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Why are supernovae in our Galaxy so frequent?

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ABSTRACT
We show that if the observed surface density function of supernovae in external spiral galaxies is used to calibrate the historical data on the supernova rate in the solar neighbourhood, then the calculated total supernova rate for the Galaxy is abnormally high. This can be explained if Galactic supernovae are not uniformly distributed over the Galactic disc, but tend to be localized near spiral arms and star-forming regions. Such a distribution would be consistent with evidence for an association of Type Ib/c and Type II supernovae with H II regions in late-type galaxies. It seems that we occupy a privileged position in the Milky Way – one which gives us the impression of a considerably higher supernova rate than is valid for the Galaxy as a whole.

Key words: supernovae: general –solar neighbourhood – galaxies: spiral – galaxies: stellar content.

1 INTRODUCTION

The methods used to estimate the supernova (SN) rate in our Milky Way Galaxy may conveniently be classified into three groups, according to whether they are based on extragalactic data on supernovae (SNe), Galactic data related to SNe or historical data on SNe in the solar neighbourhood. Without attempting a comprehensive review, we set the scene by briefly summarizing relatively recent rate estimates resulting from the various methods, under the following groupings.

1.1 Group A: from extragalactic SN discoveries

The SN rates determined by SN discoveries in external galaxies are generally expressed in 'supernova units' (SNu), meaning SNe century^{-1} per 10^{10} solar blue luminosities ($L_{B\odot}$). Because the calculated luminosity of a galaxy varies as the square of its distance, rates so expressed scale as the square of H_0, the present value of the Hubble constant.

To estimate the SN rate in the Milky Way Galaxy from external galaxy rates requires knowledge of H_0 and of the spiral classification and luminosity of our Galaxy. Primary classification criteria – the relative size of the Galactic bulge and the patchiness of the spiral arms – indicate the Milky Way to be of spiral subtype Sbc, but secondary criteria indicate Sb (Tammann, Löffler & Schröder 1994, p. 490). The Galaxy's blue luminosity has been estimated by van den Bergh (1988) to be 2.3 ± 0.6 in units of 10^{10} $L_{B\odot}$. We use 75 km s^{-1} per Mpc for H_0 or, equivalently, $h = 0.75$ for H_0 in units of 100 km s^{-1} Mpc^{-1}. We shall now summarize SN rate estimates from four studies that analysed data from systematic extragalactic SN searches, and another which utilized a distance-limited set of galaxies, and consider their implied Milky Way rates for $h = 0.75$ and 2.3 ± 0.6 units of blue luminosity.

(1) Cappellaro et al. (1993) analysed data from the two independent long-term photographic searches carried out by Asiago Astrophysical Observatory and the Crimean Station of the Sternberg Astronomical Institute, Moscow. They retained data for 2461 searched galaxies for which redshift, morphological type, axial ratio and blue magnitude were available, from which they selected 65 SNe. To reduce statistical error in estimating an implied Milky Way rate, they used the SN rate that they found for spirals of subtypes Sab, Sb and Sbc, namely $(0.85 \pm 0.2)(h/0.75)^2$ SNu. For $h = 0.75$ and a Galactic blue luminosity of 2.3 ± 0.6 units, this implies a Milky Way rate of 2.0 ± 1.1 SNe century^{-1}.

(2) Evans' (1994, 1997) visual search of 855 Shapley–Ames galaxies provided 24 SNe (largely discovered by him) from 74 678 observations over the period 1980–88. His data were analysed by van den Bergh & McClure (1994; see also van den Bergh 1993), who found a total SN rate of $(1.3 \pm 0.3)(h/0.75)^2$ SNu for galaxies of types Sab–Sd. Of these events, 80–90 per cent were core-collapse SNe (Types Ib/c and II) and 10–20 per cent were of Type Ia. On using $h = 0.75$ and a Galactic blue luminosity of 2.3 ± 0.6 units, this result yields an implied rate for the Milky Way of 3.0 ± 1.5 SNe century^{-1} – perhaps too high because of the inclusion of the later type (Sc–Sd) galaxies, which studies have shown to have relatively high SN rates.

(3) The Berkeley automated CCD-based search operated from 1986 to 1991, detecting 20 SNe (17 discoveries and 3 others). It yielded a SN rate of $(3.1 \pm 0.7)h^2$ SNu for galaxies of types Sbc–Sd (Muller et al. 1992). On taking the Milky Way's blue luminosity to be 2.3 ± 0.6 units and using $h = 0.75$, this result implies a rate of 4.0 ± 2.0 SNe century^{-1} for our Galaxy – probably too high because of the inclusion of the later type (Sc–Sd) galaxies and absence of Sb or earlier ones.

