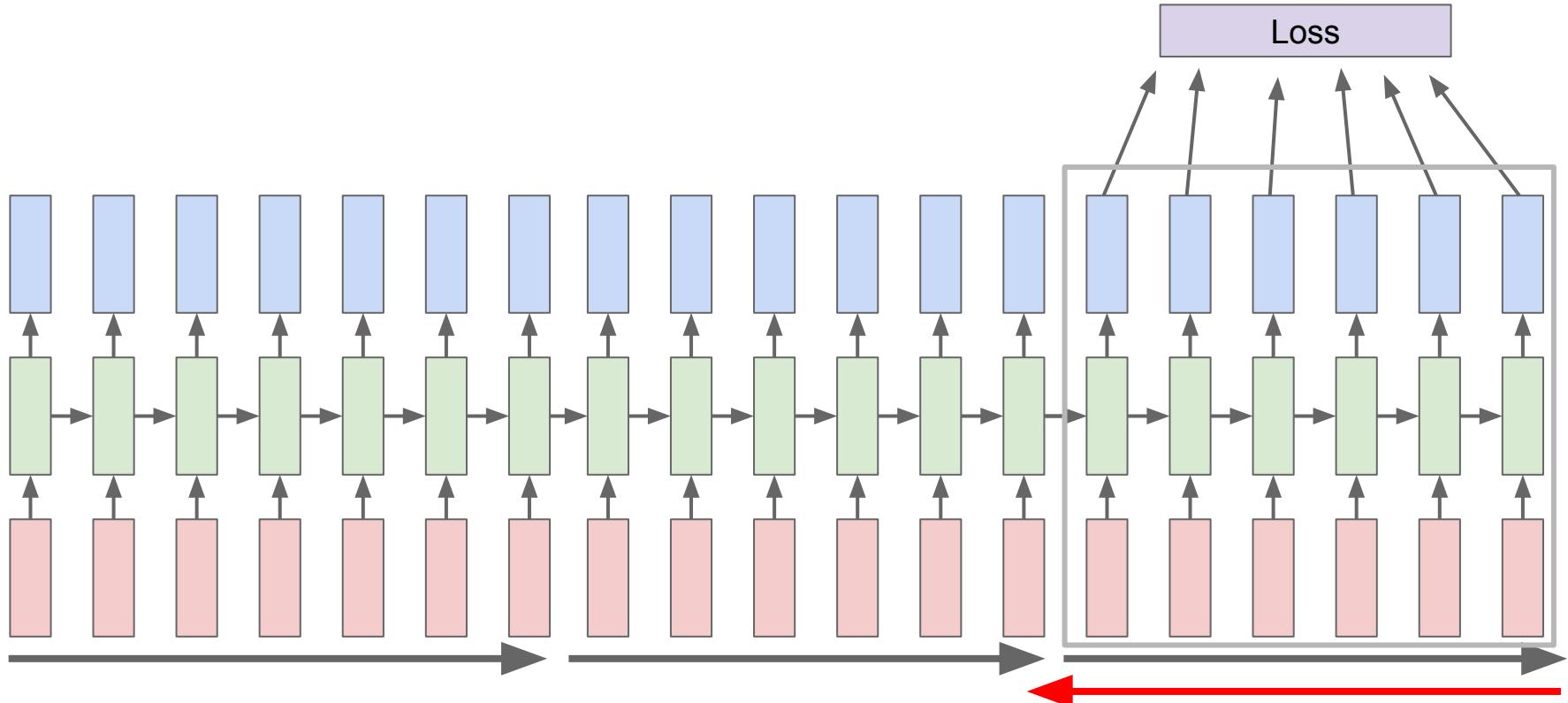


Image Captioning

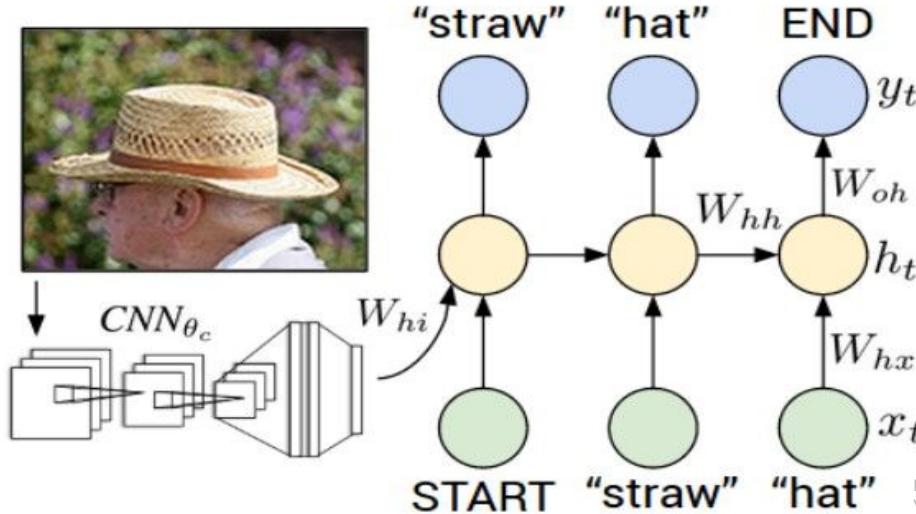


Figure from Karpathy et al, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015; figure copyright IEEE, 2015.
Reproduced for educational purposes.

Explain Images with Multimodal Recurrent Neural Networks, Mao et al.

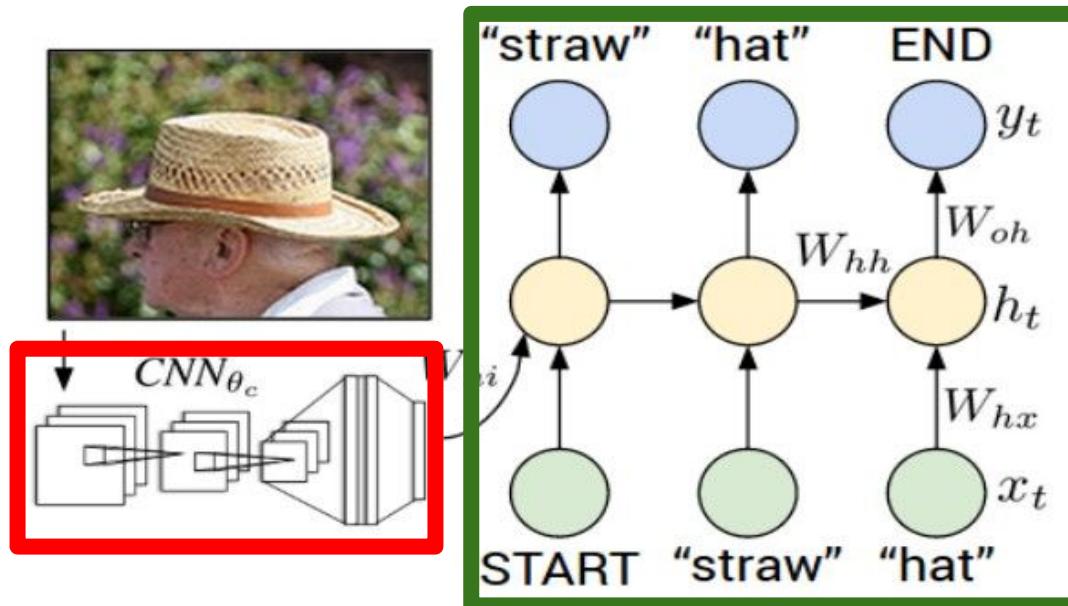
Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei

Show and Tell: A Neural Image Caption Generator, Vinyals et al.

Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al.

Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick

Recurrent Neural Network



Convolutional Neural Network



test image

[This image is CC0 public domain](#)

image



test image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

maxpool

FC-4096

FC-4096

FC-1000

softmax

image



test image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

maxpool

FC-4096

FC-4096

FC-1000

softmax

Fe

X Justin Johnson & Serena Yeung

Lecture 10 -

May 2, 2019

image



test image

conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

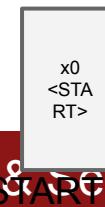
conv-512

conv-512

maxpool

FC-4096

FC-4096



image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

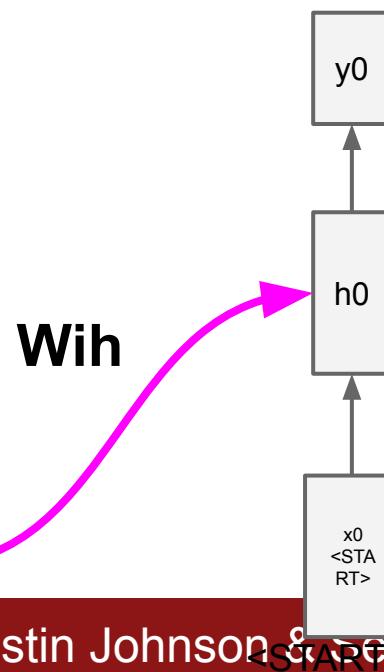
maxpool

FC-4096

FC-4096



test image



before:

$$h = \tanh(W_{xh} * x + W_{hh} * h)$$

now:

$$h = \tanh(W_{xh} * x + W_{hh} * h + W_{ih} * v)$$

image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

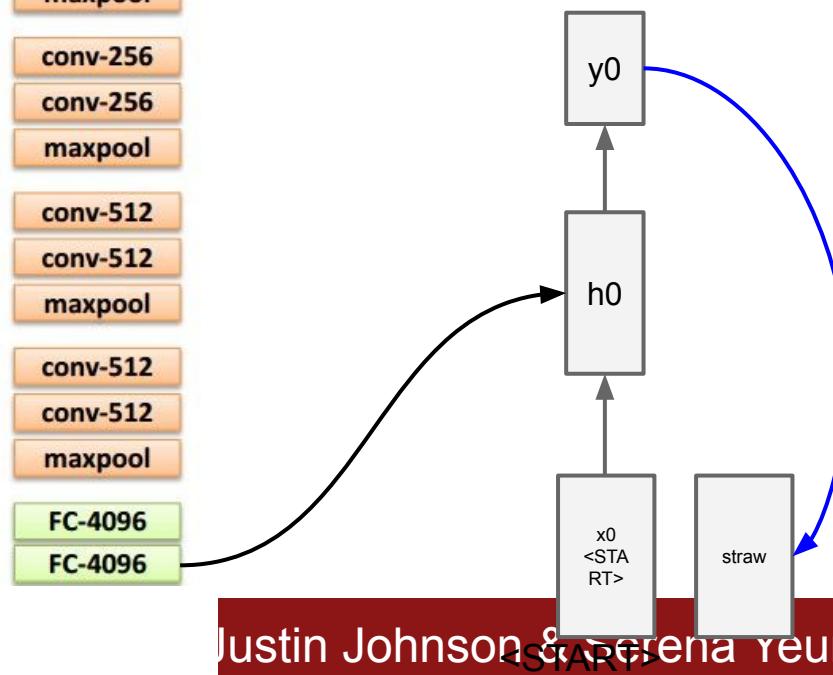
maxpool

FC-4096

FC-4096



test image



image



test image

conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

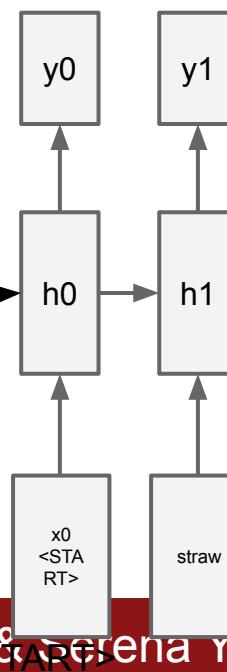
conv-512

conv-512

maxpool

FC-4096

FC-4096



image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

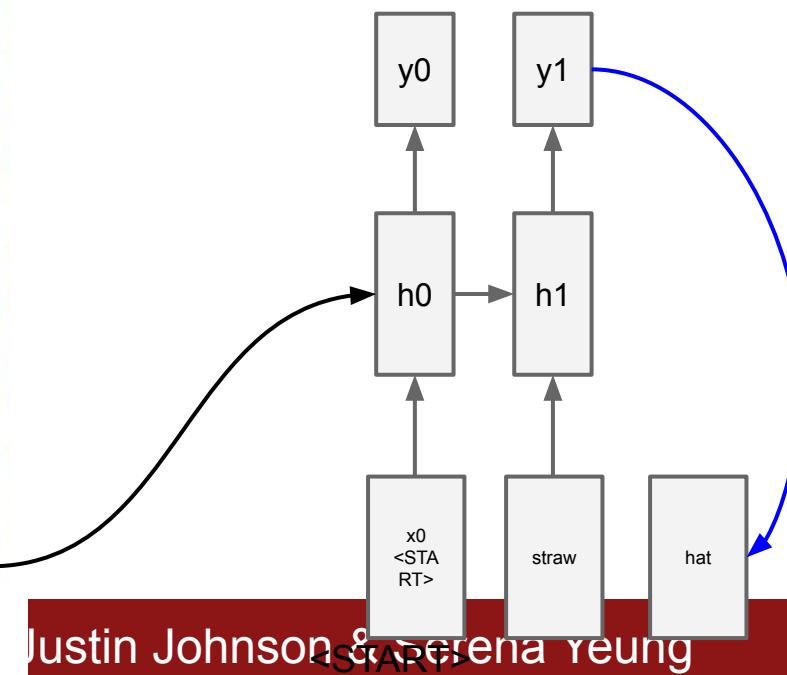
maxpool

FC-4096

FC-4096



test image



image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

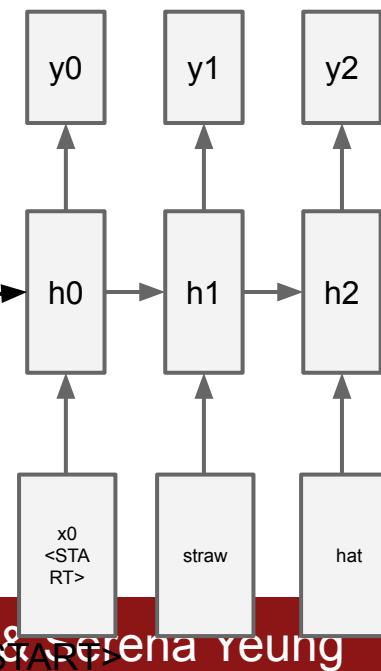
maxpool

FC-4096

FC-4096



test image





test image

sample
<END> token
=> finish.

