Theory of Computing
CSE - 203

Course Objective:

- Introduce concepts in automata theory and theory of computation
- Identify different formal language classes and their relationships
- Design grammars and recognizers for different formal languages
- Prove or disprove theorems in automata theory using its properties
- Determine the decidability and intractability of computational problems

Why should I study CSE-203?

- Regular expressions are used in many systems, Like UNIX.
- Context-free grammars are used to describe the syntax of essentially every programming language.
- When developing solutions to real problems, we often confront the limitations of what software can do. CSE-203 gives you the tools.

Syllabus

1. Introduction to Automata 2. Deterministic Finite Automata 3. Nondeterministic Finite Automata 4. Regular Expressions 5. Context-Free Languages	6. Non Context Free Language (NCFL) 7. Parse Trees 8. PushDown Automata (PDA) 9. Turing Machines 10. Undecidable Problems

Text Book

Reference	Introduction to the Theory of Computation Michael Sipser: 3rd Edition
Further Reading	Introduction to automata theory, languages and computation (JE Hopcroft, R Motwani and JD Ullman) Addison Wesley/ Pearson; 3rd Edition

Marks Distribution

S.L	Exam	Mark	Syllabus
1	Midterm	30	1 – 5
2	Assignment	10	A-1, A-2
5	Final	40	6 - 10
4	Teacher Assessment	10	Class attendance, Class Performance
Total		100	

Assignment policy:

- Assignment must be submitted in class as hardcopy on the due date mentioned in the homework
- All Assignment must be done individually.
- No late submissions will be allowed on any Assignment.

Introduction to Automata

Automata – What is it?
Study of abstract computing devices, or *"machines"*

The term "Automata" is derived from the Greek word "αὐτόματα" which means "selfacting". An automaton (Automata in plural) is an abstract self-propelled computing device which follows a predetermined sequence of operations automatically. An automaton with a finite number of states is called a **Finite Automaton** (FA) or **Finite State Machine** (FSM).

Alphabet
An alphabet is any finite, non-empty set of symbols. We use the symbol Σ (sigma) to denote an alphabet. Examples:

- Binary: Σ = {0,1}
- All lower case letters: Σ = {a, b, c, ..z}
- Alphanumeric: Σ = {a-z, A-Z, 0-9}
- DNA molecule letters: Σ = {a,c,g,t}

Where 'a', 'b', 'c', and 'd' are *symbols*.

String
A *string* is a finite sequence of symbols taken from Σ.
Example: 'cabcad' is a valid string on the alphabet set Σ = {a, b, c, d}

Length of a String
It is the number of symbols present in a string. (Denoted by **|S|**).
Examples:

- If S='cabcad', |S|= 6
- If |S|= 0, it is called an empty string (Denoted by λ or **ε**)

Kleene Star
The Kleene star, **Σ***, is a unary operator on a set of symbols or strings, **Σ**, that gives the infinite set of all possible strings of all possible lengths over **Σ** including **λ**.

Representation: $\Sigma^* = \Sigma_0 \cup \Sigma_1 \cup \Sigma_2 \ldots$. Where Σ_p is the set of all possible strings of length p.
Example: If Σ = {a, b}, Σ*= {λ, a, b, aa, ab, ba, bb, ……}

Kleene Closure / Plus
The set Σ^+ is the infinite set of all possible strings of all possible lengths over Σ excluding λ.
Representation: $\Sigma^+ = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \ldots\ldots$

$$\Sigma+ = \Sigma^* - \{ \lambda \}$$

Example: If Σ = {a, b } , Σ+ ={ a, b, aa, ab, ba, bb,……..}

Language
A language is a subset of Σ* for some alphabet Σ. It can be finite or infinite.
Example: If the language takes all possible strings of length 2 over Σ = {a, b},
then L = { ab, bb, ba, bb}

Finite Automaton can be classified into two types −

- Deterministic Finite Automaton (DFA)
- Non-deterministic Finite Automaton (NDFA / NFA)

Deterministic Finite Automaton (DFA)

In DFA, for each input symbol, one can determine the state to which the machine will move. Hence, it is called Deterministic Automaton. As it has a finite number of states, the machine is called Deterministic Finite Machine or Deterministic Finite Automaton.

