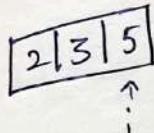


4/12/25

Recursion & Backtracking

Combination sum

arr[] = {2, 3, 5} target = 8



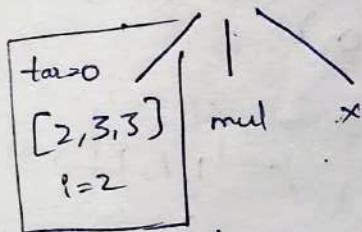
arr[i]
↓
inc single =
inc multiple =
exclude =

i=0

tar=6 / | tar=6 | tar=8

[2] [2] []

tar=3 / | tar=3 | tar=6
[2, 3] [2, 3] [2]
i=2 i=1 i=2



pseudocode/logic:

void combSum(arr[], i, combin, ans, tar)

if (i == n)
return;

if (tar == 0)

ans.PB(combin) } ans
return

combin.PB(arr[i])

Single → CS(arr, i+1, combin, ans, tar - arr[i])

Mul → CS(arr, i, combin, ans, tar - arr[i])

combin.popback() → Backtracking

CS(arr, i+1, combin, ans, tar)

Code:

```
def combination(self, candidates, target):
    res = []
    def backtrack(remaining, comb, ind):
        if remaining == 0:
            res.append(list(comb))
            return
        if remaining < 0 or ind == len(candidates):
            return
        comb.append(candidates[ind])
        backtrack(remaining - candidates[ind], comb, ind)
        comb.pop()
        backtrack(remaining, comb, ind + 1)
    backtrack(target, [], 0)
    return res
```

$\leftarrow [1, 1, 1] \rightarrow$

2. combination sum-II

arr[] = [1, 1, 1, 2, 2]

0 1 2 3 4

target = 4

$\xrightarrow{2^n}$

T.C $\rightarrow 2^n \times k$

S.C $\rightarrow k \times k$

$f(0, 4, 1)$

$f(1, 3, 1) \times f(4, 2, 2)$

$f(2, 2, 1) \times f(4, 1, 2) \times f(5, 0, 2)$

$f(2, 1, 1) \times f(4, 0, 2)$

if (tar == 0)
ans.add(comb)

if (ind, tar, ans)

loop(ind->n-1) -> i

i -> add(arr[i])

f(i+1, tar - arr[i], c)

c.remove(_)

code:

(candidates.sortc)

res = []

```
def backtrack(tar, comb, start):
```

If $\tan = 0$:

```
res.append(Comb[:])
```

Zebuan

```
for i in range(start, len(candidates)):
```

if $i > \text{start}$ and $\text{candidates}[i] == \text{candidate}[i-1]$:

Continue

If $\text{candidates}[i] > \text{target}$:

Continue

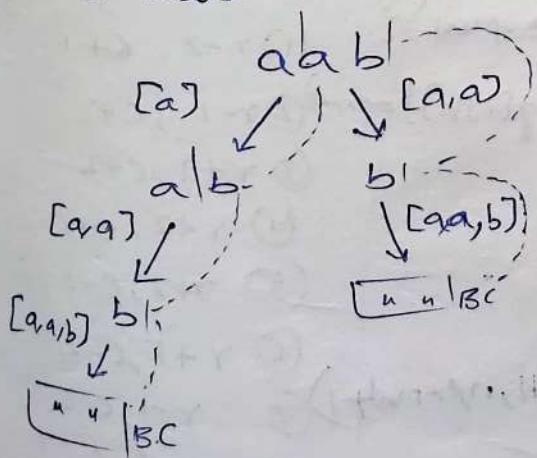
(comb. append (candidates[?]))

backtrack (tar-candidate[i], comb, i+1)

comb. POPC)

backslash (\backslash , $[$, $]$, \circ)

Palindrom
 $s = "aab"$



```
void getAparts(S, partition, ans) {
```

if (S.size() == 0) {

ans. pb(partition)
return;

Rec

```
-for(i=0; i<n; i++) {
```

String part S.substr(0, i+1)

f(c) is palin (part)) {

positions. $\text{pb}(\text{part})$

(S-substr(p+1),
partition)

partitions. popback();

```

ans = []

def getAllParts(s, partitions):
    if not s:
        ans.append([list(partition) for partition in partitions])
        return

    for i in range(0, len(s)):
        part = s[0:i+1]
        if part == part[::-1]:
            partitions.append(part)
            getAllParts(s[i+1:], partitions)
            partitions.pop()
        else:
            getAllParts(s[i:])
    return ans

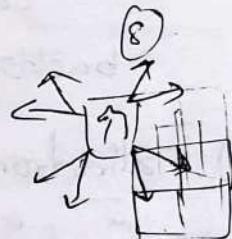
```

4-(25/96) Check Knight Tour Configuration

0 to $n^2 - 1$

(0)	1	3	6
5	(8)	10	11
2	7	12	13
14	15	16	(17)

exp > 1
false



```

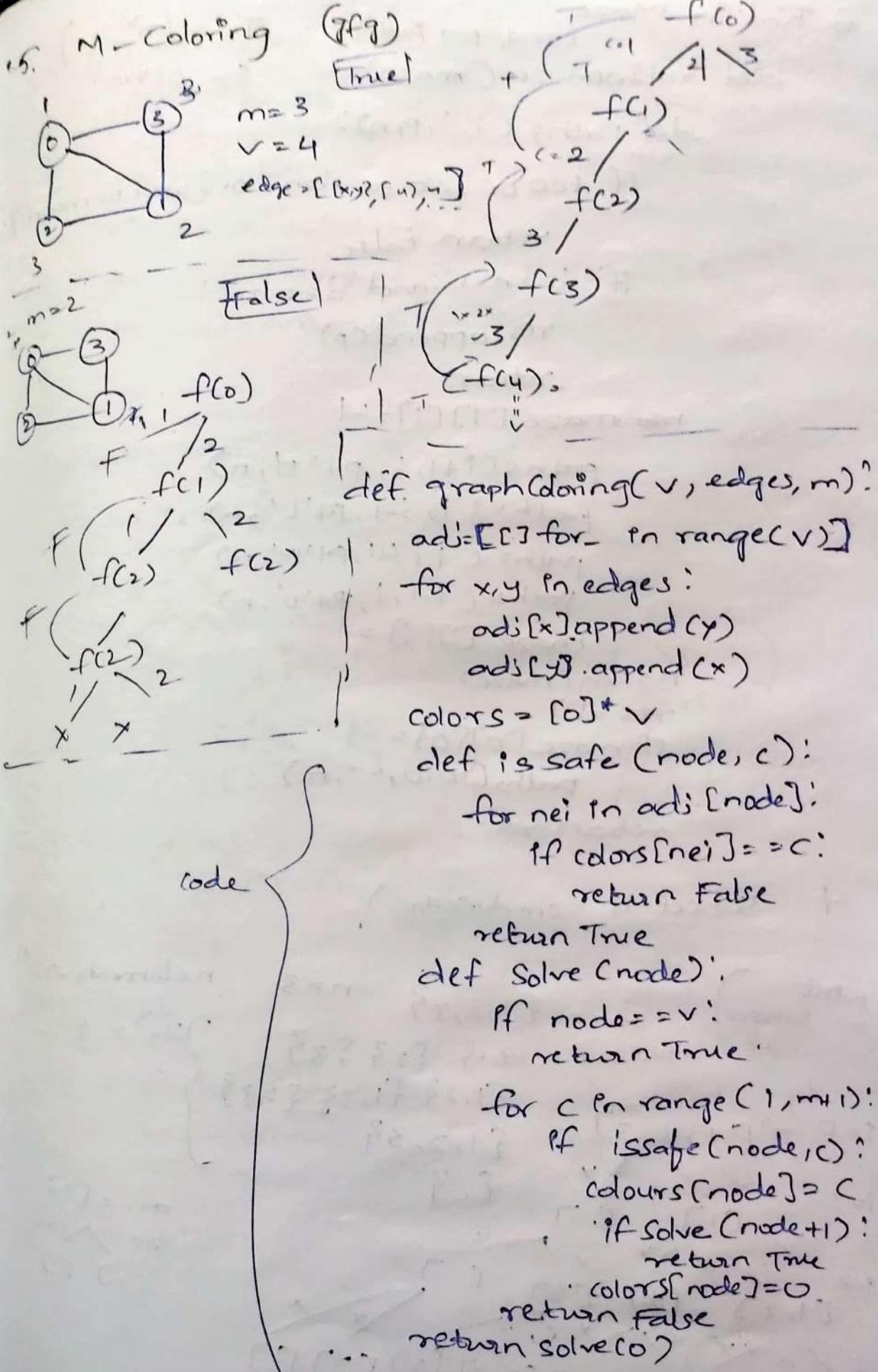
bool isValid(grid, r, c, n, expval) {
    if(r < 0 || c < 0 || r >= n || c >= n || grid[r][c] != expval) return false
    return true;
}

if (expval == n * n) return true

ans1 = isValid(grid, r - 2, c + 1, n, expval)
ans2 = isValid(grid, r - 1, c + 2, n, expval)
ans3 = isValid(grid, r + 1, c + 2, n, expval)
ans4 = isValid(grid, r + 2, c + 1, n, expval)
ans5 = isValid(grid, r + 2, c - 1, n, expval)
ans6 = isValid(grid, r + 1, c - 2, n, expval)
ans7 = isValid(grid, r - 1, c - 2, n, expval)
ans8 = isValid(grid, r - 2, c - 1, n, expval)

return ans1 || ans2 || ans3 || ... || ans8

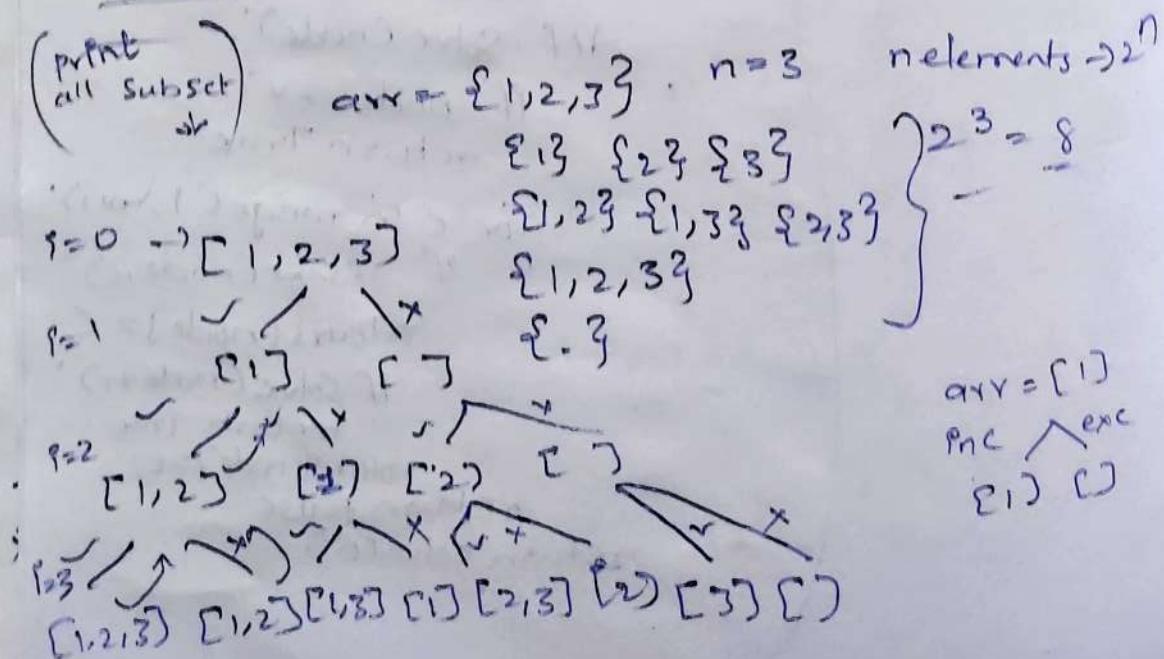
```



6. Rat in maze : (no. of poss paths)

def ratInMaze(maze):
 def paths(i, j, p, n):
 if (i < 0 or j < 0 or i == n-1 or j == n-1 or maze[i][j] == -1):
 return False
 if i == n-1 and j == n-1:
 res.append(p)
 return
 if maze[i][j] == -1:
 paths(i+1, j, p+'D', n)
 paths(i, j+1, p+'L', n)
 paths(i, j-1, p+'R', n)
 paths(i-1, j, p+'U', n)
 maze[i][j] = 1
 n = len(maze)
 res = []
 if maze[0][0] == 1:
 paths(0, 0, "", n)
 return res

7 Subsets-II (no duplicates)



void ps (arr[], ans[], i) {

if ($i == \text{arr.size()}) \Rightarrow \text{print ans}$
return;

ans.push_back (arr[i])

ps (arr, ans, i+1) } inc

ans.pop_back () \longrightarrow (Backtracking)

ps (arr, ans, i+1) } exc X

T.C \rightarrow total calls work down in each call

$$2^n \times n \times O(1)$$

(subset) only elements

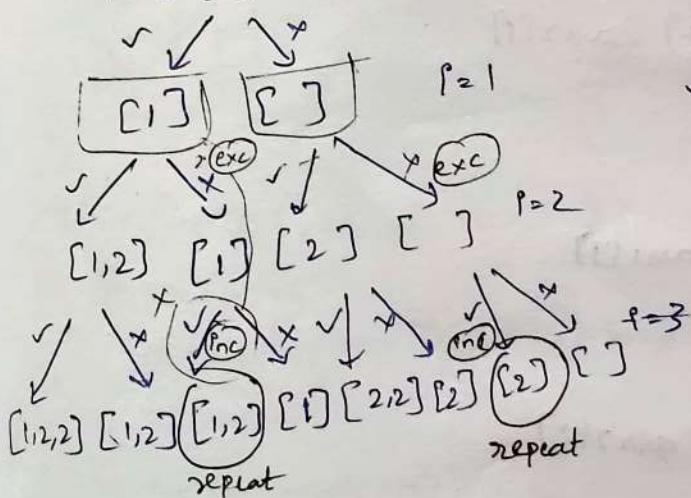
$$= O(2^n \times n)$$

// duplicates exists SS - 11

arr = {1, 2, 2}

[1 1 2] X X
 $i = i+1$

$i = 0 []$



arr [p]
✓ ✓ ✗ =
arr [p+1]

- ① Sort the arr
- ② calc subsets

nums.sort()

allSubsets = []

def getAllSubsets (pdx, ss):

If $i == \text{len}(\text{nums})$:

allSubsets.append (ss[:])

return

include

ss.append (nums[i])

getAllSubsets (i+1, ss)

ss.pop()

pdx

{
 pdx = i+1
 while $i < \text{len}(\text{nums})$ and $\text{nums}[pdx] == \text{nums}[pdx-1]$:
 i += 1

exclude

getAllSubsets (i, ss)

getAllSubsets (0, [])

return (allSubsets)

Print

8. (q12) Sort An Array

T.C $\rightarrow N \log N$
 $\therefore n \log \{ SC \rightarrow O(n) + O(n) \}$
 [1, 2, 3, 5]

def merge(nums, s, m, e):

$n_1 = m+s+1$

$n_2 = e-m$

$arr1 = \text{nums}[s:m+1]$

$arr2 = \text{nums}[m+1:e+1]$

$i=0$

$j=0$

$k=s$

while $i < n_1$ and $j < n_2$:

if $arr1[i] \leq arr2[j]$:

$\text{nums}[k] = arr1[i]$

$i+=1$

else:

$\text{nums}[k] = arr2[j]$

$j+=1$

$k+=1$

while $i < n_1$:

$\text{nums}[k] = arr1[i]$

$i+=1$

$k+=1$

while $j < n_2$:

$\text{nums}[k] = arr2[j]$

$j+=1$

$k+=1$

def mergesort(nums, s, e):

if $s > e$:

$m = s + (e-s)/2$

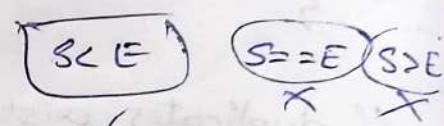
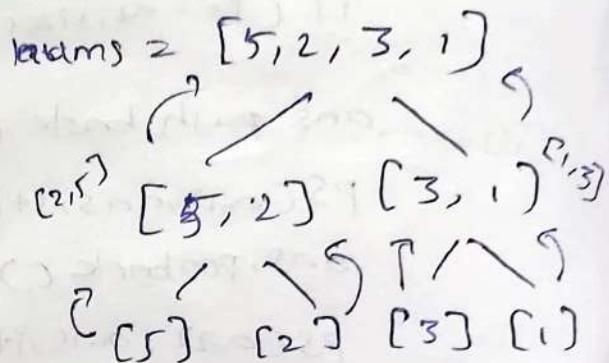
mergesort(nums, s, m)

mergeSort(nums, m+1, e)

merge(nums, s, m, e)

mergeSort(nums, 0, len(nums)-1)

print(nums)



$$mid = s + (e-s)/2$$

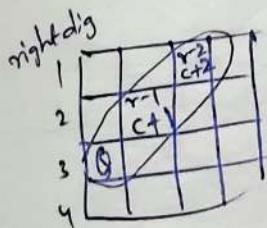
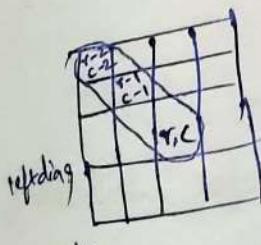
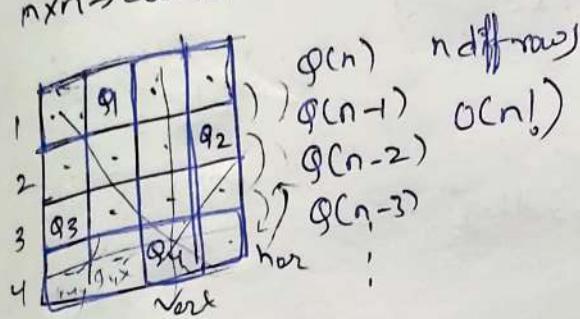
$$0 + (3-0)/2$$

$$= 1$$

9. (51) NQueens

$n=4$

$n \times n \rightarrow \text{board}$



def nQueens(board, row, n):

if row == n:

ans.append(''.join(r for r in board))

return

for col in range(n):

if isSafe(board, row, col, n):

board[row][col] = 'Q'

nQueens(board, row+1, n)

board[row][col] = '.'

ans = []

boards = [['_' * n for _ in range(n)]]

nQueens(board, 0, n)

print(ans)

$n=4$

def isSafe(board, row, col, n):

hor

for j in range(n):

if board[i][col] == 'Q':
return F

ver

for i in range(n):

if board[i][col] == 'Q':
return F

Left diag

i, j = row, col

while i >= 0 and j >= 0:

if board[i][j] == 'Q':
return F

i -= 1

j += 1

i = row, j = col

while i >= 0 and j < n:

if board[i][j] == 'Q':
return False

i -= 1

j += 1

return True

10. (37) Sudoku Solver

```

def isSafe(board, row, col, dig):
    for j in range(9):
        if board[row][j] == dig:
            return False
    for i in range(9):
        if board[i][col] == dig:
            return False
    sr = (row // 3) * 3
    sc = (col // 3) * 3
    for i in range(sr, sr + 3):
        for j in range(sc, sc + 3):
            if board[i][j] == dig:
                return False
    return True

```

```
def helper(board, row, col):
```

```
    if row == 9:
```

```
        return True
```

```
    if col == 9:
```

```
        nextRow = row + 1
```

```
        nextCol = 0
```

```
    else:
```

```
        nextRow = row
```

```
        nextCol = col + 1
```

```
    if board[row][col] != '.':
```

```
        return helper(board, nextRow, nextCol)
```

```
    for dig in "123456789":
```

```
        if isSafe(board, row, col, dig):
```

```
            board[row][col] = dig
```

```
            if helper(board, nextRow, nextCol):
```

```
                return True
```

```
                board[row][col] = '.'
```

```
    return False
```

```
helper(board, 0, 0)
```

TLE) (empty) & T_{is}

rows = [set for n in range(9)]

G = {n}

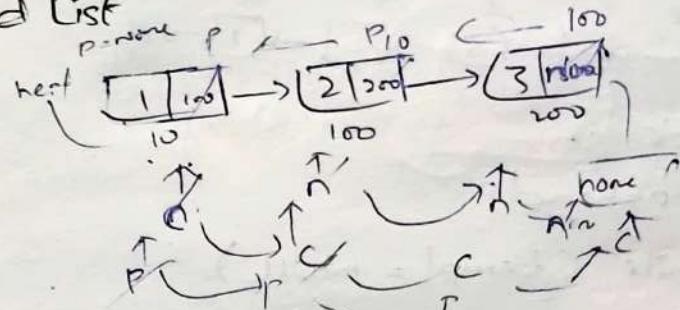
boxes = {4}

Linked List

1) Reversed Linked List

```

P = None
C = head
while C:
    n = C.next
    C.next = P
    P = C
    C = n
return P
  
```



2) middle of a LL

Slow = head

fast = head even odd

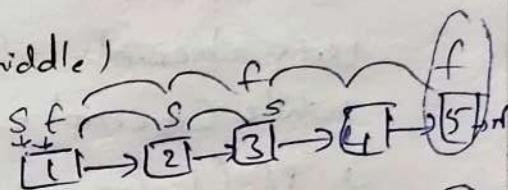
while(fast != null & fast->next != null)

{ Slow = slow->next

fast = fast->next->next

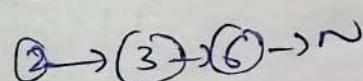
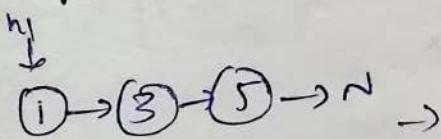
return slow;

(1) brute force
(2) slow-fast even - (2 middle)

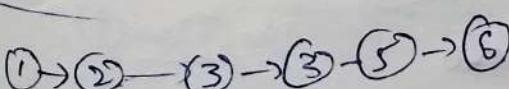


SC : O(1)
TC : O(n)

3) merge two sorted lists



h_2

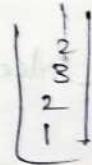
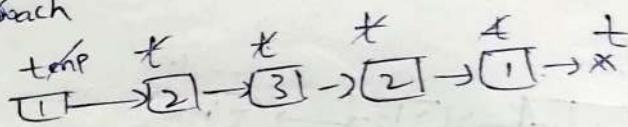


T.C = O(n+m)

if ($h_1 = \text{null}$) || ($h_2 = \text{null}$)
 $h_1 \rightarrow \text{val} < h_2 \rightarrow \text{val} \rightarrow \text{case 1}$
 $h_1 \rightarrow \text{next} = \text{merge}(h_1 \rightarrow \text{next}, h_2)$
return h_1
else
 $h_2 \rightarrow \text{next} = \text{merge}(h_1 \rightarrow \text{next}, h_2 \rightarrow \text{next})$
 $h_1 \rightarrow \text{next} = h_2 \rightarrow \text{next}$

10 (1) (28u) palindrome linked list

① approach



Stack st

temp = head

while (temp != NULL)

{ st. Push (temp->data) }

temp = temp->next;

} temp = head

while (temp != NULL)

{ if (temp->data != st. top()) return false;

temp = temp->next

} st.pop();

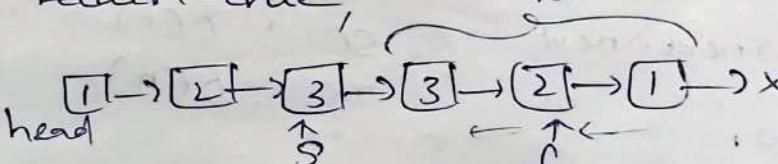
} step 1:
O(n)

- O(n)

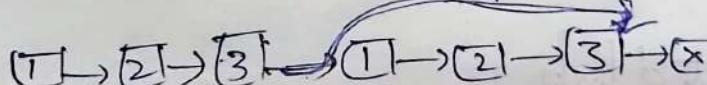
return true;

[TC → O(2n)
SC → O(n)]

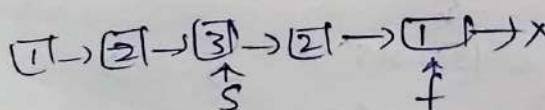
②



even



odd



slow = head fast = head

while (fast->next != NULL && fast->next->next != NULL)

{ slow = slow->next

} fast = fast->next->next

node * newnode = reverse (slow->next)

first = head second = newhead

while (second != NULL)

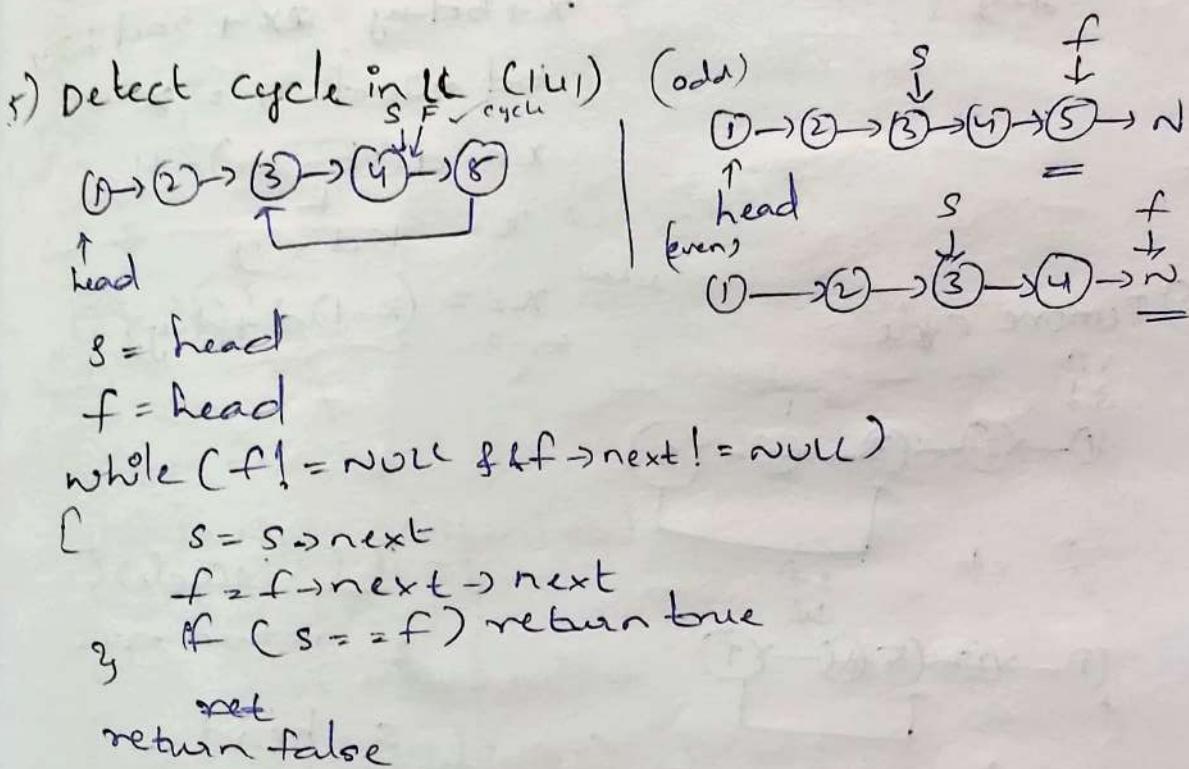
{ if (first->data != second->data)

~~reverse (newhead)~~)

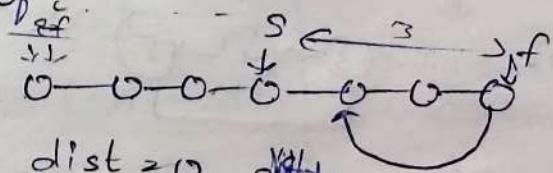
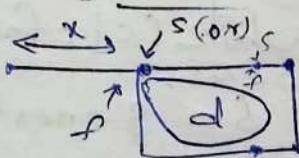
return -false;

3)

~~reverse (newhead)~~ - $f \leftarrow f \rightarrow \text{next}$
 $s \leftarrow s \rightarrow \text{next}$
 return true

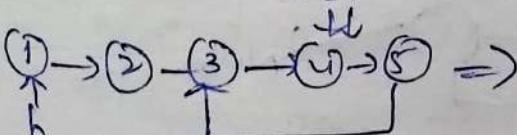


Mathematical proof



6) (L1) LL-II

- ① start node
 ② remove node



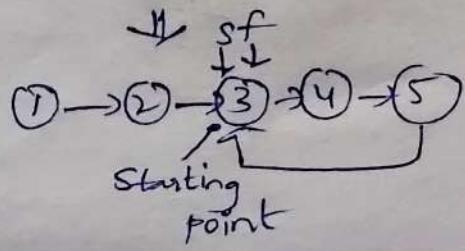
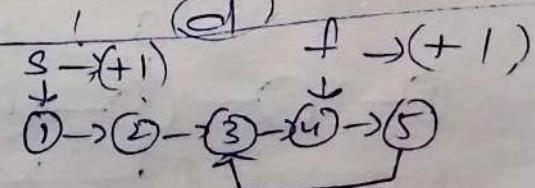
- (i) find a cycle exists or not
 (ii) $s \geq h$ if [$s == f$]:

while ($s \neq f$) {

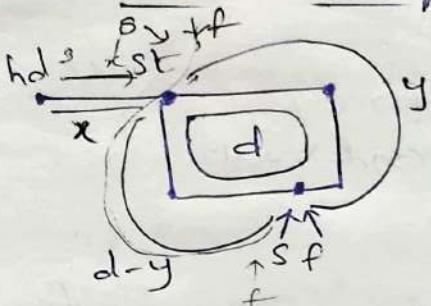
$s = s \rightarrow \text{next}$

$f = f \rightarrow \text{next}$

} return



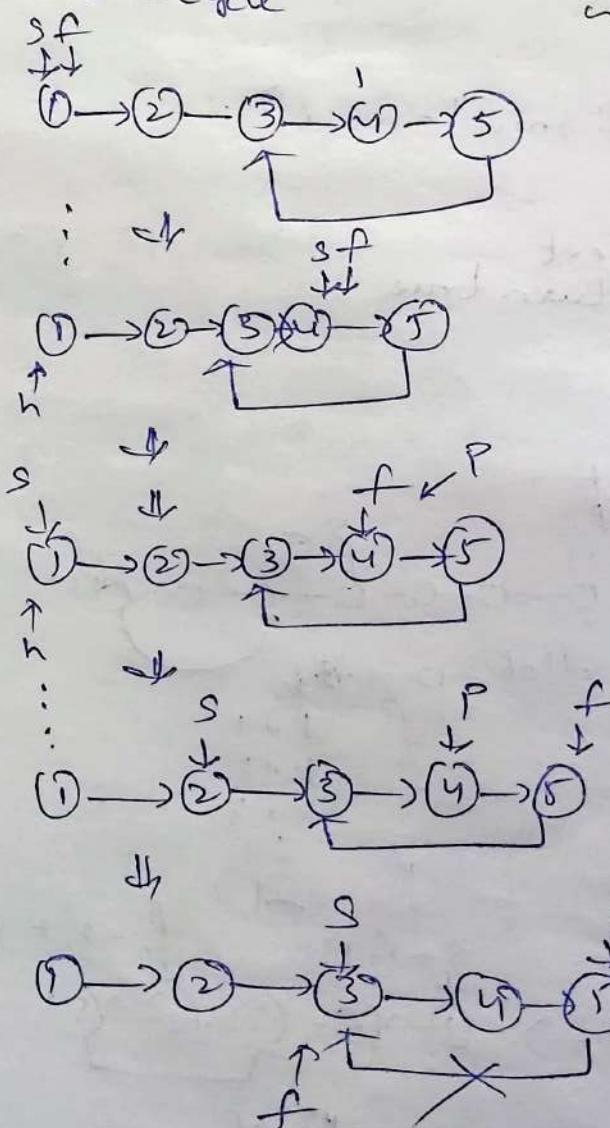
Mathematical proof



(from mind)

$$\begin{aligned} \text{slow dist} &= x + \text{abd} + y \\ \text{fast dist} &= x + bxd + y \\ \text{fast} &= 2 \times \text{slow} \\ x + bxd + y &= 2 \times (x + ad + y) \\ x + bd + y &= 2x + 2ad + 2y \\ bd - 2ad - y &= x \\ x &= d(b - 2a) - y \\ x &= kd - y \\ x &= \underbrace{(k-1)d}_{s} + \underbrace{(d-y)}_{f} \end{aligned}$$

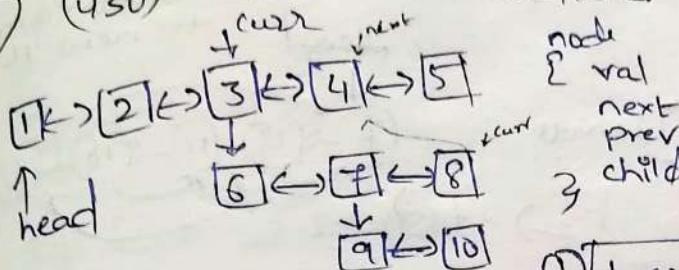
(2) Remove cycle



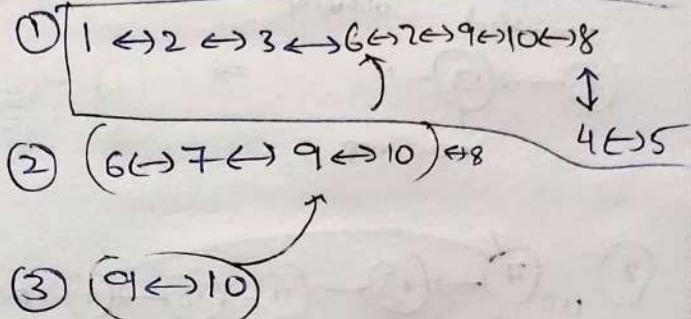
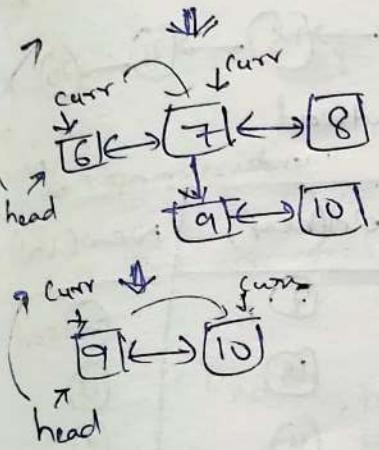
If (!iscycle)?
{ return NULL }

$s = \text{head}$
 $p = \text{NULL}$
 $\text{while } (s \neq \text{fast})$
{
 $s = s \rightarrow \text{next}$
 $p = f$
 $f = f \rightarrow \text{next}$
}
 $p \rightarrow \text{next} = \text{NULL}$ remove cycle
return s ,

#7 (430) → Flatten a multilevel Doubly LL



node
 val
 next
 prev
 child



Flatten (Node* head) {

head = NULL → return head

Node* curr = head

while (curr != NULL)