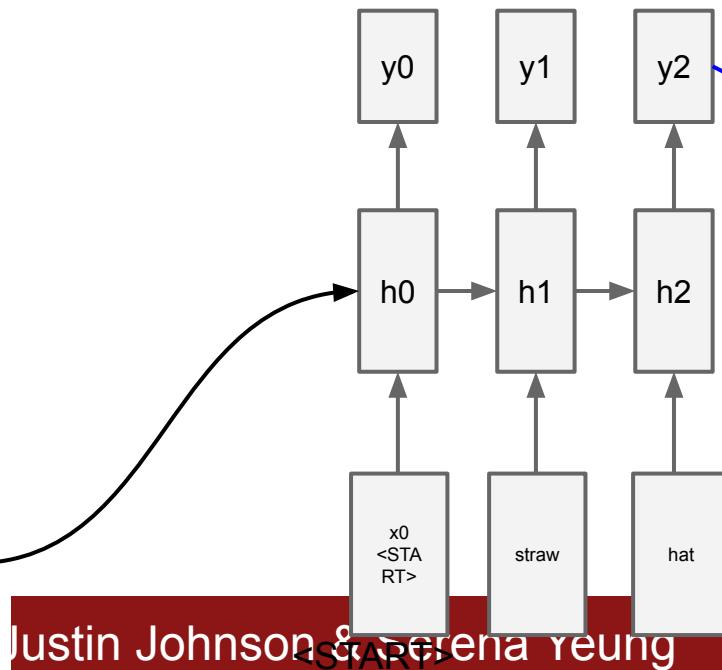


Image Captioning: Example Results



A cat sitting on a suitcase on the floor



A cat is sitting on a tree branch



A dog is running in the grass with a frisbee



A white teddy bear sitting in the grass



Two people walking on the beach with surfboards



A tennis player in action on the court



Two giraffes standing in a grassy field



A man riding a dirt bike on a dirt track

Image Captioning: Failure Cases



A woman is holding a cat in her hand



A person holding a computer mouse on a desk



A woman standing on a beach holding a surfboard



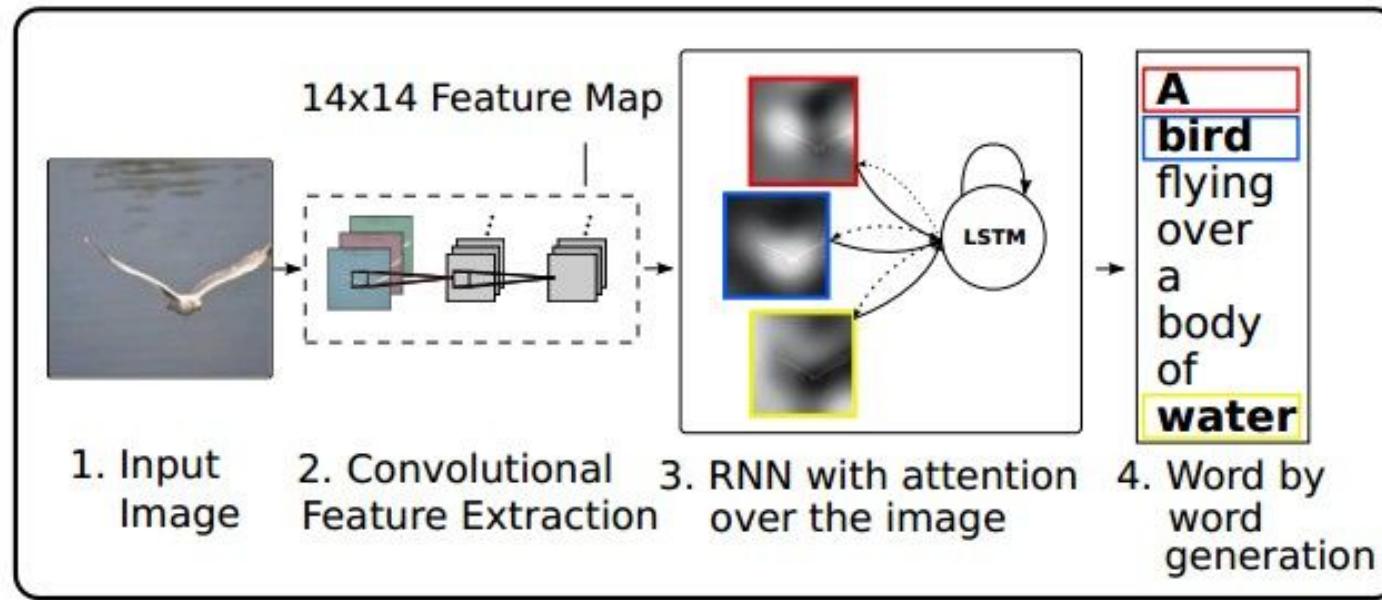
A bird is perched on a tree branch



A man in a baseball uniform throwing a ball

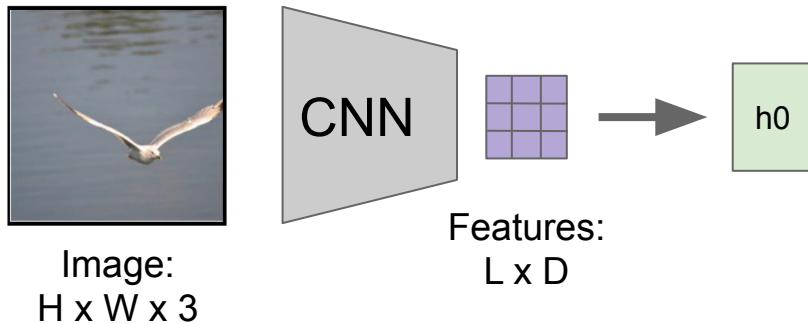
Image Captioning with Attention

RNN focuses its attention at a different spatial location when generating each word



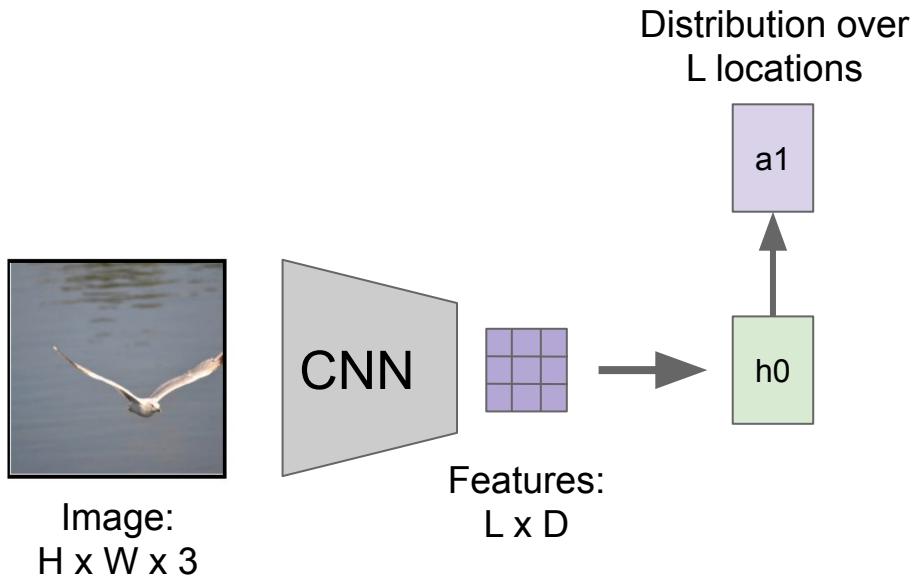
Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015
Figure copyright Kelvin Xu, Jimmy Lei Ba, Jamie Kiros, Kyunghyun Cho, Aaron Courville, Ruslan Salakhutdinov, Richard S. Zemel, and Yoshua Bengio, 2015. Reproduced with permission.

Image Captioning with Attention



Xu et al, "Show, Attend and Tell: Neural
Image Caption Generation with Visual
Attention", ICML 2015