Formal Definition of a DFA

A DFA can be represented by a 5-tuple $(Q, \sum, \delta, q_0, F)$ where −

- **Q** is a finite set of states.
- **∑** is a finite set of symbols called the alphabet.
- **δ** is the transition function where $\delta: Q \times \sum \rightarrow Q$
- **q_0** is the initial state from where any input is processed ($q_0 \in Q$).
- **F** is a set of final state/states of Q ($F \subseteq Q$).

Graphical Representation of a DFA

A DFA is represented by digraphs called **state diagram**.

- The vertices represent the states.
- The arcs labeled with an input alphabet show the transitions.
- The initial state is denoted by an empty single incoming arc.
- The final state is indicated by double circles.

Example

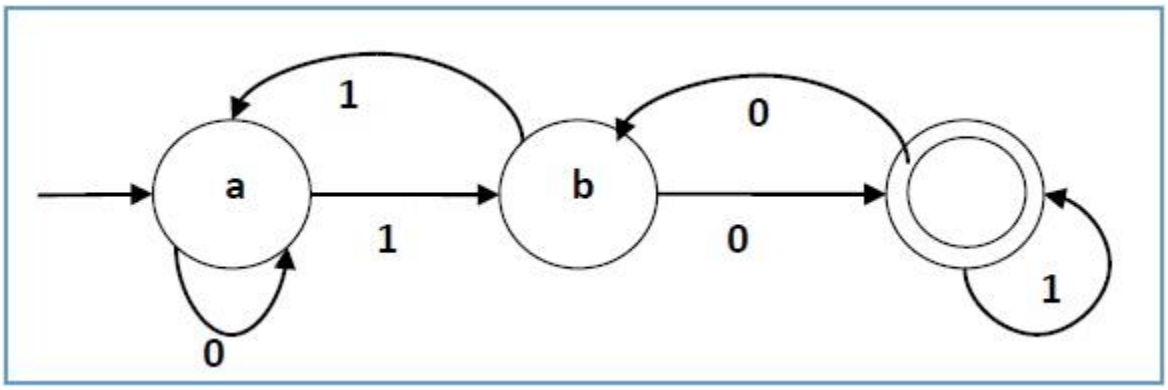
Let a deterministic finite automaton be →

- Q = {a, b, c},
- ∑ = {0, 1},
- q_0 = {a},
- F = {c}, and

Transition function δ as shown by the following table −

Present State	Next State for Input 0	Next State for Input 1
a	a	b
b	c	a
c	b	c

Its graphical representation would be as follows −



Finite Automata

Some Applications

- Software for designing and checking the behavior of digital circuits
- Lexical analyzer of a typical compiler
- Software for scanning large bodies of text (e.g., web pages) for pattern finding
- Software for verifying systems of all types that have a finite number of states (e.g., stock market transaction, communication/network protocol)

Example

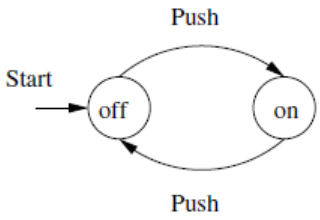


Fig: A finite automation modeling an on/off switch

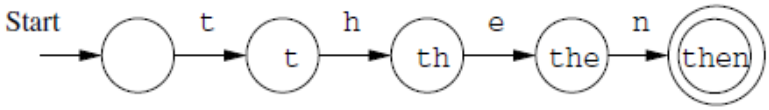


Fig: A finite automation modeling recognition of "then"

Structural Representations

There are two important notations that are not automaton like, but play an important role in the study of automata and their applications.

1. **Grammars** are useful models when designing software that processes data with a recursive structure. The best known example is a "parser" the component of a compiler that deals with the recursively nested features of the typical programming language, such as expressions, arithmetic, conditional, and so on.

2. **Regular Expressions** also denote the structure of data especially text strings.
The UNIX style regular expression "[A-Z] [a-z]*[ ] [A-Z] [A-Z]"represents capitalized words followed by a space and two capital letters. This expression represents patterns in text that could be a city and state, e.g., Ithaca NY.