{ if (curr->child != NULL)

{ Node* next = curr->next

curr->next = flatten (curr->child)

curr->next->prev = curr

curr->child = NULL

① flatten the child nodes

while (curr->next != NULL)

curr = curr->next

② find tail

if (next != NULL)

{ curr->next = next

next->prev = curr

③ attach tail with next ptr

{ curr = curr->next

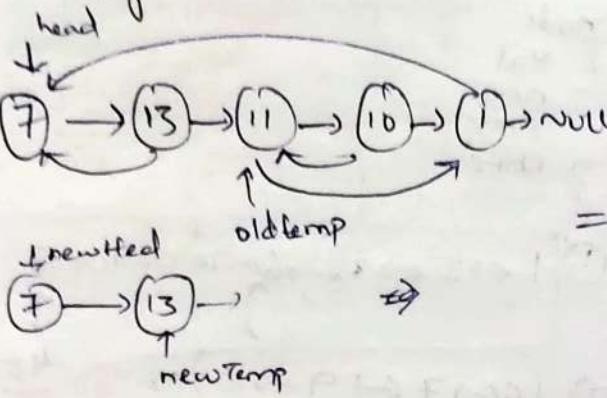
} return head

8) Copy List with Random pointer

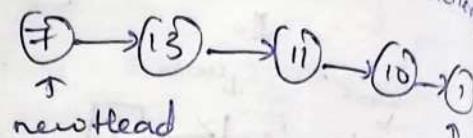
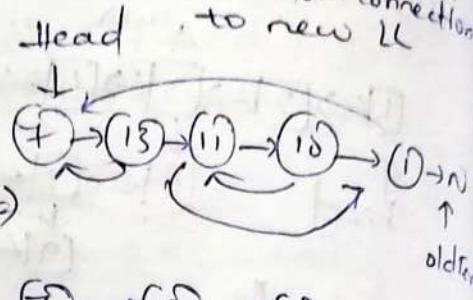
(None)

① Simple copy

② Random connection
to new LL

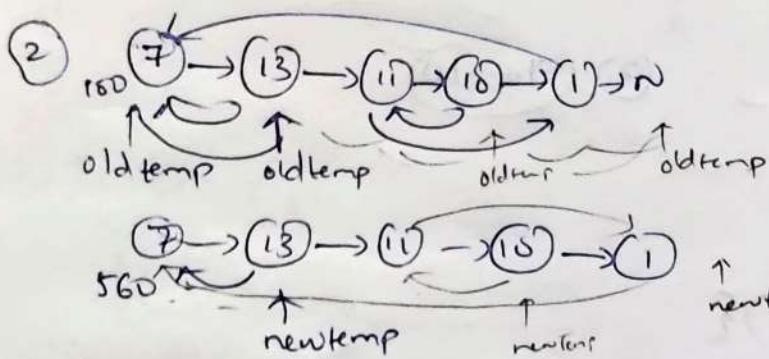


=> -----



unordered_map<int, int>

old(key)	new(val)
7	560
13	13
11	11
10	10
1	1



logic
unordered_map<N, N>m;

node* newHead = newNode(head->val)

oldTemp = head → next

newTemp = newHead

m[head] = newHead

while (oldTemp != NULL)

{ node* copyNode = newNode(oldTemp->val)

m[oldTemp] = copyNode;

newTemp → next = copyNode

oldTemp = oldTemp → next

3 newTemp = newTemp → next

oldTemp = head; newTemp = newHead;

while (oldTemp != NULL)

{ newTemp → random = m[oldTemp → random]

oldTemp = oldTemp → next

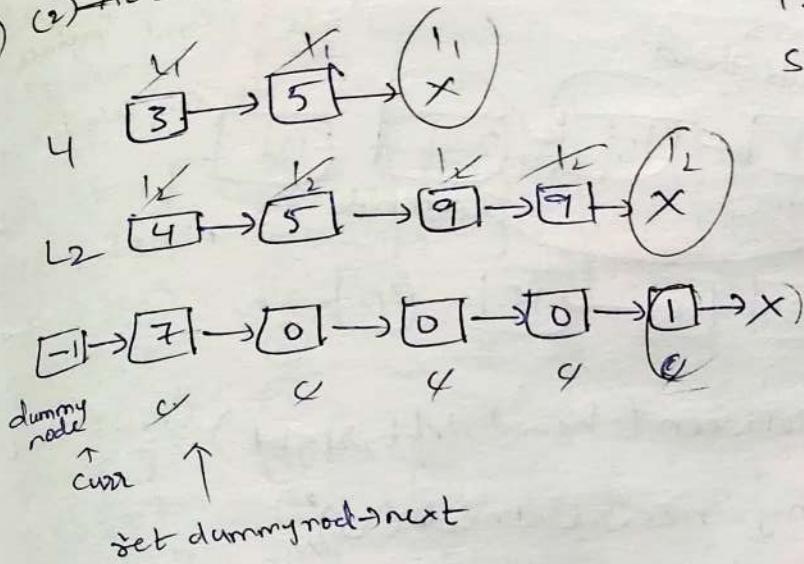
3 newTemp = newTemp → next

return newHead

①

②

9) (2) Add 2 numbers



logic

func (head1, head2)

 t1 = head1, t2 = head2

 dummyNode = newNode(-1)

 curr = dummyNode

 while ($t_1 \neq \text{NULL}$ & $t_2 \neq \text{NULL}$)

 { sum = carry

 if (t_1) sum = sum + $t_1 \rightarrow \text{data}$

 if (t_2) sum = sum + $t_2 \rightarrow \text{data}$

 newNode = newNode($\text{sum} \% 10$)

 carry = $\text{sum} / 10$.

 curr \rightarrow next = newNode;

 curr = curr \rightarrow next

 if (t_1) $t_1 = t_1 \rightarrow \text{next}$

 if (t_2) $t_2 = t_2 \rightarrow \text{next}$

 if (carry)

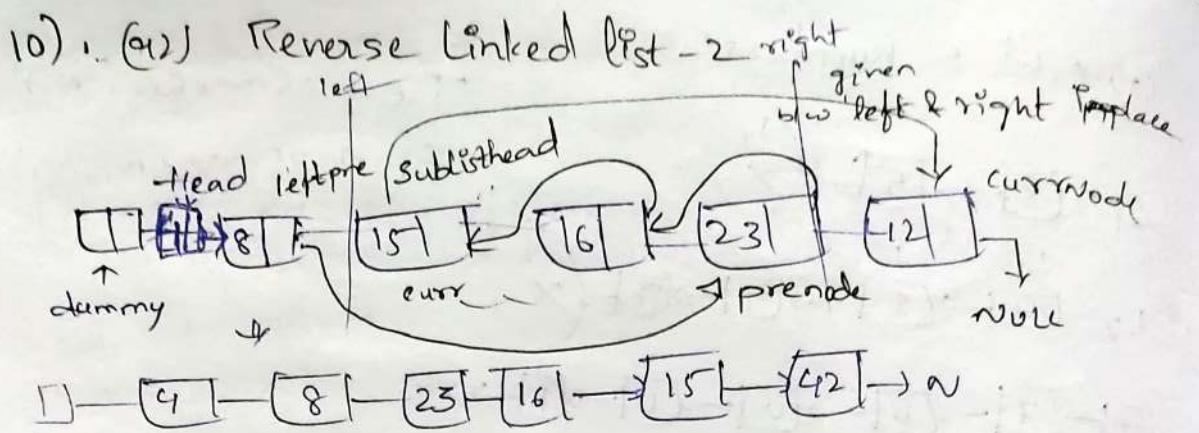
 { newNode = newNode(carry)

 curr \rightarrow next = newNode;

 return dummyNode \rightarrow next

T.C $\rightarrow O(\max(N_1, N_2))$

SC $\rightarrow O(\max(N_1, N_2))$



Listnode ReverseBetween (head, left, right)

{ Listnode dummy = new Listnode(0);

 dummy.next = head;

 Listnode leftpre = dummy; // useful when left is head

 Listnode currNode = head;

 for (int p=0; i<left-1; i++)

 { leftpre = leftpre.next;

 3 currNode = currNode.next;

 Listnode subListHead = currNode; // marker - where we start reversing

 Listnode preNode = null;

 for (int p=0; i<=right-left; i++) // s-l+1 times

 { Listnode nextNode = currNode.next;

 currNode.next = preNode;

 preNode = currNode;

 currNode = nextNode;

 3 "Join the pieces,"

 leftpre.next = preNode;

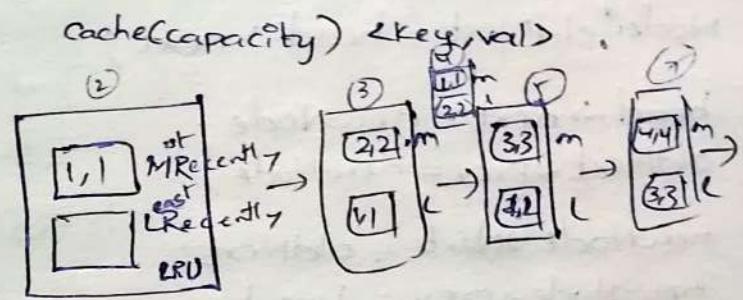
 subListHead.next = currNode;

 return dummy.next;

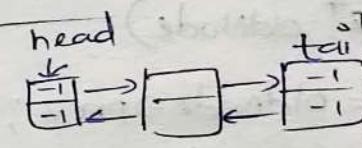
3

Design an LRU Cache

- 1) LRU Cache (2)
- put (1,1) ①
- put (2,2) ②
- get [1] ③
- put (3,3) ④
- get [2] ⑤
- put (4,4) ⑥
- get [1] ⑦
- get [3] ⑧
- get [4] ⑨



1 - 2 - 1 3 4

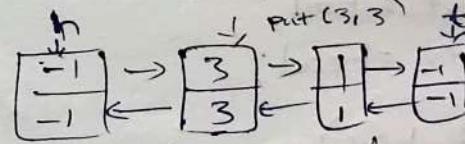
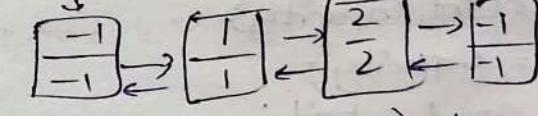


Class node {

Node* next, prev;

{ int key, val;

head get(1);



T.C → O(1)

Code:

class Node{

Node* next, prev;

int key, val;

node (sdf, k, v):

{ key = K

val = V

prev = next = NULL;

Node * head = new Node(-1, -1);

Node * tail = new Node(-1, -1);

unordered_map<int, Node*> m;

int limit;



```

void addNode (Node* newNode)          o(1)
{
    Node* oldNext = head->next;
    head->next = newNode;
    oldNext->prev = newNode;
    newNode->next = oldNext;
}
newNode->prev = head;
}

void delNode (Node* oldNode)
{
    Node* oldPrev = oldNode->prev;
    Node* oldNext = oldNode->next;
    oldPrev->next = oldNext;
}
oldNext->prev = oldPrev;
}

```

LRUCache (int capacity)

```

{
    limit = capacity;
    head->next = tail;
}
tail->prev = head;

```

int get (int key)

```

{
    if (m.find (key) == m.end ()) // if key doesn't exist
        return -1;
}

```

Node* ansNode = m[key];

int ans = ansNode->val;

m.erase (key);

delNode (ansNode);

addNode (ansNode);

```

}
m[key] = ansNode;
return ans;

```

void put (int key, int val)

```

{
    if (m.find (key) != m.end ()) // exist
}

```

Node* oldNode = m[key];

delNode (oldNode);

m.erase (key);

if (m.size () == limit) // delete LRU data

{ m.erase (-tail->prev->key);

delNode (-tail->prev);

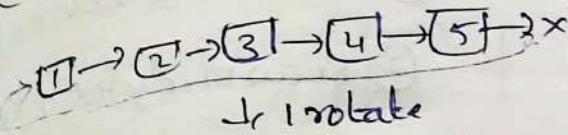
Node* newNode = new Node (key);
 addNode (newNode);
 m[key] = newNode;

3

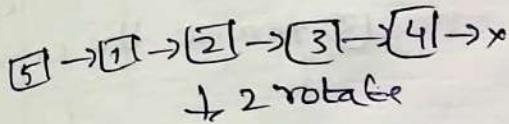
3

/

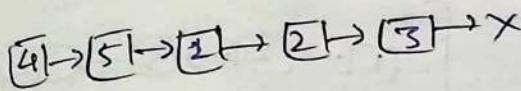
R(61) Rotate a LL



$$k = (\text{len}) \text{ multiple - no change}$$
$$5 + 5 + 5 = 15$$
$$\begin{array}{cccc} 5 & 5 & 5 \\ \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 \end{array}$$



$$5 + 5 + 4 = 14$$
$$\boxed{k \% \text{ len}}$$



$$15 \% 5 = 0$$

$$14 \% 5 = 4$$

$$12 \% 5 = 2$$

Node* findNode (temp, int k)

```
{ int cnt = 1;
  while (temp != NULL)
  { if (cnt == k) return temp;
    cnt++;
    temp = temp->next;
  }
  return temp;
```

Node* rotate (node* head, int k)

```
{ if (head == NULL || k == 0) return head;
```

Node* tail = head;

int len = 1;

while (tail->next != NULL)

```
{ tail = tail->next;
```

```
len++;
```

```
if (k % len == 0) return head;
```

$k = k \% \text{ len}$

"attach tail to head"

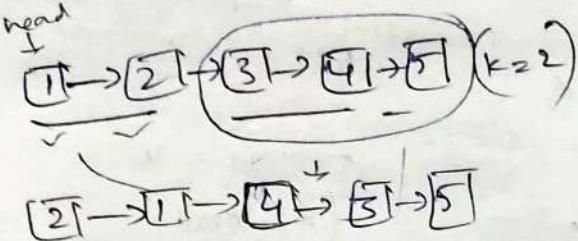
$\text{tail} \rightarrow \text{next} = \text{head}$

Node* newLastNode = findNode (head, len - k)

```
head = newLastNode->next;
```

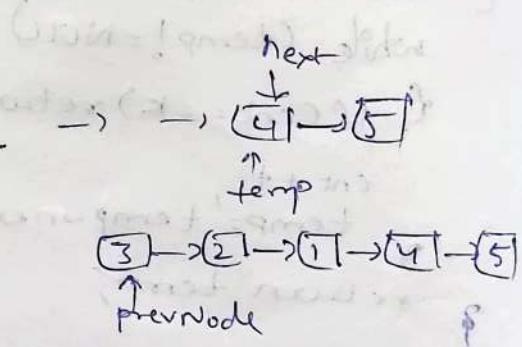
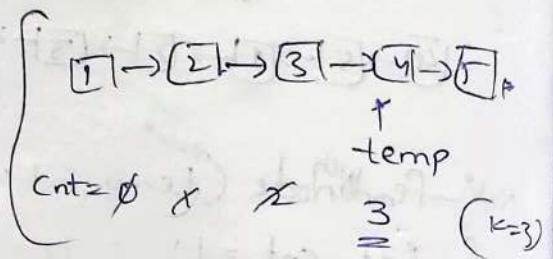
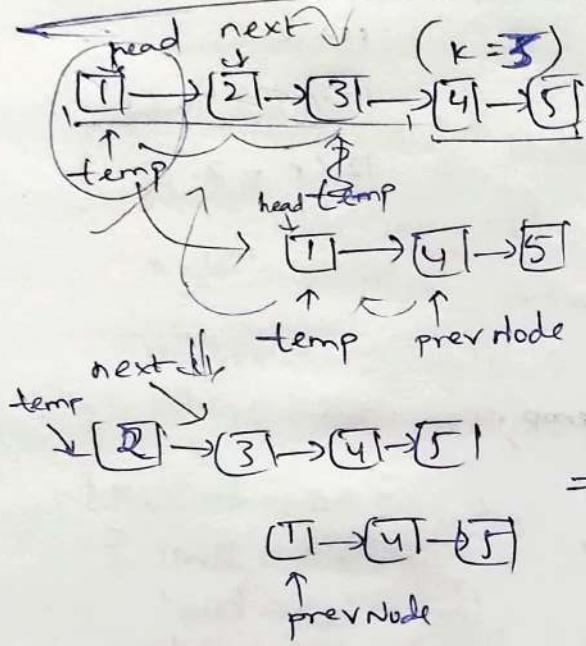
```
newLastNode->next = NULL;
```

13. (25) Reverse k-Group LL



(rev_k K. k < k keep it,)

- ① check if k nodes exist
- ② Recursively rest of the LL
- ③ reverse the current group



Node* reverse (head, k)

{ Node* temp = head

cnt = 0

while (cnt < k) {

if (temp == NULL)

return head;

temp = temp->next

prevNode = reverse (temp, k)

temp = head; cnt = 0

while (cnt < k)

{ Node* next = temp->next

temp->next = prevNode

prevNode = temp

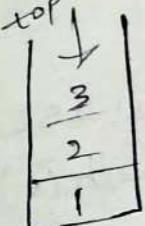
} temp = next
cnt++ ; return prevNode

new head of LL

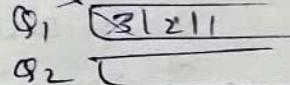
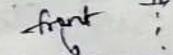
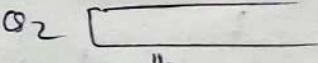
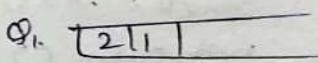
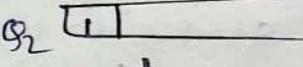
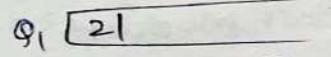
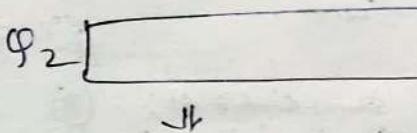
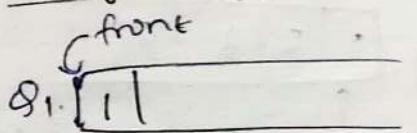
+

Stack & Queue

9/2/21
Q2121
(225) Stack using queues



stack
LIFO



class MyStack {

queue<int> q1;
queue<int> q2;

mystack() {

}

void push(int x) {

while (!q1.empty()) {

q2.push(q1.front());

} q1.pop();

q1.push(x);

while (!q2.empty()) {

q1.push(q2.front());

} q2.pop();

int pop() {

int ans = q1.front();

q1.pop();

return ans;

push

pop

top

empty

061

① q1 elements copy q2

② q1.push(data)

③ q2 elements copy back q1

top = q1.front

pop = q1.pop

int top() {

return q1.front();

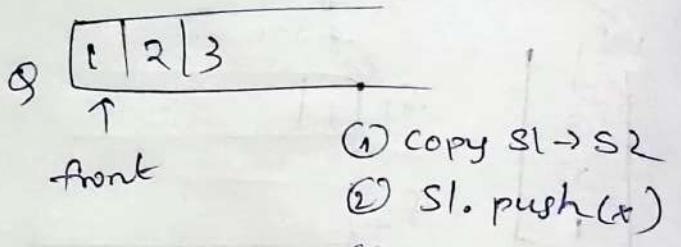
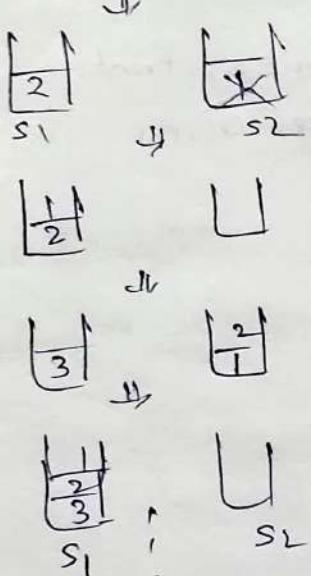
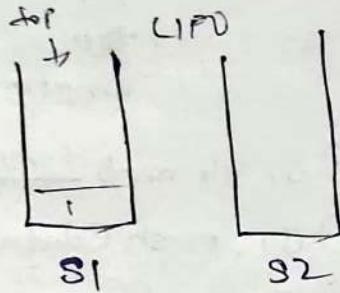
}

bool empty() {

return q1.empty();

}

2-(232) Implement Queue using stacks



Stack<int> s1;
Stack<int> s2;

Class MyQueue {

Stack<int> s1;

Stack<int> s2;

MyQueue() {

}

void push(int x) {

 s2.push(s1.top());

 s1.pop();

 s1.push(x);

 while(!s2.empty()) {

 s1.push(s2.top());

 s2.pop();

 } }

 int pop() {

 int ans = s1.top();

 s1.pop();

 } }

 int peek() {

 return s1.top();

 bool empty() {

 return s1.empty();

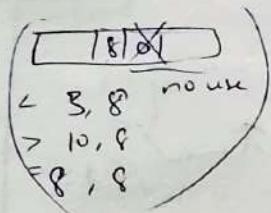
 }

3. Next Greater Element

$\text{arr} = [6, 8, 0, 1, 3]$

8, -1, 1, 3, -1

6
8
0
X
3



Stack $\leftarrow \text{int} > S;$

vector $\leftarrow \text{int} \rightarrow \text{NG} \quad (\text{ans}, \text{arr.size()})$

for ($p=n-1; i>s0; i--$)

{ while ($S.size() > 0$ & $S.top() \leq \text{arr}[p]$)

{ $S.pop();$

}

if ($S.empty()$) $\rightarrow -1$

else $\rightarrow \text{NG}(S.top())$

$S.push(\text{arr}[p])$.

}

(496)

$\text{nums1} = [4, 1, 2] \quad \text{nums2} = [1, 3, 4, 2] \quad \downarrow$

$\text{ans2} = [-1, 3, -1]$

① $\text{nums2} \rightarrow \text{Next Greater}$

```
def nextGreaterElement(nums1, nums2):
    stack = []
    NG = nextGreater = [-1]
    for i in reversed(nums2):
        while stack and stack[-1] < i:
            stack.pop()
        if not stack:
            NG[i] = -1
        else:
            NG[i] = stack[-1]
        stack.append(i)
    return [NG[x] for x in nums1]
```

map -

$\text{nums2}[i]$	NG
1	3
3	4
4	-1
2	-1

②

$\text{nums1} \rightarrow$ ↗ search

map[$\text{nums1}[i]$]

"NG"

Q(20) valid parenthesis

$\left(\left\{ \left[\right] \right\} \right)$

$\left\{ \right\}$ $\left\{ \right\}$ $\left[\right]$ $\left[\right]$

$\left(\left\{ \left[\right] \right\} \right)$

Stack - LIFO

open > closing :-

$C \left[\left\{ \right\} \right]$ $\left[\right]$

return st.size == 0.

closing > opening:

$\left[\right] \left[\right]$

if st.size == 0
ret False

bool isValid (str)

{ stack <char> st;

for (p=0 to n)

{ if (str[p] == opening)

{ st.push(ch)

}

else :

{ if (st.size() == 0)

return false }

close opening

if (str[p] match with st.top())

st.pop()

else

return false;

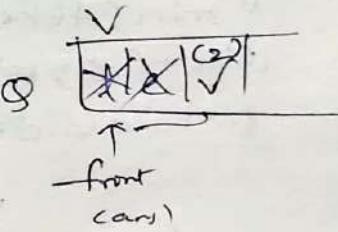
}

return st.size() == 0; } open > close

5. (387) First unique character in a string

str = "level"

0	1	2	3	4
l	e	v	e	l

Q: 

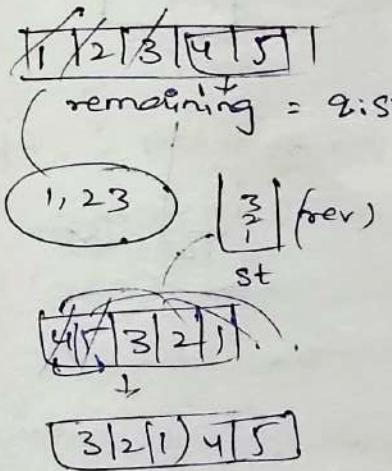
```

unordered_map<char,int> m;
for (int i=0 to n)
    if (m.find[S[i]] == m.end())
        Q.push(i);
    m[S[i]]++;
while (Q.size() > 0)
    if (m[Q.front()] > 1)
        Q.pop();
return Q.empty() ? -1 : Q.front();

```

6. Reverse first k elements of Queue (gfg)

$$k=3, \quad Q = [1|2|3|4|5]$$



① Container

② revers container

③ push them in queue

④ Remaining part

```

def reverseK(Q, k):
    if k > len(Q) or k == 0:
        return Q
    Stack = []
    for i in range(k):
        Stack.append(Q.popleft())
    while Stack:
        Q.append(Stack.pop())
    return Q

```

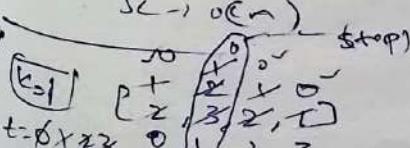
7. (2073) Time Needed to Buy Tickets

$k=1$ $(2, 3, 2, 1)$ no. of tickets to buy
 0 1 2 3 person

$$T = 0 + 2 + 3 + 2 + 1 = 8$$

$T \in O(n)$

$S \in O(n)$



```

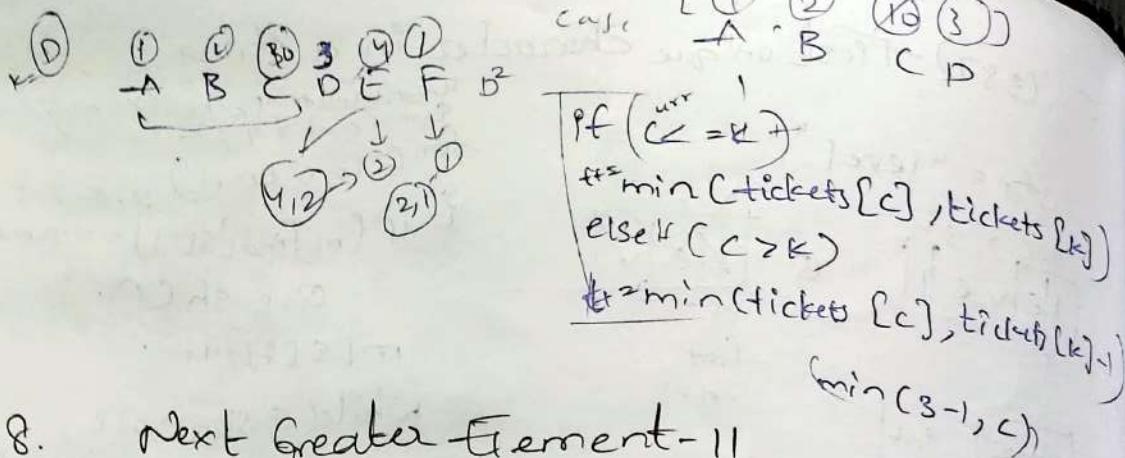
    Q.append(Cstack.pop())
    rem = len(Q) - k
    for i in range(rem):
        Q.append(Q.popleft())
    return Q

```

```

func (tickets, k)
{
    int time = 0;
    for (int i=0; i<tickets.length; i++)
    {
        if (i <= k)
            time = time + min(tickets[i], tickets[k]);
        else
            time = time + min(-tickets[i], -tickets[i-k] - 1);
    }
    return time;
}

```



8. Next Greater Element - II

(Circular)

nums = $\left[\begin{matrix} 1, 2, 3, 1, 3 \\ 3, 6, 5, 4, 2 \end{matrix} \right]$

nums:

3	6	5	4	2	3	6	5	4	2
0	1	2	3	4	5	6	7	8	9

ans:

6	-1	-1	-1	-1
0	1	2	3	4

0 (3)
1 (6)

→ next
green

1 (6)
2 (3)

vector<int> ans(n, -1)

stack<int> s

for (2n-1 to 0)

{ while(s.size() > 0 & nums[s.top()] \leq nums[i%n])

{ s.pop(); } } } }

ans[i%n] = s.empty() ? -1 : nums[s.top()];

s.push(i); "valid id"

return ans;

q. previous smaller element

TC - O(n)

SC - O(n)

arr[i] → immediately smaller
& ts on left

arr = [3, 1, 0, 8, 6]

→ -1 -1 -1 0 0



Stack

vector<int> ans

stack<int> s;

for (i=0; i<n; i++)

{ while (s.size() > 0 & s.top() >= arr[i])

{ s.pop(); }

? if (s.empty()) → -1

else → s.top()

? s.push(arr[i]);

C1856) Maximum Subarray Min-product:

nums

(integer array)

↳ subarray

minproducts = $\{ \text{sum}(\text{subarray}) \times \text{min}(\text{subarray}) \}$

mar (min product)

[1|2|3|2].

(sum * min)

$$[1] \Rightarrow 1 \cdot 1 = 1$$

$$[2] \Rightarrow 2 \cdot 2 = 4$$

$$[3] \Rightarrow 3 \cdot 3 = 9$$

$$[2] \Rightarrow 2 \cdot 2 = 4$$

$$[1, 2] \Rightarrow 3 \cdot 1 = 3$$

$$[2, 3] \Rightarrow 5 \cdot 2 = 10$$

$$[3, 2] \Rightarrow 5 \cdot 2 = 10$$

$$[1, 2, 3] \Rightarrow 6 \cdot 1 = 6$$

$$[2, 3, 2] \Rightarrow 7 \cdot 2 = 14$$

$$[1, 2, 3, 2] \Rightarrow 8 \cdot 1 = 8$$

max

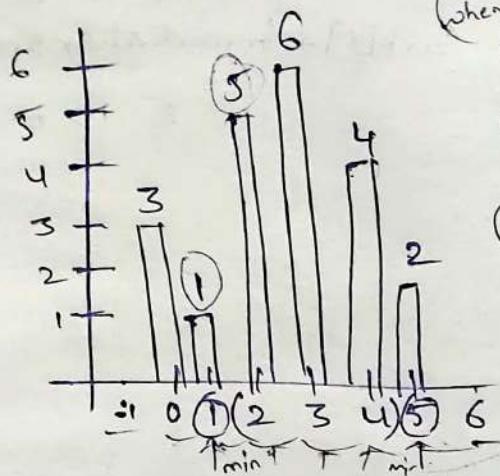
A B C D

$$4 + 3 + 2 + 1$$

$$n + (n-1) + (n-2) + \dots + 1$$

$$\frac{n(n+1)}{2} \Rightarrow O(n^2)$$

Time - O(n)



(when m is assumed as ∞)

$$\textcircled{3} [3] \Rightarrow 3 \cdot 3 \Rightarrow 9$$

$$\textcircled{1} [3, 1, 5, 6, 4, 2] \Rightarrow (4, 2) \Rightarrow (2)$$

$$\textcircled{5} [5, 6] \Rightarrow 5 - 11 \Rightarrow \underline{\underline{5}}$$

$$\textcircled{6} \quad [6] \Rightarrow 6, 6 \Rightarrow \underline{\underline{36}}$$

$$\textcircled{4} \quad [5, 6, 4] = 120 \rightarrow \textcircled{80}$$

$$\textcircled{2} \quad [5, 64, 2] \Rightarrow 2 \cdot 17 \Rightarrow 34$$

num =

3	1	5	6	4	2
0	1	2	3	4	5

presum =

0	3	4	9	15	19	21
0	1	2	3	4	5	6

$$\text{SmallerOnLeft} = \boxed{\begin{matrix} & 1 & 3 & 4 \\ \textcircled{-1} & -1 & 1 & 2 & 1 & 1 \end{matrix}}$$

Smaller On Right

$$\text{Sum} \Rightarrow \text{Resum } [rr] - \text{Resum } [r]$$

$$19 - 4 \Rightarrow 15$$

class sol{

```
public int maxSumMinProduct(int[] nums) {
```

```
{ int n = num.length;
```

long PSum [] = new long [n+1];

```
for(int i=0;i<n;i++)
```

$$\left\{ \begin{array}{l} \text{Psum}[i+1] = \text{Psum}[i] + \text{nums}[i] \end{array} \right.$$

int smallerOnleft[] = NSEL [num]

`int SmallerOnRight[] = NSER(nums);` (next smallest element on left) } (2)

int max = long. MIN_VALUE;

```
-for (int i=0 ; i<n ; i++)
```

```
int min = nums[0];
```

```
int lr = smallerOrEqual[i];
```

```
int rr = smallerOrRight[i];
```

```

long sum = psum[rr] - psum[lr+1];
long minProduct = min * sum;
max = Math.max(minProduct, max);
}
return (int)(max / 1000000007); // result10^9 + 7
}
}

```

-lit in 64 bit signed int

```

public static int[] NSEL(int[] nums)
{
    Stack<Integer> st = new Stack<>();
    int res[] = new int[nums.length];
    for (int i = 0; i < nums.length; i++)
    {
        while (st.size() > 0 & nums[st.peek()] >= nums[i])
        {
            st.pop();
        }
        if (st.size() == 0)
        {
            res[i] = -1;
        }
        else
        {
            res[i] = st.peek();
        }
        st.push(i);
    }
    return res;
}
}

```

```

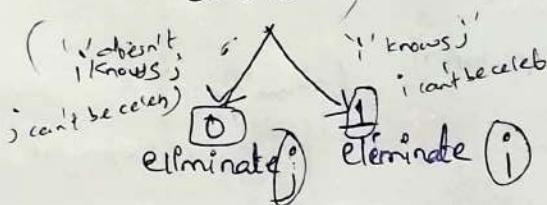
public static int[] NSER(int[] nums)
{
    for (int i = nums.length - 1; i >= 0; i--)
    {
        while (st.size() > 0 & nums[st.peek()] >= nums[i])
        {
            st.pop();
        }
        res[i] = st[-1] if a err (n) (x)
    }
    st.append(i);
}
}

```

10. ^{1nd} Celebrity problem

$$\text{err} = [0, 1, 0, 1, 0, 0, 0]$$

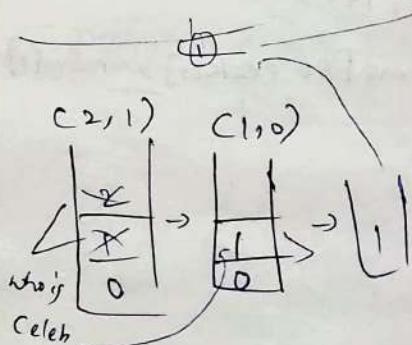
`arr[1][j] =`



0, 1, 2

arr[0][1]

`arr[1][2]`



	0	1	2
0	0	1	0
1	0	0	0
2	0	1	0

① everyone should know celeb)

② arr[celeb][i] = 0

Celebs shouldn't know anyone)

Tc : o(n)

Sci: Oct

```

stack<int>s;
for (i=0 to n-1)
    s.push(i)
while (s.size() > 1)
{
    i = s.top(); s.pop();
    j = s.top(); s.pop()
    if (arr[i][j] == 0)
        s.push(i)
    else
        s.push(j)
}