Image Captioning with Attention

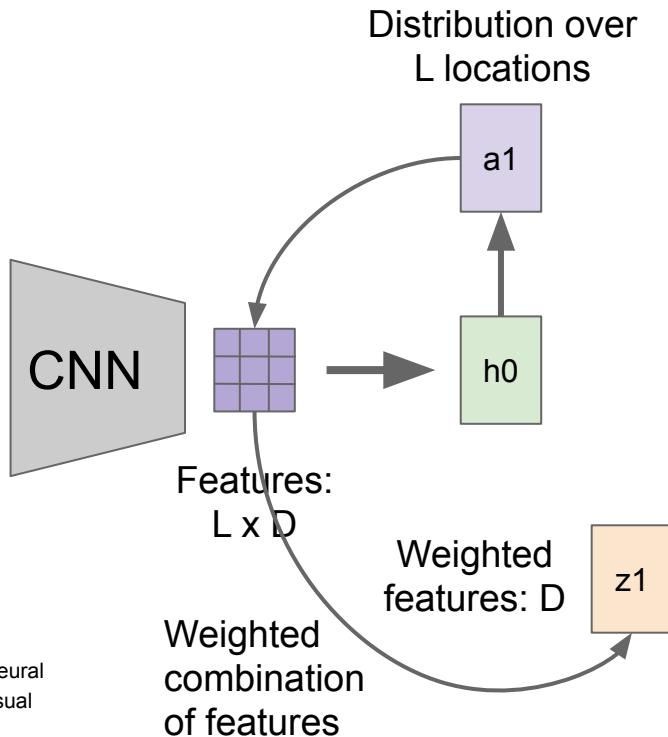


Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention



Image:
 $H \times W \times 3$



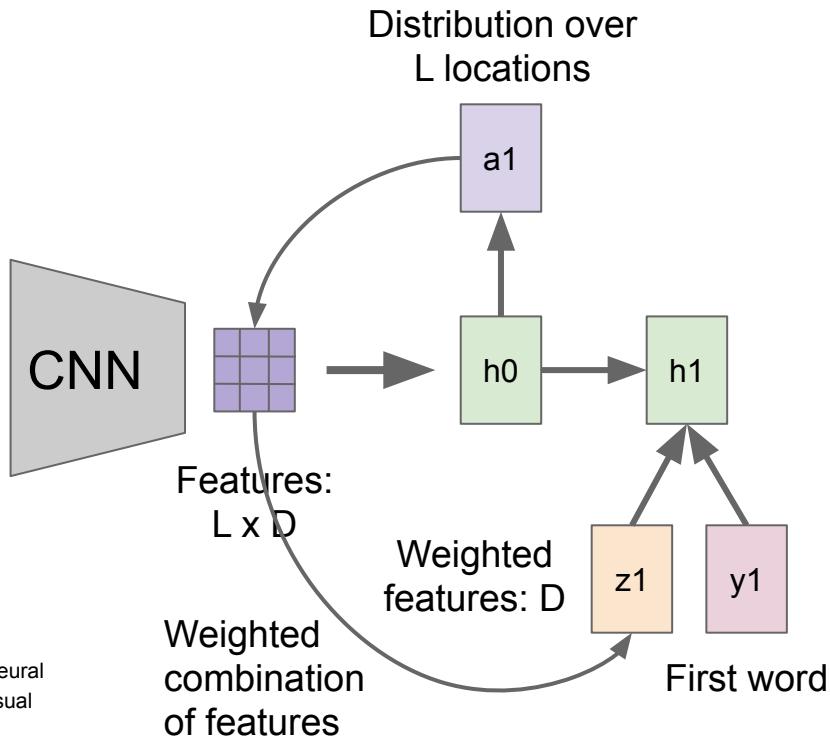
$$z = \sum_{i=1}^L p_i v_i$$

Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention



Image:
 $H \times W \times 3$

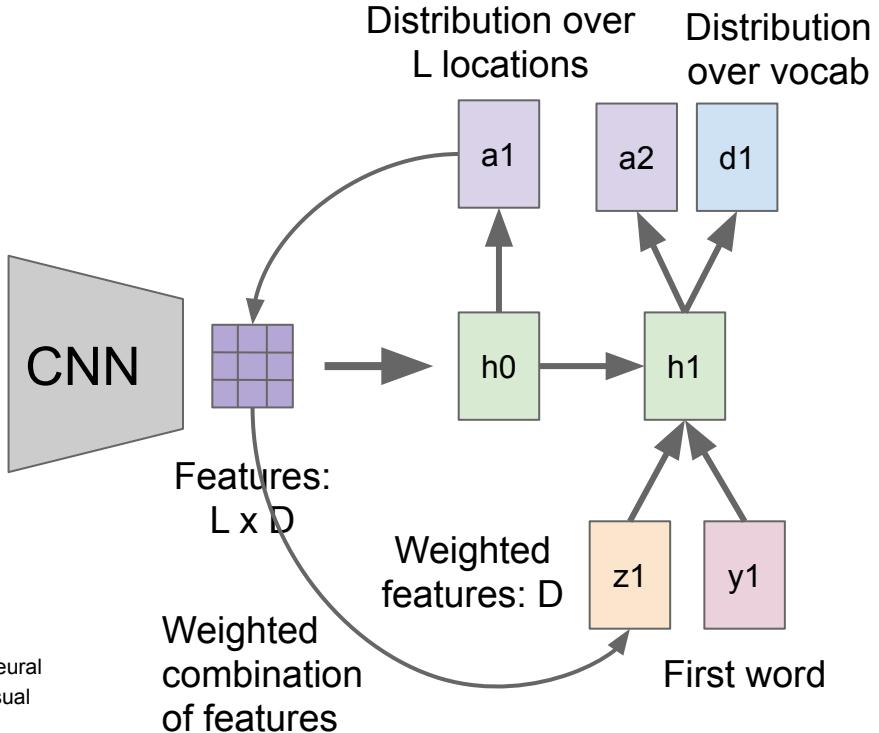


Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention



Image:
 $H \times W \times 3$

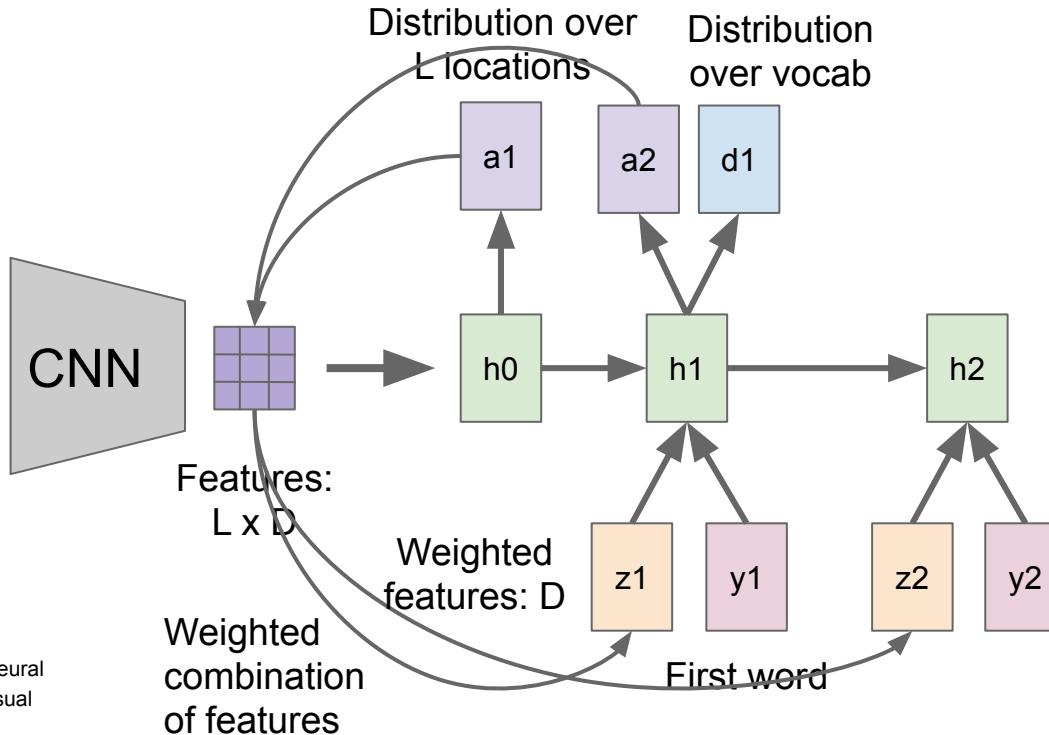


Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention

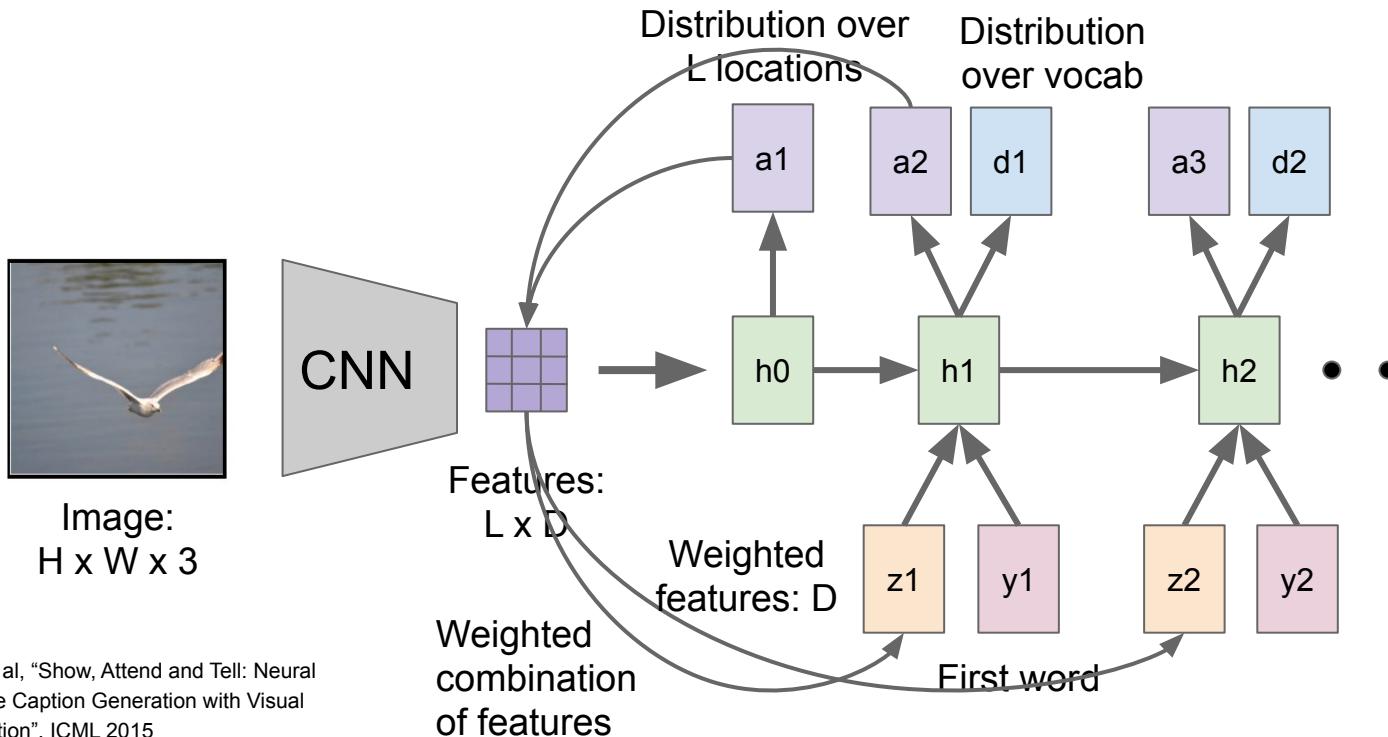


Image:
 $H \times W \times 3$



Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention



Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention

Soft attention



Hard attention



A

bird

flying

over

a

body

of

water

.

Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Figure copyright Kelvin Xu, Jimmy Lei Ba, Jamie Kiros, Kyunghyun Cho, Aaron Courville, Ruslan Salakhutdinov, Richard S. Zemel, and Yoshua Bengio, 2015. Reproduced with permission.

Image Captioning with Attention



A woman is throwing a frisbee in a park.



A dog is standing on a hardwood floor.



A stop sign is on a road with a mountain in the background.



A little girl sitting on a bed with a teddy bear.



A group of people sitting on a boat in the water.



A giraffe standing in a forest with trees in the background.

Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015
Figure copyright Kelvin Xu, Jimmy Lei Ba, Jamie Kiros, Kyunghyun Cho, Aaron Courville, Ruslan Salakhutdinov, Richard S. Zemel, and Yoshua Bengio, 2015. Reproduced with permission.

Visual Question Answering



Q: What endangered animal is featured on the truck?

- A: A bald eagle.
- A: A sparrow.
- A: A humming bird.
- A: A raven.



Q: Where will the driver go if turning right?

- A: Onto 24 1/4 Rd.
- A: Onto 25 1/4 Rd.
- A: Onto 23 1/4 Rd.
- A: Onto Main Street.



Q: When was the picture taken?

- A: During a wedding.
- A: During a bar mitzvah.
- A: During a funeral.
- A: During a Sunday church service

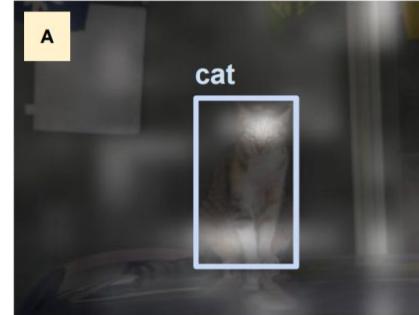
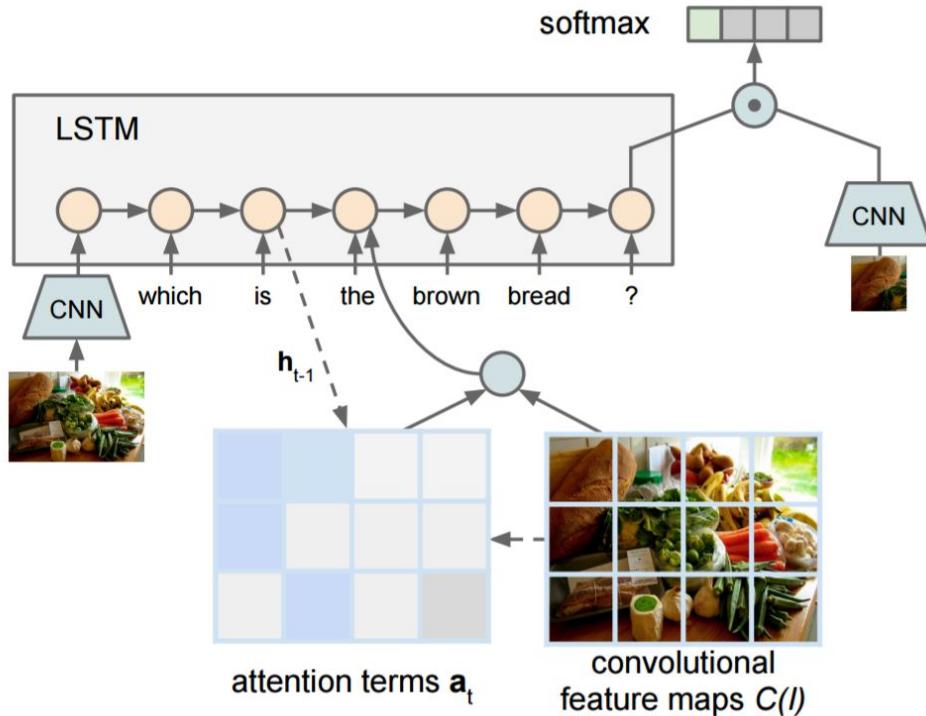


Q: Who is under the umbrella?

- A: Two women.
- A: A child.
- A: An old man.
- A: A husband and a wife.

Agrawal et al, "VQA: Visual Question Answering", ICCV 2015
Zhu et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2016
Figure from Zhu et al, copyright IEEE 2016. Reproduced for educational purposes.

Visual Question Answering: RNNs with Attention



What kind of animal is in the photo?
A **cat**.



Why is the person holding a knife?
To cut the **cake** with.

Zhu et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2016
Figures from Zhu et al, copyright IEEE 2016. Reproduced for educational purposes.

Multilayer RNNs

$$h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$h \in \mathbb{R}^n \quad W^l [n \times 2n]$$

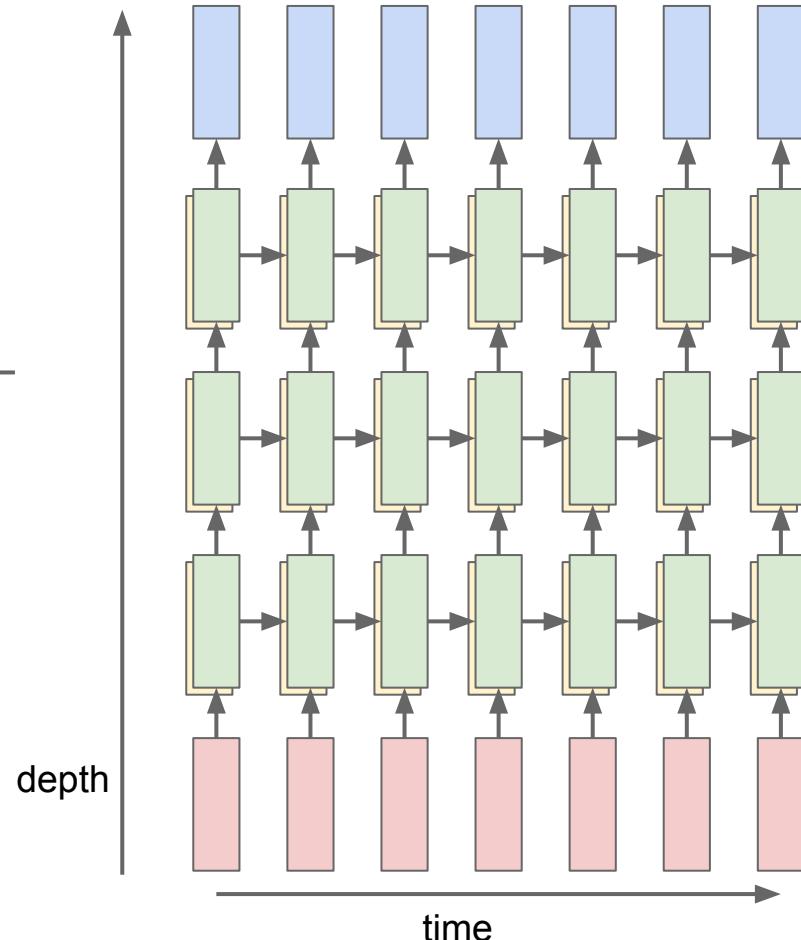
LSTM:

$$W^l [4n \times 2n]$$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

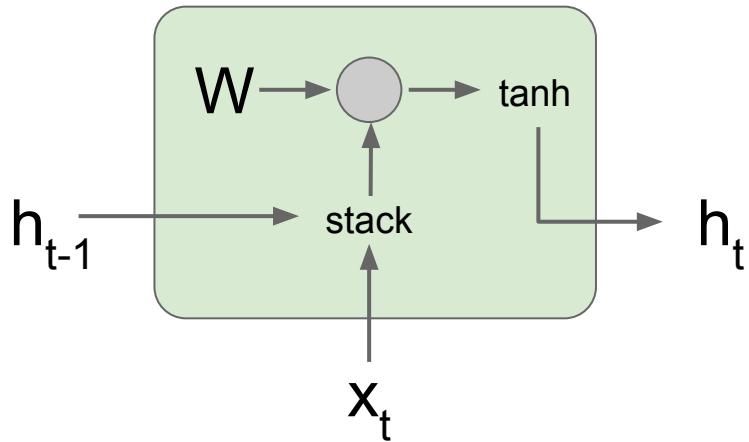
$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$



Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

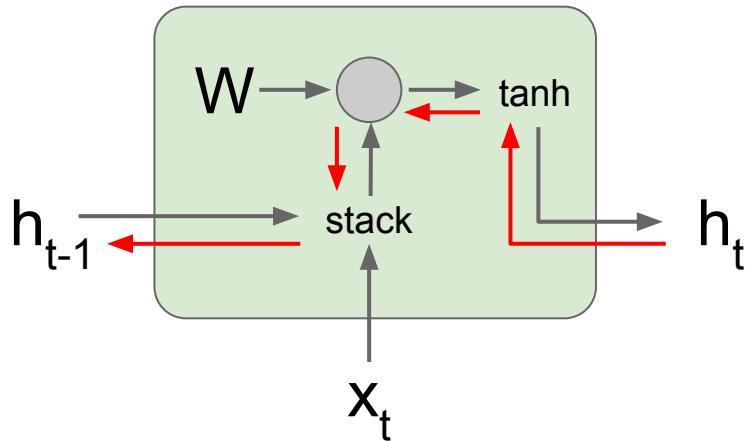


$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\ &= \tanh \left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \\ &= \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \end{aligned}$$

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

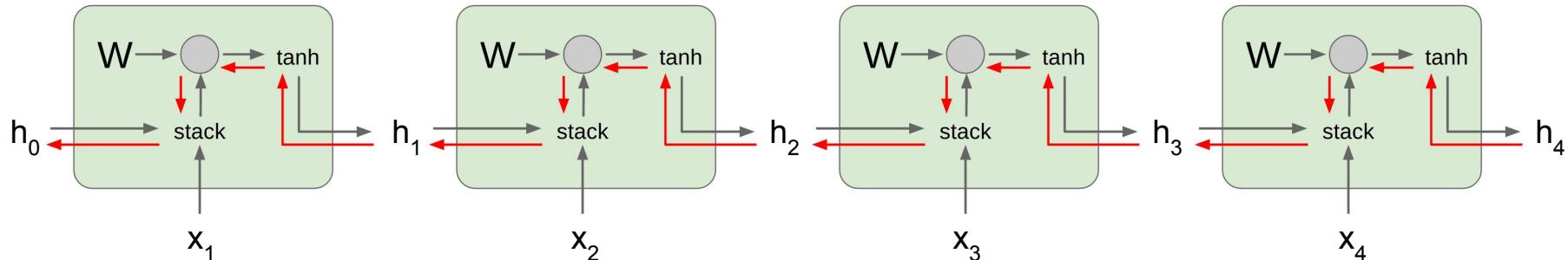
Backpropagation from h_t
to h_{t-1} multiplies by W
(actually W_{hh}^T)



$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\ &= \tanh \left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \\ &= \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \end{aligned}$$

Vanilla RNN Gradient Flow

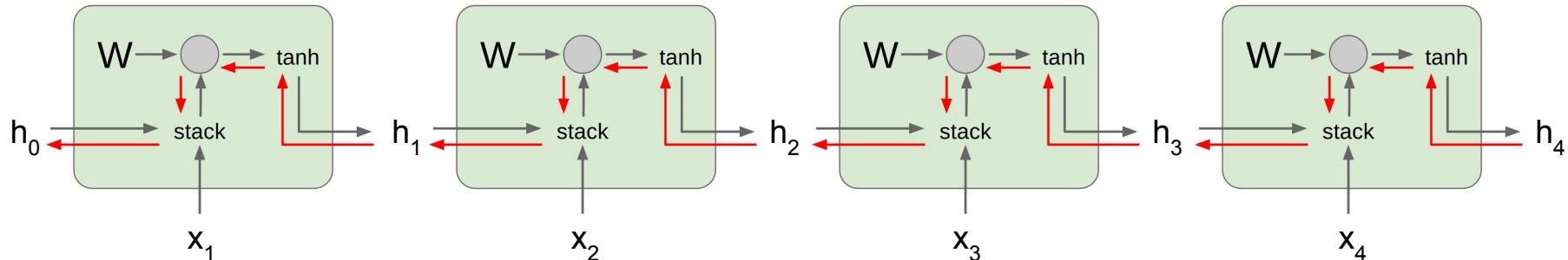
Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient
of h_0 involves many
factors of W
(and repeated tanh)

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



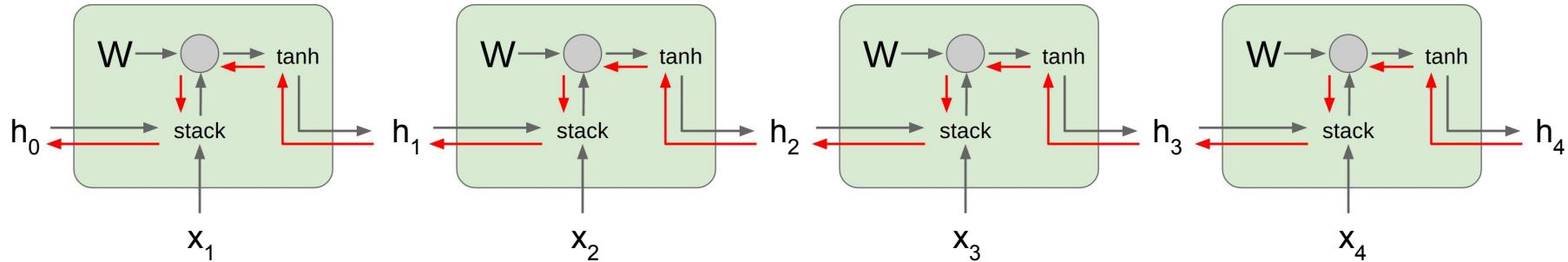
Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1 :
Exploding gradients

Largest singular value < 1 :
Vanishing gradients

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated \tanh)

Largest singular value > 1 :
Exploding gradients

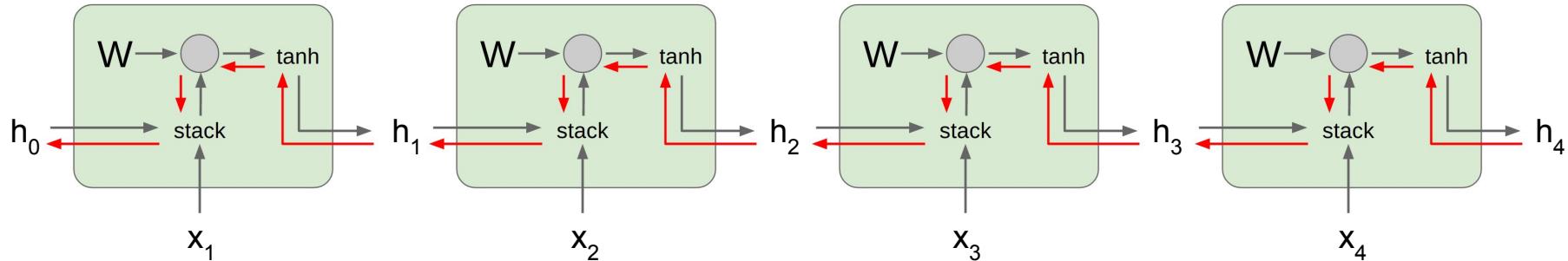
Largest singular value < 1 :
Vanishing gradients