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(4) Tammann and coworkers (Tammann 1994; Tammann et al. 1994) have tackled the problem of the small-number statistics of the systematic searches by using a wider sample of SN discoveries to establish the relative rates of SNe of different types in several bins of galaxy type, and then obtaining absolute rates by using results of systematic searches for calibration. They selected 330 galaxies with reliably known parameters from A Revised Shapley–Ames Catalogue of Bright Galaxies, out to about the distance of the Virgo cluster. These produced 96 known SNe, from the original extragalactic event, SN 1885 in M31, to 1990 March 1. After calibration using the results of the Asiago and Evans searches and taking H_0 to be $50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, they found absolute rates for all SN types of 0.53 SNu for Sab–Sb spirals and 1.29 SNu for Sbc–Sd spirals (among other results – see Tammann 1994, table 5, or Tammann et al. 1994, table 2). We take our Galaxy to be an Sb/Sbc spiral and use the simple mean of these two rates, namely $3.64h^2$ SNu. On taking the Milky Way's luminosity to be 2.3 units and using $h = 0.75$, this implies a (seemingly rather high) rate of ≈ 4.7 SNe century^{-1} for the Galaxy (which would be replaced by about 2.1 SNe century^{-1} if we were to adopt Tammann's preferred value of 0.5 for h).

(5) Cappellaro et al. (1997) have improved SN statistics by joining the logs of five searches to obtain a sample of 110 SNe. Four photographic searches – those of Asiago, Sternberg–Crimea, Calán–Tololo and the Observatoire de la Côte d'Azur (OCA) – were used, together with Evans' visual search. The authors retained 7773 galaxies which had been observed at least once in the five surveys and for which the recession speed, morphological type, luminosity and (for spirals) axial ratio were available. This left 110 from the total number of 211 SNe discovered in the five searches in the periods considered. They found SN rates of $0.78h^2$ SNu for galaxies of types Sab–Sb and $1.19h^2$ SNu for galaxy types Sbc–Sc (among other results – see their table 3). Again regarding our Galaxy as an Sb/Sbc spiral, we use the simple mean of those two rates, namely $(0.99 \pm 0.43)h^2$ SNu, with an error estimated from their table 4 in which broader bins of galaxy type are used. On taking the Milky Way's luminosity to be 2.3 ± 0.6 units and using $h = 0.75$, this implies a rate of $\approx 1.3 \pm 0.9$ SNe century^{-1} for the Galaxy.

The above rate estimates for the Milky Way Galaxy, based on SN discoveries in external galaxies, and scaled to $h = 0.75$ and a Milky Way blue luminosity of 2.3 ± 0.6 units, are listed as the first five entries in Table 1. [Another systematic CCD SN search, at Perth Observatory in Western Australia, has yielded 10 confirmed SN discoveries to date and some preliminary statistics (Williams 1997).]

We note that Cappellaro et al. (1993) included a substantial (distance-dependent) correction for the loss of SNe caused by the overexposure of the central regions of galaxies because of the low dynamic range of photographic plates – a serious limitation on photographic searches, as compared with visual and CCD ones. As had been found for other photographic surveys, those authors also found the 'inclination effect' to be very important – a loss of SNe in highly inclined spirals, which seems to set in abruptly for inclinations to the plane of the sky $i > 30°$. They found the effect to be more severe in late-type (Sbc–Sd) spirals than in early ones, and adopted a correction factor of about 4 for the Sbc–Sd spirals with $i > 30°$ (more precisely, 2 for the relatively scarce Type Ia events and 4 for the others) and half this value for earlier spirals with $i > 30°$.

For the nearby galaxies forming the set of Tammann and coworkers, the loss of SNe in central regions appeared to be very small and was neglected. However, they found the inclination effect to be very large for $i > 30°$, for which they applied correction factors of 4 for Sbc–Sd and 2.5 for Sab–Sb spirals (and 1.2 for S0/a–Sa ones).

Cappellaro et al. (1997) corrected for the loss of SNe in the central regions of galaxies in the four photographic searches (invoking completeness factors ranging from 0.85 for the Crimean observations of nearby spirals to 0.55 for the OCA and Calán–Tololo observations of more distant ellipticals), but not in Evans' visual search of nearby galaxies. Their corrections for the inclination effect are substantial – increasing the total SN rates by factors of about 1.5 for Evans' search and about 2 for the four photographic searches – but smaller than those made by Cappellaro et al. (1993). Both nuclear and inclination corrections were estimated by taking the average of factors obtained from two different methods.

The inclination effect may be caused by absorption in the parent galaxy or by the greater surface brightness of inclined spirals. It may be a problem only for photographic searches, since it was not found in Evans' visual search (van den Bergh 1993) or in the

Table 1. Supernova rate estimates for the Milky Way Galaxy. The group A rates have been standardized to $h = 0.75$ and a Milky Way blue luminosity of 2.3 ± 0.6 in units of $10^{10}\,\mathrm{L_{B\odot}}$.