```

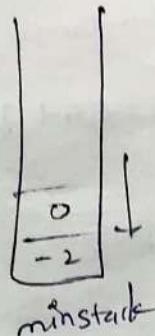
```

int celeb = s.topc ;
for (pk=0 ; i < n ; i++)
{
    if (pi == celeb) &&
        (arr[pi][celeb] == 0 || arr[celeb][pi] == 1)
    {
        return -1 ;
    }
}
return celeb ;
}

```

11. (155) min stack

push -2
 push 0
 push -3
 getMin
 pop
 top
 getMin



-3
0
-2

(top push)
S: O(2xn)

① $\text{structed pair } \langle \text{val}, \text{minval} \rangle$

S;

minstack() { }

-3 void push(end val)

-2 { if (S.empty()) }

{ S.push({val, val}); }

else

{ int minval = min(val),

S.top().second);

S.push({val, minval});

3 }

void pop()

{ S.pop(); }

3 void top()

{ return S.top().first; }

3 void getMin()

{ return S.top().second; }

3 }

SL: O(n)

②

min val = -2 -3
old new

push val < min val

$$\text{formula: } \boxed{\text{val}' = 2 \times \text{val} - \text{minval}}$$

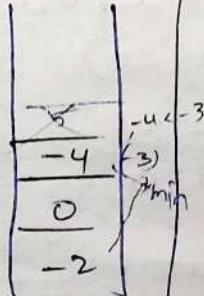
$$\text{minval} = \text{val} \quad \left(= 2(-3) - (-2) \right)$$

pop

$$\boxed{\text{old minval} = 2 \times \text{minval} - \text{val}'}$$

$$= 2 \times (-3) - (-4)$$

= E2



{ int getMin() noC1)

{ return minval;

3 }

void push(val)

{ if (S.empty())

 → S.push(val)

 → minval = val

else if (val < minval)

{ S.push(2 * val - minval)

 → minval = val

else

{ S.push(val)

3 }

void pop()

{ if (S.top() < minval)

{ minval = 2 * minval - S.top(); }

3 S.pop()

void top()

{ S.top() < minval

 → return minval

else

 → return S.top();

3 }

Q. (B4) Gas Station .
 gas = $\begin{bmatrix} \times & \times & \times & \times \\ 1, 2, 4, 5, 6 \end{bmatrix}$
 cost = $\begin{bmatrix} 3, 4, 1, 10, 1 \end{bmatrix}$

$$(i+1).start = TC[i][n] \\ currGas = SC[i][0](1) \\ CG = CG + gas[i] - cost[i]$$

→ station idx
 Surplus amt of Gas

- ① total gas < total cost $\Rightarrow -1$
 $21 < 19 \times$ (transpossible)

$$g = [2, 3, 4], c = [3, 4, 3] \\ g < c \quad \checkmark \text{ (not possible)}$$

② start=0; CG=0
 for(i=0 to n)

$$\{ currGas += (gas[i] - cost[i])$$

if (currGas < 0)

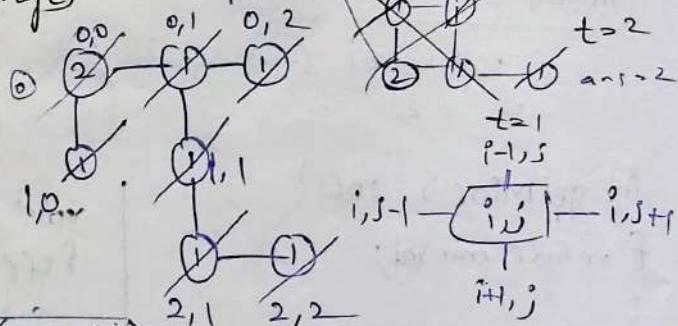
{ start=i+1

currGas = 0

return start;

13. ~ Rotting Oranges

rotten	2	1	1
fresh	1	1	0
empty	0	0	1



Q. $\langle \text{Pair} \langle \text{Pair} \langle \text{int}, \text{int} \rangle, \text{int} \rangle \rangle$ while (Q.size > 0)
 for (p=0 to n) {
 for (j=0 to m) {
 if (grid[i][j] == 2) {
 Q.push((i, j), 0);
 vis[i][j] = true;
 ans = 0 + 2 * 3^j;
 }
 }
 }

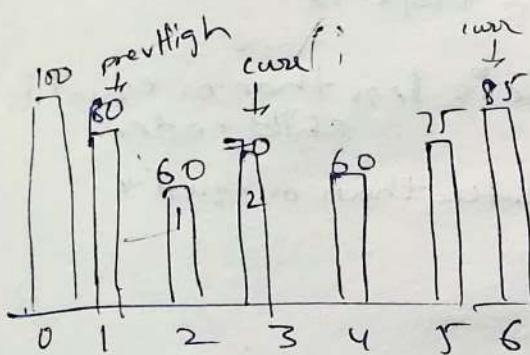
while (Q.size > 0)
 {
 i, j, time = Q.pop();
 ans = max(ans, time);
 if (p-1 >= 0 && !vis[p-1][j]) {
 if (grid[i-1][j] == 1) {
 Q.push((i-1, j), time+1);
 vis[i-1][j] = true;
 if (j+1 < m) right++;
 if (i+1 < n) bottom++;
 if (j-1 >= 0) left++;
 }
 }
 }
 // check for fresh oranges
 for (i=0 to n) {
 if (i==0 to m) {
 if (grid[i][j] == 1) {
 return -1;
 }
 }
 }
 return ans;
 }

ii) Stock Span (栈和数组)

price = [100, 80, 60, 70, 60, 75, 85]

Span = max no. of consecutive days from today

price[i] = today's price
include all day price



i=0 1
i=1 1
i=2 1
i=3 2
i=4 1
i=5 4
i=6 6

prevHigh -
↓
immediate greater value
for any day

stack
↑ prev values
greater values
for today (down)

$$Span = i - prevHigh$$

$$= 4 - 3 = 1$$

$$= 5 - 1 = 4$$

$$= 6 - 0 = 6$$

[1, 1, 1, 2, 1, 4, 6]

vector<int> ans;

Stack<int> S;

for (i=0; i < n; i++)

{ while (S.size() > 0 && Price[S.top()] <= Price[i])

{ S.pop();

if (S.size() == 0) \Rightarrow i + 1

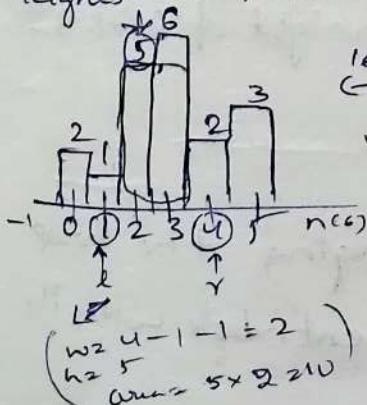
else \Rightarrow i - S.top()

S.push(i);

return ans;

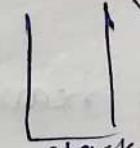
iii) Largest Rectangle in Histogram

heights = [2, 1, 5, 6, 2, 3]



left smaller nearest

width = r - l + 1



(1) right smaller nearest \rightarrow next greater

(2) left α \rightarrow previous smaller

right = [1 | -1 | 4 | 4 | -1 | -1 / end]

-1 | -1 | 1 | 2 | 1 | 4 |

ans = 0

for (i=0 to n)

{ currArea = height[i] * (r-l+1)

ans = max (currArea, ans)

return ans

right smaller

for (i=n-1 to 0):

{ while S && h[S.top()] >= h[i]):

{ S.pop();

if [i] == S.empty() ? n : S.top();

S.push(i);

while (S.empty() && S.pop());

left smaller

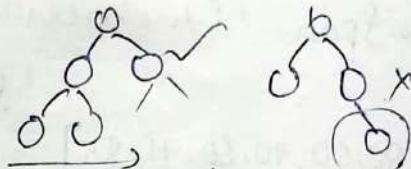
13/12/25

Heaps

Heaps

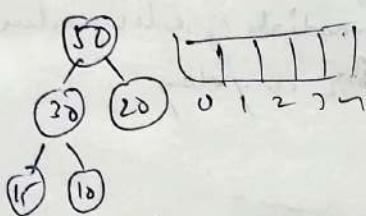


Complete Binary Tree + Heap property



① min heap \rightarrow value of node is less than or equal to its child nodes.

② max heap \rightarrow u is greater than or equal to



convert array into tree nodes:

A) If parent node is at i^{th} index

\hookrightarrow left child would be at $[2*i+1]$

\hookrightarrow right u u u $[2*i+2]$

B) If child node is at i^{th} index:

\hookrightarrow parent will be at $\left[\frac{N}{2}\right]-1$; (ceil)

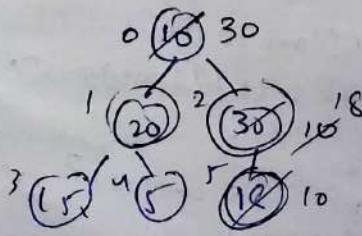
C) If height of a tree is h , max nodes in a complete binary tree will be $(2^h)-1$ [h always start from 0]

D) If there are N nodes, then height of Comp.B.T is $\lceil \log_2 N \rceil$

Heap operations

\rightarrow Heapify \rightarrow The process of rearranging the heap by recursively comparing a parent node with its child node.

\hookrightarrow Heapify given tree for the index 0 (maxheap)



$$\rightarrow 10(5 \overbrace{20, 6, 11}^8) \quad 8$$

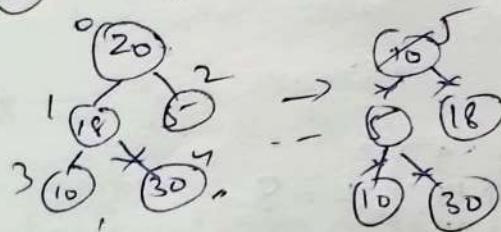
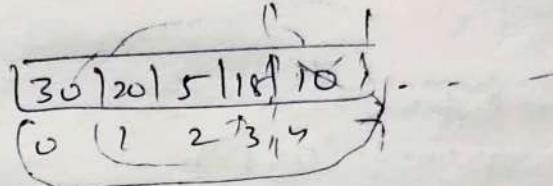
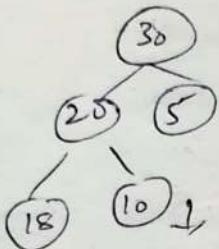
$\begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 5/2 = 2 \end{matrix}$

- | | |
|--|---|
| 1. Heaps
2. Heapsify
3. HeapSort
4. Min Heap
5. Max Heap | 6. Increase key
7. Decrease key
8. Insert in heap
9. Delete from heap
10. Complete Tree |
| | |

\rightarrow heapSort \rightarrow use heap property to sort an array

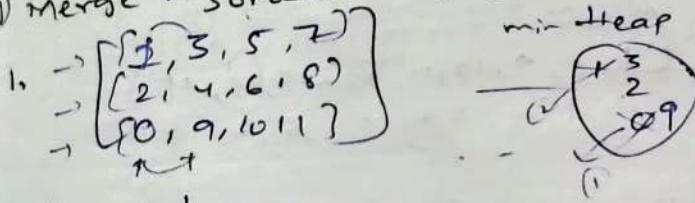
\hookrightarrow Build heap

\hookrightarrow use extract max (store at the end)



5, 10, 18, 20, 30

(q) Merge K Sorted Arrays:



import heapq

def mergeArrays(mat):

n = len(mat)

heap = []

res = []

for i in range(n):

if mat[i]:

- heapq.heappush(heap, (mat[i][0], i, 0))

while heap:

val, ri, ci = heapq.heappop(heap)

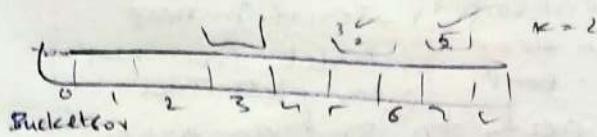
res.append(val)

if (ci+1) < len(mat[ri]):

- heapq.heappush(heap, (mat[ri][ci+1], ri, ci+1))

return res

(347)
2. k most frequent elements:



→ heap
• max/
• min/
• bucket

3. Heap Sort

Heapify nums = {5, 2, 3, 1}
whilenum: top(i)

/ mergesort

4) k-th smallest element

① minheap /

② Binary search

$$\begin{cases} x < k-1 \\ \geq k-1 \end{cases} \quad \begin{matrix} m \text{ R} \\ m \text{ L} \end{matrix} \quad \begin{matrix} s = m+1 \\ E = m-1 \end{matrix}$$

$$\text{mat} = \begin{pmatrix} 2 & 5 & 9 \\ 10 & 11 & 13 \\ 12 & 13 & 15 \end{pmatrix} \quad k=8$$

$$\begin{matrix} \min \text{ for extract!} \\ \text{top(i)} \end{matrix} \rightarrow \begin{pmatrix} 1, 5, 9, 10, 11, 12, 13, 13, 15 \\ 0 \ 1 \ 2 \ 3 \ 4 \ r \ 6 \ 7 \ 8 \end{pmatrix}$$

$$③ 8 \text{ is } \frac{8+(r-8)}{2} = 11 \quad \begin{matrix} x = 13 \\ 1 \quad 1 \end{matrix}$$

$$\begin{cases} x < k \\ x \geq k \end{cases} \quad \begin{matrix} 1 \\ 1 \end{matrix} \quad \begin{matrix} 1 \\ 1 \end{matrix}$$

min heap

Binary search

$$T$$

2

$$O(k \log n)$$

$O(n)$

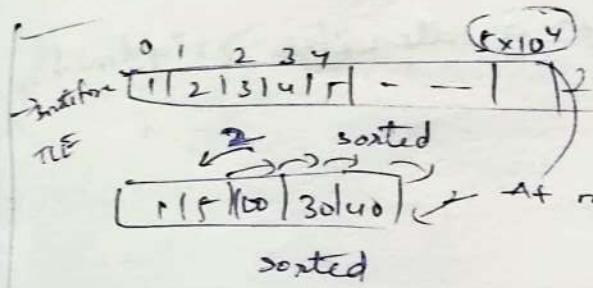
$$O(n \log \text{range})$$

$O(1)$

g) (295) Find median from Data Stream

$$arr = \{2, 3, 4\} \quad n$$

1, 3, 25 ← addnum & findmedian (ordered 1, 2)

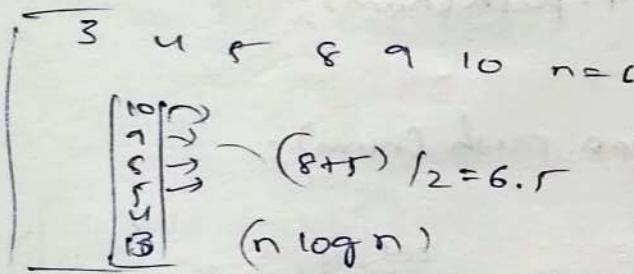


$$\text{even} = \left(\frac{n}{2} + \frac{n}{2} - 1 \right) / 2$$

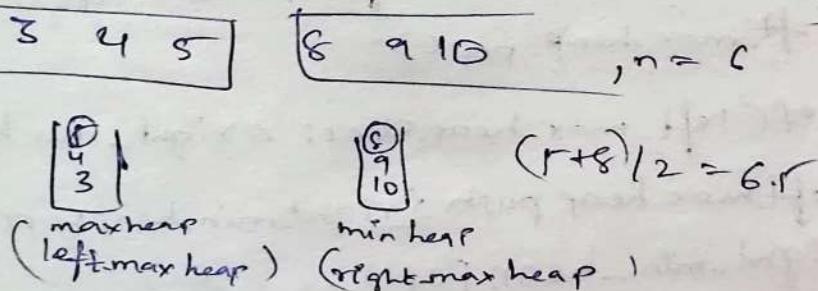
$$\text{odd} = \frac{n}{2} \Rightarrow \text{arr} \left[\frac{n}{2} \right] \rightarrow \text{ans}$$

at most 5×10^4 calls will be made to addnum & findmedian

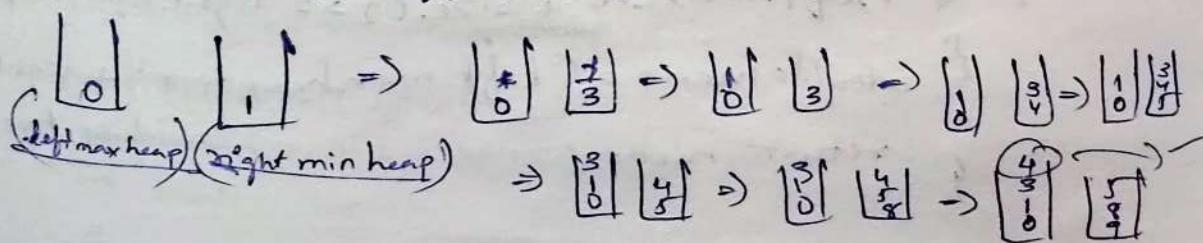
→ Heap ← (Need DS that can keep your data sorted, you don't need to sort it, just)



⇒ optimal approach



0 3 4 5 8 9, n=7



class medianFinder

public:

priorityqueue<int> left_max_heap;

priorityqueue<int>, vector<int>, greater<int> right_min_heap;

MedianFinder()

{

}

void addnum(int num)

{ if (left_max_heap.empty() || num < left_max_heap.top())

{ left_max_heap.push(num);

}

else

{ right_min_heap.push(num);

}

if (left_max_heap.size() - right_min_heap.size() > 1)

{ right_min_heap.push(left_max_heap.top());

left_max_heap.pop();

else if (left_max_heap.size() < right_min_heap.size(),

{ left_max_heap.push(right_min_heap.top());

right_min_heap.pop();

double findmedian()

{ if (left_max_heap.size() == right_min_heap.size(),

{ double mean = (left_max_heap.top() + right_min_heap.top()) / 2,

return mean;

return left_max_heap.top();

,

,

6) (632) Smallest Range covering Elements from k lists
 (Naive approach)
 [{4, 10, 15, 24, 26}, {0, 9, 12, 20}, {5, 18, 22, 30}]

[0, 4, 15, 9, 10, 12, 15, 18, 20, 22, 24, 26, 30]

(0, 4), (0, 5)

$c_{i,r} \times c_{i,a}$

$c_{i,b}$

$(20, 22) \rightarrow 20, 21, 22 \times (20, 24) \rightarrow 20, 21, 22, 23, 24$

$(20, 24) \rightarrow 6$

Improving -

(The lists are sorted \rightarrow we didn't use any benefit out of it)

[4 min max
 0 5] $c_{i,b} \Rightarrow (0, 5)$

{4 10 15 24 26},
 {0 9 12 20},
 {5 18 22 30} x and

0 1 2
 i j k

4 10 15 20 24 26 28 30
 0 9 12 20 21 22 23 24
 [0, 5] $\rightarrow [5, 10] \rightarrow [10, 15]$

cif '5' is decreased 3rd arr we can't find
 so increase min element next

min max 5-0=5

T.C = $O(n \times k)$ S.C = $O(k)$

↓
 SmallestRange (nums)

minElk =
 minElkIdx =
 maxElk =

```
int k = nums.size();
vector<int> vec(k, 0); // {0, 0, 0, ...}
```

[10, 20]
 ↓ (5)
 [20, 24]
 ↓ (2)

```
vector<int> resultRange = {-10000, 10000};
```

```
int minElk = INT_MAX;
```

```
int maxElk = INT_MIN;
```

```
int minElkListIdx = 0;
```

```
for (int i = 0; i < k; i++)
```

```
    int listIdx = i;
```

```
    int elIdx = vec[i];
```

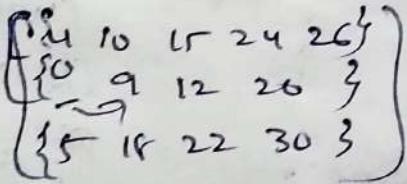
```
    int element = nums[listIdx][elIdx];
```

```

if (element < minEl)
{
    minEl = element
    minElListIdx = lIdx;
}
maxEl = max(maxEl, element);
l-range
if ((maxEl - minEl) < (resultRange[1] - resultRange[0]))
{
    resultRange[0] = minEl;
    resultRange[1] = maxEl;
    // shorten this range by moving minEl idy
    int nextIndex = vec[minElListIndex] + 1;
    if (nextIndex >= nums[minElListIndex].size())
    {
        break;
    }
    vec[minElListIndex] = nextIndex;
}
return resultRange;

```

heap - T.C = O(n x log k)



maxEl = 5

minEl = 0

listIdx = 1

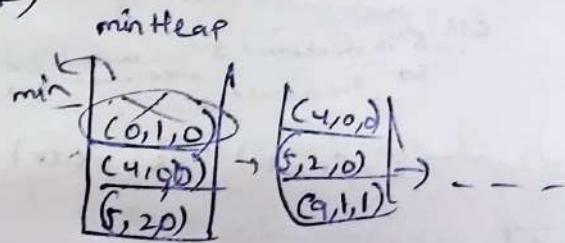
idx = 0

((idx+1) < nums[listIdx].size())

{ nextEl = nums[listIdx][idx+1];

PQ.push(nextEl, listIdx, idx+1);

} maxEl = max(maxEl, nextEl);



14/12/2005

Tries

1. (a) Longest common prefix

strs = ["flower", "flow", "flight"]

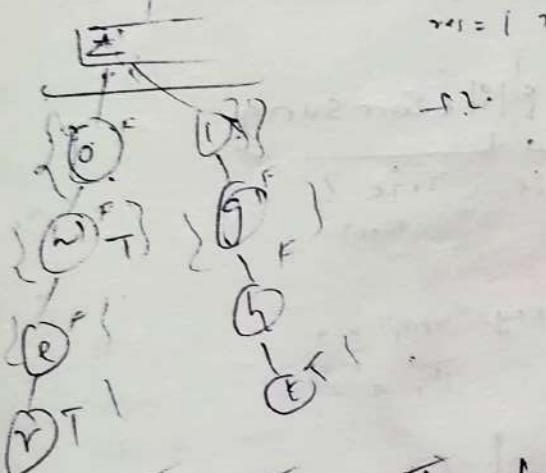
str = fl

Trade? self.data = 23

? self.end = false

(- str.sort(),
- compareStrs[0]
[] str[-1] 2-17
flight - flower, flower)

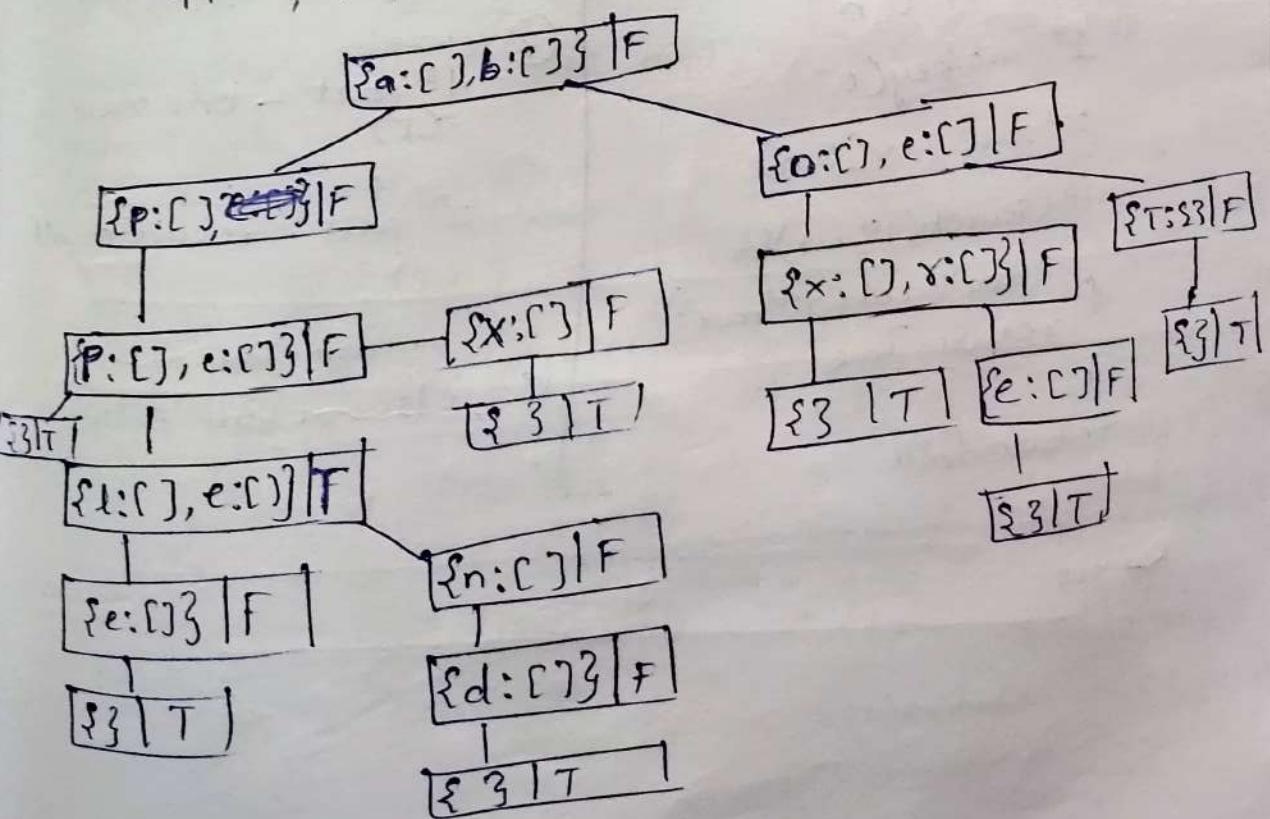
[{e: []} | F]



(dict) self.data (boolean)
self.next

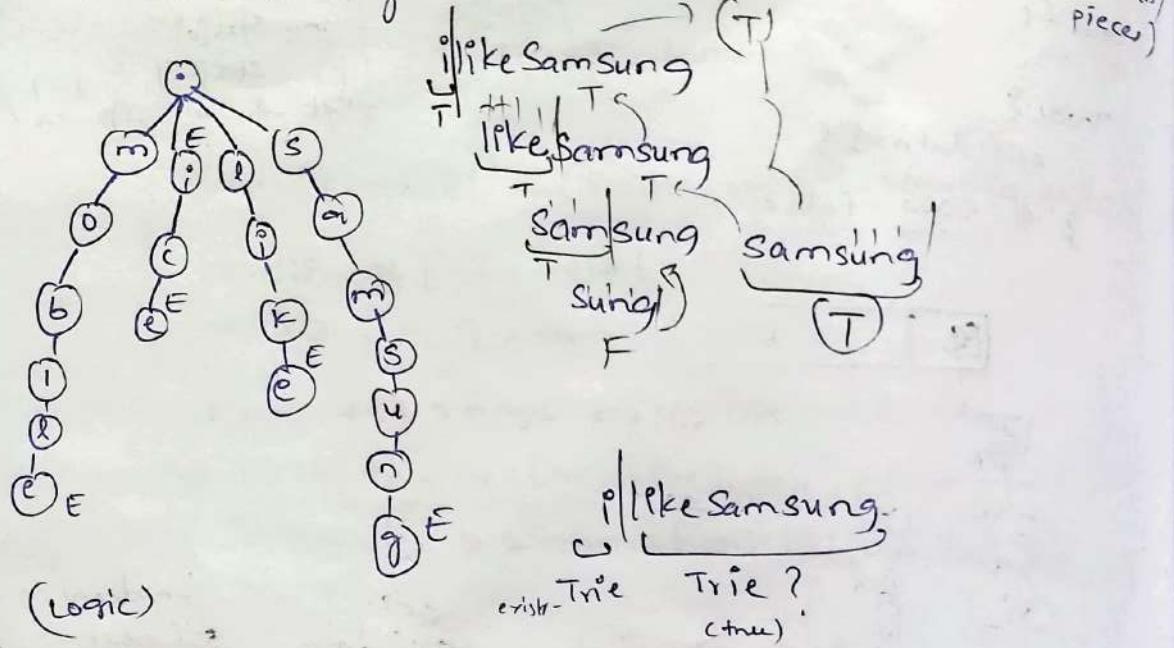
[{ch: 'f'} | F]

- apple, app, apex, bet, append, ape, box, bone



2. Word Break problem

word = { i, like, Sam, Samsung, mobile, ice }
 key = "likeSamsung" (An possible substr. piece)



```
bool wordBreak(string key) // key = "apple"
{ if(key.length == 0)
    return true;
```

```
for (int i=1 to n)
{ 1st part key(0,i)
   2nd part key(i,n)
   if (Search(1st part))
       wordBreak(2nd part))
   return true
}
return false
```

↑ me

use memo={ } else

- ① all possible parts string
- ② 1st part
1st part - trie search (T)
- ③ 2nd part +
2nd part - recursive call (T)
- ④ return True
- ⑤ else return False

memo={ }
 if(memo[i])
 { if (1st part
 return memo[i]) }

phone directory

3. Insert all contacts into Trie

for each char in contact
move/create child node
mark end of word

For each character in search-string's

Extend prefix

If prefix exists in Trie

Traverse to that node

collect all words below it using DFS

else

store ["0"] (no match)

Return all prefix results.

4 Implement a Trie

- Insert

- Search

- startsWith (prefix)

root.child[?]

↓

!null

↓

EOW = True

① temp = char

② compare temp & ans

③ recursive child

char) p + 'a'
↓
+ 'a' → b

5. Longest word with all prefixes.

\rightarrow (node.end = True)
words = ["ai", "banana", "app", "appl", "ap", "apply", "apple"]

ans = "apple"

apple app(?)
lexicographic (ascending)

longestword (root, $\text{string}^{\text{temp}}$)

static string ans = " " if (root == null)
{ return; }

{ for (int i = 0; i < 26;)

{ if (root.children[i] != null &&
root.children[i].eow == T)

{ temp.append((char)(p + 'a'))

if (temp.length() > ans.length())

{ ans = temp

longestword (root.children[i], temp)

temp.deleteCharAt (temp.length() - 1)

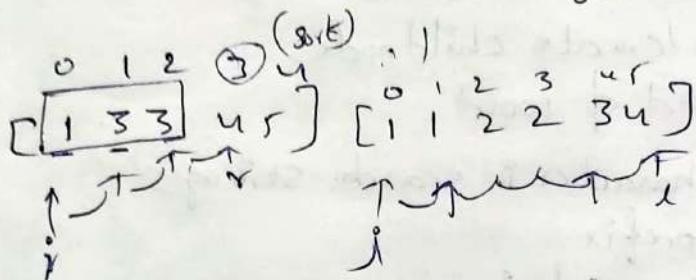
2012

Greedy

1. (15) Assign cookies

$$\text{greed} = [1 \ 5 \ 3 \ 3 \ 4] \quad s = [4 \ 2 \ 1 \ 2 \ 1 \ 3]$$

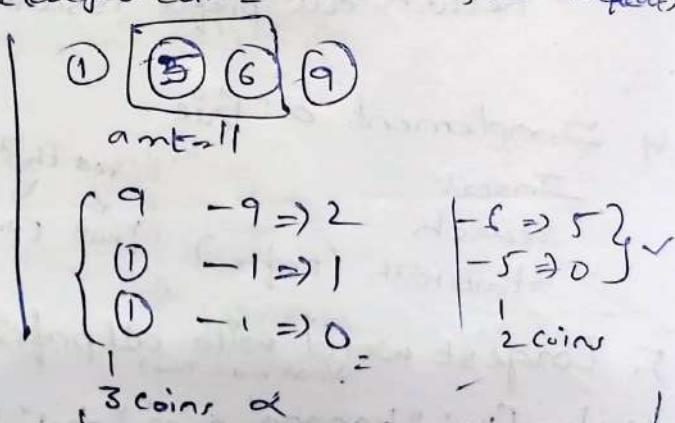
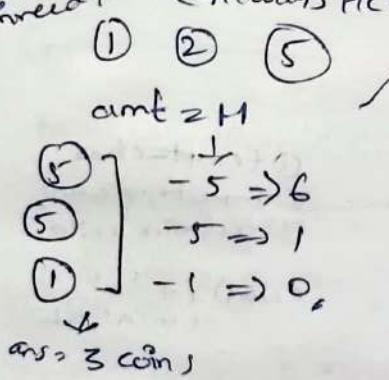
$O(n \log n + m \log m + m)$



return: $g[i] \leq s[i] \quad i <= 1 \quad \text{ans} = 3$

$3 \leq 3 \quad \text{can satisfy 3 child}$
 $3 \leq 4 \quad \text{can satisfy 4 child}$

2. (322) Coin change

(greedy) — (Always pick the largest coin \leq remaining amt. Repeat)

PP
amt = 11

[1 5 6 9]

This demands dp for optimality.

amt	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
Min coins needed		•	•	• • •	•	•	•	•	• •	•	• •	• •

$\left\{ \begin{array}{l} \text{amt to make} = 3 \\ \text{coin choice} = 1 \\ \text{rem} = 2 \end{array} \right. \quad \left. \begin{array}{l} 1+2 = 3 \\ 1+1+1 = 3 \end{array} \right.$

• represents the coin we pick + how many more coins we need

$\left\{ \begin{array}{l} \text{amount to make} = 5 \\ \text{coin choice} = 1 \\ \text{remainder} = 4 \end{array} \right. \quad \left. \begin{array}{l} 1+4 = 5 \\ 1+1+1+1+1 = 5 \end{array} \right.$

$\left. \begin{array}{l} 1+1+1+1 = 4 \\ 1+1+1+1+1 = 5 \end{array} \right. \quad \left. \begin{array}{l} 1+1+1+1 = 4 \\ 1+1+1+1+1 = 5 \end{array} \right.$

$5 > 1$

$$\begin{array}{l}
 \text{amt to make} = 8 \\
 \text{coin choice} = 1+2 = 3 \\
 \text{remainder} = 7
 \end{array}
 \quad
 \left| \begin{array}{c}
 5+3=4 \\
 3 \\
 2
 \end{array} \right| \quad
 \left| \begin{array}{c}
 6+2=3 \\
 2
 \end{array} \right|$$

$$\begin{array}{l}
 \text{amt to make} = 9 \\
 \text{coin choice} = 1+3 = 4 \\
 \text{remainder} = 8
 \end{array}
 \quad
 \left| \begin{array}{c}
 5+4=9 \\
 4 \\
 3
 \end{array} \right| \quad
 \left| \begin{array}{c}
 6+3=9 \\
 3
 \end{array} \right| \quad
 \boxed{9}$$

$$\begin{array}{l}
 \text{amt} = 11 \\
 \text{choice} = 1+2 = 3 \\
 \text{choice} = 2+10
 \end{array}
 \quad
 \left| \begin{array}{c}
 5+1=6 \\
 6
 \end{array} \right| \quad
 \left| \begin{array}{c}
 6+1=7 \\
 5
 \end{array} \right| \quad
 \left| \begin{array}{c}
 7+2=9 \\
 2
 \end{array} \right| \quad :$$

int coinchange(int[] coins, int amt)
 { if (amt < 1) return 0;
 // create DP array
 int[] mincoinsDP = new int[amt+1];

for (int i=1; i <= amt; i++)