Gradient clipping: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
    grad *= (threshold / grad_norm)
```

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1 :
Exploding gradients

Largest singular value < 1 :
Vanishing gradients

→ Change RNN architecture

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right)$$

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

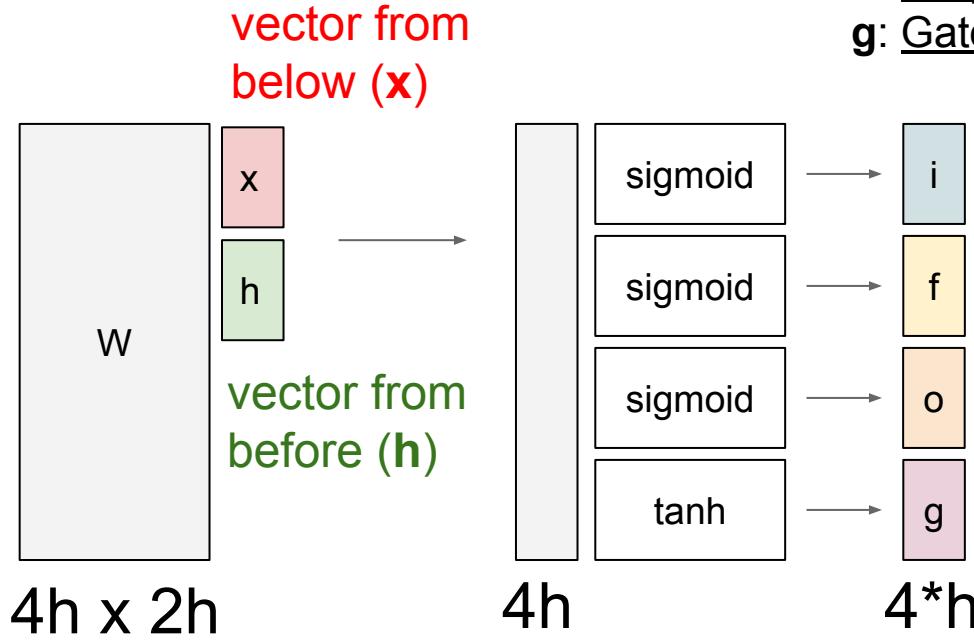
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation
1997

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



- i: Input gate, whether to write to cell
- f: Forget gate, Whether to erase cell
- o: Output gate, How much to reveal cell
- g: Gate gate (?), How much to write to cell

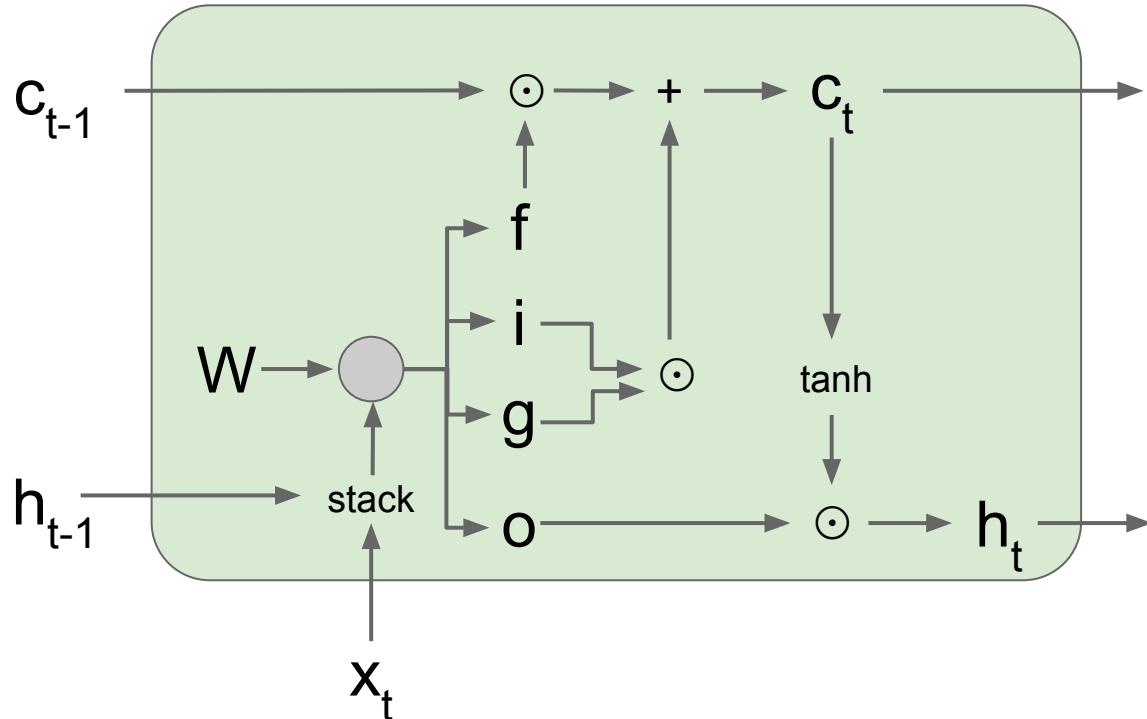
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



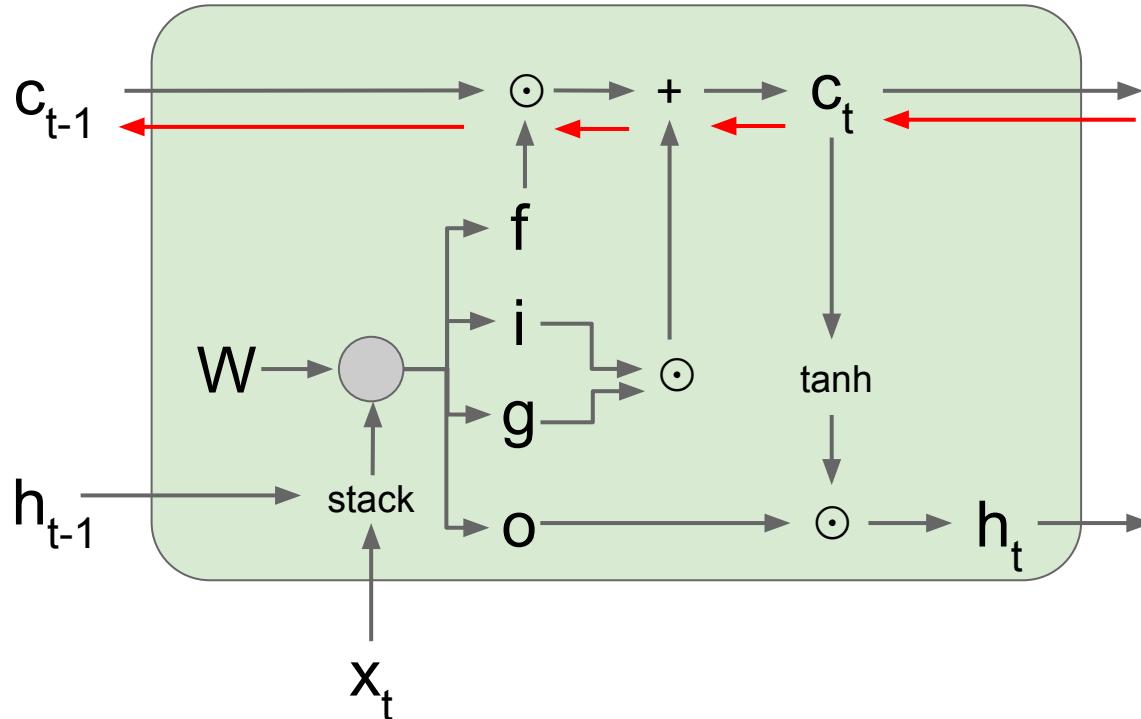
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]



Backpropagation from c_t to c_{t-1} only elementwise multiplication by f , no matrix multiply by W

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

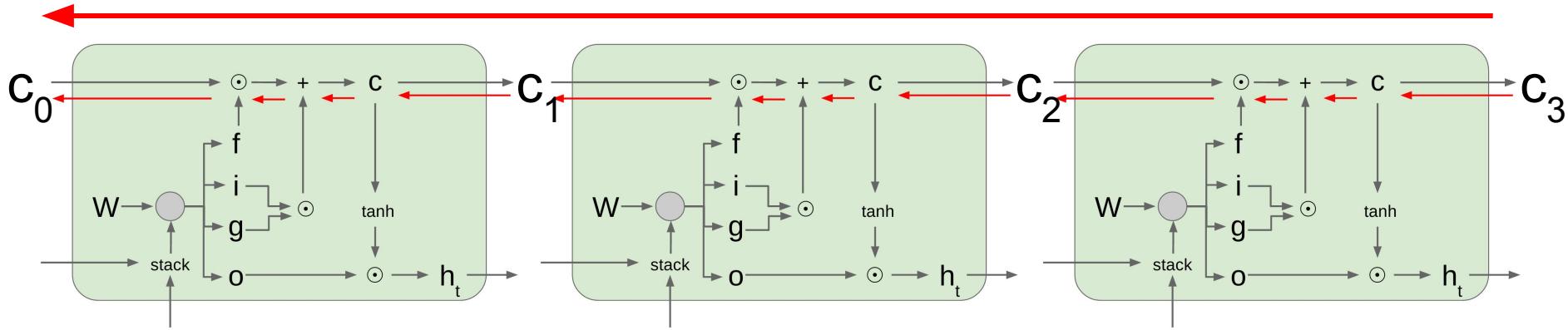
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

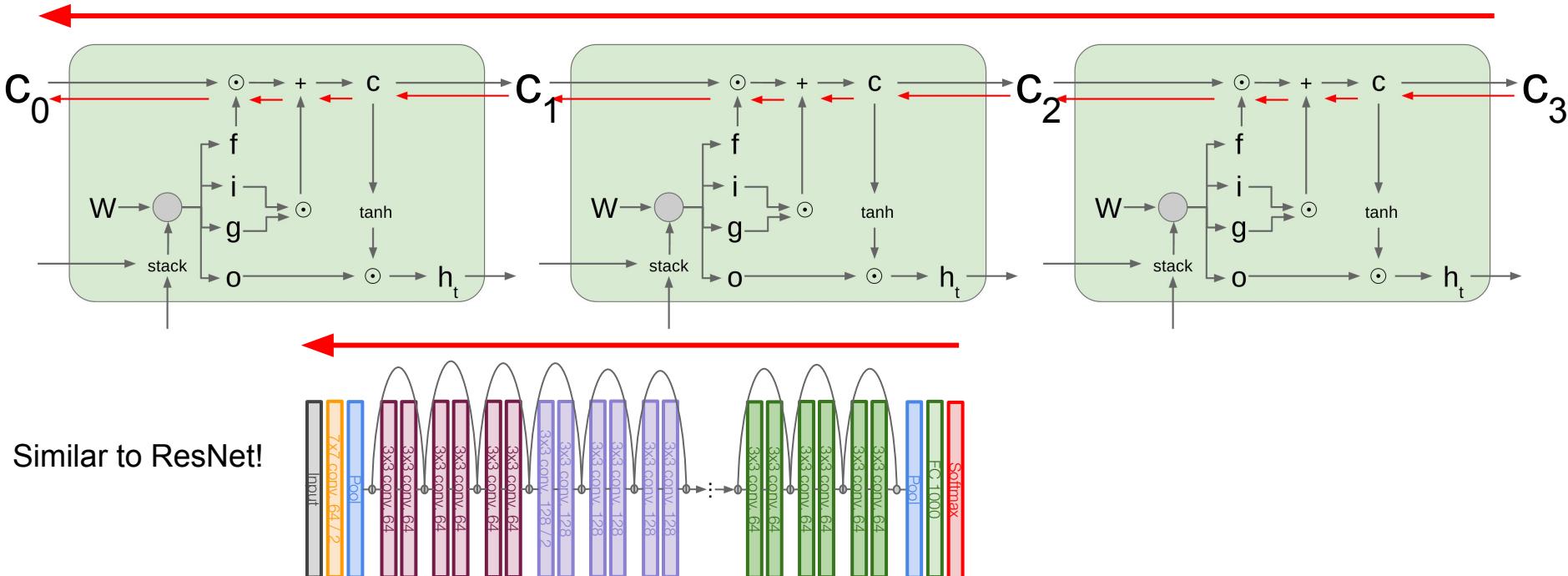
Uninterrupted gradient flow!



Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

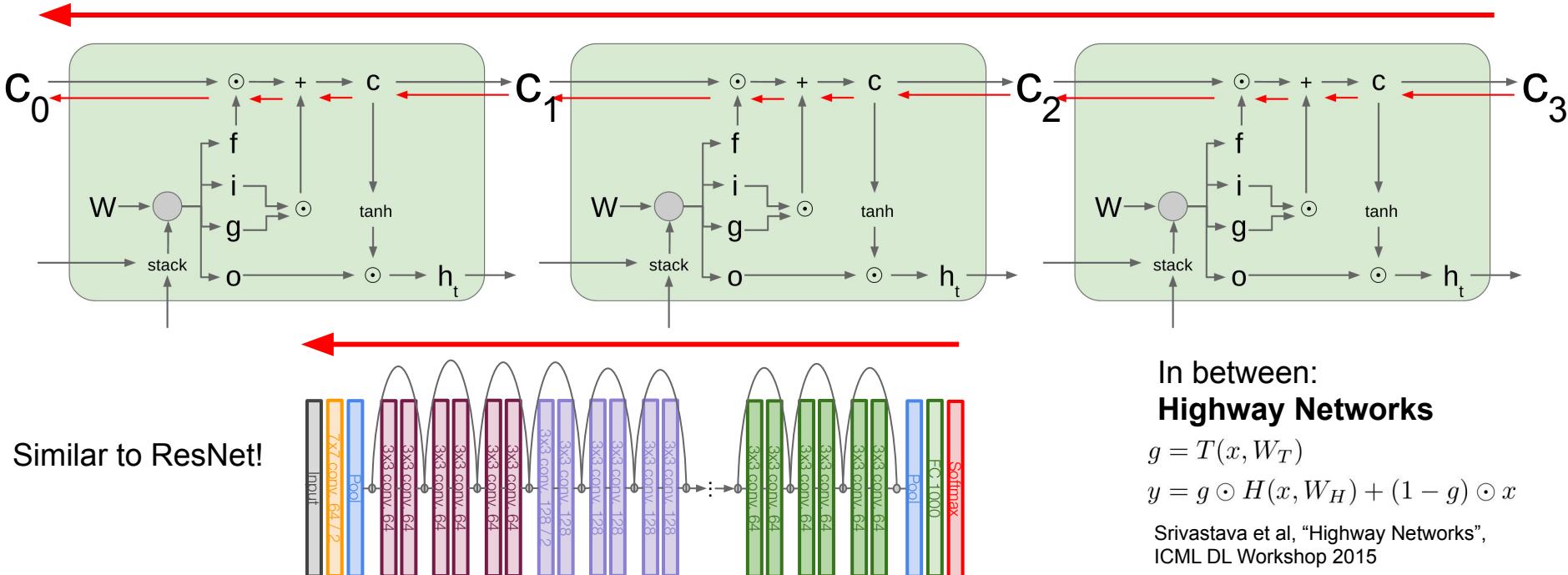
Uninterrupted gradient flow!



Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



Other RNN Variants

GRU [*Learning phrase representations using rnn encoder-decoder for statistical machine translation*, Cho et al. 2014]

$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$

$$z_t = \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z)$$

$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h)$$