Method	Rate (SNe century^{-1})	Reference
A: From extragalactic SN discoveries ($h = 0.75$)		
(1) Asiago & Sternberg (photographic)	2.0 ± 1.1	Cappellaro et al. (1993)
(2) Evans (visual)	3.0 ± 1.5	van den Bergh (1993)
(3) Berkeley (CCD)	4.0 ± 2.0	Muller et al. (1992)
(4) Distance-limited galaxies	≈ 4.7	Tammann (1994)
(5) Five searches (4 photographic, 1 visual)	1.3 ± 0.9	Cappellaro et al. (1997)
B: From SN-related Galactic data		
(1) SNR observations	–	–
(2) massive star formation	$1.2^{+1.7}_{-0.7}$	Ratnatunga & van den Bergh (1989)
(3) chemical evolution	1.9, 2.5	Matteucci & François (1989)
C: From the historical record		
(1) sector – constant distribution	5.8 ± 2.4	Tammann (1982)
(2) horizon – constant distribution	3^{+2}_{-1}	Strom (1990)
(3) sector + SNRs – constant distribution	2.2 ± 1.3	van den Bergh (1991)
(4) horizon – exponential distribution	11–13	Blair (1987)
(5) horizon + SNRs – exponential distribution	5.7 ± 1.7	Strom (1994)
(6) horizon – exponential distribution ($h = 0.75$)	8.4 ± 2.8	this paper

Berkeley CCD search. Its reality has been questioned – it might be an artefact of either a tendency for face-on spirals to be photographed or the fact that 11 SNe have been discovered in two nearby face-on spirals (van den Bergh 1993). If surface brightness is a factor, then the effect should depend on the focal ratio of the telescope used in the survey (Tammann 1994, p. 10).

1.2 Group B: from SN-related Galactic data

The SN rate in the Galaxy has been estimated from modern Galactic data in several ways, including observations of Galactic supernova remnants (SNRs), theoretically from the mass spectrum of star formation, and theoretically from modelling the chemical evolution of the Galaxy.

(1) The birthrate of Galactic radio SNRs is poorly known, because of the problems of completeness and determining distances and ages, a particular difficulty being the dependence of a remnant's lifetime on the density of the ambient interstellar medium and so on its distance from the Galactic plane (Tammann et al. 1994). The last point is shown by the small scaleheight (60 pc) of SNRs from the Galactic plane, which is less than that of their massive O and B star progenitors (90 pc) and much less than the scaleheight (210 pc) of the historical SNe (Tammann 1994, pp. 25–6). It seems that SN rates cannot yet be deduced directly from SNRs, although SNRs can be used in assessing the completeness of the historical observations (see below).

(2) Ratnatunga & van den Bergh (1989) used the mass spectrum of star formation to calculate the rate of massive star collapse (leading to Types Ib/c and II SNe) in the Milky Way. Their work gave a rate of $1.0^{+1.5}_{-0.6}$ century^{-1} for a minimum stellar mass of 8 $M_{\odot}$ for core collapse (or $2.2^{+2.7}_{-1.6}$ century^{-1} for a minimum of 5 $M_{\odot}$). Taking core-collapse SNe, occurring at $1.0^{+1.5}_{-0.6}$ century^{-1}, to constitute 85 per cent of all SN events, the other 15 per cent being of Type Ia, implies a total SN rate for the Galaxy of $1.2^{+1.7}_{-0.7}$ century^{-1}.

(3) Matteucci & François (1989) developed models of the chemical evolution of the Galaxy, which included calculation of the time variation of the abundances of eight metals and their present abundance variation with radial distance from the Galactic Centre. This work yielded SN rates of 1.9 and 2.5 SNe century^{-1} at the present epoch in different models (see their table 2, p. 893). Although these total rates are consistent with ones yielded by other methods, Matteucci & François found only 58 and 64 per cent (in the respective models) of the SNe to be core-collapse events, the remainder having been modelled as carbon-deflagration events of white dwarfs in binary systems – extragalactic surveys suggest a larger proportion of core-collapse events.

These two rate estimates for the Milky Way Galaxy, based on SN-related Galactic data with theoretical modelling, are listed as the second group of entries in Table 1. The only extragalactic information used here is on the relative rate of core-collapse and non-core-collapse events, which is independent of distance and hence of H_0. Consequently, the Group B SN rate estimates do not depend on H_0.

1.3 Group C: from the historical record

The most direct way to calculate the SN rate for the Milky Way Galaxy is to use historical records of SN sightings. Two basic methods have been used to calculate the rate from historical data. One method is to assume that the historical sightings – after corrections for gaps in the historical record and for extinction toward the centre of the Milky Way have been applied – are reasonably complete out to a horizon at 4 or 5 kpc from the Solar system; comparison of the surface area within this horizon with that of the Galactic disc enables the rate to be calculated. The other method is to assume that the historical records are essentially complete for a sector of the Galaxy, containing the Solar system more or less symmetrically, in which the historical sightings occurred; the ratio of the apex angle of the sector, of about 50° at the Galactic centre, to 360° (i.e. about 1/7) is used to find the rate.

In refinements of these approaches, modern data on SNRs are invoked either to assess the completeness of the historical sightings within the horizon or to extrapolate from the sector just mentioned.

(1) Using the sector method, Tammann (1982) calculated a rate of 5.8 ± 2.4 century^{-1} (van den Bergh & Tammann 1991).

(2) Using the horizon method, Strom (1990) reported a rate of 3^{+2}_{-1} SNe century^{-1}.

(3) By using the fraction of bright Galactic radio SNRs which lie in the Galactic anticentre direction (Galactic longitudes in the range 100° to 260°) – in order to extrapolate from the number of historical SNe observed in that range during the last two millennia – van den Bergh (1991) obtained a Galactic SN rate of 2.2 ± 1.3 century^{-1}.