{ mincoinsDP[i] = Integer.MAX_VALUE;

// Try each coin

for (int coin : coins)

{ if (coin <= i && mincoinsDP[i-coin] != Integer.MAX_VALUE)

{ mincoins[i] = Math.min(mincoinsDP[i], mincoinsDP[i-coin] + 1);

}

if (mincoinsDP[amt] == Integer.MAX_VALUE)

 return -1;

 return mincoinsDP[amt];

}

Fractional knapsack

problem

$$w=15 \\ n=7$$

max prof/order

$$\textcircled{I} \quad 47.25$$

min weight order

$$\textcircled{II} \quad 4.6$$

Profit/weight order

$$\textcircled{III} \quad 51$$

(Max prof/wt ratio)

$$\begin{array}{r} 5 \\ 6 \\ 4 \\ 3 \\ 2 \\ 1 \end{array} \begin{array}{r} 15,4 \\ 3,4 \\ 12,4 \\ 3,4 \\ 1,4 \\ 0,4 \end{array}$$

$\frac{15}{3} = 5$

$\frac{12}{3} = 4$

$\frac{3}{3} = 1$

$\frac{1}{1} = 0$

$\frac{0}{0} = 0$

$\text{gcd}(9,6) = 3$

$\text{gcd}(3,4) = 1$

$\text{gcd}(1,4) = 1$

$\text{gcd}(0,4) = 0$

objects - 1 2 3 4 5 6 7

profit - 5 10 15 7 8 9 4

weight - 1 3 5 4 1 3 2

P/w - 5 3.3 3 1.75 8 3 2

Objects	profit (P)	weight (w)	Remaining wt
5	8	1	15 - 1 = 14
1	5	1	14 - 1 = 13
2	10	3	13 - 3 = 10
3	15	5	10 - 5 = 5
6	9	3	5 - 3 = 2
7	4	2	2 - 2 = 0
51			

pairs, sort (key = lambda x: x[1])(sort by second value)

4. max length of pair chain

5. Activity Selection (greedy)

Same as Job scheduling

without overlapping

activities = sorted(zip(start, finish), key=lambda x: x[1])

start, finish = zip(*activities)

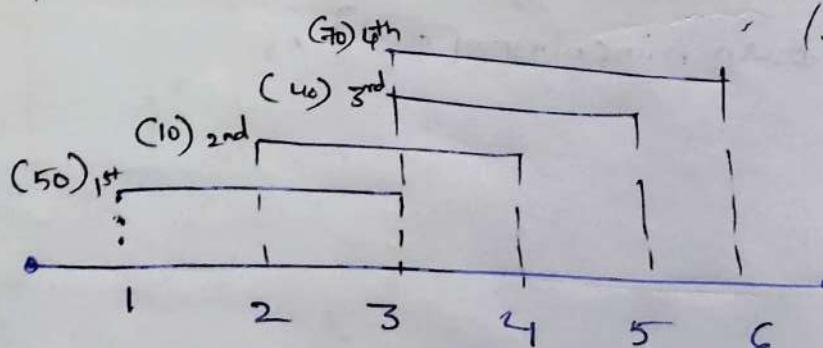
(1, 2) (3, 5) (6, 8)

(2, 3) (4, 6) (5, 7)

(3, 4) (5, 8) (6, 9)

(4, 5) (6, 7) (7, 9)

6. (1235) Maximum profit in Job Scheduling



50 + 40 | 50 + 70 | 10 | 40 | 70
90 | 120

{ s, e, pg } \rightarrow "cost based on 's'

{ {1, 3, 50}, {2, 4, 10}, {3, 5, 40}, {3, 6, 70} }

↑ 3,2 ×

(50)

③ {1, 3, 50}
taken / \ not taken

next job?

getnextjob
(currJobEnd <=
nextJobSt)

④ {3, 5, 40} {3, 6, 70} (70)

(+) taken = 50 + solve(nextJob)

+ 90 / / \ \ (120) (2) not taken = solve(nextJob)

class Solution

```
{ public:  
    int n; int memo[5000];  
    int l; int currentJobEnd;  
    int getNextIndex (vector<vector<int>>&array, int l,  
                      int currentJobEnd)  
    { int r = n-1;  
        int result = n+1;  
        while (l <= r)  
        { int mid = l + (r-l)/2;  
            if (array[mid][0] >= currentJobEnd) // we can make  
                // this task  
            { result = mid;  
                r = mid-1;  
            } else {  
                l = mid+1  
            }  
        }  
        return result;  
    }  
    int solve (vector<vector<int>>&array, int i)  
    { if (i >= n)                                          // if memo[i] != -1  
        return 0;                                                  return memo[i];  
    int next = getNextIndex(array, next, array[i][1]);  
    int taken = array[i][2] + solve(array, next);  
    int notTaken = solve(array, i+1);  
    return max(taken, notTaken);  
}
```

```

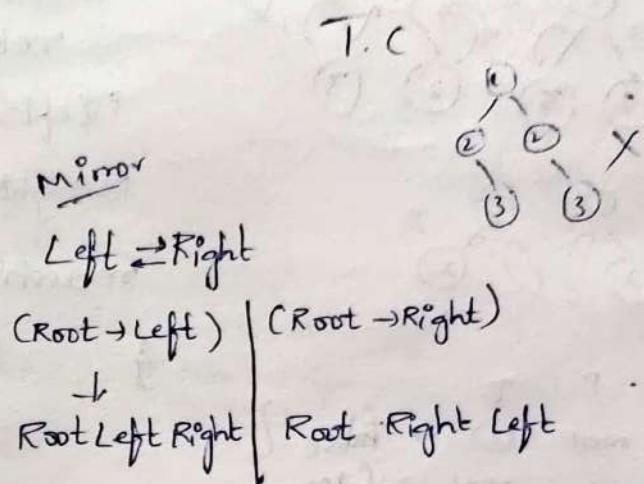
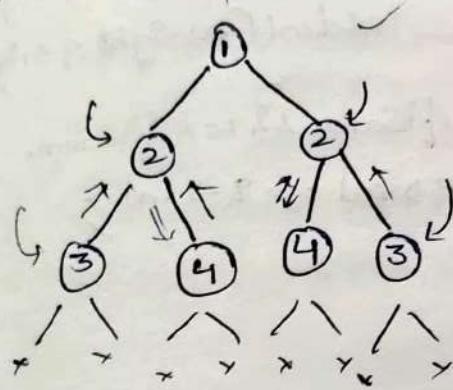
int jobscheduling(vector<int>& startTime, vector<int>& endTime,
                  vector<int>& profit)
{
    n = startTime.size()
    vector<vector<int>> array(n, vector<int>(3, 0));
    for(int i=0; i < n; i++)
    {
        array[i][0] = startTime[i];
        array[i][1] = endTime[i];
        array[i][2] = profit[i];
    }
    auto comp = [&](auto& vec1, auto& vec2)
    {
        return vec1[0] <= vec2[0];
    };
    sort(begin(array), end(array), comp);
    return solve(array, 0);
}

```

25/12/25

Binary Trees

- Inorder, preorder, postorder

4. Symmetric Tree

```
bool isSymmetric (TreeNode* root)
```

```
{ return root == NULL || isSymmetricHelp (root->left, root->right)
```

}

```
bool isSymmetricHelp (TreeNode* left, TreeNode* right)
```

```
{ if (left == NULL || right == NULL)
```

```
    return left == right;
```

```
    if (left->val != right->val) return false
```

```
    return isSymmetricHelp (left->left, right->right)
```

```
} } && isSymmetricHelp (left->right, right->left)
```

5. Min Distance b/w BST nodesBST: sorted \rightarrow Inorder

left subtree

root \rightarrow , prev = root

right subtree

TreeNode* prev = NULL

int mindist (root)

{ if (root == NULL)

return INT_MAX;

ans = INT_MAX;

if (root->left)

{ left_min = mindist (root->left) } - ①

{ ans = min (ans, left_min); }

if (prev != NULL)

{ ans = min (ans, root->prev) } - ②

prev = root;

if (root->right)

{ right_min = mindist (root->right) } - ③

{ ans = min (ans, right_min); }

prev = NULL

52

52

82

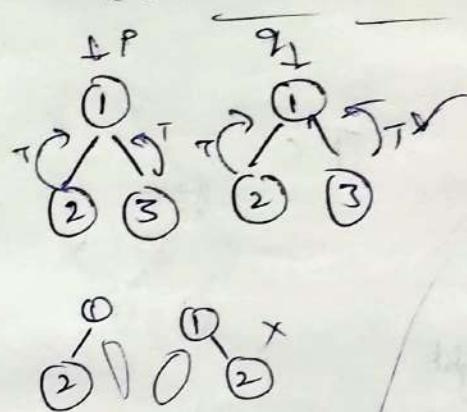
82

83-88

3

{ ret ans; }

6. (100) Same Tree (Identical Tree)



bool IsIdentical (P, Q)

? If ($P == \text{NULL}$ || $Q == \text{NULL}$)

return $P == Q$

IsLeftSame = IsIdentical ($P \rightarrow \text{left}$, $Q \rightarrow \text{left}$)

IsRightSame = IsIdentical ($P \rightarrow \text{right}$, $Q \rightarrow \text{right}$)

return IsLeftSame & IsRightSame

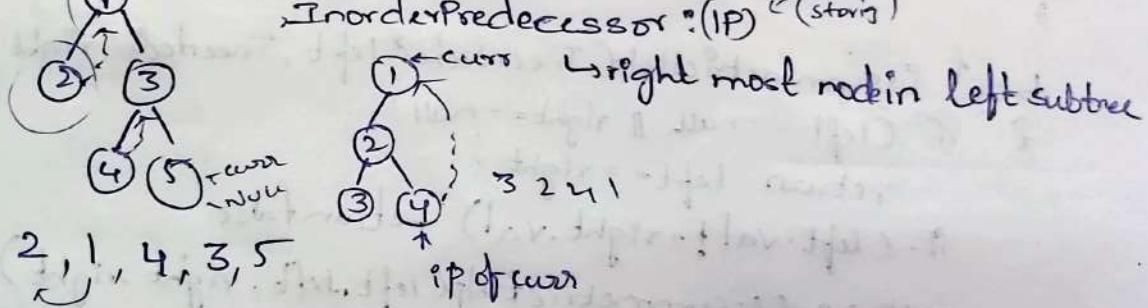
& $P \rightarrow \text{val} == Q \rightarrow \text{val}$;

P	Q
NULL	$1 \rightarrow \text{false}$
1	$\text{NULL} \rightarrow \text{false}$
NULL	$\text{NULL} \rightarrow \text{True}$

F. Morris Inorder Traversal

recursion ✗
iteration ✓

InorderPredecessor : (IP) (Stack)



while ($\text{curr} \neq \text{NULL}$)

if ($\text{curr} \rightarrow \text{left} == \text{NULL}$)

IP Point curr

curr = curr \rightarrow right

else

find InorderPredecessor (IP)

if ($IP \rightarrow \text{right} == \text{NULL}$)

- IP \rightarrow right = curr // create thread

curr = curr \rightarrow left

else

IP \rightarrow right = NULL // delete thread

// Point curr / res.append(curr.val)

curr = curr \rightarrow right

{
 IP = curr \rightarrow left
 while ($IP \rightarrow \text{right} \neq \text{NULL}$)
 if ($IP \rightarrow \text{right} \neq curr$)
 IP = IP \rightarrow right;
 }
}

8. Diameter of Binary Tree

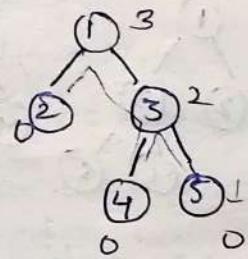
ans = 6; 3;

int height(root)

{ if (root == null) return 0;

leftHt = ht (root->left);

rightHt = ht (root->right)



ans = max(LH+RH, ans)

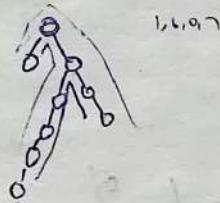
/* extra ht func
for diameter

return max(LH, RH)+1;

}

height(root)

return ans



ans = 6; 3;

int height(root)

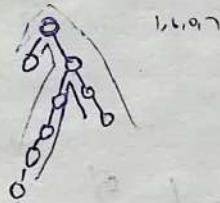
{ if (root == null) return 0;

leftHt = ht (root->left);

rightHt = ht (root->right)

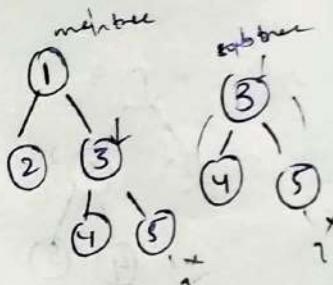
int height(root)

return ans



10) ^(Q2)

Subtree Tree of Another Tree



bool IsSubtree(TreeNode* root, TreeNode* subRoot)

```
{
  if (root == NULL || subRoot == NULL)
    return root == subRoot;
}
```

if (root->val == subRoot->val) {

isIdentical(root, subRoot);
 return true;

return IsSubtree(root->left, subRoot);

|| IsSubtree(root->right, subRoot);

bool isIdentical(TreeNode* p, TreeNode* q){

```
{
  if (p == NULL || q == NULL)
    return p == q;
}
```

return p->val == q->val

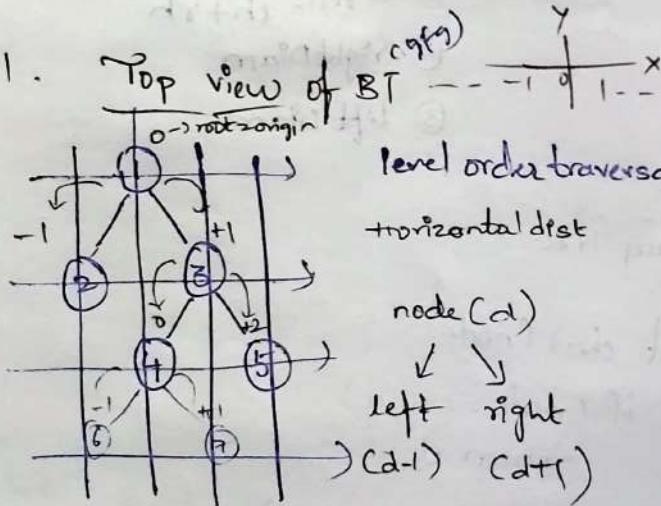
&& isIdentical(p->left, q->left)

&& isIdentical(p->right, q->right)

3

11.

Top view of BT



level order traversal
horizontal dist

node (cd)

left right
(cd-1) (cd+1)

$O(n \times \log m)$ map finds()
in cur

HD	Node
0	1
-1	2
+1	3
+2	5

void topView(TreeNode* root)

```
{
  queue<pair<TreeNode*, int>> q;
  map<int, int> m;  // HD, node var.
  while (q.size() > 0) {
    TreeNode* curr = q.front().first;
    int currHD = q.front().second;
    q.pop();
}
```

if (m.find(currHD) == m.end())

m[currHD] = curr->data;

if (curr->left != NULL) {

q.push({curr->left, currHD - 1});

if (curr->right != NULL) {

q.push({curr->right, currHD + 1});

for (auto it : m)
 cout << it.second << "

12) (513) Find bottom-left Tree value

CDFS)

Approach 1 :

root, depth = 0

maxDepth = $\max\{1, 2\}$

if (depth > maxDepth)

maxDepth = depth;

result = root->val;

T.C = O(n)

S.C = Aux.S.P = O(1)

$O(\log n)$ $O(\text{depth} \times 2^{\text{depth}})$

Approach - 2 :

normal

(BFS)

Approach - 3 : (Level By Level processing)

Level-0 [2]

L-1 [3] [1]

L-2 [4] [5] [6]

① root \rightarrow right

② root \rightarrow left

int maxDepth;

int bottomLeft;

void dfs(TreeNode* root, int curDepth)

{ if (!root) { return; }

if (curDepth > maxDepth)

{ maxDepth = curDepth;

bottomLeft = root->val;

dfs(root->left, curDepth+1);

dfs(root->right, curDepth+1);

dfs(root, 0);

return bottomLeft;

while (!que.empty())

{ node = que.front(); que.pop();

res = node->val;

if (node->right)

que.push(node->right);

if (node->left)

que.push(node->left);

int bottomLeft;

que.push(root);

while (!que.empty())

{ int n = que.size();

while (n--)

{ TreeNode* node = que.front();

que.pop();

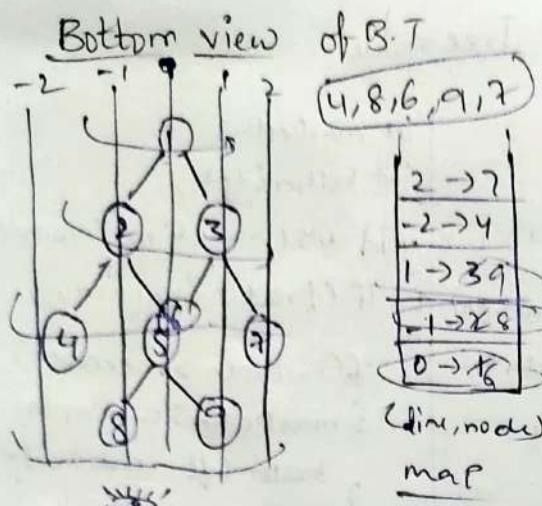
bottomLeft = node->val;

if (node->right)

que.push(node->right);

if (node->left)

que.push(node->left);



5	1
(9, +)	
(8, -1)	
(7, 2)	
(6, 0)	
(5, 0)	
(4, -2)	
(3, +1)	
(2, -1)	
(1, 0)	

node = 1 X 3 X 4 P 6 X 8 9
4 X 0 P 8 X 2 X 1 + 1

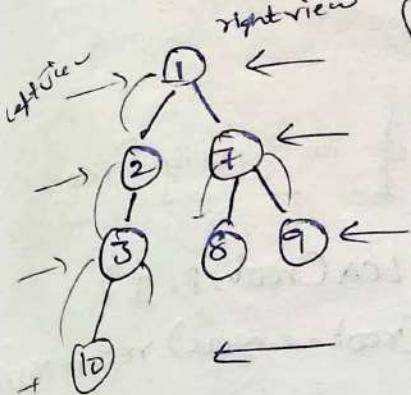
{
 └ bottomview(node root)

```

ArrayList<Integer> ans = new ArrayList<>();
if (root == null) return ans; ret.hd = 0
Map<Integer, Integer> map = new TreeMap<>();
Queue<Node> q = new LinkedList<Node>();
root.hd = 0;
q.add(root);
while (!q.isEmpty())
{
    Node temp = q.remove();
    int hd = temp.hd;
    map.put(hd, temp.data);
    if (temp.left != null)
    {
        temp.left.hd = hd - 1;
        q.add(temp.left);
        if (temp.right != null)
        {
            temp.right.hd = hd + 1;
            q.add(temp.right);
        }
    }
}
for (Map.Entry<Integer, Integer> entry : map.entrySet())
{
    ans.add(entry.getValue());
}
ret ans;
    
```

3 3

Right view



rightview { 1 7 9 10 }

L: 1 <= i <= n rightsideview (TreeNode root)

```

    List<Integer> res = new ArrayList<>();
    if (root == null)
        return res;
    Queue<TreeNode> queue = new LinkedList<>();
    queue.offer(root);
    //bfs
    while (!queue.isEmpty())
    {
        int size = queue.size();
        for (int i = 0; i < size; i++)
        {
            TreeNode node = queue.poll();
            res.add(node.val);
            if (node.left != null)
                queue.offer(node.left);
            if (node.right != null)
                queue.offer(node.right);
        }
    }
    return res;
}

```

Left view

```

    if (size > 0) {
        node node = queue.peek();
        res.add(node.val);
    }
}

```

TC → O(n)

SC → O(m) m = max

x

Left view

3 3 3 3

retres

```

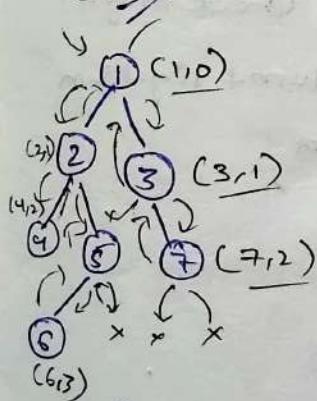
        size--;
        if (size == 0)
    {
        res.add(node.val)
    }
}

```

}

rec
start

Right view/Left view



3

0

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

21

22

23

24

25

26

27

28

29

30

31

32

33

34

35

36

37

38

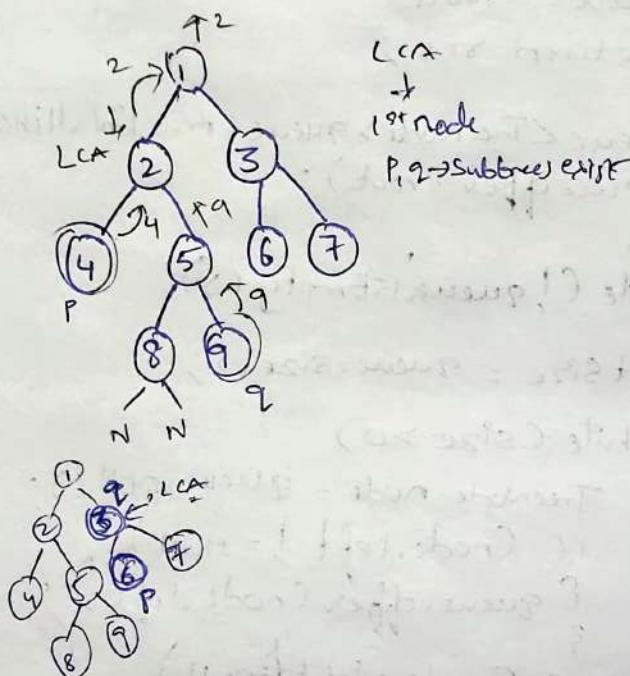
39

40

14) (102) Level order traversal

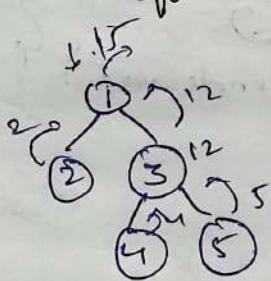
15) (1611) Maximum Level sum of a BT

16) (236). Lowest Common Ancestor of a Binary Tree



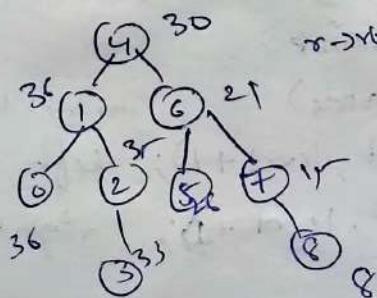
```
Node* LCA(Croot, P, Q)
{
    if (Croot == NULL) return NULL;
    if (Croot == P || Croot == Q) return Croot; "LCA"
    leftLCA = LCA(Croot->left, P, Q);
    rightLCA = LCA(Croot->right, P, Q);
    if (leftLCA != rightLCA)
        return root;
    else if (leftLCA != NULL)
        return leftLCA;
    else
        return rightLCA;
}
```

17) Transform to Sumtree



```
int sumTree(Croot)
{
    if (Croot == NULL) return 0;
    leftSum = sumTree(Croot->left);
    rightSum = sumTree(Croot->right);
    Croot->data += leftSum + rightSum;
    return Croot->data;
}
```

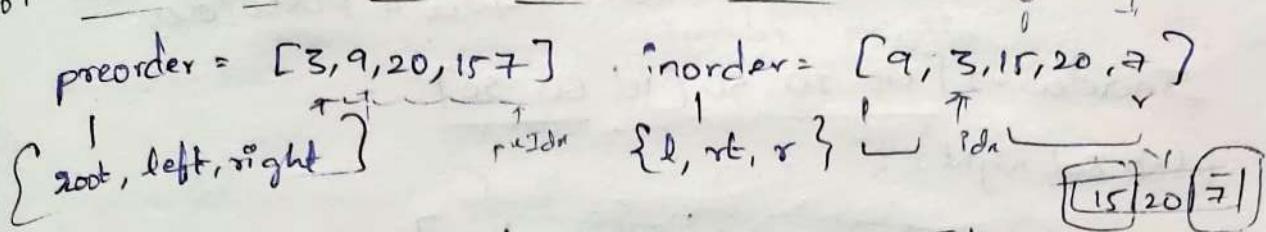
(cost) BST to GBT (greater sum tree)



→ V → l. BT To GBT (root)

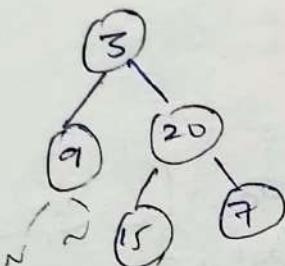
```
{ Self.total = 0
def dfs(node):
    if not node: return
    dfs(node.right)
    self.total += node.val
    dfs(node.left)}
```

MB. 1105) Build Tree from preorder & Inorder



left : l to inIdx - 1 inPdx

right : inIdx + 1 to r



buildTree (&preorder[], &inorder[], &preIdx, left, right)
{ Node* root = preorder[preIdx]; if (left > right) return NULL;
 preIdx++

 inIdx = search (preorder, left, right, preorder[preIdx])

 root->left = BT (preorder, inorder, preIdx, left, inIdx - 1)

 root->right = BT (preorder, inorder, preIdx, inIdx + 1, right)

 return root;

return BT (preorder, inorder, preIdx, 0, inorder.size() - 1);

int search (vector<int>& inorder, int left, int right, int val)

{ for (int i = left; i <= right; i++)
 { if (inorder[i] == val)
 { return i;
 } }

(preIdx, left, right)

(0, 0, 4)

 └─┐ ┌─┘
 (1, 0, 0) (2, 2, 4)

 └─┐ ┌─┘ └─┐ ┌─┘
NULL NULL (3, 2, 2) (4, 4, 4)

(b) (106) Build tree from Inorder & Postorder

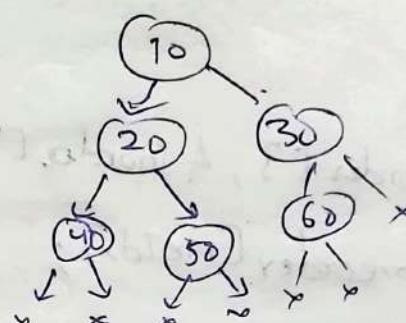
Inorder $\rightarrow [40 \underset{\text{is}}{20} 50] \underset{\text{ie}}{[10 \underset{\text{left}}{60} 30]}$

(Left Root Right) 0 1 2 3 4 5

Postorder $\rightarrow [40 \underset{\text{left}}{50} 20] \underset{\text{right}}{[60 \underset{\text{left}}{30} 10]}$

(Left Right Root) $ps \uparrow pe-1 \uparrow pe$

In $[40] \underset{\text{left}}{20} [50]$



Inorder $\rightarrow [10 \underset{\text{left}}{20} 30]$

Postorder $\rightarrow [40 \underset{\text{left}}{50} 20] \underset{\text{right}}{[60 \underset{\text{left}}{30} 10]}$

map

$Tc \rightarrow \Theta(n) \times (\Theta(n))$

$S \rightarrow \Theta(n)$

In $\rightarrow 60 30$

Post $\rightarrow 60 30$

$\frac{co}{lo}$

buildTreeNode (Inorder, postorder)

```
{ if (Inorder.size() != postorder.size())
    return NULL;
```

map<int, int> hm;

```
for (int i = 0; i < Inorder.size();)
```

hm[Inorder[i]] = i;

return buildTreePostIn (Inorder, 0, Inorder.size() - 1,

postorder, 0, postorder.size() - 1, hm);

}

buildTreePostIn (Inorder, 0, InRoot - 1, postorder, ps, pe, hm)

```
{ if (ps > pe || ps > pe) return NULL;
```

root = TreeNode (postorder[pe]);

int inRoot = hm[postorder[pe]];

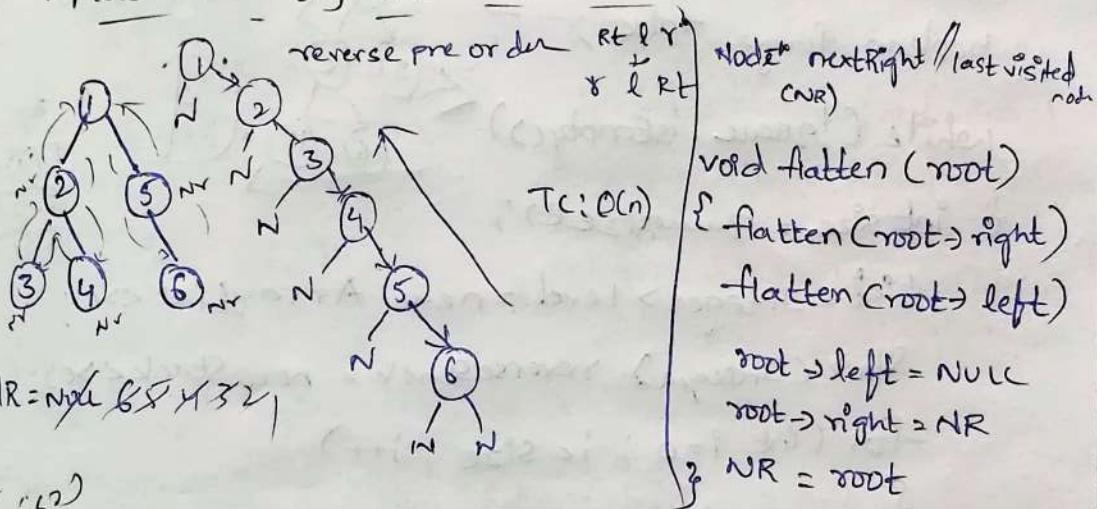
int numLeft = inRoot - ps;

root->left = buildTreePostIn (Inorder, ps, inRoot - 1,

postorder, ps + numLeft - 1, hm);

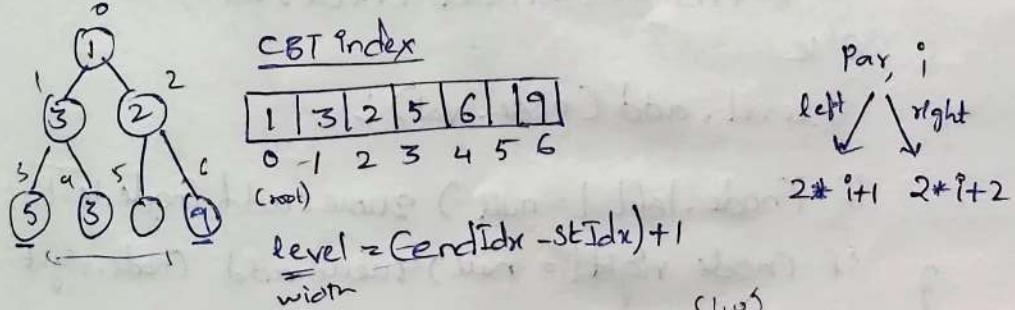
$\text{root} \rightarrow \text{right} = \text{buildTreePostInC}(p\text{order}, p\text{Root}+1)$, i.e.,
 return root;
 postorder, $p\text{order} + \text{numLeft}$, $p\text{Root}$, height

20) flatten Binary Tree to linked list



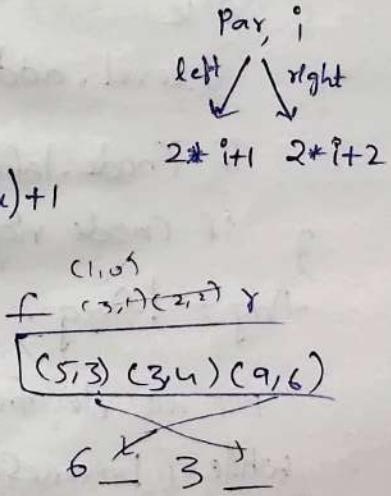
21) Maximum width of Binary Tree

(longest length b/w the end-nodes where the null nodes b/w the end-nodes - that would be present in a Complete BT (CBT))



```

int maxwid (root)
{
    Queue<pair<Node*, int>> q;
    q.push (root, 0);
    maxwid = 0;
    while (q.size() > 0)
    {
        startIdx = q.front().second;
        endIdx = q.back().second;
        maxwid = max (maxwid, endIdx - startIdx);
        for (p = 0 to curr.size())
        {
            currNode = q.pop() -> pIdx;
            if (left) &gt; currNode->left, 2 * pIdx + 1;
            if (right) &gt; currNode->right, 2 * pIdx + 2;
        }
    }
}
    
```



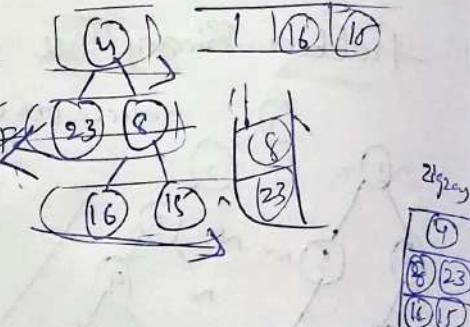
$$\text{max width} = 124$$

103. Zigzag Traversal $O(n)$

```

List<List<Integer>> zigzag = new ArrayList<>();
if (root == null) return zigzag;
Queue<TreeNode> queue = new LinkedList<TreeNode>;
queue.add(root);
boolean flag = false;
while (!queue.isEmpty())
{
    int size = queue.size();
}

```



```

List<Integer> level = new ArrayList<>();
Stack<Integer> reverseStack = new Stack<>();
for (int i=0; i<size; i++)
{
    TreeNode node = queue.poll();
}

```

```

if (flag)
    reverseStack.add (node.val);
else
    level.add (node.val);
if (node.left != null) queue.add (node.left);
if (node.right != null) queue.add (node.right);
flag = !flag;
// pop all ele from stack
while (!reverseStack.isEmpty())

```

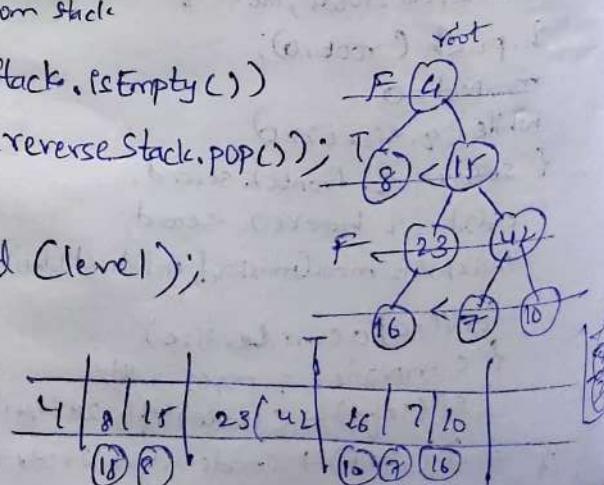
```

level.add (reverseStack.pop());

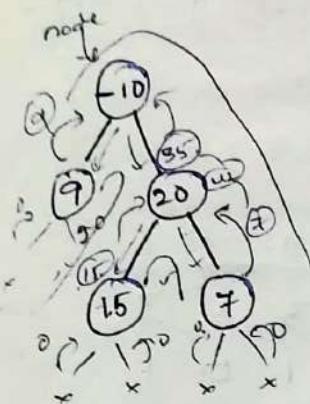
```

