Theory of Computing

CSE-203

Mealy & Moore machine, Chomsky Normal Form, Ambiguity

Finite automata may have outputs corresponding to each transition. There are two types of finite state machines that generate output −

- Mealy Machine
- Moore machine

Mealy Machine

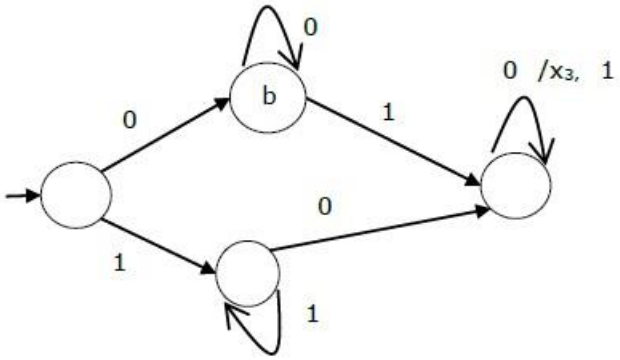
A Mealy Machine is an FSM whose output depends on the present state as well as the present input. It can be described by a 6 tuple (Q, ∑, O, δ, X, q_0) where –

- **Q** is a finite set of states.
- **∑** is a finite set of symbols called the input alphabet.
- **O** is a finite set of symbols called the output alphabet.
- **δ** is the input transition function where δ: Q × ∑ → Q
- **X** is the output transition function where X: Q × ∑ → O
- **q_0** is the initial state from where any input is processed (q_0 ∈ Q).

The state table of a Mealy Machine is shown below −

Present state	Next state			
	input = 0		input = 1	
	State	Output	State	Output
→ a	b	x_1	c	x_1
b	b	x_2	d	x_3
c	d	x_3	c	x_1
d	d	x_3	d	x_2

The state diagram of the above Mealy Machine is −



Moore Machine

Moore machine is an FSM whose outputs depend on only the present state.

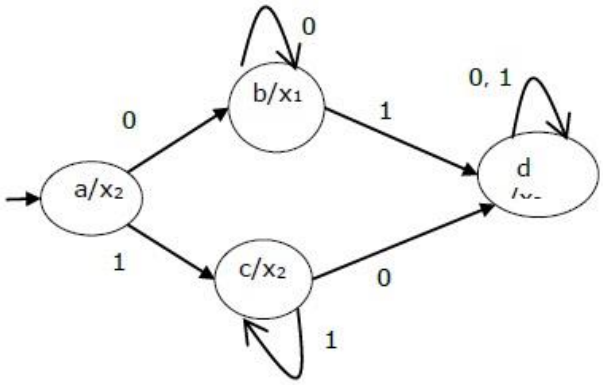
A Moore machine can be described by a 6 tuple (Q, ∑, O, δ, X, q_0) where −

- **Q** is a finite set of states.
- **∑** is a finite set of symbols called the input alphabet.
- **O** is a finite set of symbols called the output alphabet.
- **δ** is the input transition function where δ: Q × ∑ → Q
- **X** is the output transition function where X: Q → O
- **q_0** is the initial state from where any input is processed (q_0 ∈ Q).

The state table of a Moore Machine is shown below –

Present state	Next State		Output
	Input = 0	Input = 1	
→ a	b	c	x_2
b	b	d	x_1
c	c	d	x_2
d	d	d	x_3

The state diagram of the above Moore Machine is –



Mealy Machine vs. Moore Machine

The following table highlights the points that differentiate a Mealy Machine from a Moore Machine.

Mealy Machine	Moore Machine
Output depends both upon present state and present input.	Output depends only upon the present state.
Generally, it has fewer states than Moore Machine.	Generally, it has more states than Mealy Machine.
Output changes at the clock edges.	Input change can cause change in output change as soon as logic is done.
Mealy machines react faster to inputs	In Moore machines, more logic is needed to decode the outputs since it has more circuit delays.