(4) Blair (1987) made use of data from McCarthy (1974) and Tammann (1982) to calibrate the Galactic SN rate with an exponential SN surface density function, declining with increasing radial distance from the Galactic centre following the distribution of pulsars, with a scalelength of 7.5 kpc (Lyne et al. 1985). He calculated a rate of 11–13 SNe century^{-1}, which is considerably larger than the estimates obtained by other authors (see also Blair & Williams 1992).

(5) Strom (1994) invoked SNRs that show optical nebulosity and have reasonably well-determined distances. Although radio emission from SNRs can often be observed at great distances, extinction and intrinsic faintness mean that their optical emission cannot be seen much beyond 5 kpc. By comparing this SNR sample with Chinese, Japanese and Korean SN sightings over the 2000-year period 200 BC to 1800 AD, Strom estimated the Oriental records to be 70 per cent complete, with the observers recording 8 out of an estimated 11.4 ± 3 SNe in that period. Following Ratnatunga & van den Bergh (1989), he took an exponential radial distribution of stars in the Galactic disc with a scalelength of 4 kpc, which meant that his horizon at about 5 kpc encompassed about 10 per cent of disc stars. In this way, Strom deduced a Galactic rate of 5.7 ± 1.7 SNe century^{-1}.

These five rate estimates for the Milky Way Galaxy based on the historical record, supported in some cases by SNR data, are listed in the third group of entries in Table 1. As they are based on Galactic data alone, these five values do not depend on H_0.

Many estimates of the SN rate from the historical SNe – exceptions being those of Blair (1987) and Strom (1994) – have been based on the assumption that SNe are distributed uniformly across the Galactic plane. This is in spite of the fact that numerous studies of SNe in external spiral galaxies have shown that the surface density as a function of radius is definitely not constant. Iye & Kodiara (1975) performed a thorough analysis of the radial distribution of SNe, and more up-to-date results using a much larger sample have been obtained by Bartunov et al. (1992).

In this paper we re-calibrate the Galactic SN rate following Blair's method. After including the substantial number of external SN sightings from 1987 to 1993, we first show that the radial distribution of SNe in spiral galaxies can be approximated very well with an exponential function. Assuming that the SN distribution is axisymmetric, we then use this function to calibrate the historical SN rate for the Milky Way. As extragalactic data is used to determine the scalelength of the radial distribution, our rate estimate (the sixth Group C entry in Table 1) depends on H_0.

2 METHOD AND ANALYSIS

2.1 The method

To test the exponential model of the distribution of SN surface density as a function of radial distance from galaxy centres, data on sightings of SNe in spiral galaxies were collected from IAU Circulars for the period 1977 to early 1993. The SNe are reported as being at certain angular distances from the centre of the host galaxy. The angular major and minor axis measurements of the host galaxy taken from various galaxy catalogues were used to correct for the inclination of the plane of the galaxy disc to the plane of the sky. We normalized the radial angular distance of each SN to the angular radius of its host, as corrected for inclination. Altogether, 218 sightings had enough information to be included in this analysis.

The resulting data were sorted into radius bins, and converted into surface density information by dividing the number of SNe in each bin by the surface area of that bin. This was plotted to give Fig. 1. Fitting an exponential curve to the data gives a correlation coefficient of 0.98, which convincingly shows that the SN surface density in spiral galaxies declines exponentially with radial distance in the disc from the centre of the galaxy.

We can therefore take the surface density of SNe in spiral galaxies to follow

$$\sigma(r) = \sigma_c \exp(-r/R_{scl}), \tag{1}$$

with r the radial distance in the plane of the galaxy from its centre, σ_c the surface density at the centre of the galaxy, and R_{scl} the exponential scalelength. This surface density function is a number per unit area for the time interval during which the data were collected. It is also dependent upon the number of galaxies in which the SNe were sighted. In reality, therefore, it is a surface density per unit time, multiplied by the number of galaxies in the sample. This dependence on time interval and number of galaxies will manifest in the constant σ_c, which drops out in the SN rate calculation. The constant R_{scl} will not change because of the time interval or the number of galaxies observed, apart from fluctuations caused by statistical error. We now use this exponential surface density of SNe in estimating the SN rate in our Galaxy from historical records.

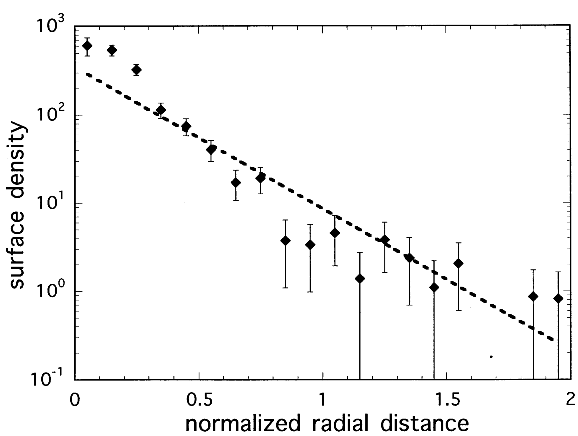

Figure 1. The surface density of SNe in external spiral galaxies as a function of normalized radial distance from the host galaxy centre. The curve fitted has the equation $\sigma(r) = 350 \exp(-r/0.27)$ and gives a correlation coefficient of 0.98.