~~flag~~
g zigzag.add (level);

return zigzag;



2nd) Maximum Path Sum in Binary Tree



$$15 + 7 + 20 = 42$$

$$9 + (-10) + 35 = 34$$

$$\text{Max}^i = \emptyset \ 9 \ 15 \ 42$$

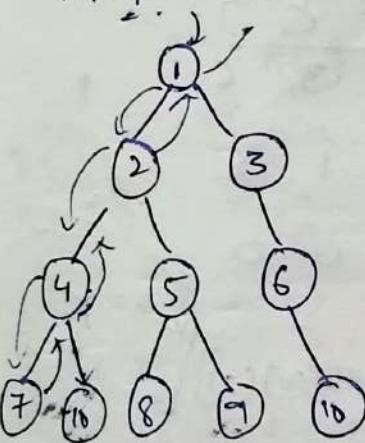
```

int maxPath (node, maxi)
{
    if (node == null)
        return 0;
    leftSum = maxPath (node->left)
    rightSum = maxPath (node->right)
    maxi = max(maxi, leftSum + rightSum
                + node->val)
}
return (node->val) + max(leftSum,
                           rightSum);
    
```

2nd) (1483) k^{th} Ancestor of a Tree Node

$T_k = 4/32 + 0$ (k^{th} ancestor above given node in BT)

val = 7



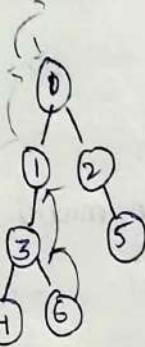
```

Node* helper (Node* root, int& val, int& k)
{
    if (!root).return NULL;
    Node* rv = NULL;
    if (root->data == val)
        or (rv = helper (root->left, val, k))
    or (rv = helper (root->right, val, k))
    if (k == 0)
        cout << "ans:" << root->data << endl;
    return NULL;
    k--;
}
return rv ? rv : root;
}
return NULL;
    
```

```

void getkthAncestor (Node* root, int val, int k)
{
    Node* parent = helper (root, val, k);
    if (parent) {
        cout << "-1" << endl;
    }
}
    
```

k^{th} Ancestor | Binary lifting



Parent [0 0 0 1 3 2 3]
 \oplus 1 0 1 2 3 4 5 6
 pair of $i = P[i]$

	0	1	2	3	4	5	6
0	0	0	0	1	3	2	3
1	0	0	0	0	1	0	1
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0

(0 1 1 0 0 0 1 1 0 1 0 1 0 0 0 0)

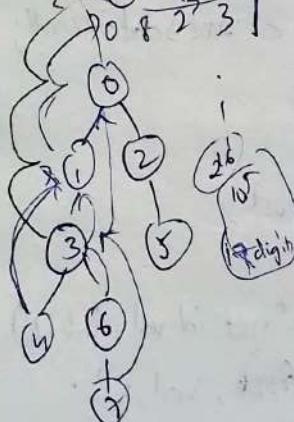
$$10^5 \leq 65536$$

$$\begin{matrix} k \\ 9 \end{matrix} \rightarrow \frac{1}{2^3} \quad \frac{0}{2^2} \quad \frac{0}{2^1} \quad \frac{1}{2^0} \quad \frac{14-1}{8+2+1} \quad \frac{1}{2} \quad \frac{0}{2^1} \quad \frac{1}{2^0}$$

	0	1	2	(3)	4	5	6	7
0	0	0	0	1	3	2	3	6
1	0	0	0	0	1	0	1	3
2	0	0	0	0	0	0	0	0

$$\text{table}[i][j] = \text{table}[i-1][\text{table}[i-1][j]]$$

$\frac{1}{2^m}$ (2^m points)



$$\begin{matrix} 10 \\ 2^3 \\ 2^2 \\ 2^1 \\ 2^0 \end{matrix} \quad \begin{matrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{matrix} \quad \begin{matrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{matrix} \quad \begin{matrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{matrix} \quad \begin{matrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{matrix}$$

now

$$\log_2 Cn + 1$$

$a \rightarrow k$ 3 0

$k \rightarrow 3 \rightarrow 10$

$\text{query}(a, k)$

{ for ($i=0$; $i < \text{max}$; $i++$)

$\text{mask} = (1 \ll i);$

$\text{if } (a \& \text{mask}) > 0,$

$\{ a = \text{table}[i][a]; \}$

no op spin
 $\log n \rightarrow 1/n$

```

class Main
{
    int MAX = 17;
    static int table [][] ;
    static void build (int P[])
    {
        int n = P.length;
        table = new int[MAX][n];
        for (int i=0; i<n; i++)
        {
            table[0][i] = P[i];
        }
        for (int i=1; i<MAX; i++)
        {
            table[0][i] = fff; table[i-1][table[i-1][0]];
        }
    }
    for (int
}
static int query (int a, int k)
{
    for (int i=0; i<MAX; i++)
    {
        int mask = (1 << i);
        if (k & mask) > 0
        {
            a = table[i][a];
        }
    }
}

```

```

void main( args ) —
{
    int n;
    parent [];
    q;
    build (parent);
    while (q-- > 0)
    {
        String input = read.readLine().split(" ");
        int a = Integer.parseInt (read.readLine());
        int q = Integer.parseInt (read.readLine());
    }
}

```

28/12

Binary Search Trees

1. ($\Theta(n)$) k^{th} Largest element in a stream.

$[3, [4, 5, 8, 2], [3], [5], [10], [9], [4]]$

$K=3 \rightarrow [4, 5, 8, 2] \quad | \quad | \quad |$
 $[2, 3, 4, 5, 8] \rightarrow 4$

$[2, 3, 4, 5, 8] \rightarrow 5$

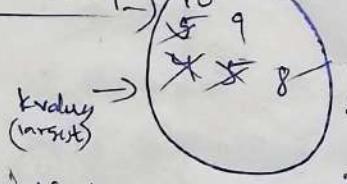
$[2, 3, 4, 5, 8, 6, 9, 10] \rightarrow 8$

Bruteforce
 $\{ 4, 5, 8, 2, 3 \rightarrow \text{sort} \}$
 $\text{arr.get}(n-k)$

$n, n+1, n+2, \dots$

opt \rightarrow heap
add $\{ 4, 5, 8, 2, 3 \}$
 $\{ 3, 5, 10, 9, 4 \}$

minheap



$K=3$

$S = 8 + 2 \times 3$

$= 2 \times 4$

T.C $\rightarrow O(k) \log k$

$\rightarrow O(k) + 10 \log k$

S.C $\rightarrow O(k)$

public k^{th} largest(k , $\text{nums}[]$) // constructor

{
 this. $k = k$

$PQ = \text{new priorityQueue} < ()$;

for (int num : nums) {
 minheap

 if add(num);
 $(3) \leftarrow \text{key}$

3 3

public void add(int val)

{
 if ($PQ.\text{size}() < k$ || $val > PQ.\text{peek}()$)

 {
 $PQ.\text{offer}(val)$;

 if ($PQ.\text{size}() > k$)

 {
 $PQ.\text{poll}()$;

3 3 3

2. (108) Sorted Array to Balanced BST

$\text{arr} = [-10, -3, 0, 5, 9]$

$st=0$

mid

$end=4$

$\text{left} < \text{par} < \text{right}$

Node^* helper(nums[], st, end)

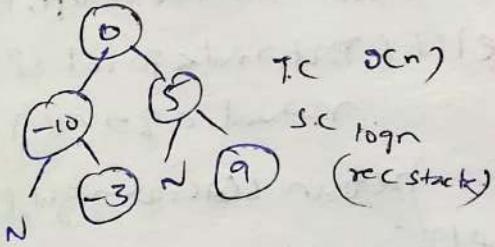
{ $mid = st + (end-st)/2$

$\text{root} = \text{newNode}(\text{nums}[mid])$

$\text{root} \rightarrow \text{left} = \text{helper}(\text{nums}, st, mid-1)$

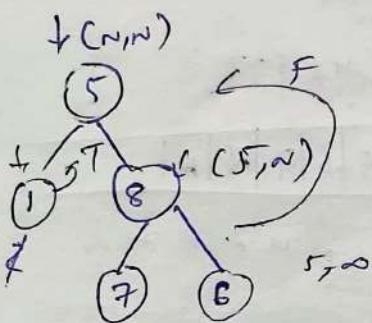
$\text{root} \rightarrow \text{right} = \text{helper}(\text{nums}, mid+1, end)$

return root;



3

3. (18) Validate BST



bool isBST (root, min, max)

{ if (root == NULL) return true;

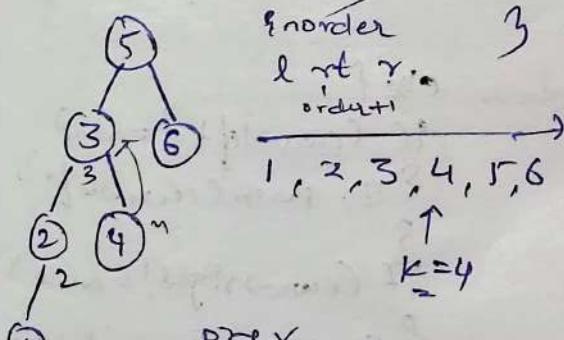
if (min != NULL & root->val <= min->val) return false;

if (max != NULL & root->val >= max->val) return false;

return isBST(root->left, min, root)

&& isBST(root->right, root, max);

4. k^{th} Smallest in BST



3

k^{th} smallest (root, k)

{ if (root == NULL) return -1

left { if (root->left) { leftAns = kthSmallest(root->left); - if (leftAns != -1) k } return leftAns }

root { if (preOrder+1 == k) return root->val; preOrder = preOrder+1; }

right { if (root->right) { rightAns = kthSmallest(root->right); - if (rightAns != -1) k } return rightAns }

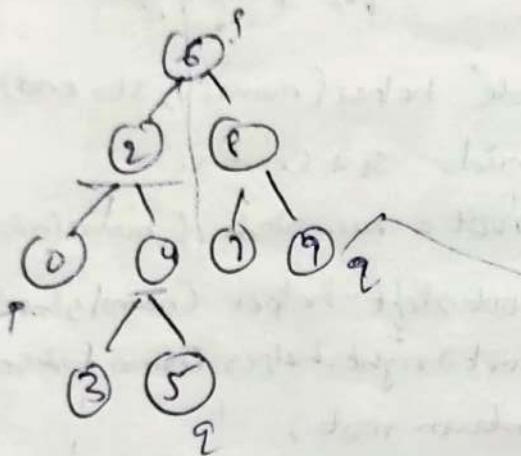
1, 2, 3, n, r, b

(min) (max) (n)

order = x

order = K

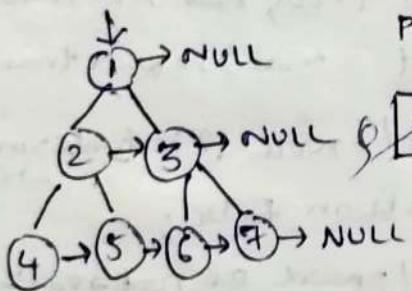
5. Lowest Common Ancestor (LCA) (23r)



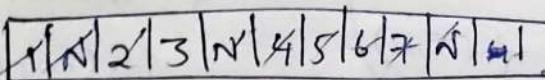
```

node* LCA (root, p, q)
if (root->val > p->val & root->val < q->val)
    return LCA (root->left, p, q)
else if (root->val < p->val & root->val < q->val)
    return LCA (root->right, p, q)
else
    return root
  
```

6. (11c) populate Next Right Pointers



prev = 1, n = 2, 3, 4, 5, 6, 7



curr = n
 if prev != N
 prev->next = curr
 prev = curr

Node* connect (node* root)

```
{ if (root == NULL || root->left == NULL)
```

```
{ return root;
```

```
}
```

```
queue <node*> q;
```

```
q.push(root);
```

```
q.push(NULL); "end of level"
```

```
node* prev = NULL;
```

```
while (q.size() > 0)
```

```
{ node* curr = q.front();
```

```
q.pop();
```

```
if (curr == NULL)
```

```
{ if (q.size() == 0)
```

```
{ break;
```

```
q.push(NULL);
```

```
}
```

```
else {
```

```
if (curr->left != NULL)
```

```
{ q.push(curr->left);
```

```
}
```

```
if (curr->right != NULL)
```

```
{ q.push(curr->right);
```

```
}
```

```
if (prev != NULL)
```

```
{ prev->next = curr;
```

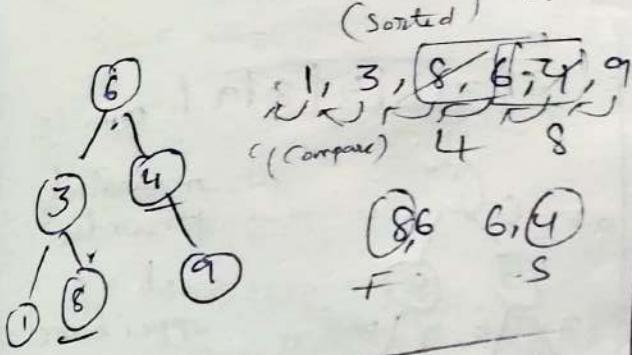
```
}
```

```
prev = curr;
```

```
return root;
```

```
}
```

(a9) Recover BST (inorder) {prev, next}



first = &8

sec = &6 4

prev = &x 3 8 6 4 9

int temp = ->val
f->val => s->val
s->val = f->temp

prev = NULL, first = NULL, sec = NULL

inorder(root)

if (root == NULL) → return;

inorder(root->left)

if (prev != NULL && root->val < prev->val)

{ if (!first) {

[first = prev], s

[sec = root], f, s

prev = root

} inorder(root->right)

swap f & s

→ To optimize S.C to O(1) use

Morris Inorder Traversal

(Iterative)

Tc O(n)

S.C O(1)

{ if (prev != NULL && prev->val > root->val)

{ if (first == NULL)

{ first = prev

{ second = root.

prev = root;

IP->right = NULL,

root = root->right,

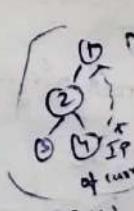
} }

if (first != NULL && second != NULL)

t->val = f->val,

f->val = s->val,

s->val = t->val;



void recoverTree(TreeNode* root)

{ TreeNode* first = NULL;
 sec = NULL;
 prev = NULL;

while (root != NULL)

{ if (root->left == NULL)

{ if (prev != NULL && prev->val > root->val)

{ if (first == NULL)

{ first = prev;

{ second = root;

prev = root;

} root = root->right;

else

{ TreeNode* IP = root->left;

while (IP->right != NULL && IP->right != root)

{ IP = IP->right;

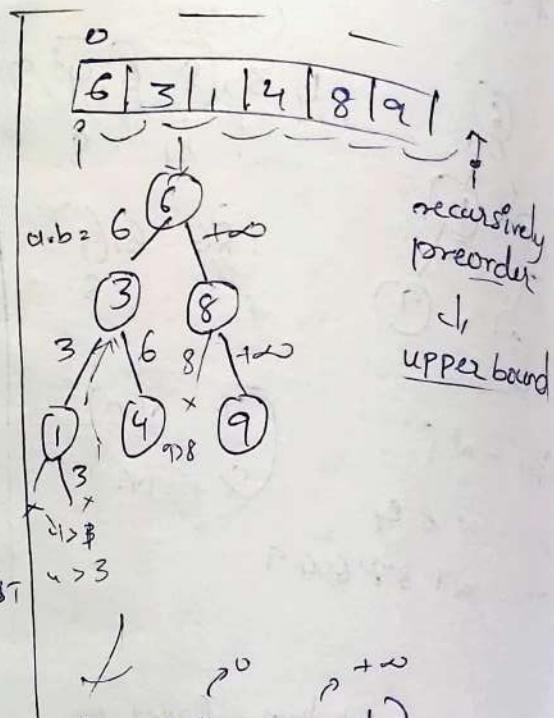
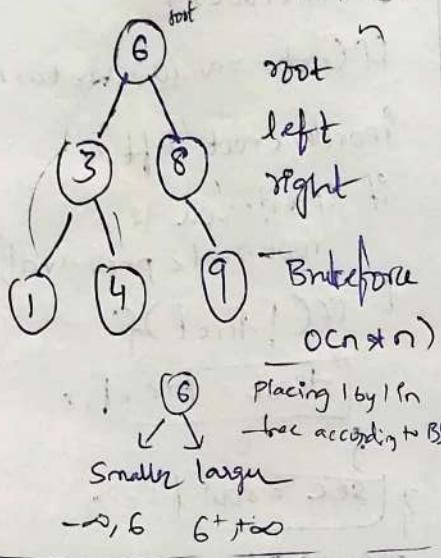
if (IP->right == NULL)

{ IP->right = root->left;

} root = root->left;

8. (100%) Construct BST from PreOrder

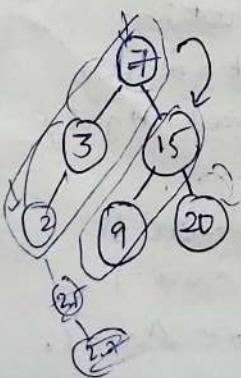
Preorder = [6, 3, 1, 4, 8, 9]



Node* buildBST (preorder, &i, bound)

```
{
    if (i >= preorder.size() || preorder[i] > bound)
        return null;
    Node* root = new Node (preorder[i]);
    i++;
    if (left = buildBST (preorder, i, root->val))
        if (right = buildBST (preorder, i, bound))
            return root;
}
```

9. (17%) BST Iterator



inorder → left, root, right

↳ 5, 7 (9), 15, 20

next() 5

next() 7

next() 9

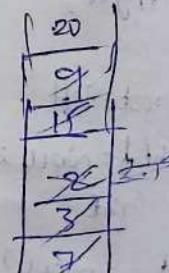
hasNext() true

next() 15

2, 3, 5, 7, 9, 15, 20

TC: O(1)

SC: O(h)



Stack

① constructor
storeLeftNodes()

② next()

ans = S.top();
S.pop();

if (ans->right)

storeLeftNodes(ans->right);

③ hasNext()

return S.size() > 0;

```
void storeLeftNodes(root)
{
    while (root != null)
    {
        S.push(root);
        root = root.left;
    }
}
```

10. Flatten BST to sorted list

prev = None

head = None

def inorder(root):

if not root: return None
inorder(root.left)

if prev:

prev.right = root
prev.left = None

else:

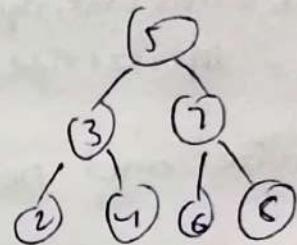
head = root

prev = root

inorder(root.right)

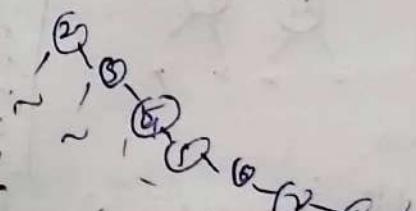
If prev: prev.left = None
return head

IP = [5, 3, 7, 2, 4, 6, 8]

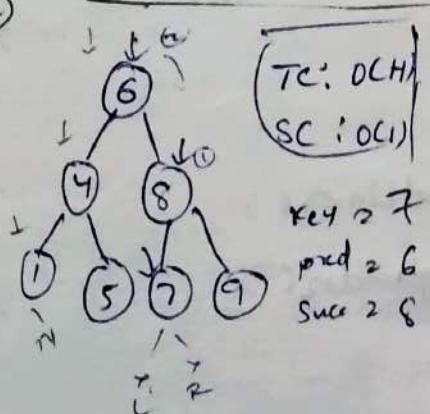


DPS = [2, N, 3, N, 4, N, 5, N]

6, N, 7, N, 8



11) Inorder Successor & Inorder Predecessor



TC: O(1)
SC: O(1)

key = 7
pred = 6
succ = 8

key = 3

pred = 1

succ = 6, 4

① key < curr->data

Succ = curr

curr = curr->left

② key > curr->data

Pred = curr

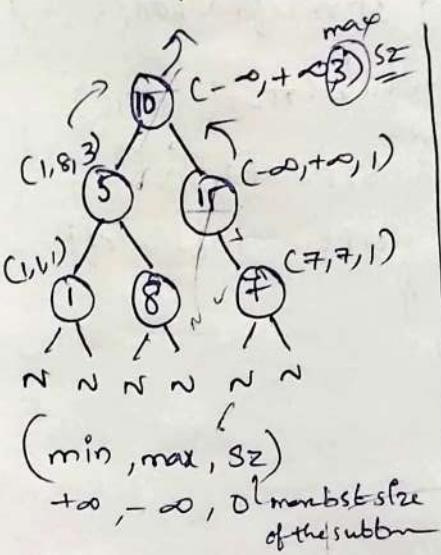
curr = curr->right

③ key == curr->data

↳ Inpred = right most in left subtree

↳ Insucc = left most in right subtree
break!

13. Largest BST in BT



helper(root)

BASECASE → return info(+∞, -∞, 0)

info.left = helper(root→left)

info.right = helper(root→right)

if (root→data > left.max) &

root→data < right.min) // valid BST

return info(currMin, currMax, currSz)

else

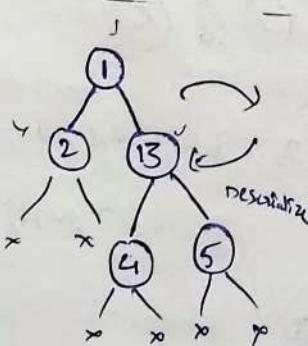
return (-∞, +∞, max(left_sz, right_sz))

$$\text{currMin} = \min(\text{root}, \text{leftMin})$$

$$\text{currMax} = \max(\text{root}, \text{rightMax})$$

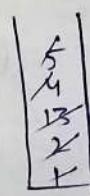
$$\text{currSz} = \text{leftSz} + \text{rightSz} + 1$$

14. Serialize and Deserialize Binary Tree



Serialize → Level order

String → 1, 2, 13, #, #, 4, 5, #, #, #, #,



Q → empty
return root

T.C → O(n)

S.C → O(n)

s

root → string
d

public class Codec

{ public String serialize(TreeNode root).

{ if (root == null) return "#".

Queue<TreeNode> q = new LinkedList<>();

StringBuilder res = new StringBuilder();

q.add(root);

while (!q.isEmpty())

{ TreeNode node = q.poll();

if (node == null)

{ res.append("#");

y continue;

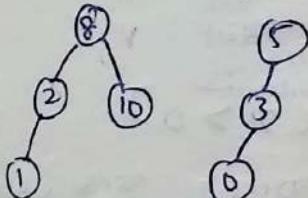
```

    res.append(node.val + " ");
    q.add(node.left);
    q.add(node.right);
}

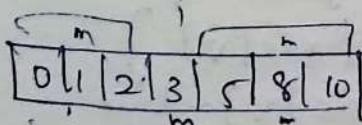
public TreeNode deserialize(String data)
{
    if (data == "") return null;
    Queue<TreeNode> q = new LinkedList<>();
    String[] values = data.split(" ");
    TreeNode root = new TreeNode(Integer.parseInt(values[0]));
    q.add(root);
    for (int i = 1; i < values.length; i++)
    {
        TreeNode parent = q.poll();
        if (!values[i].equals("n"))
        {
            TreeNode left = new TreeNode(Integer.parseInt(values[i]));
            parent.left = left;
            q.add(left);
        }
        if (!values[i+1].equals("n"))
        {
            TreeNode right = new TreeNode(Integer.parseInt(values[i+1]));
            parent.right = right;
            q.add(right);
        }
    }
    return root;
}

```

15. Merge 2 BSTs



arr1 [1 2 8 10] arr2 [0 3 5]



node* merge(node1, node2)

{ inorder(node1, arr1) }

inorder(node2, arr2)

vector<int> temp;

while(i < arr1.size() && j < arr2.size())

{ if (arr1[i] < arr2[j])

temp.push_back(arr1[i++])

else temp.push_back(arr2[j++])

while (i < arr1.size()) → temp.push_back(arr1[i++])

while (j < arr2.size()) → temp.push_back(arr2[j++])

buildBST(temp)

return

T.C - O(m+n)

30/12/25

Graphs

1. (733) — Flood fill

0	1	2
1	3	3
2	2	3

newcolor = 3

sr = 2 sc = 0

Initial color = 2

DFS(2, 0)

X X

DFS(1, 0) DFS(2, 1)

/

DFS(1, 1)

/

DFS(2, 1)

DFS(2, 2)

(r-1, c+0)

(row, col) → (r+0, c+1)

(r+1, c)

$$\Delta \text{row} = \{-1, 0, +1, 0\}$$

$$\Delta \text{col} = \{+0, +1, +0, -1\}$$

Void dfs (int row, int col, vector<vector<int>>&ans, vector<int>&image,

{ ans[r][c] = newColor;

n = image.size(),

m = image[0].size();

for (i = 0; i < 4; i++)

{ int nrow = row + delRow[i];

int ncol = col + delCol[i];

if (nrow >= 0 && nrow < n && ncol >= 0 && ncol < m &&

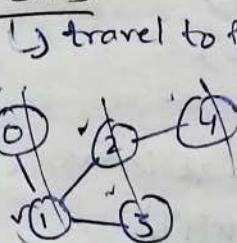
image[nrow][ncol] == initColor && ans[nrow][ncol] ==

dfs(nrow, ncol, ans, image, newColor, delRow[i]);

3 3 3

BFS

src(E)



T _F				
0	1	2	3	4

0	1	2	3	4
---	---	---	---	---

u = 0 1 2 3 4 O/P: {0 1 2 3 4}

v = 5
src → 4
dest → v

v's (0, 1, ..., n)

o. push(v); vis[v] = true

while (Q.size() > 0)

int u = Q.pop(); src

cout << u; if it's written in reverse

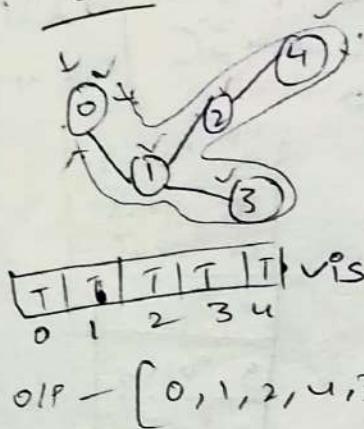
for (v in neighbours of u) {

if (!vis[v]) {

① vis[v] = true

② Q.push(v)

DFS



```

for (int i = 0; i < v; i++)
{
    if (!vis[i])
        { dfsHelper(i, vis);
        }
}
cout << endl;
    
```

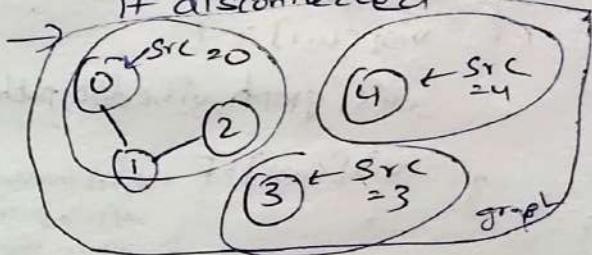
dfs(u, vis)

```

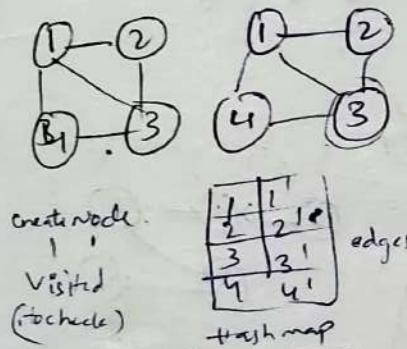
{
    cout << u;
    vis[u] = true;
    for (v : l[u])
    {
        if (!vis[v])
            { dfs(v, vis);
            }
    }
}
    
```

3 3 3

PF disconnected



2. (133) Clone Graph (Deep copy - connections same, memory diff.)



Node* cloneUtil(Node* node, map)

```

{
    Node* newNode = new Node(node->val);
    mp[node] = newNode;
    for (auto neighbour : node->neighbours)
    {
        if (mp.find(neighbour) == mp.end())
        {
            // Create the neighbor's clone
            (newNode->neighbours).push_back(
                cloneUtil(neighbour, mp));
        }
        else
        {
            (newNode->neighbours).push_back(
                mp[neighbour]);
        }
    }
    return newNode;
}
    
```

Node* cloneGraph (Node* node)

```

{
    if (!node) return NULL;
    unordered_map<Node*, Node*> mp;
    return cloneUtil(node, mp);
}
    
```

All paths from source to target

```

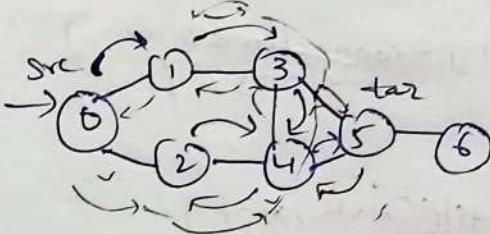
dfs(graph, vis[], curr, String path, tar)
{
    if (curr == tar)
        print(path)
    return
}
    
```

for (int i=0 to graph[curr].size())

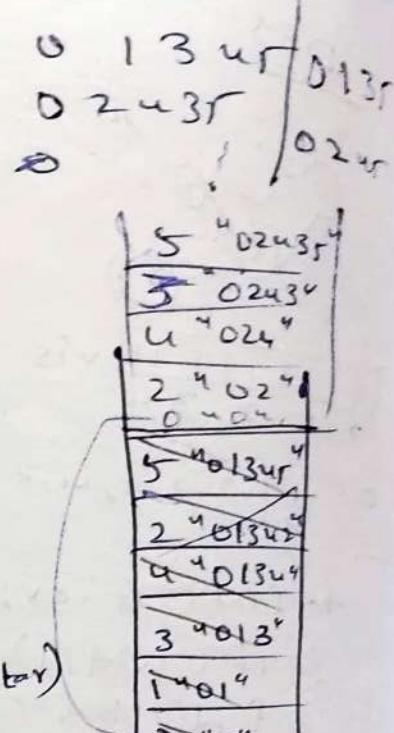
```

    {
        Edge e = graph[curr].get(i)
        if (vis[e.dest] == F)
            vis[curr] = T
            dfs(graph, vis, e.dest, tar)
    }
}
    
```

3 3 3 vis[curr] = F → (not mandatory to set like
dfs as it can be traversed again for path)



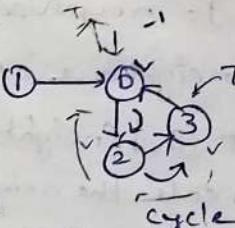
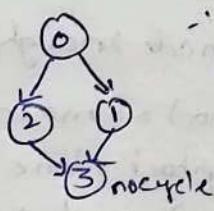
T	T		T	T	T	T	T	T	vis
0	1	2	3	4	5	6			



Rec Stack

Cycle detection

weighted/unweighted
directed / undirected



True
Rec.stack

```

dfs(graph, vis[], curr, rec[])
    
```

vis[curr] = T
rec[curr] = T

```

    for (int i=0 to graph[curr].size())
    
```

```

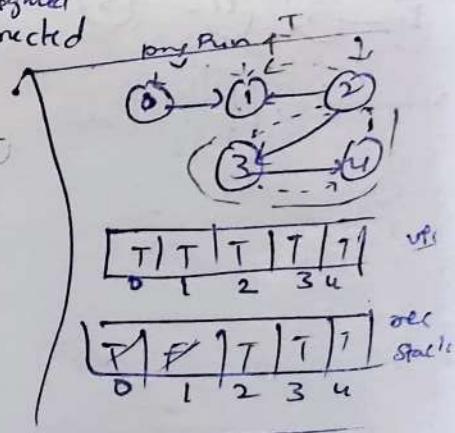
        {
            Edge e = graph[curr].get(i)
            if (rec[e.dest] == T)
                return T
        }
    }
}
    
```

else if (!vis[e.dest])

```

        if (dfs(graph, vis, e.dest, rec))
            return true
    }
}
    
```

rec[curr] = F



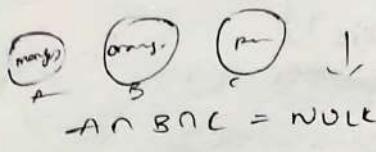
(directed)

```

for (int i=0; i<v; i++)
{
    if (!vis[i])
        ...
}
    
```

!vis[i] = No cycle
vis[i] = Yes cycle
graph, vis, curr, rec, s.o.p(isCycle), break;

Disjoint Set Union (DSU) / Union-Find



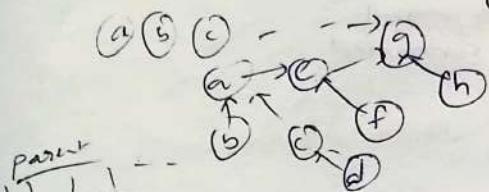
op^r-① → Combine two given sets

op^v-② → Tell me if two members

(b,g) belong to same set or not
union(a,b), (g,d)
union(a,c); union(e,c)

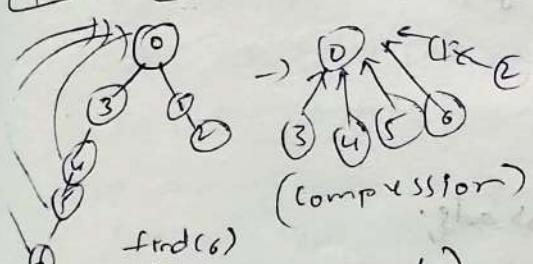
find(c) → g
find(g) → g

DSU → find
Union → Union



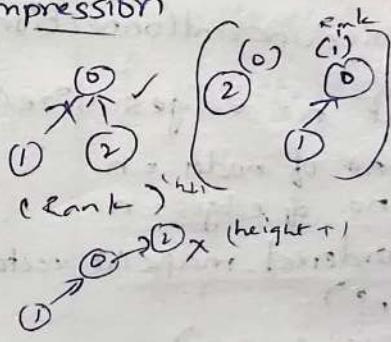
To make height minimum

Union by Rank & Path compression



find(6)
find(int i, parent)
{ if (parent[i] == i)

return i;
3 return not find(parent[i], parent);
Parent(P)=i



void Union(int x, int y, vector<int>& parent, vector<int>& rank)

{ int x-parent = find(x, parent);

int y-parent = find(y, parent);

if (x-parent == y-parent)
return

if (rank[x-parent] > rank[y-parent])

{ Parent[y-parent] = x-parent;

else if (rank[x-parent] < rank[y-parent])