It misses multiword city names such as **Palo Alto CA** which could be captured by the more complex expression

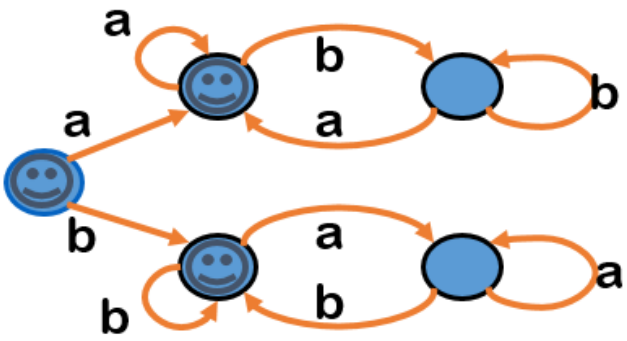
"[A-Z] [a-z]*([ ][A-Z][a-z]*)*[ ][A-Z][A-Z]"

Automata and Complexity

Automata are essential for the study of the limits of computation. There are two important issues.

- What can a computer do at all? This study is called "decidability", and the problems that can be solved by computer are called "decidable".
- What can a computer do efficiently? This study is called "intractability", and the problems that can be solved by a computer using no more time than some slowly growing function of the size of the input are called "tractable".

Meet "ABA" The Automaton!



Input String	Result
aba	Accept
aabb	Reject
aabba	Accept
ε	Accept

How Machine M operates

- M "reads" one letter at a time from the input string (going from left to right)
- M starts in state q_0.
- If M is in state q_i reads the letter a then

  If $\delta(q_i, a)$ is undefined then CRASH. Otherwise M moves to state $\delta(q_i, a)$

Finite Automation

Finite set of states		$Q = \{q_o, q_1, q_2, \ldots, q_k\}$
A start state		q_o
A set of accepting states		$F = \{q_{io}, q_{i1}, q_{i2}, \ldots, q_{ir}\}$
A finite alphabet	*a, b, #, x, 1*	∑
State transition instructions		&: Q x ∑ -> Q $d(q_i, a) = q_j$

Let *M* = (*Q, ∑, F,* &) be a finite automaton.

M accepts the string *x* if when *M* reads *x* it ends in an accepting state.

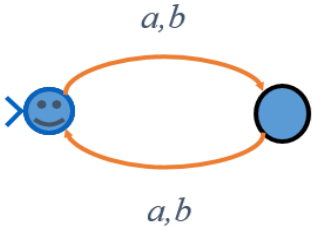
M rejects the string *x* if when *M* reads *x* it ends in a non-accepting state.

M crashes on *x* if *M* crashes while reading *x*.

What is the language accepted by this machine?

L = {*a,b*}* = all finite strings of *a*'s and *b*'s

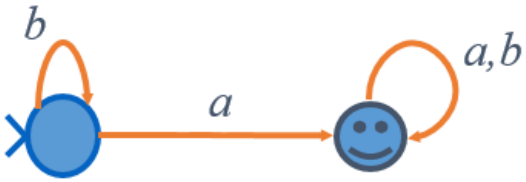
What is the language accepted by this machine?



L = all even length strings of a's and b's

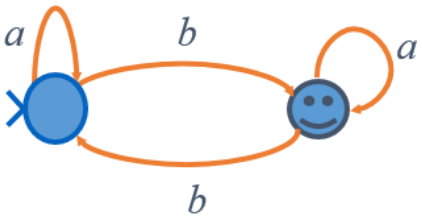
What machine accepts this language?

L = all strings in {*a,b*}* that contain at least one *a*

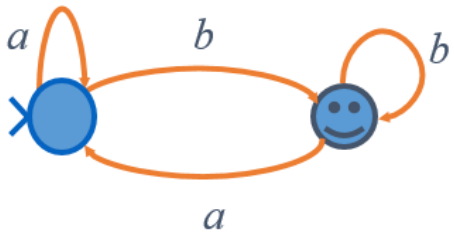


What machine accepts this language?