Assuming the SN surface density distribution to be rotationally symmetric about the centre of the Milky Way Galaxy at $r = 0$, the number of SNe occurring within a given area A of the Galactic disc, in some time interval, is given by

$$n_A = \iint_A \sigma(r)\, r\, dr\, d\theta, \tag{2}$$

with r and θ the plane polar cooordinates in the disc. To calibrate the historical SN rate within the Milky Way, this integral has to be evaluated for the whole Galaxy, yielding n_{Gal}, and from the Sun out to the horizon of the historical SNe, yielding n_{loc}, the number of SNe visible locally during the same time interval.

For the 'whole Galaxy' integral, substituting the exponential form (1) into (2) and integrating out to the cut-off radius R_{cut} of the Galactic disc, beyond which the SN distribution is negligible, yields

$$n_{Gal}/\sigma_c = 2\pi R_{scl}^2[1 - (1 + R_{cut}/R_{scl}) \exp(-R_{cut}/R_{scl})]. \tag{3}$$

For $R_{cut}/R_{scl} > 5$, the approximation

$$n_{Gal}/\sigma_c \approx 2\pi R_{scl}^2 \tag{4}$$

holds, with an error of 4 per cent or less: R_{cut} becomes effectively infinite.

The 'local' integral may be evaluated numerically, integrating from the Sun out to the horizon radius R_{hor} for the historical SNe, or approximately by replacing the surface density function in the vicinity of the Solar system by the constant $\sigma_c \exp(-D_\odot/R_{scl})$, the value of $\sigma(r)$ at our distance $D_\odot$ from the Galactic Centre, yielding

$$n_{loc}/\sigma_c \approx \pi R_{hor}^2 \exp(-D_\odot/R_{scl}). \tag{5}$$

The approximate results (4) and (5) give the simple formula

$$n_{Gal}/n_{loc} \approx 2(R_{scl}/R_{hor})^2 \exp(+D_\odot/R_{scl}) \tag{6}$$

for the ratio of SN numbers Galaxy-wide and seen locally, in a given time interval.

In this way, the Galactic SN frequency may be calculated from

$$\nu = (n_{Gal}/n_{loc})N_{SN}/10 \text{ SNe century}^{-1}, \tag{7}$$

where N_{SN} is the number of local SNe (within the horizon) in a millennium.

2.2 The parameters

The rate obtained from equation (7) depends upon five parameters: the distance $D_\odot$ of the Sun from the centre of the Galaxy, two local parameters and two Galactic parameters. In a recent review, Reid (1993) found that $D_\odot = 8.0 \pm 0.5$ kpc, which we adopt. The local parameters are the horizon radius R_{hor} for the historical SNe and the number N_{SN} of local SNe per millennium obtained from historical records, supplemented by SNR data. The two Galactic parameters are the scalelength R_{scl} of the SN surface density distribution across the Galaxy and the cut-off radius R_{cut} of the Galactic disc, beyond which the SN distribution is negligible.

2.2.1 Local parameters

Estimated distances from the Solar system of the historical SNe seen in the last millennium are as follows (Strom 1990):

SN 1006	1.4 kpc,
SN 1054	2 kpc,
SN 1181	2.6 kpc,
SN 1572	2.5 kpc,
SN 1604	4.2 kpc,
Cas A	2.9 kpc,
MSH 11−54	3.7 kpc, from Tammann (1982).

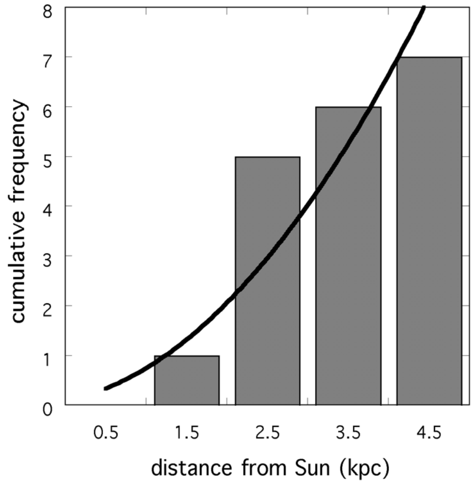

Figure 2. The cumulative frequency of historical SN sightings, as a function of distance from the Sun. The curve fitted has the equation $N(R) = \sigma(D_\odot)\pi R^2$, and gives a correlation coefficient of 0.9.