{ Parent[x-parent] = y-parent

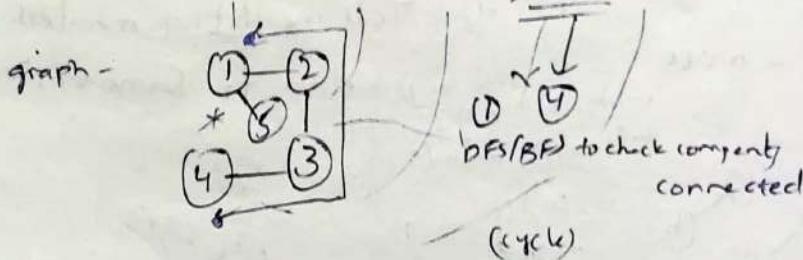
else

{ Parent[x-parent] = y-parent;

3 rank[y-parent] += 1;

3. Redundant graph : (684)

edges = $\{(1,2), (2,3), (3,4), (1,4), (4,5)\}$

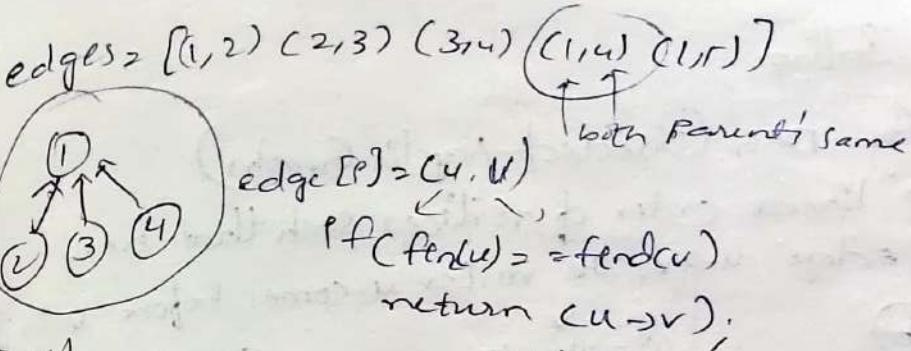


```

f int n = edges.size();
    // no. of nodes = n
    // no. of edges = n
unordered_map<int, vector<int>> adj;
// O(n^2)
// space : O(n)
for(int i=0; i<n; i++) // O(n)
{
    int u = edges[i][0];
    int v = edges[i][1];
    vector<bool> visited(n, false);
    if(adj.find(u) != adj.end() && adj.find(v) != adj.end()
        && !dfs(adj, u, v, visited))
    {
        return edges[i]; // cycle exists
    }
    adj[u].push_back(v);
    adj[v].push_back(u);
}
return {};
    
```

```

bool dfs (&adj, u, v, vis)
{
    vis[u] = true;
    if (u == v)
    {
        return true;
    }
    for (int nqbr : adj[u])
    {
        if (vis[nqbr]) continue;
        if (dfs(adj, nqbr, v, vis))
        {
            return true;
        }
    }
    return false;
}
    
```



DSU

```
{
    findRedundantConnection(edges)
    {
        n = edges.size();
        DSU dsu(n);
        // T.C: DSU = alpha(n)
        // T.C: O(n + alpha(n))
        for (auto &edge : edges) // O(n)
        {
            int u = edge[0];
            int v = edge[1];
            if (dsu.find(u) == dsu.find(v))
            {
                return edge;
            }
            dsu.union(u, v);
        }
        return {};
    }
}
```

class DSU

parent;

rank;

DSU() {

parent.resize(n+1);

rank.resize(n+1);

for (int i = 1; i < n; i++)

{ parent[i] = i;

rank[i] = 0;

3 } }

int find(int x)

{ if (x == parent[x])
 { return x;
 }

return parent[x] = find(
 parent[x]);
 } }

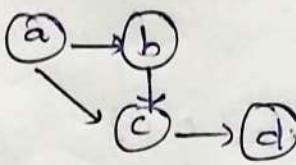
"path compression"
 parent[x])

void Union (int x, int y)

```
{
    int x_parent = find(x);
    int y_parent = find(y);
    if (x_parent == y_parent)
    {
        return;
    }
    if (rank[x_parent] > rank[y_parent])
    {
        parent[y_parent] = x_parent;
    }
    else if (rank[y_parent] > rank[x_parent])
    {
        parent[x_parent] = y_parent;
    }
    else
    {
        parent[y_parent] = x_parent; // we need x as parent
        rank[x_parent]++;
    }
}
```

Topological Sorting

- ↳ works for DAG (Directed Acyclic Graphs)
- ↳ It is a linear order of vertices such that every directed edge $u \rightarrow v$, the vertex u comes before v in the order.



$$a \rightarrow b \quad ①$$

$$c \rightarrow d \quad ②$$

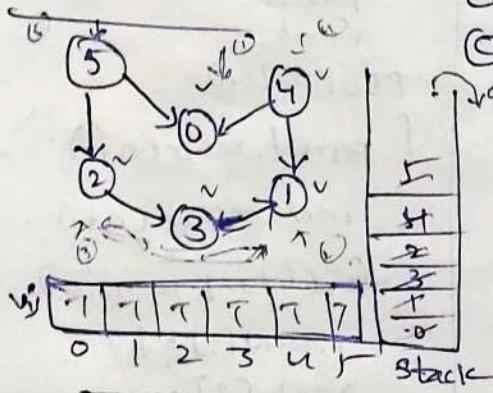
$$a \rightarrow c \quad ③$$

$$c \rightarrow a \quad ④$$

$$b \rightarrow d \quad ⑤$$

DAG

$$a \rightarrow b$$



$$v[0:p] = [5\ 4\ 2\ 3\ 1\ 6]$$

`topsort(graph, vis, cur, stack)`

$$\{ \rightarrow \text{vis}[curr] = T$$

for (int p=0 to graph[curr].size())
 {

Edge e = graph[curr].get(i)

if (!vis[e.dest])

topsort(graph, vis, dest, stack)

stack.push(curr)

Cycles in Graph

Undirected graph →

DFS

BFS

DSU (Disjoint Set Union)

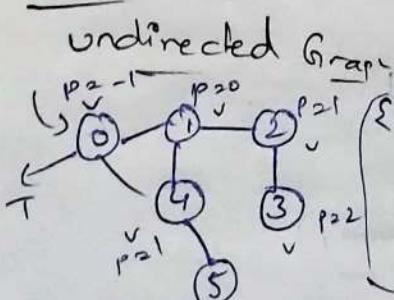
Directed graph →

DFS

BFS

Topological sorting

Graph coloring



(modified DFS) `dfs(graph, vis[], curr, par)`

$\{ \text{vis}[curr] = T$

for (int i=0 to graph[curr].size())
 {

Edge e = graph[curr].get(i) //e.dest

if (vis[e.dest] == T && par != e.dest)
 return true

if (!vis[e.dest])
 return true

if (dfs(graph, vis, e.dest, curr) == T)
 return true

① $\text{vis}[n] = T \Rightarrow \text{cycle}(\text{true})$

par.x

② $\text{vis}[n] = T \Rightarrow \text{do nothing}$

par.y

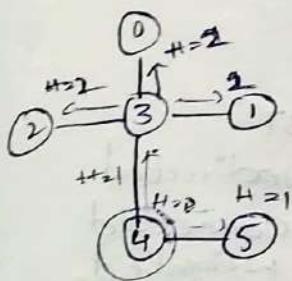
③ $\text{vis}[n] = F \Rightarrow \text{visit } \frac{\text{true}}{\text{for}}$

Minimum height trees

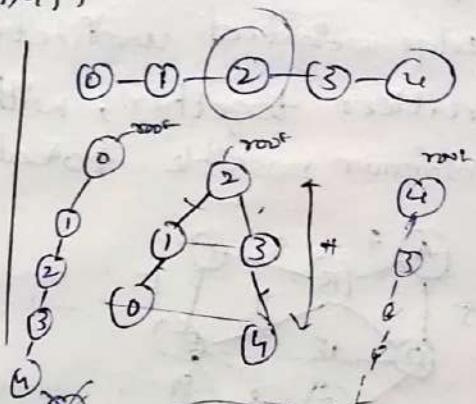
4.

$n=6$ edges = $\{[3,0], [3,1], [3,2], [3,4], [5,4]\}$

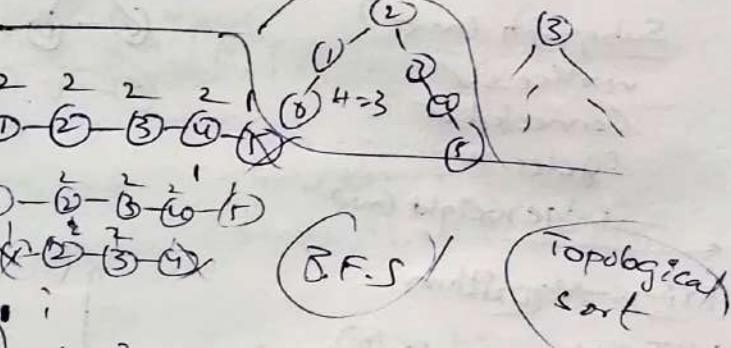
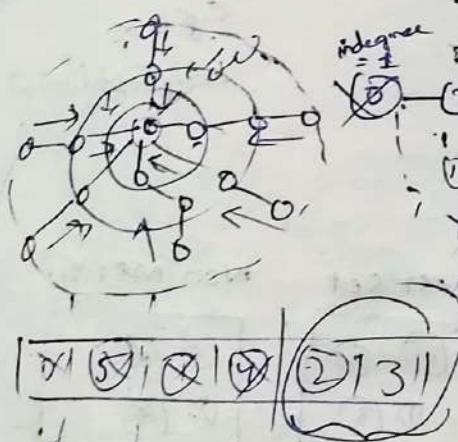
O/P - $[3,4]$



$\text{root} = 0 \rightarrow H = 3$
 $\text{root} = 2 \rightarrow H = 2$
 $\text{root} = 4 \rightarrow H = 2$
 $\text{root} = 5 \rightarrow H = 3$
 (T-L.E)
 B.F.S DFS



- central nodes
- answers can be almost 2 roots



```

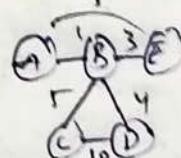
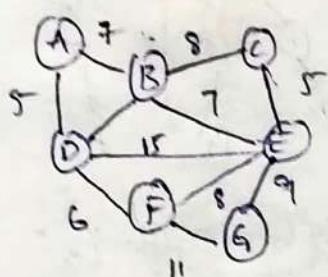
    -> findMinHeightTree(n, edges)
    {
        unordered_map<int, vector<int>> adj;
        vector<int> indegree(n);
        if(n==1) return 0;
        for (auto &edge : edges) {
            int u = edge[0];
            int v = edge[1];
            indegree[u]++;
            indegree[v]++;
            adj[u].push_back(v);
            adj[v].push_back(u);
        }
        vector<int> result;
        queue<int> que;
        for (int i=0; i<n; i++) {
            if (indegree[i]==1)
                que.push(i);
        }
    }
  
```

```

    while (n > 2) {
        int size = que.size();
        n -= size;
        while (size--) {
            int u = que.front();
            que.pop();
            for (int &v : adj[u]) {
                indegree[v]--;
                if (indegree[v] == 1)
                    que.push(v);
            }
        }
        while (!que.empty()) {
            result.push_back(que.front());
            que.pop();
        }
    }
    return result;
}
  
```

Minimum Spanning Tree

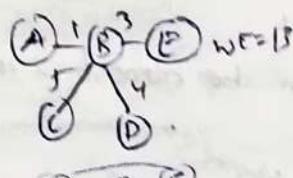
↳ (MST) is a subset of the edges of a connected, edge-weighted undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.



Subgraph

MST

undirected
connected
weighted



Subgraph (MST)

vertices ✓

Connected ✓

Cycles ✗

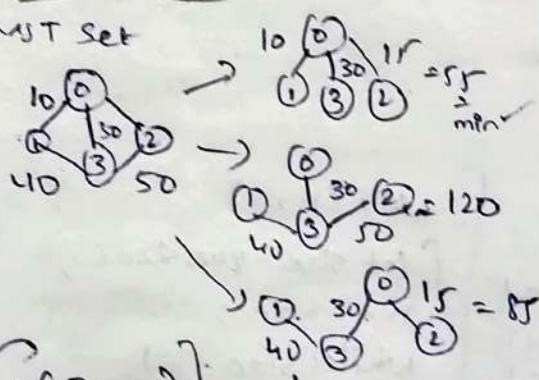
edge weight (min)

MST
nodes ✓ + connected
cycles ✗
edge wt (min)



Prims Algorithm

MST Set



$\Theta(E \log E)$

pair.add(pair(0, 0))

boolean vis[] = F

while (!pq.isEmpty())

{ pair curr = pq.remove()

if (!vis[curr.node])

{ vis[curr.node] = T

cost += curr.cost

for (int i=0; i<graph[curr.node].size(); i++)

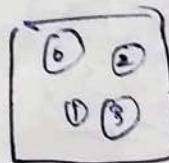
{ Edge e = graph[curr.node].get(i)

if (!vis[e.dest])

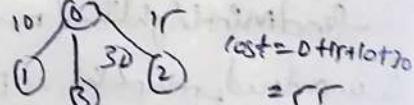
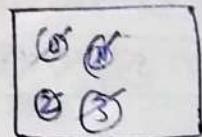
pq.add(e.dest, e.wt)

3 3 9

MST Set



Non-MST Set



Pair (node, cost)

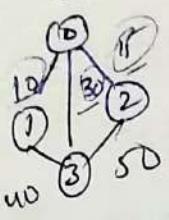
(mincost)

↓

PSU

pq (non-MST)

vis[] (MST)



$\text{Par}(n, \text{cost})$

 $(0, 0) \Rightarrow +0$
 $(1, 10) \Rightarrow +10$
 $(2, 15) \Rightarrow +15$
 $(3, 30) \Rightarrow +30$
 $(3, 40) \Rightarrow \times$

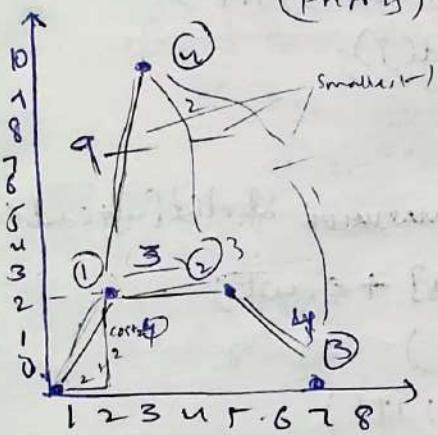
already visited
(3, 50) $\Rightarrow \times$.

PQ sort automatically

(1, 10)	(2, 15)	(3, 30)	(3, 40)	(3, 50)
---------	---------	---------	---------	---------

$T | T | T | T$ vis
0 1 2 3

I.C. 5.
O(log n)
S.C.
O(n²)



Dist.
 Δx
 Δy .
manhattan dist

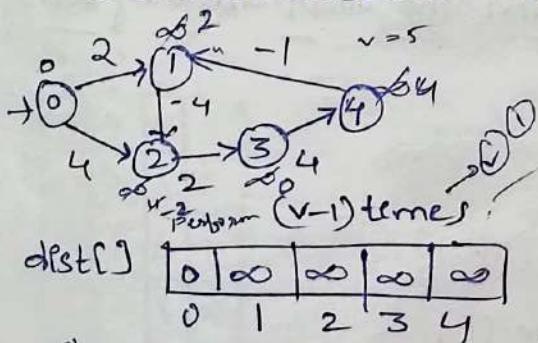
H.S. \Rightarrow Vis [0, 1, 2, 3, 4] = len/no of nodes
(Heap + [dis, point])
[0, 0] (0, 1) (2, 3) (3, 3) (6, 4)
(7, 0) (7, 1) (13, 2) (10, 7, 2) (9, 2)

cost: 0 + 1 + 3 + 4 + 9 =

(dijkstra) Dijk Bellman O(V+E log V) O(V.E)
works only for positive (+ve) weights Greedy DP

Bellman Ford Algorithm

↳ shortest distance from the source to all vertices



dist[]

0	∞	∞	∞	∞
0	1	2	3	4

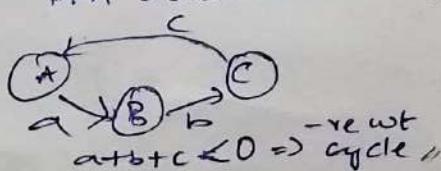
1st 0 2 -2 0 4
2nd 0 2 -2 0 4
3rd 0 2 -2 0 4
4th 0 2 -2 0 4

$\Theta(V \cdot E)$
for all edges (u, v)
{ if $\text{dist}[u] + \text{wt}(u, v) < \text{dist}[v]$
 $\text{dist}[v] = \text{dist}[u] + \text{wt}(u, v)$

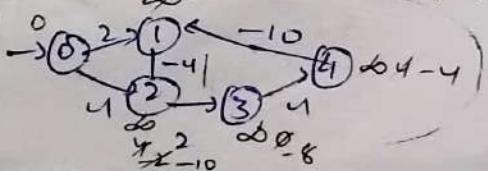
for (int k=0 to v-1) -①

② { for (int i=0 to v)
for (int j=0 to graph[i].size())
edge e = graph[i].get(j)

→ BFA doesn't work for negative cycles



$a+b+c < 0 \Rightarrow$ cycle //



```

void bellmanford( graph[], int src, int v)
{
    int dist[] = new int[v];
    for (int i=0; i<v; i++)
    {
        if (i!=src)
            dist[i] = Integer.MAX_VALUE;
        for (int k=0; k<v-1; k++)
        {
            for (int p=0; p<v; p++)
            {
                for (int s=0; s<graph[p].size(); s++)
                {
                    Edge e = graph[p].get(s);
                    int u = e.src;
                    int v = e.dest;
                    if (dist[u] != Integer.MAX_VALUE & dist[u]+e.wt < dist[v])
                        dist[v] = dist[u] + e.wt;
                }
            }
        }
    }
}

```

↑↑↑ (detect -ve wt cycles)

for (int i=0; i<dist.length; i++)
 System.out.println(dist[i]);

6. 1282. Group the people Given the Group Size they belong
 $\{0, 1, 2, 3, 4, 5, 6\}$
 $\{3, 3, 3, 3, 3, 1, 3\}$

map

SIZE	PEOPLE
3	{0, 1, 2} {3, 4, 5}
1	{5}

res = [0, 1, 2] [3] [3, 4, 5]

L = groupSizes[i]

if (mp[L].size() == 3)
 res.append(mp[L])

Floyd Warshall Algorithm

↳ Multi-Source Shortest Path

T.C - $O(CV^3)$

S.C - $O(CV^2)$

(DP over intermediate nodes)

$$dist[i][j] = \min(dist[i][j], dist[i][k] + dist[k][j])$$

use → All pair shortest path

→ handles negative edges (x not neg cycles)

→ Dense graphs, small-medium v

code $n = len(dist)$

for k in range($0, n$) // intermediate node

 for i in range(n):

 for j in range(n):

 if $dist[i][k] != \text{INF}$ and $dist[k][j] != \text{INF}$:

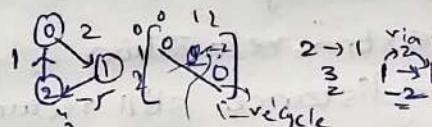
$$dist[p][j] = \min(dist[p][j], dist[i][k] + dist[k][j])$$

Negative cycle, optional,

for p in range(n):

 if $dist[p][p] < 0$:

 print ("neg cycle exist")

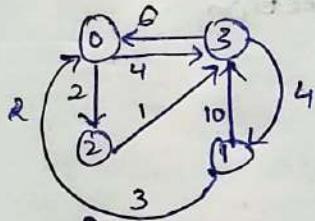


	0	1	2	3
0	0	∞	2	4
1	3	0	∞	10
2	∞	∞	0	1
3	6	4	∞	0

	0	1	2	3
0	0	∞	2	4
1	3	0	5	7
2	∞	∞	0	1
3	6	4	8	0

transitive dependency
 $a \rightarrow b, b \rightarrow c \Rightarrow a \rightarrow c$

$$i=1, j=2 \\ 1 \xrightarrow{3} D + 0 \xrightarrow{4} 3 \\ 1 \xrightarrow{3} 3 = 7$$



	0	1	2	3
0	0	7	2	3
1	3	0	5	6
2	7	5	0	1
3	6	4	8	0

	0	1	2	3
0	0	∞	2	3
1	3	0	5	6
2	∞	∞	0	1
3	6	4	8	0

	0	1	2	3
0	0	∞	2	4
1	3	0	5	7
2	∞	∞	0	1
3	6	4	8	0

(1) 0 through
 (2) Directly
 (3) Not connected

① ordering doesn't matter $[(A \xrightarrow{B} \xrightarrow{C} C)] = s$

Dijkstra :

Position

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

Dijkstra vs Bellman-Ford vs Floyd-Warshall

Algorithm Negative edges Detect neg cycle works with Neg edges

Dijkstra	X	X	X Breaker
Bellman-Ford	✓	✓	X reports cyl.
Floyd-Warshall	✓	✓	X reports cyl

why?

Dijkstra:

- ↳ Assumes once a node is finalized, it can't get shorter
- ↳ Negative edge violates this manipulation assumption
- ↳ Fails silently → wrong answer

Bellman-Ford:

- ↳ Relaxes edges $n-1$ times
- ↳ one extra relaxation
 - if distance still reduces → negative cycle exists

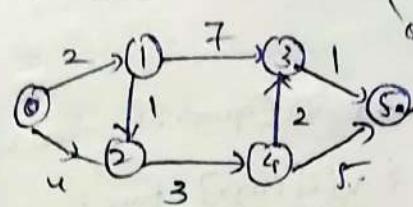
Floyd-Warshall:

- ↳ Computes all-pairs shortest paths
- ↳ After completion:
 - if $\text{dist}[i][j] < 0 \rightarrow$ negative cycle reachable from i

- If $E > V^2 / \log V \rightarrow$ treat as dense
- Else → sparse
- Sparse → few nodes, smart shortcuts
- Dense → everything connects, brute force wins

shortest dist from SIC to all vertices) $T.C \Theta(E + E \log N)$

Dijkstra's Algorithm



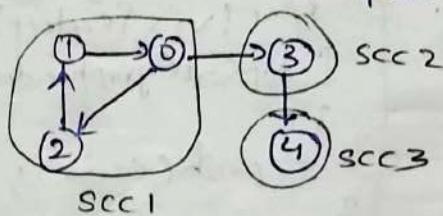
PQ (min-heap) (sort based on weight)

dist	0	2	3	8	6	9	ans
	0	1	2	3	4	5	
0	3	5	3	8	6	7	
1							
2							
3							
4							
5							

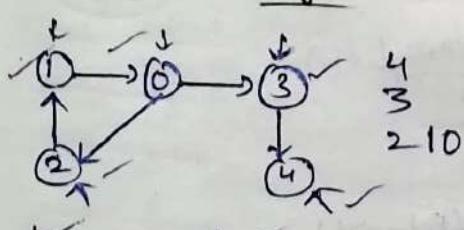
vis [] = {0, 1, 2, 3, 4, 5}

Strongly Connected Component

↳ SCC is a component in which we can reach every vertex of the component from every other vertex in that component.



Kosaraju's Algorithm



works for only directed graph,

DFS

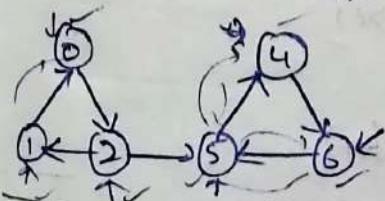
(emerges diff algorithm into 1 alg.)

Reverse DFS

(for SCC) (go from last)

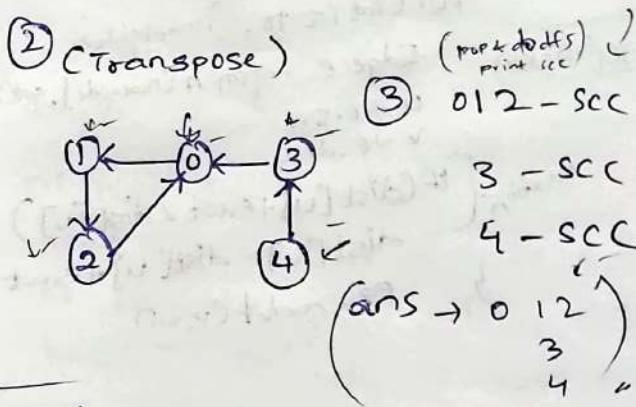
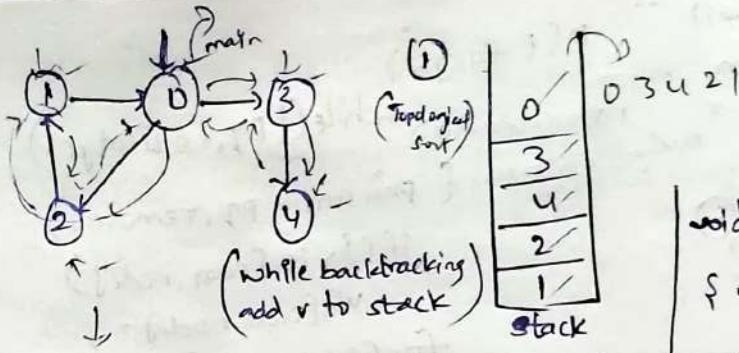
If no i/n in rev vis
it is complete in 1 alg.
no SCC

6 5 4 SCC
2 1 0 SCC



Steps:

- Get nodes in stack (topological sort).
- Transpose the graph (reverse edges b/w vertices)
- Do DFS according to stack nodes on the transpose graph.



```
void kosarajuAlgo(Graph graph[], int v)
{
    // Step-1 - O(V+E)
    Stack<Integer> s = new Stack<>();
    boolean vis[] = new boolean[v];
    for (int i=0; i<v; i++)
    {
        if (!vis[i])
            topSort(graph, i, vis, s);
    }
    // Step-2 - O(V+E)
}
```

```
ArrayList<Edge> transpose[] = new ArrayList[v];
for (int i=0; i<graph.length; i++)
{
    vis[i] = false;
    transpose[i] = new ArrayList<Edge>();
    for (int j=0; j<graph[i].size(); j++)
    {
        Edge e = graph[i].get(j); // e.src(i) > e.dest
        transpose[e.dest].add(new Edge(e.dest, e.src));
    }
}
```

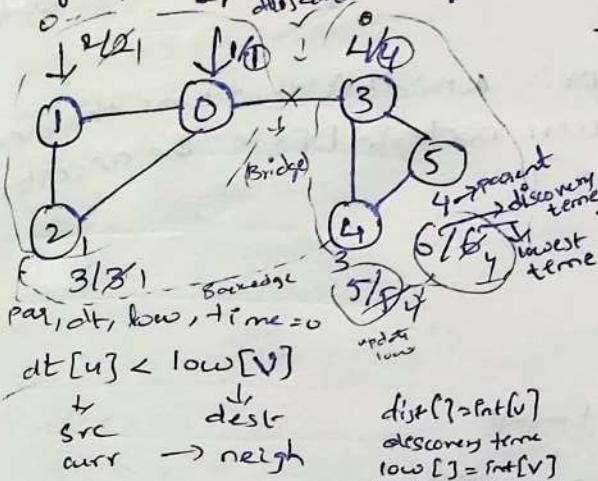
```
// Step-3 - O(V+E).
while (!s.isEmpty())
{
    int curr = s.pop();
    if (!vis[curr])
        dfs(transpose, curr, vis);
}
```

```
void dfs(Graph graph[], int curr, boolean vis[])
{
    vis[curr] = true;
    S.O.P(curr);
    for (int p=0; p<graph[curr].size(); p++)
    {
        Edge e = graph[curr].get(p);
        if (!vis[e.dest])
            dfs(graph, e.dest, vis);
    }
}

void topSort(Graph graph, int curr, boolean vis[])
{
    vis[curr] = true;
    for (int p=0; p<graph[curr].size(); p++)
    {
        Edge e = graph[curr].get(p);
        if (!vis[e.dest])
            topSort(graph, e.dest, vis);
    }
    s.push(curr);
}
```

Bridge in Graphs (using Tarjan's Algorithm)

Bridge is an edge whose deletion increases the graph's number of connected components.



Algorithm - DFS

vis[curr] = T

dt[curr] = low[curr] = ++time

for (int p=0 to neigh)

Edge e ..

① Par → continue

② ! vis[e.dest] → dfs

low[curr] = min(low[curr], low[neigh])

if (dt[curr] < low[neigh])

print(curr + neigh)

③ vis[e.dest]

low[curr] = min(low[curr], dt[neigh])

(if vis[e.dest] even connection exist if we remove so it is not bridge)

```

void dfs (graph[], int curr, bool vis[], 
        int dt[], int low[], int time, par)
{
    vis[curr] = true;
    dt[curr] = low[curr] = ++time;
    for (int i=0; i<graph[curr].size(); i++)
    {
        Edge e = graph[curr].get(i);
        if (e.dest == par)
            continue;
        else if (!vis[e.dest])
        {
            dfs(graph, e.dest, vis, dt, low, time, curr);
            low[curr] = Math.min (low[curr], low[e.dest]);
            if (dt[curr] < low[e.dest])
                System.out.println(curr + " --- " + e.dest);
        }
        else
            low[curr] = Math.min (low[curr], dt[e.dest]);
    }
}

```

void getBridge (graph[], w, v)

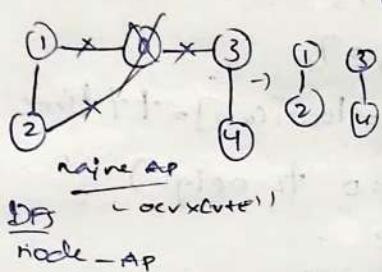
```

{
    int dt[] = new int [v];
    int low[] = new int [v];
    int time = 0;
    boolean vis[] = new boolean [v];
    for (int i=0; i<v; i++)
    {
        if (!vis[i])
            dfs(graph, i, vis, dt, low, time, -1);
    }
}

```

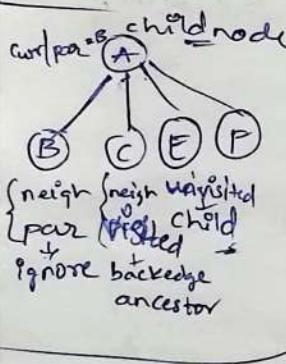
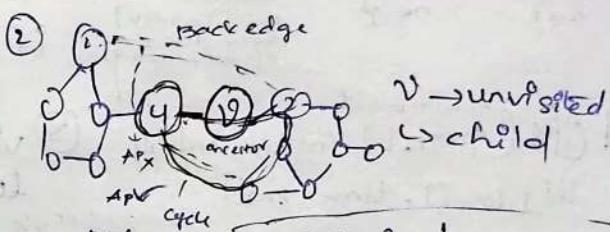
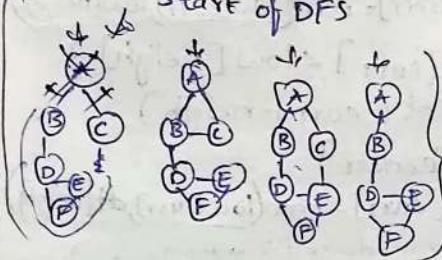
Articulation point

A vertex in an undirected connected graph is an articulation point (or cut vertex) if removing it (and edges through it) disconnects the graph.