Moore Machine to Mealy Machine

Algorithm 1

Input − Moore Machine

Output − Mealy Machine

Step 1 − Take a blank Mealy Machine transition table format.

Step 2 − Copy all the Moore Machine transition states into this table format.

Step 3 − Check the present states and their corresponding outputs in the Moore Machine state table; if for a state Q_i output is m, copy it into the output columns of the Mealy Machine state table wherever Q_i appears in the next state.

Example 1:

Let us consider the following Moore machine −

Present State	Next State		Output
	a = 0	a = 1	
→ a	d	b	1
b	a	d	0
c	c	c	0
d	b	a	1

Now we apply Algorithm 4 to convert it to Mealy Machine.

Step 1 & 2 −

Present State	Next State			
	a = 0		a = 1	
	State	Output	State	Output
→ a	d		b	
b	a		d	
c	c		c	
d	b		a	

Step 3 −

Present State	Next State			
	a = 0		a = 1	
	State	Output	State	Output
=> a	d	1	b	0
b	a	1	d	1
c	c	0	c	0
d	b	0	a	1

Mealy Machine to Moore Machine

Algorithm 2

Input − Mealy Machine

Output − Moore Machine

Step 1 − Calculate the number of different outputs for each state (Q_i) that are available in the state table of the Mealy machine.

Step 2 − If all the outputs of Qi are same, copy state Q_i. If it has n distinct outputs, break Q_i into n states as Q_{in} where **n** = 0, 1, 2.....

Step 3 − If the output of the initial state is 1, insert a new initial state at the beginning which gives 0 output.

Example 2: Let us consider the following Mealy Machine −

Present State	Next State			
	a = 0		a = 1	
	Next State	Output	Next State	Output
→ a	d	0	b	1
b	a	1	d	0
c	c	1	c	0
d	b	0	a	1

Here, states ‘a’ and ‘d’ give only 1 and 0 outputs respectively, so we retain states ‘a’ and ‘d’. But states ‘b’ and ‘c’ produce different outputs (1 and 0). So, we divide **b** into $\mathbf{b_0}$, $\mathbf{b_1}$ and **c** into $\mathbf{c_0}$, $\mathbf{c_1}$.

Present State	Next State		Output
	a = 0	a = 1	
→ a	d	b_1	1
b_0	a	d	0
b_1	a	d	1
c_0	c_1	C_0	0
c_1	c_1	C_0	1
d	b_0	a	0

Chomsky Normal Form

A CFG is in Chomsky Normal Form if the Productions are in the following forms −

- A → a
- A → BC
- S → ε

where A, B, and C are non-terminals and **a** is terminal.

Algorithm to Convert into Chomsky Normal Form −

Step 1 − If the start symbol **S** occurs on some right side, create a new start symbol **S’** and a new production **S’→ S**.

Step 2 − Remove Null productions. (Using the Null production removal algorithm discussed earlier)

Step 3 − Remove unit productions. (Using the Unit production removal algorithm discussed earlier)

Step 4 − Replace each production $\mathbf{A \rightarrow B_1 \ldots B_n}$ where $\mathbf{n > 2}$ with $\mathbf{A \rightarrow B_1 C}$ where $\mathbf{C \rightarrow B_2 \ldots B_n}$. Repeat this step for all productions having two or more symbols in the right side.

Step 5 − If the right side of any production is in the form **A → aB** where a is a terminal and **A, B** are non-terminal, then the production is replaced by **A → XB** and **X → a**. Repeat this step for every production which is in the form **A → aB**.