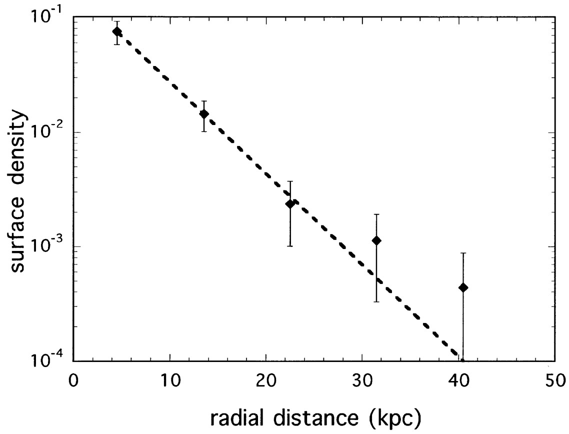

Figure 3. The surface density of SNe in Sb–Sbc external galaxies as a function of absolute radial distance from the host galaxy centre, for $H_0 = 75$ km s^{-1} Mpc^{-1}. The curve fitted has the equation $\sigma(r) = 0.17\exp(-r/R_{\rm scl})$ with $R_{\rm scl}$ = 5.4 kpc, and gives a correlation coefficient of 0.99.

Strom's list includes Cas A, which was not observed as a SN but has been dated to 1658 ± 3 (van den Bergh & Kamper 1983). We have added the southern remnant MSH 11–54, which has an age of under 1000 yr and is quite close (Tammann 1982) – it is important as a representative of the southern hemisphere, for which no records of Galactic SN sightings exist.

Strom (1990) argued that the number of historical sightings is very probably incomplete for distances greater than 3 kpc from the Sun, but we believe that the historical SNe for the last millennium are an adequate sample out to 4 kpc or so. Strom (1990) plotted the frequency of historical SNe versus radial distance from the Sun, and superimposed a parabola, in order to come to his conclusion about the sample. However, a parabolic function represents the *total* number of SNe within that radius: if we assume that the surface density function in the vicinity of the Solar system is approximately constant, equal to $\sigma(D_\odot)$, then the number of SNe within radius R of the Sun is $\sigma(D_\odot)\pi R^2$ in the given time interval. Hence the *cumulative* frequency of historical SNe must be compared with the parabola, as we do in Fig. 2. Fitting a parabola to the data in the above list for the last millennium gives a correlation coefficient of 0.9. Even though there are not many data points, this suggests that the five historical SNe for that interval, together with the two supplementary remnants, form a reasonably complete sample, out to 4 or 5 kpc.

We therefore take the horizon radius $R_{\rm hor}$ to be 4.5 ± 0.5 kpc, and the number of local SNe in the last millennium to be 7, as in the above list. We take the error in this number to be the statistical error, $\sqrt{7}$, giving $N_{\rm SN} = 7.0 \pm 2.6$ in a millennium. As observers recorded five SNe, our estimate corresponds to 70 per cent completeness of the historical records for the last millennium – the same figure as Strom (1994) obtained from his detailed consideration of optical SNRs in conjunction with sightings over the two millennia 200 BC to 1800 AD.

2.2.2 *Galactic parameters*

To determine the scalelength of the SN surface density in galaxies like our own, we selected 36 SNe in Sb–Sbc galaxies from our catalogue of sightings, and calculated the distances of these SNe from the nuclei of their host galaxies. We have corrected these distances for the inclinations of the host galaxies to the plane of the sky but, for the following reasons, we have not corrected for the host galaxy position angles (see, for example, Iye & Kodiara 1975).

After comparing the traditional measurement of galaxy inclinations based on axial ratios (Hubble 1926) with independent measurements given in the Uppsala General Catalogue and the Morphological Catalogue of Galaxies, which were obtained from the plates of the sky surveys from which the catalogues were derived, we have come to the conclusion that axial ratios are not a reliable measure of the inclinations of galaxies, particularly for galaxies that are highly inclined; these findings will published later.

The errors in inclination corrections exceed the size of position angle corrections and, furthermore, the position angle corrections require knowledge of the inclinations.

For these reasons, we have assumed each galaxy has a position angle of zero. This leads to an error in the calculated distance of a SN from the centre of its host galaxy, which is related to the inclination angle (see the Appendix). Calculating a weighted average of this error, with the weights being the relative frequencies of galaxies in our sample at that inclination angle, leads to a mean random error of 24 per cent.

The surface density has been calculated as already described, and a function of the form in equation (1) has been fitted using the least-squares method, as shown in Fig. 3. The scalelength and the associated error then follow from this curve fit, giving $R_{\rm scl}$ = 5.4 ± 0.1 kpc for $h = 0.75$; adding the 24 per cent error leads to an estimate of

$$R_{\rm scl} = (5.4 \pm 1.4)(0.75/h)\ {\rm kpc}. \qquad (8)$$

This value of 5.4 kpc, obtained directly from measured SN positions in external Sb–Sbc galaxies, is intermediate between the 7.5-kpc scalelength of the pulsar distribution across our Galaxy used by Blair (1987) and the 4-kpc scale used by Strom (1994). (Allowing for errors, $R_{\rm scl}$ and the pulsar scale may in fact agree, particularly if h is somewhat less than 0.75.)

From our data, SNe are seen out to more than 40 kpc (for $h = 0.75$) from the centres of Sb–Sbc galaxies (Fig. 3): $R_{\rm cut}/R_{\rm scl} > 5$, and we can take $R_{\rm cut}$ to be effectively infinite and use the approximate result $n_{\rm Gal}/\sigma_{\rm c} \approx 2\pi R_{\rm scl}{}^2$ with negligible error.