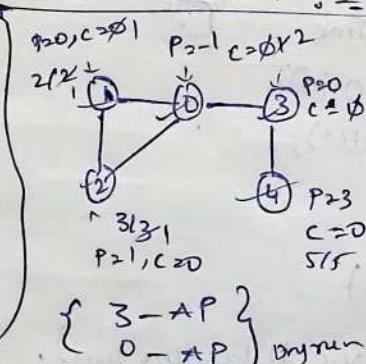
ancestor: - a node A that was discovered before curr node in DFS is the ancestor of curr

① node - corner node disconnected
 $\text{par} = -1 \wedge \text{children} > 1 \rightarrow \text{AP}$
 start of DFS



```

void dfs(graph*, curr, par, dt, low, vis, tme, ap)
{
    vis[curr] = true
    dt[curr] = low[curr] = ++tme
    cout << "curr" << curr << endl
    for (int i=0; i<graph[curr].size(); i++)
    {
        Edge e = graph[curr].get(i)
        cout << "e" << e << endl
        cout << "neigh" << e.dest << endl
        if (par == neigh)
            continue;
        else if (vis[neigh])
        {
            low[curr] = min(low[curr],
                dt[neigh]);
        }
        else
        {
            dfs(graph,neigh,curr,dt,low,vis,tme,ap);
            low[curr] = min(low[curr],low[neigh]);
            if (dt[curr] <= low[neigh] && par != 1)
                ap[curr] = true;
            children++;
        }
    }
}
    
```



Tarjan's Algorithm

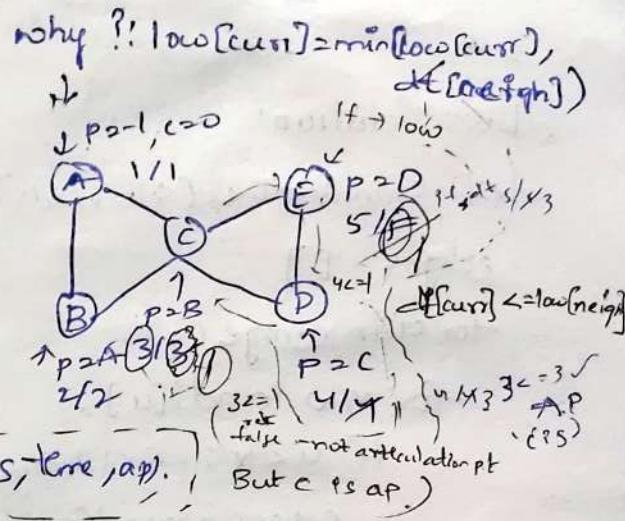
```

vis[curr] = t
dt[curr] = low[curr] = ++tme
children = 0
for (r.neigh)
    Edge e.(src->dest)
    ① if (neigh == par) → ignore
    ② if (neigh - vis)
        low[curr] = min(low[curr],
            dt[neigh])
    ③ if (neigh - unvis)
        dfs(neigh)
        low[curr] = min(low[curr],
            low[neigh])
        if (dt[curr] <= low[neigh] && par != 1)
            curr = AP
            child++
            if (par == -1 && children > 1)
                curr = AP
            if (par == -1 && children > 1)
                ap[curr] = true;
            children++;
    }
}
    
```

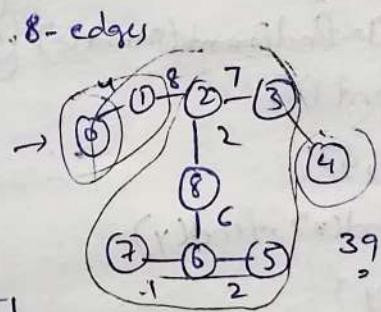
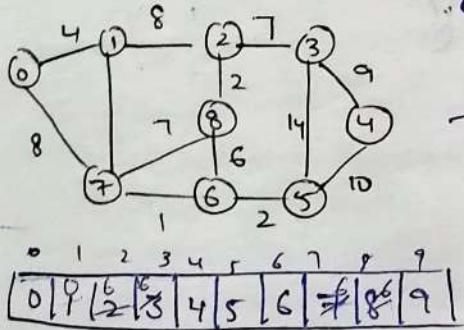
```

void getAPC(graph[], v)
{
    int dt[] = new int[v];
    int loco[] = new int[v];
    int time = 0;
    boolean vis[] = new boolean[v];
    boolean ap[] = new boolean[v];
    for (int i = 0; i < v; i++)
    {
        if (!vis[i])
            {dfs(graph, i, -1, dt, loco, vis, time, ap);
            }
    }
    for (int p = 0; p < v; p++)
    {
        if (ap[p])
            {S.O.P("AP! " + ap[p])}
    }
}

```

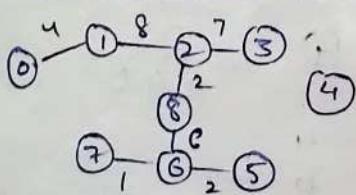


Kruskals Algorithm



Disjoint set

- 1
- $O(1)$ constant
- $O(n)$



Different set

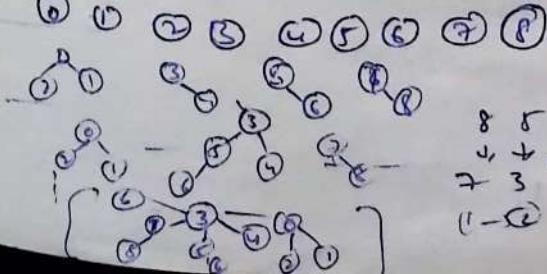
Connect them

8 c
6 8
2-change
6-change

(0,1)	Ultimate parent?
(1,2)	Parent
(3,4)	
(5,6)	
(7,8)	
(3,6)	which set to merge to which
(0,1)	which

Union by rank/siz

- Find the ultimate parent of u & v
- Find the rank of ultimate (p_u, p_v)
- Merge the smaller set into larger set (rank)



8 5
7 3
(1-2)

- Find parent
 - Union by Rank
- $O(n \alpha(n))$ constant
Path compression

Code:

dsu \rightarrow T.C. $O(E \log E)$
Sorting edge,

class Solution:

def SpanningTree(self, v, adj):

edges = []

for i in range(v):

for v, w in adj[i]:

if u < v: // avoid duplicates,

edges.append((w, u, v))

edges.sort()

parent = [i] * v

rank = [0] * v

def find(x):

if parent[x] != x:

parent[x] = findParent(parent[x])

return parent[x]

def union(x, y):

rx, ry = find(x), find(y)

if rx == ry:

return False

if rank[rx] < rank[ry]:

parent[rx] = ry

elif rank[rx] > rank[ry]:

parent[ry] = rx

else:

parent[ry] = rx

rank[rx] += 1

return True

mst_weight = 0

edges_used = 0

for w, u, v in edges:

if union(u, v):

mst_weight += w

edges_used += 1

If edges_used == N-1:

return mst_weight

$O(E, \alpha(v)) = O(E)$

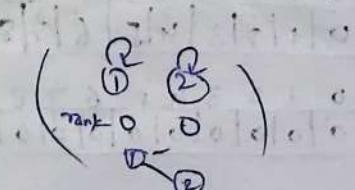
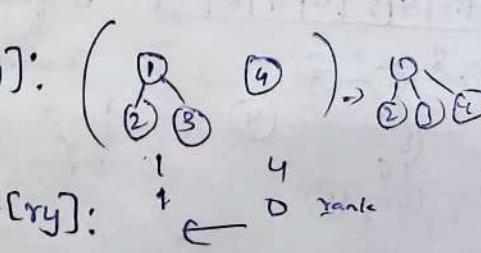
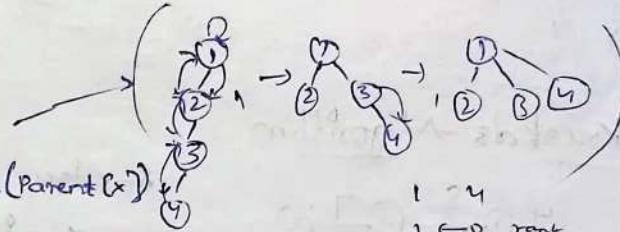
\rightarrow S.C. $O(V+E)$

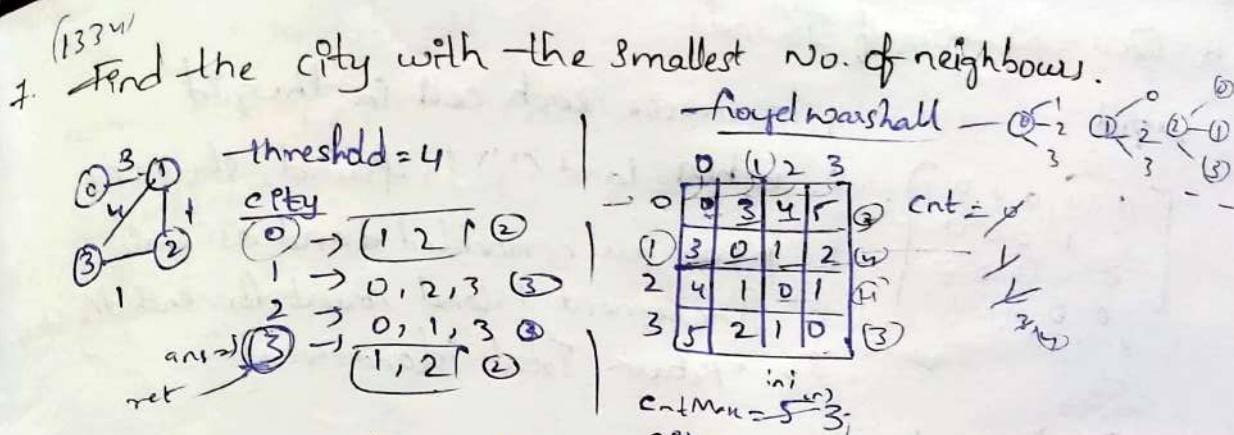
T Edgelist - $O(E)$

DSU(Parent+rank) - $O(V)$

(Kruskal \rightarrow edge based, sparse graph)
Prim \rightarrow node based, dense graph)

(edges, sort(key=lambda x: x[2])) / sort by weight





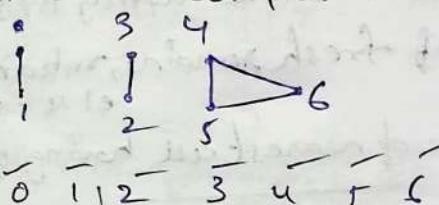
Floyd-Warshall - adj^0

0	1	2	3
0	0	3	4
1	3	0	1
2	4	1	0
3	5	2	1

cnt = 0
cntMin = 5 → 3
City = 1 0 3

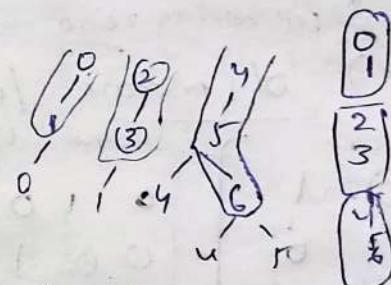
1. Initialize $n \times n$ dist matrix with ∞
2. Fill edge weights and set $d[0][i] = 0$.
3. Apply floyd-warshall to get all pairs shortest path
4. count cities reachable within the threshold for each city.
5. return the cities with min count (tie → larger index).

8. connected components in an undirected Graph.

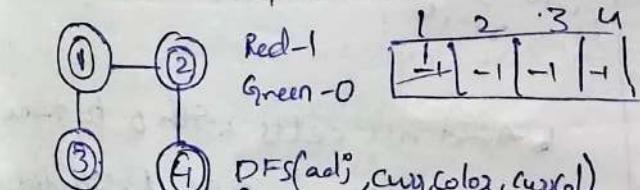


viii) Algo steps:

1. Build adj list from edges
2. Maintain vis arr
3. Run DFS from each unvisited node
4. collect nodes as one connected component
5. Continue until all nodes are vis.



9. (28v) Is Graph Bi-partite?



adj:
 1 → 2, 3
 2 → 1, 4
 3 → 1
 4 → 2

```

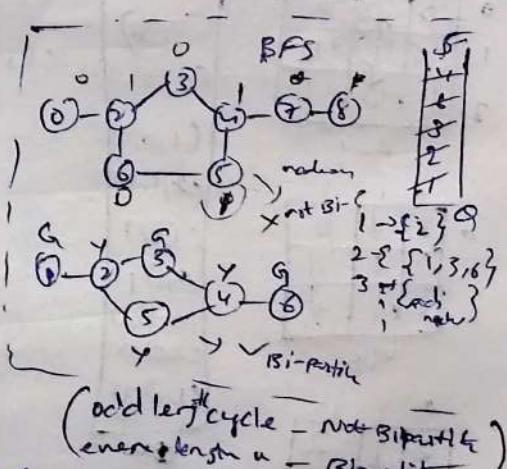
DFS(adj, currColor, currCol)
{
  color[curr] = currColor;
  for (int v : adj[curr])
  {
    if (color[v] == color[curr])
      return false;
    if (color[v] == -1) // never colored
      color[v] = 1 - currColor;
    else if (color[v] != currColor)
      return false;
  }
  return true;
}
  
```

bool isBiPartite(v, adj)

vector<int> color(N, -1);

for (int i = 0; i < N; i++)
 if (color[i] == -1)

return !dfs(adj, N, i, 1);



(odd length cycle - not bipartite)
(even length n - bipartite)

if (dfs(adj, N, i, 1)) return false;

10. (Q20) Number of Islands

2D grid

1	1	0	0	0
1	0	0	0	0
0	0	1	0	0
0	0	0	1	1

3x

1. Traverse each cell in the grid
2. When land ('1') is found, start DFS
3. Mark all connected land as water
4. Increment island count for each off stack
5. Return Total island count

11. (994) Rotting oranges

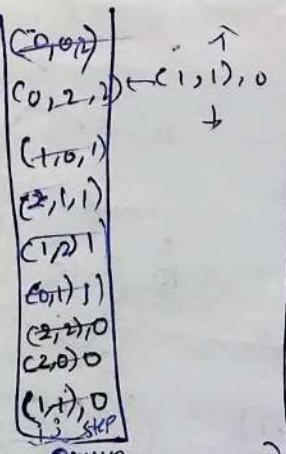
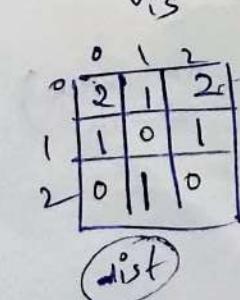
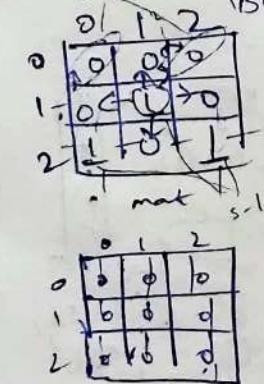
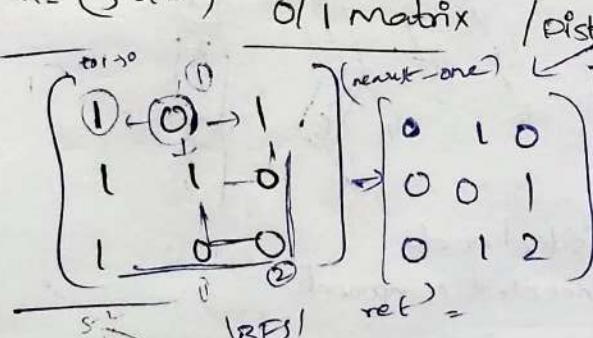
rotten	fresh
2	1
0	1
1	0
-1	=

second minutes?

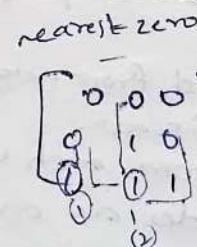
1. Push all initially rotten orange into queue

2. count - fresh oranges
3. BFS level by level to rot neigh's
4. Track time using BFS depth
5. If fresh remains, return -1
else return

12. (5421) 0/1 matrix



1. push({minu,nlo},gps+1)

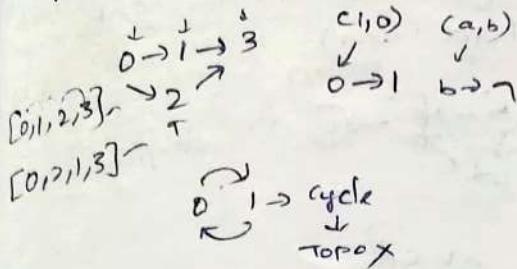


↑
↑
↑
↑
↓
↓
↓
↓

1. Add all cells with 0 to queue
2. Mark them visited with dist 0
3. Run BFS in 4 directions
4. Update distance using BFS level
5. Return distance matrix.

Course Schedule - 11

13. $\text{numCourses} = 4$, pre-req = $[[1,0], [2,0], [3,1], [3,2]]$



$n \rightarrow V$ & pre-edges

(graph)
DAG
TOPO

cycle

T.C. $O(V+E)$

S.C. $O(V+E)$

Code

```
bool isCyclicDFS(Src, vis, recPath, edges)
```

```
{
    vis[src] = true;
    recPath[src] = true;
    for (int i=0; i<edges.size(); i++)
        if (int v = edges[i][0]; !v->u)
            int u = edges[i][1];
            if (src == u)
                if (!vis[v])
                    if (isCyclicDFS(v, vis, recPath, edges))
                        return true;
                else if (recPath[v]) // Backedge cycle
                    return true;
            }
        }
    }
    recPath[src] = false;
    return false;
}
```

```
void topoOrder(Src, vis, s, edges)
{
    vis[src] = true;
    for (int i=0; i<edges.size(); i++)
        if (int v = edges[i][0];
            int u = edges[i][1];
            if (src == u)
                if (!vis[v])
                    topoOrder(v, vis, s, edges);
            }
        }
}
```

$s.push(src)$

```
vector<int> findOrder(n, edges)
{
    vector<bool> vis(n, false);
    vector<bool> recPath(n, false);
    vector<int> ans;
```

for (int i=0; i<n; i++)
 if (!vis[i])

{ if (!vis[i])

{ isCycleDFS(i, vis, recPath, edges);
 { return ans;
 }

}

/* Topological sorted order

Stack<int> s;

vis.assign(n, false);
 for (int i=0; i<n; i++)
 { if (!vis[i])

{ topoOrder(i, vis, s, edges);
 }

}

while (s.size() > 0)

{ ans.push_back(s.top());
 s.pop();
}

return ans;

}

14. (QFG) - Alien Dictionary

baa
abcd
abca
cab
cad

normal dict - abc...xyz
abca
abcd
b-a
bab
cad

- Find out alien order?

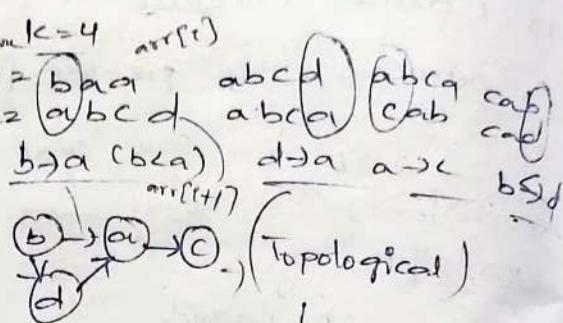
{bdac}

cycle \rightarrow Toposort X



$\{abc\}^k$ error fix - $\{ab, cd\}^k$

shortest



1. Initialize graph, vis, recStack
2. build graph from ads words
3. check invalid prefix condition
4. Run DFS for topological sort
5. Detect cycle using rec stack
6. Reverse DFS order for result

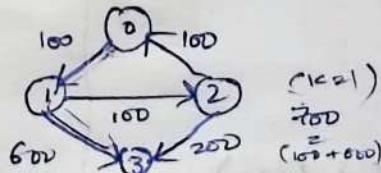
15. (TSP) Cheapest flights within k stops

$n = 4$, flights = $\{\{0, 1, 100\}, \{1, 2, 100\}, \{2, 0, 100\}, \{1, 3, 600\}, \{2, 3, 200\}\}$
 (costs) \rightarrow cost

$$src = 0$$

$$dst = 3$$

$$k = 1 \text{ (stops)}$$

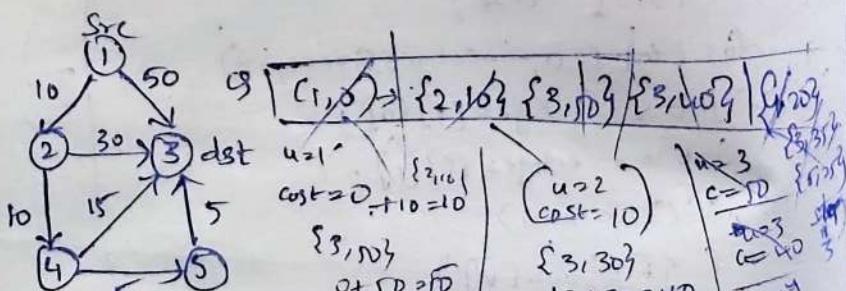
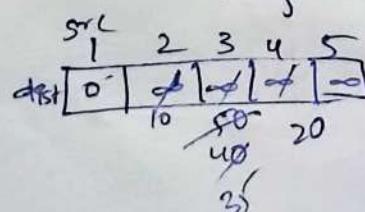


adj
 1 $\rightarrow \{2, 10\}, \{3, 600\}$
 2 $\rightarrow \{3, 30\}, \{0, 10\}$
 3 $\rightarrow \{0\}$
 4 $\rightarrow \{3, 15\}, \{5, 15\}$
 5 $\rightarrow \{3, 5\}$

$$src = 1$$

$$dst = 3$$

$$k = 2$$

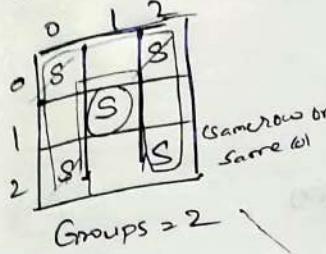


while (!que.empty()) {

Steps <= k {
 src = que.front();
 while (size > 0) {
 if (src == dst) {
 cout << src << endl;
 return;
 }
 size--;
 Step++;

16. (Aug) most stones removed with same row or column

stones = $\{(0,0), (0,2), (1,1), (2,0), (2,2)\}$



$$\begin{aligned} & G_1 \quad G_2 \quad G_3 \dots \\ & x_1 \quad x_2 \quad x_3 \dots \\ & \text{removed} = (x_1 - 1) + (x_2 - 1) + \dots \\ & = (x_1 + x_2 + x_3) - 1 - 1 - 1 \\ & = (x_1 + x_2 + x_3 + \dots) - (1 + 1 + 1 \dots) \end{aligned}$$

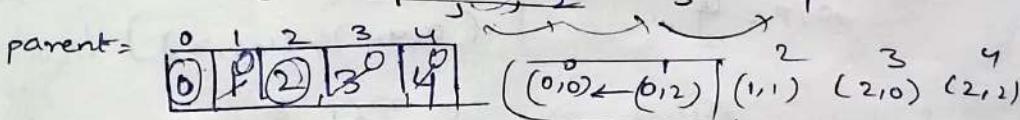
Approaches
DFS
DSU (find union)

$G_1 \quad G_2$

$4 - 1 \quad 1 - 1$

$$2^3 + = 0 \Rightarrow 3 \text{ will be removed} \quad 5 - 2 = 3.$$

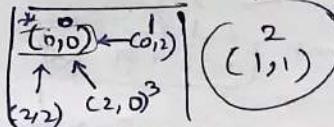
stones = $\{(0,0), (0,2), (1,1), (2,0), (2,2)\}, n=5$.



$$\text{Group} = 1 + 1.$$

$$= 2$$

$$n - (\text{no. of groups}) = 5 - 2 = 3$$



remove stones (stones)

{ n = stones.size();
parent.resize(n); };

rank.resize(n);

for (int i=0; i<n; i++)

{ parent[i] = i; }

rank[i] = 1;

11 S.C: $\Theta(n)$

(dsu)

{ find(i) }
3

Union(i, j)
3

for (int i=0; i<n; i++) // T.C: $O(n^2 \times \alpha(\phi)) \rightarrow O(n^2)$

{ for (int j=i+1; j<n; j++) }

(with small)

{ if (stones[i][0] == stones[j][0]) {

|| stones[i][1] == stones[j][1]) { compare if 2 rows or 2 col same
{ Union(i, j); // $\alpha(\phi)$ with remaining in stones } } }

3 3 3

int groups=0;

for (int p=0; p<n; p++)

{ if (parent[p] == p)

groups++;

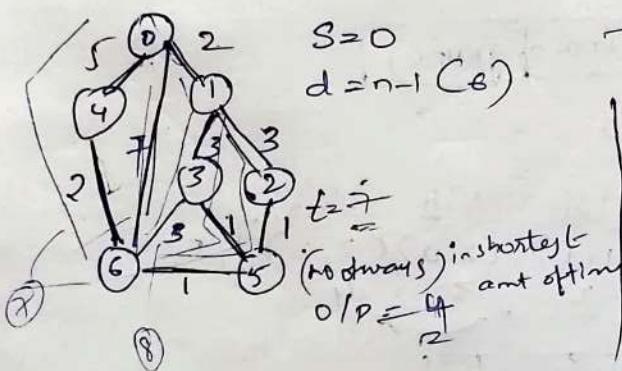
3 return n-groups;

17. No. of provinces (ruge)

$$\text{iscore} = \{1, 1, 0, 1, 1, 0, 1, 0, 1\}$$

$$O/P = 2$$

18) (1996) Number of ways to arrive at destination



Dijkstra's algo

(Single source shortest path)

$$n = 7$$

$$edges = \{(0, 1, 2), (0, 2, 1), (1, 2, 1),$$

$$(1, 3, 3), (6, 3, 3), (3, 5, 1), (6, 5, 1), (2, 5, 1)$$

$$(0, 4, 1), (4, 6, 2)\}$$

result	0	1	2	...	$n-1$
(0, 2, 0)	0	1	2	...	3

$s \rightarrow (n-1)$

count $\begin{matrix} 0 & 1 & 2 & \dots & n-1 \\ 0 & 1 & 3 & \dots & 5 \end{matrix}$

$\text{res}[n-1]$

Count $[n-1]$

$\xrightarrow{\text{ngbr} = (n-1)} \text{ (u, v, t)}$

if (current time + 2) \geq res[ngbr]

{ res[ngbr] = current time + 2.

} PQ.push((res[y], ngbr)).

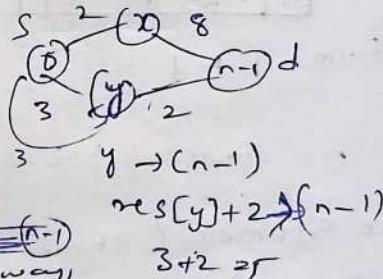
else if (current time + 2) \geq countPath[ngbr],

{ current time + 2 = res[ngbr]

countPath[ngbr] + = countPath[y];

curr node.

return pathCount[n-1];



19. (32a) Longest Increasing path in a Matrix.

matrix

	1	2	3	4
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6
4	4	5	6	7
5	5	6	7	8
6	6	7	8	9
7	7	8	9	10
8	8	9	10	11
9	9	10	11	12
10	10	11	12	13

LIS

T.C. $O(mn)$ S.C. $O(mn)$

LIP

(DP)

1. Use DFS from each cell to explore increasing paths

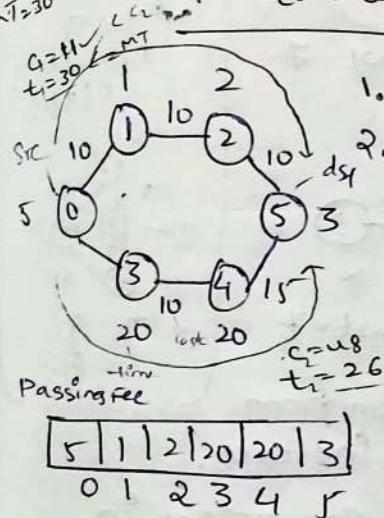
2. Stop DFS if out of bound, or value not increasing

3. Use memorization to avoid recomputation

4. Try all 4 directions from each cell.

5. Return the max value stored in DP.

20. (32b) min time to reach Destination in Time

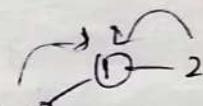


1. minimum possible cost

2. time \leq maxTime

{ simple Dijkstra }

{ Cost, t, neighbor }



cost will be same
time diff
edge relaxation will
be for time.

int dfs(maxTime, edges, passingFee)

{ "make adjacency list"

" node \rightarrow {neigh, time}"

vector<int> minTime(n, INT_MAX);

Priority-queue <vector<int>, vector<vector<int>>, greater<vector<int>> PQ;

PQ.push({passingFee[0], 0, 0}); // {cost, time, node}

minTime[0] = 0

while (!PQ.empty())

{ auto top = PQ.top();

PQ.pop();

int cost = top[0];

int time = top[1];

int node = top[2];

if (node == n-1)

: return cost;

for (auto &neigh : adj[node])

{ int nextTime = time + t;

PQ.push({newCost, nextTime, neigh});

int newCost = cost + passingFee[neigh].

If (newTime \leq maxTime && newTime < minTime[neigh])

{ minTime[neigh] = newTime.

PQ.push(newCost, newTime, neigh);

} }

return -1;

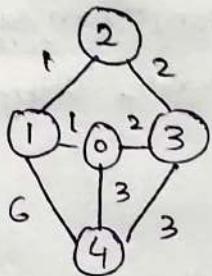
21. (1489). - Find Critical and Pseudo-critical edges in MST

Ex: $n = 5$

edges = $\{ \{0,1,1\}, \{1,2,1\}, \{2,3,2\}, \{0,3,2\}, \{0,4,3\}, \{3,4,3\}, \{1,4,6\} \}$

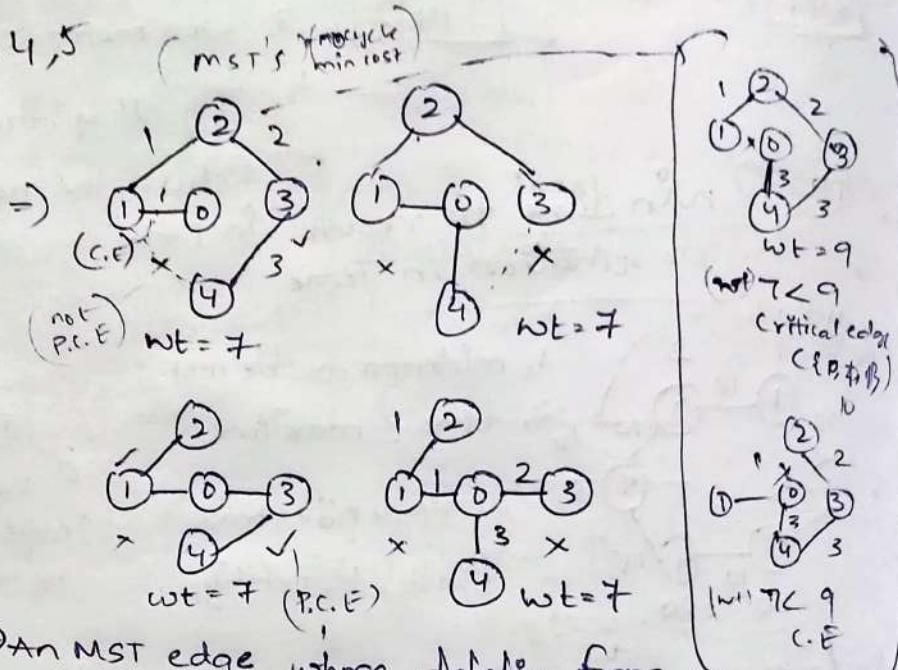
C.E = {1, 0}

P.C.E = {2, 3, 4, 5}



wt = 21

V = 5



Critical Edge → An MST edge whose deletion from the graph would cause MST weight to increase.

Pseudo-Critical Edge → is that which can appear in some MST's but not all.

1. Sorted wt & we added idx.

$\{ \{0,1,1,0\}, \{1,2,1,1\}, \{2,3,2,3\}, \{2,3,2,2\}, \{0,4,3,4\}, \{3,4,3,5\}, \{1,4,6,6\} \}$

$w = 0$
 $v = 1$
 $wt = 1$
 $idx = 0$

Skipwt = knruskal(skip=i);

if (skipwt > mst-wt) - skip

if critical, push-back(idx);

else if = = mst.wt - force include

Pseudo.critical

① push original idx

② sort based on wt

③ find mst.wt → knruskal

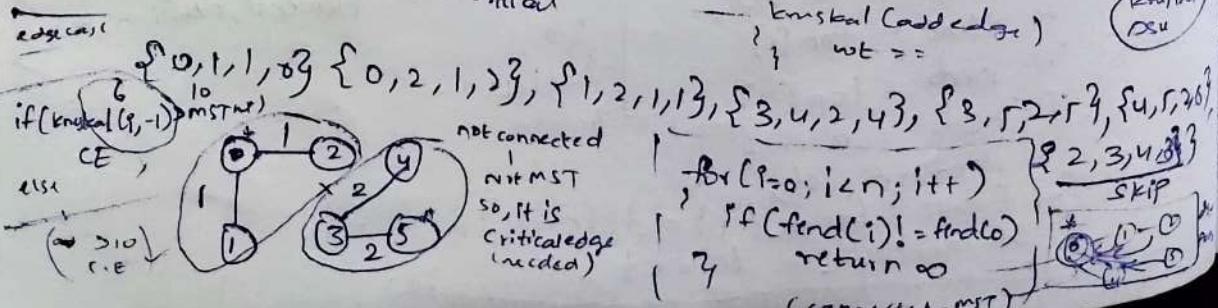
④ checked edge

⑤ skip (CE)

⑥ add (PCE)

kruskal (add edge)
wt ==

Kruskal
DSU



code

```

int kruskal(vec, skip_edge, add_edge)
{
    int sum = 0;
    UnionFind uf(n);
    if (add_edge != -1) // PC
    {
        uf.Union(vec[add_edge][0], vec[add_edge][1]);
        sum += vec[add_edge][2];
    }
    for (int i = 0; i < vec.size(); ++i)
    {
        if (i == skip_edge) continue;
        int u = vec[i][0];
        int v = vec[i][1];
        int wt = vec[i][2];
        int parent_u = uf.find(u);
        int parent_v = uf.find(v);
        if (parent_u != parent_v) // extra
            { uf.Union(u, v); sum += wt; }
        for (int j = 0; j < n; ++j)
        {
            if (uf.find(j) != uf.find(0))
                return INT_MAX;
        }
        return sum;
    }
}

```

-findCriticalAndPseudo(E(vec1, vec2))

```

n = n; // add idx's + sort
for (int i = 0; i < edge.size(); ++i)
{
    Edges[i].push_back(i);
}
sort(begin(Edges), end(Edges), lambda);
auto lambda = [vec1, vec2](vector<int>& vec1, vector<int>& vec2)
{
    return vec1[2] < vec2[1];
}

```

```

int MST_wt = kruskal(edges, -1, -1);
cout << MST_wt << endl;
vector<int> critical;
for (int i = 0; i < edges.size(); ++i)
{
    if (kruskal(edges, i, -1) > MST_wt) // skipping
        critical.push_back(edges[i][3]); // idk
    else if (kruskal(edges, -1, i) > MST_wt) // force add by edge
        critical.push_back(edges[i][3]);
}
return critical, pseudo_critical;
}

```

public:

```

class UnionFind
{
public:
    vector<int> parent;
    vector<int> rank;
    UnionFind(int n)
    {
        parent.resize(n);
        rank.resize(n, 0);
        for (int i = 0; i < n; ++i)
            parent[i] = i;
    }
    int find(int x)
    {
        if (parent[x] == x)
            return x;
        return find(parent[x]);
    }
    bool union_(int x, int y)
    {
        int px = find(x);
        int py = find(y);
        if (px == py)
            return false;
        if (rank[px] < rank[py])
            parent[px] = py;
        else if (rank[py] < rank[px])
            parent[py] = px;
        else
            parent[px] = py;
            rank[py]++;
        return true;
    }
};

```

(graph)

using Kahn's algorithm (CBFS)

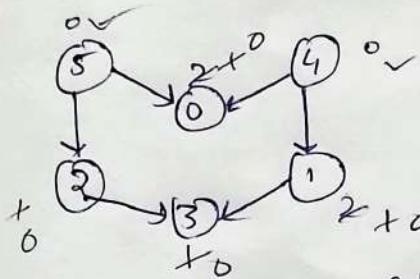
Topological Sorting

$u \rightarrow v$

indegree (no. of incoming edges to the node)

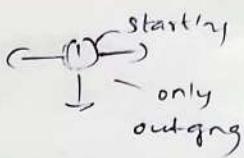
ans ->

4	5	0	2	3	1
---	---	---	---	---	---



Q

4	5	0	1	2	3	1	1
---	---	---	---	---	---	---	---



- ① calc indeg of all nodes
- ② 0 indeg $\rightarrow \emptyset$, push (start)
- ③ while ($Q.size() > 0$)
 $curr = Q.front()$
 for (neighbors of curr)
 neighbor indeg -
 \emptyset .push

Indeg

2	1	2	1	1	1	0	1	0
---	---	---	---	---	---	---	---	---

① vector <int> Indeg(v, 0)

for (i >= 0 to v)

for (int v : l[u])

Indeg[v]++

② for (i >= 0 to v)

If (Indeg[i] == 0) $\rightarrow \emptyset$.push(i)

③ while ($Q.size() > 0$)

curr = Q.pop();

sec.push_back(curr);

for (int v : l[curr])

Indeg[v]--;

If (Indeg[v] == 0) $\rightarrow \emptyset$.push(v)

};

res \checkmark $\left\{ \begin{array}{l} \text{se.size()} == v \\ \text{nodes} \rightarrow \text{cycle } x \end{array} \right\}$

5/1/26

D. Dynamic programming (DP)

1. (121) Best time to buy & sell stock.

```

min = a[0], prof = 0
for (i=1 to n)
    cost = a[i] - min
    prof = max(prof, cost)
    min = min(min, a[i])
set prof
    
```

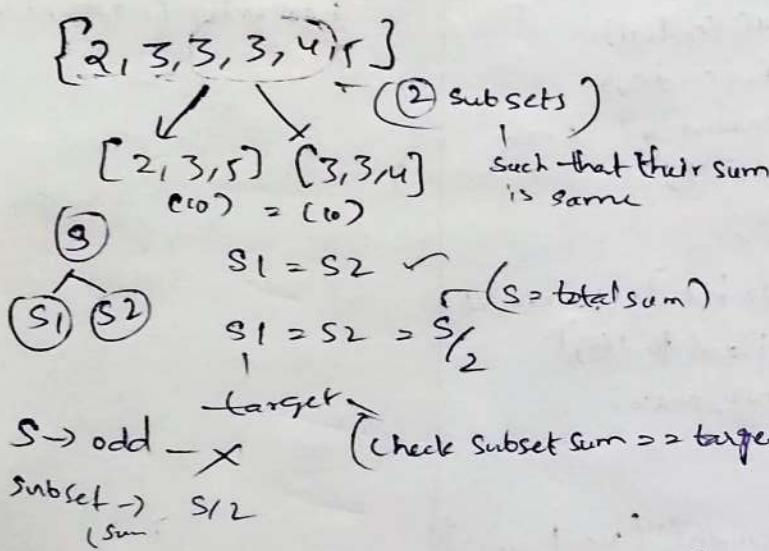
arr $\rightarrow [7, 1, 5, 3, 6, 4]$

Buy -	1
Sell -	6
	5

(only done on 5th day)