Problem 1:

Convert the following CFG into CNF

S → ASA | aB, A → B | S, B → b | ε

Solution

(1) Since **S** appears in R.H.S, we add a new state $\mathbf{S_0}$ and $\mathbf{S_0 \rightarrow S}$ is added to the production set and it becomes −

$S_0 \rightarrow S$, $S \rightarrow ASA \mid aB$, $A \rightarrow B \mid S$, $B \rightarrow b \mid \in$

(2) Now we will remove the null productions −

$B \rightarrow \in$ and $A \rightarrow \in$

After removing $B \rightarrow \varepsilon$, the production set becomes −

$S_0 \rightarrow S$, $S \rightarrow ASA \mid aB \mid a$, $A \rightarrow B \mid S \mid \in$, $B \rightarrow b$

After removing $A \rightarrow \in$, the production set becomes −

$S_0 \rightarrow S$, $S \rightarrow ASA \mid aB \mid a \mid AS \mid SA \mid S$, $A \rightarrow B \mid S$, $B \rightarrow b$

(3) Now we will remove the unit productions.

After removing $S \rightarrow S$, the production set becomes −

$S_0 \rightarrow S$, $S \rightarrow ASA \mid aB \mid a \mid AS \mid SA$, $A \rightarrow B \mid S$, $B \rightarrow b$

After removing $S_0 \rightarrow S$, the production set becomes −

$S_0 \rightarrow ASA \mid aB \mid a \mid AS \mid SA$, $S \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$A \rightarrow B \mid S$, $B \rightarrow b$

After removing $A \rightarrow B$, the production set becomes −

$S_0 \rightarrow ASA \mid aB \mid a \mid AS \mid SA$, $S \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$A \rightarrow S \mid b$

$B \rightarrow b$

After removing $A \rightarrow S$, the production set becomes −

$S_0 \rightarrow ASA \mid aB \mid a \mid AS \mid SA$, $S \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$A \rightarrow b \mid ASA \mid aB \mid a \mid AS \mid SA$, $B \rightarrow b$

(4) Now we will find out more than two variables in the R.H.S

Here, $S_0 \rightarrow ASA$, $S \rightarrow ASA$, $A \rightarrow ASA$ violates two Non-terminals in R.H.S.

Hence we will apply step 4 and step 5 to get the following final production set which is in CNF −

$S_0 \rightarrow AX \mid aB \mid a \mid AS \mid SA$

$S \rightarrow AX \mid aB \mid a \mid AS \mid SA$

$A \rightarrow b \mid AX \mid aB \mid a \mid AS \mid SA$

$B \rightarrow b$

$X \rightarrow SA$

(5) We have to change the productions $S_0 \rightarrow aB$, $S \rightarrow aB$, $A \rightarrow aB$

And the final production set becomes −

$S_0 \rightarrow$ AX | YB | a | AS | SA
S→ AX | YB | a | AS | SA
A → b A → b |AX | YB | a | AS | SA
B → b
X → SA
Y → a

Ambiguity

Problem 2:
Explain why the grammar below is ambiguous.
S → 0A | 1B
A → 0AA | 1S | 1
B →1BB | 0S | 0

Solution
The grammar is ambiguous because we can find strings which have multiple derivations:
S ⇒ 0A⇒00AA⇒001S1⇒0011B1⇒001101
S ⇒ 0A⇒00AA⇒0011S⇒00110A⇒001101

Problem 3:
Given the following ambiguous context free grammar
S → Ab | aaB
A → a | Aa
B →b

Solution
Find the string s generated by the grammar that has two leftmost derivations. Show the derivations. The string s = aab has the following two leftmost derivations
S ⇒ aaB ⇒ aab
S ⇒ AB ⇒ AaB ⇒ aaB ⇒ aab

Homework
Construct context free grammars to accept the following languages. Σ= {0, 1}
a. {w | w starts and ends with the same symbol}
b. {w | w is odd}
c. {w | w is odd and its middle symbol is 0}
d. $\{0^n1^n \mid n > 0\} \cup \{0^n1^{2n} \mid n>0\}$
e. $\{0^i1^j2^k \mid i \neq j \text{ or } j \neq k\}$
f. Binary strings with twice as many 1s as 0s.