2.3 Results

Substituting $R_{\rm scl}$ = 5.4 kpc into equation (4) gives

$$n_{\rm Gal}/\sigma_{\rm c} \approx 180\ {\rm kpc}^2. \qquad (9)$$

Substituting $D_\odot = 8.0$ kpc, $R_{\rm hor}$ = 4.5 kpc and $R_{\rm scl}$ = 5.4 kpc into equation (5) gives

$$n_{\rm loc}/\sigma_{\rm c} \approx 14.5\ {\rm kpc}^2; \tag{10}$$

numerical integration gives a value about 2.5 per cent higher. In this way, we find the ratio of SN rates Galaxy-wide and seen locally to be

$$n_{\rm Gal}/n_{\rm loc} \approx 12. \tag{11}$$

With $N_{\rm SN} = 7.0$ local SNe per millennium, the resulting Galactic frequency is

$$\nu = (n_{\rm Gal}/n_{\rm loc})N_{\rm SN}/10 \approx 8.4\ {\rm SNe\ century}^{-1}. \tag{12}$$

We determined the error in this result by an investigation of the whole-Galaxy integral, which showed it to be monotonic and well-behaved within the error range of the parameters. It is simple therefore to compute extreme values for the SN rate estimate. We assumed these to delimit a 95 per cent confidence interval for a Gaussian distribution function, and took the error to be the standard deviation of the Gaussian, yielding the estimate 8.4 ± 2.8 SNe century^{-1}.

To show the dependence of this result on H_0, we substitute $D_\odot = 8.0$ kpc, $R_{\rm hor}$ = 4.5 kpc and $R_{\rm scl}$ = 5.4(0.75/h) kpc into equation (6) to obtain

$$n_{\rm Gal}/n_{\rm loc} \approx (1.62/h)^2 \exp(1.98h/0.75). \tag{13}$$

The resulting SN rate does not scale simply with h, but does decrease with increasing h in the range $h < 1.0$, covering all h values of interest. For $h = 0.5$, $n_{\rm Gal}/n_{\rm loc} \approx 17$, giving $\nu \approx 12$ SNe century^{-1}.

3 DISCUSSION

Using data collected for extragalactic SNe, we have confirmed that the SN surface density function in spiral galaxies in general, and in galaxies that are similar to our own, can be approximated well with a decreasing radial exponential function. Using this surface density function to calculate the SN rate in the Milky Way Galaxy, from historical records, gives a SN frequency that is considerably larger than the other rates shown in Table 1, except for those calculated from historical SNe by Tammann (1982), Blair (1987) and Strom (1994).

One way to resolve the discrepancy might be to use a larger value of h than 0.75, which would have two effects. It would increase the Group A Galactic rates, based on extragalactic SN rates: the galaxies would be closer and hence less luminous, so the SN rate per unit of luminosity would be greater in proportion to h^2. Also, since the galaxies would be closer, $R_{\rm scl}$ would be smaller in proportion to 1/h, and our calculated Galactic rate would be smaller (for any $h < 1.0$). Recent studies, however, suggest that, if anything, h is smaller than 0.75, which would increase the discrepancy. Also, the Group B rates from theoretical modelling would remain discrepant.

The Group A Galactic rates would be higher if the Galaxy is, in fact, more luminous and/or of a later Hubble type than is thought to be the case. We do not know of any reason to argue for these possibilities, and the Group B theory-based rates would not lend support.

We believe that the discrepancy between our rate, other rates calculated using similar methods and rates calculated from different methods occurs because of different models of the Galactic SN surface density. Many previous authors using the historical SN method have assumed that the Galactic SN distribution is spatially constant, Blair (1987) and Strom (1994) being exceptions in using an exponential disc. In this paper, following Blair (1987), we have used an exponential and axisymmetric disc. Our results show that this improved model still does not give rates consistent with those calculated using the methods of other authors.

We suggest that the assumption that the Milky Way is axially symmetric is flawed, and that we obtain a high result for the overall SN rate from historical data because we are in a location that has excessively high numbers of SNe occurring. The *local* SN rate is not the same in different parts of the Milky Way, and hence it cannot be assumed that the Galactic SN distribution is axisymmetric for the purposes of SN rate determination.

van Dyk's (1992) observations showed that SNe of Types Ib and II are associated with H II regions, which is support for our suggestion. We suggest that further deep imaging of the spiral arms of galaxies, and correlation with SN sightings and records of SN sightings, will show further proof that the SN distribution in galaxies is not axisymmetric, but rather that SNe tend to cluster in spiral arms and star-forming regions.

The equation $\nu = (n_{\rm Gal}/n_{\rm loc})N_{\rm SN}/10$ SNe century^{-1} can be rearranged so that the local SN rate $N_{\rm SN}$ per millennium can be calculated from a given Galactic SN rate. Let us assume 2.6 SNe century^{-1} for the latter – the simple average of the rates listed under the non-historical (Group A and Group B) methods in Table 1 – and take $n_{\rm Gal}/n_{\rm loc} \approx 12$ (for $h = 0.75$) as calculated in Section 2.3 above. The local SN rate is thus calculated to be 2.2 per millennium, well below the actual number of events in the last millennium. However, our model is axisymmetric, so this is an *average* SN rate for the neighbourhood of a circle of radius 8.0 kpc about the centre of the Galaxy, not necessarily the local SN rate.