$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

[*LSTM: A Search Space Odyssey*, Greff et al., 2015]

[*An Empirical Exploration of Recurrent Network Architectures*, Jozefowicz et al., 2015]

MUT1:

$$z = \text{sigm}(W_{xz}x_t + b_z)$$

$$r = \text{sigm}(W_{xr}x_t + W_{hr}h_t + b_r)$$

$$h_{t+1} = \tanh(W_{hh}(r \odot h_t) + \tanh(x_t) + b_h) \odot z + h_t \odot (1 - z)$$

MUT2:

$$z = \text{sigm}(W_{xz}x_t + W_{hx}h_t + b_z)$$

$$r = \text{sigm}(x_t + W_{hr}h_t + b_r)$$

$$h_{t+1} = \tanh(W_{hh}(r \odot h_t) + W_{xh}x_t + b_h) \odot z + h_t \odot (1 - z)$$

MUT3:

$$z = \text{sigm}(W_{xz}x_t + W_{hz}\tanh(h_t) + b_z)$$

$$r = \text{sigm}(W_{xr}x_t + W_{hr}h_t + b_r)$$

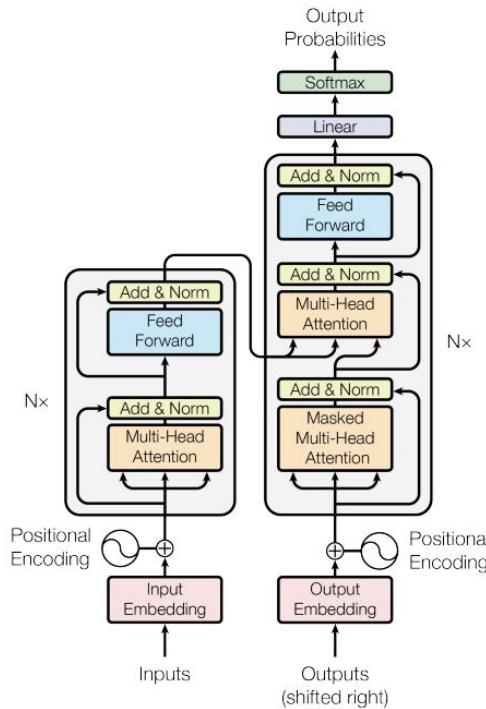
$$h_{t+1} = \tanh(W_{hh}(r \odot h_t) + W_{xh}x_t + b_h) \odot z + h_t \odot (1 - z)$$

Recently in Natural Language Processing...

New paradigms for reasoning over sequences

[“Attention is all you need”, Vaswani et al., 2018]

- New “Transformer” architecture no longer processes inputs sequentially; instead it can operate over inputs in a sequence in parallel through an attention mechanism
- Has led to many state-of-the-art results and pre-training in NLP, for more interest see e.g.
 - “BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding”, Devlin et al., 2018
 - OpenAI GPT-2, Radford et al., 2018



Summary

- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don't work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish. Exploding is controlled with gradient clipping. Vanishing is controlled with additive interactions (LSTM)
- Better/simpler architectures are a hot topic of current research, as well as new paradigms for reasoning over sequences
- Better understanding (both theoretical and empirical) is needed.