L = strings with an odd number of b's and any number of a's

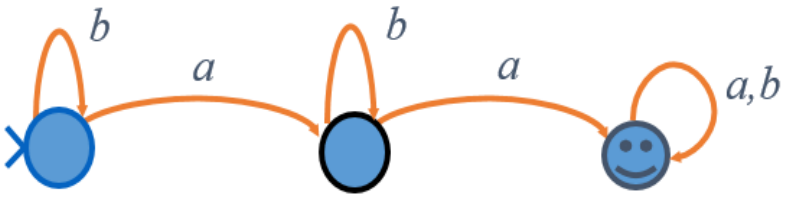


What is the language accepted by this machine?



L = any string ending with a *b*

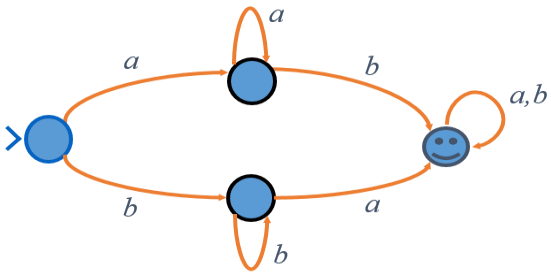
What is the language accepted by this machine?



L = any string with at least two a's

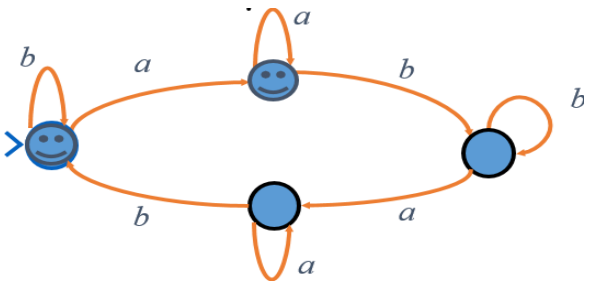
What machine accepts this language?

L = any string with an *a* and a *b*

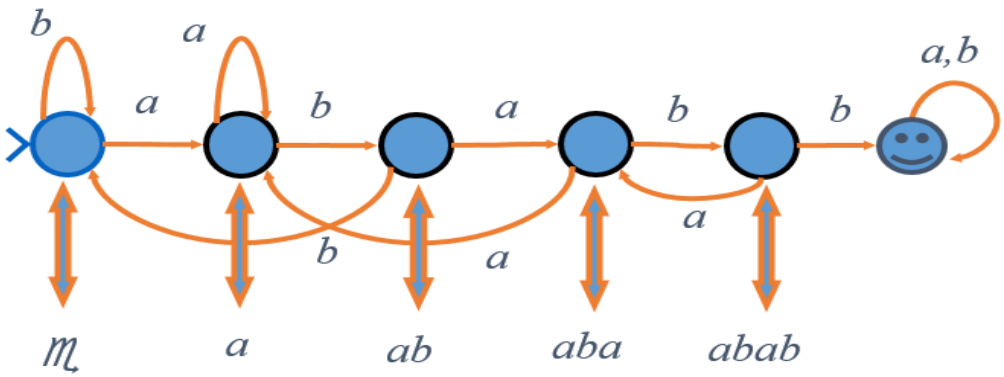


What machine accepts this language?

L = strings with an even number of ab pairs



L = all strings containing *ababb* as a consecutive substring



Formal Proofs

1. Inductive Proof
2. Deductive Proof

(Self-Study……see reference book chapter 0)

Quantifiers

- *For all"* or "*For every"*
  - Universal proofs
  - Notation = ∀
- *"There exists"*
  - Used in existential proofs
  - Notation = ∃
- Implication is denoted by =>

  E.g., "IF A THEN B" can also be written as "A=>B"

Homework

Consider the following two languages on the alphabet ∑ = {a, b}

$$L_1 = \{a^n : \ n \geq 1\}$$
$$L_2 = \{b^n : \ n \geq 1\}$$

Describe the languages below, using either the set notatin or precise definitions in English

$$L_3 = \overline{L_1^*}$$
$$L_4 = \overline{L_1}$$
$$L_5 = L_1 \cup L_2$$
$$L_6 = L_1 L_2$$
$$L_7 = (L_1{}^2)(L_2{}^2)(L_1{}^2)$$
$$L_8 = (L_1 \cup L_2)^*$$
$$L_9 = (L_1 L_2)^*$$