From the long interval since the last observed Galactic SN in 1604, it is often thought that we are overdue for another one. Clark, Andrews & Smith (1981) showed that the Poissonian distribution of arrival times, caused by the random distribution of SNe in space and time, can account for the observed wide range of intervals between visually detected SNe. They deduced that we are not seriously overdue for another Galactic SN. Further to this, our studies suggest that we occupy a privileged position in the Milky Way – one which gives us the impression of a considerably higher SN rate than is valid for the Galaxy as a whole. It seems likely that the actual rate for the whole Galaxy is about 2 per century.

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APPENDIX: CORRECTIONS FOR GALAXY ORIENTATIONS

Let ϕ and θ denote the position angles (east of north) of, respectively, a supernova and the major axis of the projection of its host galaxy on the plane of the sky. By symmetry of the assumed elliptical shape of the projection of the galaxy, $0 \le \theta \le \pi$; however $0 \le \phi < 2\pi$. The actual (not projected) distance of the SN from the centre of its galaxy, per unit projected distance in the plane of the sky, is denoted by $d(\theta, \phi, i)$ with i the inclination of the galaxy to the plane of the sky; d is given by $d^2 = \cos^2(\phi - \theta)/\cos^2 i + \sin^2(\phi - \theta)$ or

$$d^2 = \sec^2 i - \tan^2 i \sin^2(\phi - \theta). \tag{A1}$$

Position-angle correction

Let d_0 denote d for $\theta = 0$, the normalized distance uncorrected for the position angle θ of the galaxy:

$$d_0^2 = \sec^2 i - \tan^2 i \sin^2\phi. \tag{A2}$$

From equations (A1) and (A2),

$$f(\theta, \phi, i) \equiv d^2 - d_0^2 = \tan^2 i\,[\sin^2\phi - \sin^2(\phi - \theta)] \tag{A3}$$

$$= \tan^2 i\,[\sin\phi - \sin(\phi - \theta)]\,[\sin\phi + \sin(\phi - \theta)]$$

$$= \tan^2 i\,[2\sin(\theta/2)\cos(\phi - \theta/2)]\,[2\sin(\phi - \theta/2)\cos(\theta/2)]$$

$$= \tan^2 i \sin\theta \sin(2\phi - \theta). \tag{A4}$$

Equation (A4) shows that $d < d_0$ for $\phi < \theta/2$ and $d > d_0$ for $\phi > \theta/2$.

Equation (A3) shows that $\partial f/\partial\theta = \tan^2 i \sin(2\phi - 2\theta)$. Hence (or obviously from equation A3), f as a function of θ has extreme values $\tan^2 i \sin^2\phi$ for $\theta = \phi$ and $-\tan^2 i\,\cos^2\phi$ for $\theta = \phi \pm \pi/2$; i.e. for projected SN locations on the axes of the projected galaxy. Equation (A4) shows that $\partial f/\partial\phi = 2\tan^2 i \sin\theta\cos(2\phi - \theta)$. Hence f as a function of ϕ has extreme values $\pm\tan^2 i \sin\theta$ for $\phi = \theta/2 \pm \pi/4$.

Equation (A4) shows that the maximum error in normalized distance caused by neglecting the position angle of the host galaxy (i.e. by taking $\theta = 0$) is about $\tan i$. For $i = 20°$ (20° from face on) this is 0.36. We now look at averaged expected errors. Averaging equation (A4) over θ gives

$$< d^2 - d_0^2 >_\theta = -(1/2)\tan^2 i \cos 2\phi. \tag{A5}$$

The average of $|\cos 2\phi|$ over $\phi = 0$ to 2π is $2/\pi$. By averaging over θ then ϕ, we can define an rms error associated with taking $\theta = 0$:

$$\Delta \equiv <\,|< d^2 - d_0^2 >_\theta|\,>_\phi^{1/2}$$

$$= (1/\pi)^{1/2}\tan i = 0.564\tan i. \tag{A6}$$

This is 0 for face-on galaxies ($i = 0$) and 0.205 for $i = 20°$.

Inclination correction

For unit distance in the plane of the sky, d_0 is the distance corrected for the inclination i of the host galaxy but not for the position angle θ of the galaxy. To obtain a measure of the error that would occur if the correction for i were not made, we can use equation (A2) in the form

$$d_0^2 - 1 = \tan^2 i \cos^2\phi. \tag{A7}$$

Just as for the position-angle correction, therefore, the maximum error would be about $\tan i$. We can form an rms error by averaging over ϕ:

$$\delta \equiv < d_0^2 - 1 >_\phi^{1/2} = (1/\sqrt{2})\tan i = 0.707\tan i. \tag{A8}$$

Hence, from equations (A6) and (A8), $\Delta/\delta = (2/\pi)^{1/2} = 0.798$. The average error from neglect of the position-angle correction is therefore about 80 per cent of what would be incurred if the inclination correction were neglected. The average position-angle correction becomes quite large (exceeding 20 per cent) for galaxies more than 20° from face on.

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