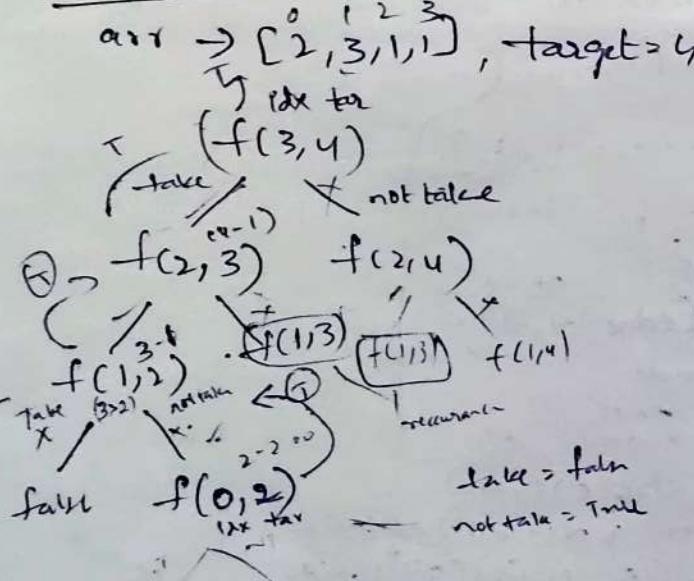
If you are selling on i^{th} day
you can buy on the min price
from $(1^{st} \text{ to } i-1)$

2. (416) Partition Equal Subset Sum



Subset - must follow order
Subset - can or cannot follow order

subset sum equals to target



recursion

① $f(\text{idx}, \text{target})$

// B. case

{ if ($\text{tar} = 0$) return true;

. if ($\text{idx} = 0$) return ($a[0] = \text{tar}$);

. if ($a[\text{idx}] + \text{tar} \leq 0$) return $f(\text{idx}-1, \text{tar})$;

bool nottake = $f(\text{idx}-1, \text{tar})$;

bool take = false;

. if ($\text{tar} > a[\text{idx}]$)

. take = $f(\text{idx}-1, \text{tar} - a[\text{idx}])$;

. dp[idx][tar];

return (take) or (nottake);

memo
 $\int_{\text{idx}}^{\text{target}}$ target $T C \rightarrow O(n \times \text{target})$ (2)
 $dp[(0^3+1) \times (0^3+1)] \quad SC \rightarrow O(n \times \text{tar}) + O(n)$
 $\text{pre}[0] = (\text{cur}[0] = \text{true})$

1) Base case
 2) Recurrence relation
 3) Aux. space
 4) Time complexity
 5) Space complexity
 6) Top-down
 7) Bottom-up

bool SubsetTo k (n, k, arr)
 {
 rec <bool> prev(k+1, 0), cur(k+1, 0);
 prev[0] = cur[0] = true;
 prev [arr[0]] = true;
 for (int idx = 1; idx < n; idx++).
 { for (int tar = 1; tar <= k; tar++)
 { bool notTake = prev [target];
 bool take = false
 if (curr [arr[idx]] <= tar) take = prev [tar - arr [idx]];
 curr [target] = take | notTake;
 prev = curr
 set prev [tar] = (P-1);
 }

S.C. $\rightarrow O(\text{tar})$

idx = 0 $\boxed{0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1}$ pre
 idx = 1 $\boxed{1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1}$ cur pre
 cur

```

④ SubsetsSumToK(n, k, arr)
{
    vec vec<bool>> dp(n, vec<bool>(k+1, 0));
    for (p=0; i < n; i++) dp[i][0] = true;
    dp[0][arr[0]] = true;
    for (int ind=1; ind < n; ind++)
    {
        for (int tar=1; tar <= k; tar++)
        {
            bool notTake = dp[ind-1][tar];
            bool take = false;
            if ((arr[ind] <= tar) && take = dp[ind-1][tar - arr[ind]]);
            dp[ind][tar] = take | notTake;
        }
    }
    return dp[n-1][k];
}

```

3. (494) Target Sum

Given an arr of arr and a target sum. Find the total no. of ways you can build an expression to reach the target. Just use '-' and '+'.

$$[1, 1, 1, 1, 1] \quad \text{target} = 3$$

0	1	2	3	4
1	1	1	1	1

$\text{dp}[\text{sum}] = \text{no_of_ways}$

$$\left\{ \begin{array}{l} +1 -1 +1 -1 +1 = 1 \\ +1 +1 +1 +1 +1 = 5 \\ -1 -1 -1 -1 -1 = -5 \\ +1 +1 +1 +1 +1 = 3 \end{array} \right.$$

① $\text{dp}[0] = 1 \xrightarrow{\substack{+1 \\ -1}} = 1 \quad \left| \begin{array}{l} \text{dp}[1] = 1 \\ \text{dp}[-1] = 1 \end{array} \right. \quad \text{T.C} \rightarrow O(n^2)$

to get sum '0' there is only one way (no ele)

$\text{dp}[1] = 1 \xrightarrow{\substack{+1 \\ -1}} = 2 \quad \left| \begin{array}{l} \text{dp}[-2] = 1 \\ \text{dp}[0] = 2 \\ \text{dp}[2] = 1 \end{array} \right. \quad \text{S.C} \rightarrow O(n)$

② $\text{dp}[1] = 1 \xrightarrow{\substack{+1 \\ -1}} = 0 \quad \left| \begin{array}{l} \text{dp}[-2] = 1 \\ \text{dp}[0] = 2 \\ \text{dp}[2] = 1 \end{array} \right.$

$\text{dp}[-1] = 1 \xrightarrow{\substack{+1 \\ -1}} = 0 \quad \left| \begin{array}{l} \text{dp}[-2] = 1 \\ \text{dp}[0] = 2 \\ \text{dp}[2] = 1 \end{array} \right.$

③ $\text{dp}[-2] = 1 \xrightarrow{\substack{+1 \\ -1}} = -1 \quad \left| \begin{array}{l} \text{dp}[-3] = 1 \\ \text{dp}[-1] = 3 \text{ ways} \\ \text{dp}[1] = 3 \\ \text{dp}[3] = 1 \end{array} \right.$

$\text{dp}[0] = 2 \xrightarrow{\substack{+1 \\ -1}} = 1 \quad \left| \begin{array}{l} \text{dp}[-3] = 1 \\ \text{dp}[-1] = 3 \text{ ways} \\ \text{dp}[1] = 3 \\ \text{dp}[3] = 1 \end{array} \right.$

$\text{dp}[2] = 1 \xrightarrow{\substack{+1 \\ -1}} = 3 \quad \left| \begin{array}{l} \text{dp}[-3] = 1 \\ \text{dp}[-1] = 3 \text{ ways} \\ \text{dp}[1] = 3 \\ \text{dp}[3] = 1 \end{array} \right.$

④ $\text{dp}[-3] = 1 \xrightarrow{\substack{+1 \\ -1}} = -2 \quad \left| \begin{array}{l} \text{dp}[-4] = 1 \\ \text{dp}[-2] = 4 \\ \text{dp}[0] = 6 \\ \text{dp}[2] = 4 \\ \text{dp}[4] = 1 \end{array} \right.$

$\text{dp}[-1] = 3 \xrightarrow{\substack{+1 \\ -1}} = 0 \quad \left| \begin{array}{l} \text{dp}[-4] = 1 \\ \text{dp}[-2] = 4 \\ \text{dp}[0] = 6 \\ \text{dp}[2] = 4 \\ \text{dp}[4] = 1 \end{array} \right.$

$\text{dp}[1] = 3 \xrightarrow{\substack{+1 \\ -1}} = 2 \quad \left| \begin{array}{l} \text{dp}[-4] = 1 \\ \text{dp}[-2] = 4 \\ \text{dp}[0] = 6 \\ \text{dp}[2] = 4 \\ \text{dp}[4] = 1 \end{array} \right.$

$\text{dp}[3] = 1 \xrightarrow{\substack{+1 \\ -1}} = 0 \quad \left| \begin{array}{l} \text{dp}[-4] = 1 \\ \text{dp}[-2] = 4 \\ \text{dp}[0] = 6 \\ \text{dp}[2] = 4 \\ \text{dp}[4] = 1 \end{array} \right.$

⑤ $\text{dp}[-4] = 1 \xrightarrow{\substack{+1 \\ -1}} = -3 \quad \left| \begin{array}{l} \text{dp}[-5] = 1 \\ \text{dp}[-3] = 5 \\ \text{dp}[-1] = 10 \\ \text{dp}[1] = 10 \\ \text{dp}[3] = 5 \\ \text{dp}[5] = 1 \end{array} \right. \quad \text{(to get target sum '3')} \\ \text{dp}[-2] = 4 \xrightarrow{\substack{+1 \\ -1}} = -1 \quad \left| \begin{array}{l} \text{dp}[-5] = 1 \\ \text{dp}[-3] = 5 \\ \text{dp}[-1] = 10 \\ \text{dp}[1] = 10 \\ \text{dp}[3] = 5 \\ \text{dp}[5] = 1 \end{array} \right. \quad \text{(we have 5 ways)} \\ \text{dp}[0] = 6 \xrightarrow{\substack{+1 \\ -1}} = 1 \quad \left| \begin{array}{l} \text{dp}[-5] = 1 \\ \text{dp}[-3] = 5 \\ \text{dp}[-1] = 10 \\ \text{dp}[1] = 10 \\ \text{dp}[3] = 5 \\ \text{dp}[5] = 1 \end{array} \right. \quad \text{(to get target sum '3')} \\ \text{dp}[2] = 4 \xrightarrow{\substack{+1 \\ -1}} = 3 \quad \left| \begin{array}{l} \text{dp}[-5] = 1 \\ \text{dp}[-3] = 5 \\ \text{dp}[-1] = 10 \\ \text{dp}[1] = 10 \\ \text{dp}[3] = 5 \\ \text{dp}[5] = 1 \end{array} \right. \quad \text{(we have 5 ways)} \\ \text{dp}[4] = 1 \xrightarrow{\substack{+1 \\ -1}} = 1 \quad \left| \begin{array}{l} \text{dp}[-5] = 1 \\ \text{dp}[-3] = 5 \\ \text{dp}[-1] = 10 \\ \text{dp}[1] = 10 \\ \text{dp}[3] = 5 \\ \text{dp}[5] = 1 \end{array} \right. \quad \text{(to get target sum '3')} \quad \text{(we have 5 ways)}$

Knapsack

0/1 Knapsack - Fractional Knapsack
by take whole or don't take

$$\text{weights} = \{3, 4, 6, 7\} \quad [n=4] \quad \sum w_i x_i \leq W$$

$$\text{profits (values)} = \{2, 3, 1, 4\} \quad [n=4]$$

		wt →							
		0	1	2	3	4	5	6	7
i	w	0	0	0	0	0	0	0	0
		1	0	0	0 ← 2	2	2	2	2
2	3	0	0	0	0 ← 3	3	3	5	5
3	4	0	0	0	2	3	4	5	6
4	5	0	0	0	2	3	4	5	6
5	6	0	0	0	0	2	3	4	5
6	7	0	0	0	0	2	3	4	5
7	8	0	0	0	0	2	3	4	5

$$m[i, w] = \max(m[i-1, w], m[i-1, w-w[i]] + p[i])$$

$$m[4, 7] = \max(m[3, 7], m[3, 7-6] + 1)$$

$$= (5, 0+1) = (5, 1) \Rightarrow \checkmark$$

$$\begin{cases} \max(4+0, 3) & \max(4+0, 5) \\ \max(4+0, 3) & \max(4+2, 5) \end{cases}$$

$$x_i = \{1, 0, 0, 1\} \quad 6-4 = (2) - 2 = 2 = 0$$

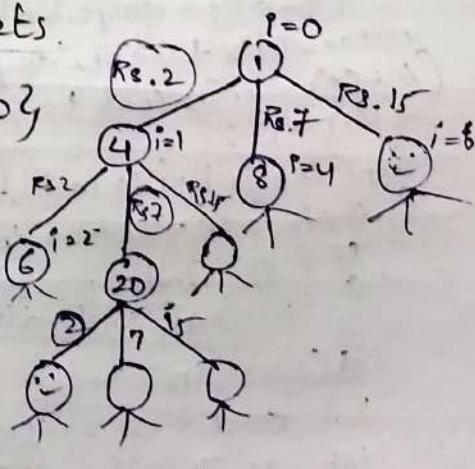
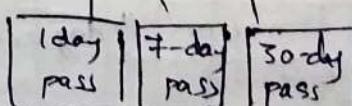
$$\begin{matrix} 3 & 4 & 5 \\ \cancel{1} & \cancel{0} & \cancel{0} \end{matrix} = 8$$

4. (b) Minimum cost for Tickets.

Input:- days = {1, 2, 3, 4, 5, 6, 7, 8, 20}

Costs = {2, 7, 11, 13}

O/P:- 11



S. (22). Coin change

T.C. = $O(\text{amt} * \text{len(coins)})$
S.C. = $O(\text{amt})$

... Longest common Subsequence

(11u3)

Given two strings, find the length of the longest common subsequence (order)

a b c d a f a b c
 a b c d a f a b d
 a c b c f a b d f "abef"
 4 - Ans

	a	b	c	d	a	f
a	0	0	0	0	0	0
b	1	1	1	1	1	1
c	1	1	2	2	2	2
d	1	2	2	2	2	2
a	1	2	3	3	3	3
f	1	2	3	3	3	4

a b c d a f
a c b c f
a, b, c, f ..

(where is this 3 coming) T.C. = $O(mn)$
S.C. = $O(mn)$

Longest Common Substring

(length + 1)
two 2 strings

"z f b c d z b"
"z b c d f"
"z f b c d z b"
"z f b c d z b"
"z f b c d z b"

"z f b b"
"b"
"b"
"x y z a b c b"
"x y z a b c b"
"x y z a b c b"
"x y z a b c b"

(4) "x y z a"

Ans \rightarrow (3) "bcd"

(If both are same add 1 to diagonal opp)
(If not put '0')

z	1	0	0	0	0	1	0
b	0	0	1	0	0	0	2
c	0	0	0	2	0	0	0
d	0	0	0	0	3	0	0
f	0	1	0	0	0	0	0

```

int longestCommonSubString (int m, int n)
{
    S1 = m. to charArray();
    S2 = n. tocharArray();
    int [ ] dp = new int [Str1.length + 1];
    int max = 0;
    // Iterate over each position in the matrix
    for (int i=1; i<=Str1.length; i++)
        for (int j=1; j<=Str2.length; j++)
            if ("Both chars are equal"
                if (Str1[i-1] == Str2[j-1])
                    { // Get the no. from diagonal opp
                        dp[i][j] = dp[i-1][j-1] + 1;
                    }
                max = math. max(dp[i][j], max);
            }
    return max;
}
  
```

Each cell is telling me the longest common subsequence ending at that particular position.

7. Max Length of Repeated Subarray

Given two integer arrays, find the len of the max subarray that appears in both of them.

$[1, 2, 3, 2, 1]$	$[0, 0, 0, 0, 0]$	$\begin{matrix} 1 & 2 & 3 & 2 \\ \cdot & \cdot & 2 & 2 \\ 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 2 \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 \end{matrix}$
$[3, 1, 2, 1, 4, 7]$	$[0, 0, 0, 0, 1]$	

Converting To
Longest Common
Substring

$a b c b a g$
(Same as)
c b a d g
longest common substring

- If both row & col characters are diff, put 0 in the matrix.
- If both are same, copy the value from top-left cell and add + 1 to it.

8. (T2) Edit Distance

Given two strings, return the min no. of operations required to convert string 1 to string 2 (insert, del, substitution)

"horse"

"ros"

remove 'r'

replace 'h' with 'r' rose

remove 'e'

(only 3 operations).

• Insert 'x' at 3rd pos) → Inxtention

• Delete first 't' → Pxvention

"intention"

"execution"

(remove 't')

(replace 'i' with 'e')

(replace 'n' with 'x')

(replace 'l' with 'c')

(insert 'u')

(5)

invention

enention

exention

execotion

execution

Brute force - "abcde f"
"a b cde"

recursively pass func after each op.
not optimal

horse
ros
3+1
(4)

" abcdef"
"a b cde"

	a	b	c	d	e	f
a	0	1	2	3	4	5
b	1	0	1	2	3	4
c	2	1	0	1	2	3
d	3	2	1	0	1	2
e	4	3	2	1	0	1
f	5	4	3	2	1	0

ab(1)

2,2

ab(2)

2,2

ab(3)

2,2

ab(4)

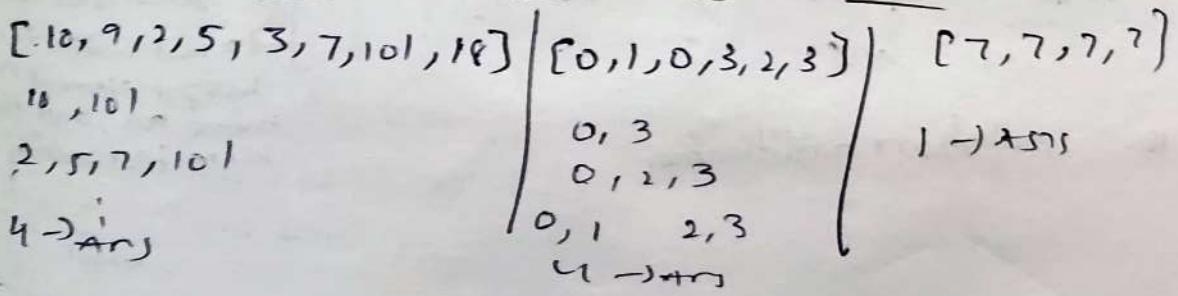
2,2

```

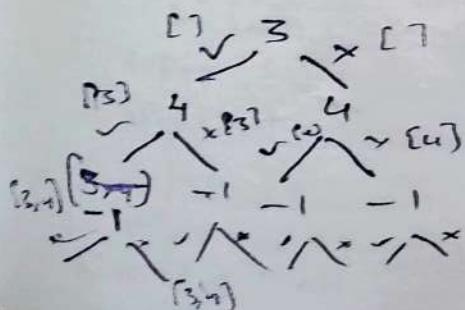
int minDistance(string w1, string w2)
{
    m = word1.length();
    n = word2.length();
    costDP[i][j] := min operations to convert word1 to word2.
    int [][]costDP = new int [m+1][n+1];
    // Initialize DP matrix
    for(int i=1; i<=m; ++i), costDP[i][0] = i;
    for(int j=1; j<=n; ++j), costDP[0][j] = j;
    for(int i=1; i<=m; ++i)
        for(int j=1; j<=n; ++j)
            if (w1.charAt(i-1) == w2.charAt(j-1))
                // copy from top left
                costDP[i][j] = costDP[i-1][j-1];
            else
                // Get min of all 3 neighbours
                int topLeft = costDP[i-1][j-1];
                int top = costDP[i-1][j];
                int left = costDP[i][j-1];
                costDP[i][j] = Math.min (topLeft, Math.min (top, left)) + 1;
    return costDP[m][n];
}

```

9. (300) Longest Increasing Subsequence

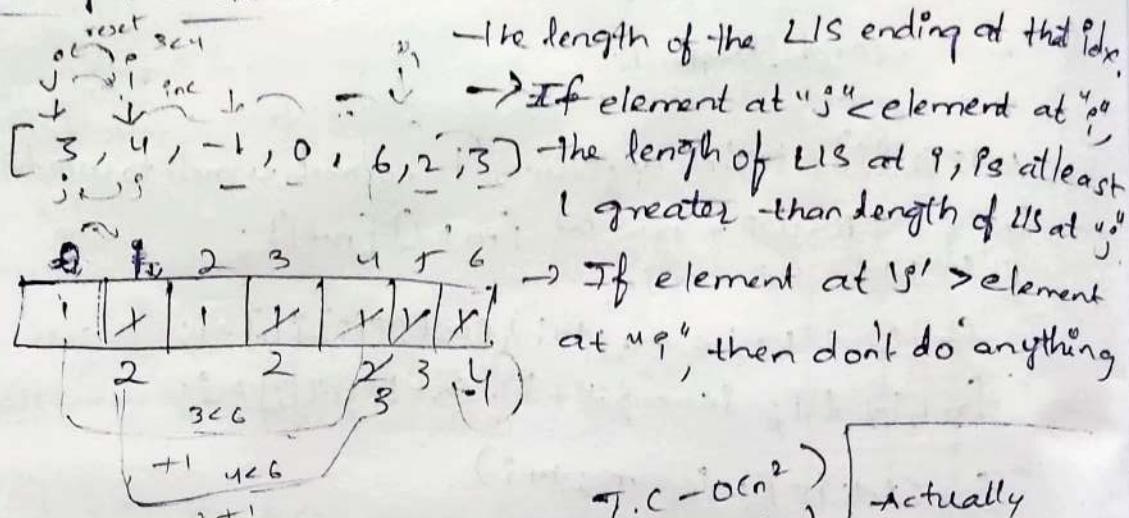


$[3, 4, -1, 0, 6, 2, 3]$



Using memorization

→ Each element in our representation



length of LIS (nums)

{ int T[] = new int[nums.length];

1. Start main ptr

for (i=1; i<nums.length; i++)

{ "start sec ptr"

for (s=0; s<i; j++)

{ if (nums[i] > nums[s])

 if (T[s]+1 > T[i])

 T[i] = T[s] + 1

send max val idx ← $\begin{cases} \text{maxIdx} = 0 \\ \text{for } (s=0 \text{ to } T[\text{len}]) \\ \quad \text{if } (T[s] > T[\text{maxIdx}]) \\ \quad \quad \text{maxIdx} = s \end{cases}$

return T[maxIdx] + 1

ans

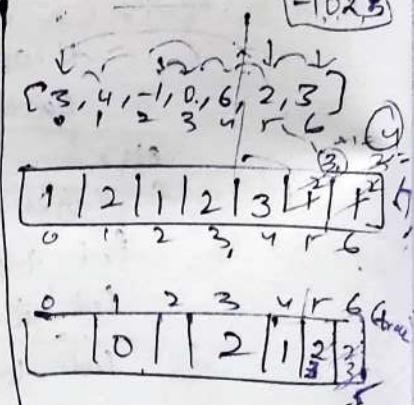
10. Palindrome partitioning

$$\begin{aligned} \text{T.C.} &= O(n^2) \\ \text{S.C.} &= O(n) \end{aligned}$$

Actually

ending the sequence

-1025



→ If we have to update the memo array, just identify the idx of first pointer, and write that in your trace array.

→ Start with the largest element in memo arr, and then keep backtracking to 0.

11. C152) Maximum product subarray
 (find contiguous sub-array with largest prod)

$[2, 3, -2, 4]$	$[-2, 0, 1]$	$[2, 3, -2, 5, 4, -1, 4]$
$\text{Ans} \rightarrow 16$	1	360

case 1 : All numbers are positive

$[2 | 3 | 1 | 2 | 1 | 5 | 1 | 6 | 1 | 4]$

case 2 : Both +ve & -ve numbers

$(2 | 3 | -2 | -5 | 6 | -1 | 4)$

↳ Even count of neg numbers

↳ Odd n n n n

case 3 : positive, negative and 0

$[2 | 3 | 0 | -5 | 6 | -1 | 4]$

$\overbrace{[2 | 3 | -2 | -5 | 6 | -1 | 4]}$

start from left

$$\begin{aligned} 2 \\ 2 \times 3 = 6 \\ 6 \times -2 = -12 \\ -12 \times -5 = 60 \\ 60 \times 6 = 360 \\ 360 \times -1 = -360 \\ -360 \times 4 = -1440 \end{aligned}$$

start from right

$$\begin{aligned} 4 \\ 4 \times -1 = -4 \\ -4 \times 6 = -24 \\ -24 \times -5 = 120 \\ 120 \times -2 = -240 \\ -240 \times 3 = -720 \\ -720 \times 2 = -1440 \end{aligned}$$

max so far

$$\begin{aligned} 4 \\ 6 \\ 6 \\ 120 \\ 360 \\ 360 \\ 360 \\ 360 \end{aligned}$$

leftprod
rightprod - y
ans 1st 6

case of zero :

$[2 | 3 | 0 | -5 | 6 | -1 | 4]$

start from left

$$\begin{aligned} 2 \\ 2 \times 3 = 6 \\ 6 \times 0 = 0 \\ 0 \times -5 = -5 \end{aligned}$$

left prod
start - 1
{-5, 0, -1}

start from right

$$4 \\ 4 \times -1 = -4$$

max so far

$$4 \\ 6 \\ ;$$

int maxProduct (int[] nums)

{ int n = nums.length;

$O(n^2)$

* leftProd = 1;

$O(1)$

* rightProd = 1;

* ans = nums[0];

for (int i=0; i<n; i++)

? leftProd = leftProd == 0 ? 1 : leftProd * (if leftProd == 1 leftProd = 1)

? rightProd = rightProd == 0 ? 1 : rightProd

? leftProd * = nums[i];

? rightProd * = nums[n-1-i];

? ans = Math.max(ans, Math.max(leftProd, rightProd));

return ans;

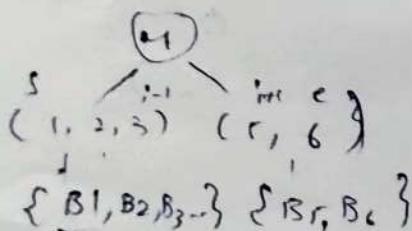
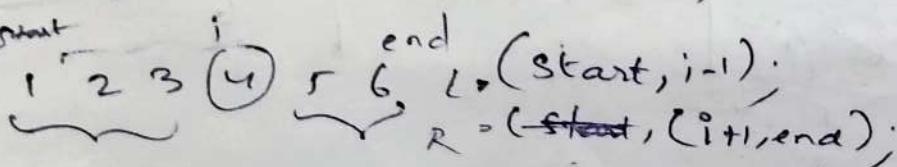
12. Ques) Unique Binary Search Tree

mp = {} map (memo)

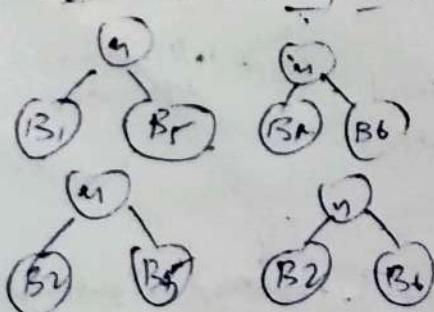
```

~ solve (int start, int end)
{ if (start > end) {
    ; return {NULL};
}
if (start == end)
{
    TreeNode* root = new TreeNode(start);
    return {root}, mp[{start, end}] = {root};
}
if (mp.find({start, end}) != mp.end())
{
    return mp[{start, end}];
}
vector<TreeNode*> result;
for (int i = start; i <= end; i++)
{
    vector<TreeNode*> left_BSTS = solve(start, i - 1);
    for (TreeNode* leftRoot : left_BSTS)
    {
        for (TreeNode* rightRoot : right_BSTS)
        {
            TreeNode* root = new TreeNode(i);
            root->left = leftRoot;
            root->right = rightRoot;
            result.push_back(root);
        }
    }
}
return result;
}

```



{ $\Theta^{\frac{n(n+1)}{2}}$ }



left_BSTS

{ for (left_BST) {
 for (right_BST) {
 ;
 }
}

13. (516) longest Palindromic Substring subsequence

(without using Lcs)
20°

(bbbab) $\xrightarrow{2+}$ (bbbab) $\xrightarrow{1+}$ (bbbab) $\xrightarrow{1+}$ (bbbab) $\xrightarrow{1+}$ (bbbab)

$$\begin{array}{r} \underline{bbbab} \\ babb \end{array} - 4$$

solve (s, i, f)

{ If ($p > j$)
 return 0;

$\rho_f(i = j)$

return

$$z \in C \neq \{p\} \cup \{q\} \Rightarrow -1$$

```
return t[i][j];
```

if ($s[p] == s[q]$)

$t^2 + 2t + 1$

182872

else $\vdash_{\text{ITP}}^{\text{DPL}}$

return many

(with CCS) - (6) check all us)

Silvano

$$S_2^{\prime }=\text{reverse}\left(S_1\right)$$

Longest Common Subsequence (s_1, s_2)

$$\{ m = \text{Stf. length} \}$$

$$\gamma = 52 \cdot \text{length}$$

Received: $t \cdot (m+1, \text{vector}\langle \text{int}(n+1) \rangle)$

for ($i = 0 \rightarrow m+1$)

{ for ($s=0 \rightarrow n+1$)

if ($i \geq 0$) $i \leftarrow 0$)

$$-(r)(s) = 0;$$

\leftarrow If ($s1[j-1] = s2[i-1]$)

-1-

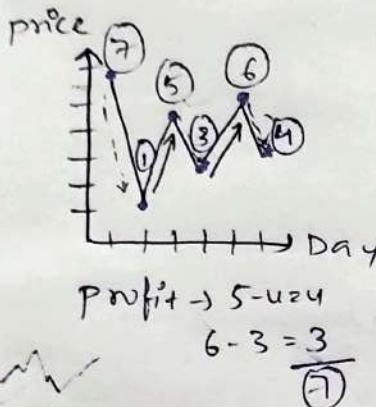
$\text{else } t[i] = \max(t[i-1], f[i-1][j]);$

3

```
return *(m)[n];
```

Q. (122) Best time to buy & sell a stock - II

Prices = $(7, 1, 5, 3, 6, 4)$ | $P = \{1, 2, 3, 4, 5\}$ | $\text{as many transaction as you like}$
 $7, 1, 5 + 3 = \{2\}$ | $\{4\}$ | $P = \{7, 6, 4, 3, 1\}$
 $\{6\}$



$T.C - O(n)$
 $S.C - O(1)$

Profit = 0
 $\text{for } i = 1 \rightarrow \text{len(prices)}$
 $\text{if } \text{prices}(i) > \text{prices}(i-1)$
 $\text{Profit} = \text{prices}[i] - \text{Profit}$
 return Profit

Recursion:

$(-1, C \rightarrow 2^n)$

(overlapping)

B $\begin{pmatrix} 0 \\ 7 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ $\xrightarrow{\text{SC} \rightarrow O(n)}$
 $\begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \end{pmatrix}$ $\xrightarrow{\text{SC} \rightarrow O(n)}$

↓ memorization
 $\begin{pmatrix} \text{ind} \\ \text{buy} \end{pmatrix}$
 $\text{dp}(N) \quad \text{P2}$

$f(\text{ind}, \text{buy})$

$i + (\text{dp}(\text{bad})[\text{buy}] \neq -1)$

return $\text{dp}(\text{bad})[\text{buy}]$

{ if ($\text{ind} = n$)
 return 0.

$\begin{pmatrix} 1, 2, 3, 4 \end{pmatrix}$

even though you buy stocks
and didn't sell prof is 0

(Tabulation)

$\rightarrow 1 \quad \text{dp}(\text{ind}+1)[0]$ (change to sell)

{ profit = max { $-\text{prices}[\text{ind}] + f(\text{ind}+1, 0)$ } // take

$0 + f(\text{ind}+1, 1)$ // not take

$\text{dp}(\text{ind}+1)[1]$ buy next day

} else

{ profit = max { $\text{prices}[\text{ind}] + f(\text{ind}+1, 1)$ } // sell and change to buy

$0 + f(\text{ind}+1, 0)$ // sell next day

return $\text{dp}(\text{ind})[\text{buy}] = \text{profit}$

$f(0, 1)$
 + / not buy

$(T.C \rightarrow O(N \times 2))$

$\text{dp}(\text{ind}+1)[0] = 0$

$f(1, 0)$
 / not buy
 + / not sell

$S.C \rightarrow O(N \times 2) + O(N)$

$\text{dp}(\text{ind}+1)[1] = 0$

$f(2, 0)$
 / not buy
 / not sell

$\text{dp}(\text{ind}+1)[0]$

$\text{dp}(\text{ind}+1)[0] = 0$

$f(3, 0)$
 / not buy
 / not sell

$\text{dp}(\text{ind}+1)[1]$

$\text{dp}(\text{ind}+1)[1] = 0$

$f(4, 0)$
 / not buy
 / not sell

$\text{dp}(\text{ind}+1)[0]$

$\text{dp}(\text{ind}+1)[0] = 0$

$f(5, 0)$
 / not buy
 / not sell

$\text{dp}(\text{ind}+1)[1]$

$\text{dp}(\text{ind}+1)[1] = 0$

$f(6, 0)$
 / not buy
 / not sell

$\text{dp}(\text{ind}+1)[0]$

$\text{dp}(\text{ind}+1)[0] = 0$

$f(7, 0)$
 / not buy
 / not sell

$\text{dp}(\text{ind}+1)[1]$

$\text{dp}(\text{ind}+1)[1] = 0$

Rec $\rightarrow 0 \rightarrow n$

reverse

1. Base case

2. $i \rightarrow (n-1 \text{ to } 0)$

buy $\rightarrow (0 \rightarrow 1)$

copy the recurrence

n-3 n-2 n-1 n-0

dp(n+1)[1]

dp(n+1)[0]

dp(n+1)[1]

dp(n+1)[0]

recursion ahead(2/10) cur(2/10)

```

getMaxProf(values, n)
{ vector<long> ahead(2, 0), cur(2, 0);
  [0]          [0 1]
  ahead[0] = ahead[1] = 0;
  for (int ind = n - 1; ind >= 0; ind--) {
    { for (int buy = 0; buy <= 1; buy++) {
      { long profit = 0;
        if (buy) {
          { profit = max ((-values[ind] + ahead[0]),
                        (0 + ahead[1]));
        } else {
          { profit = max (values[ind] + ahead[1],
                        0 + ahead[0]);
        }
        cur[buy] = profit;
      }
      ahead[buy] = cur[buy];
    }
  }
  return ahead[1];
}

```

long ahead[notBuy], ahead[Buy]
ahead[notBuy] = ahead[Buy] + p[i]

(dp) python

```

def maxProfit(prices):
  n = len(prices)
  if n == 0:
    return 0
  dp_hold = -prices[0]
  dp_free = 0
  for i in range(1, n):
    # buy or keep holding
    dp_hold = max(dp_hold, dp_free - prices[i])
    dp_free = max(dp_free, dp_hold + prices[i])
  return dp_free

```

$O(n)$
 $O(1)$

"buy or keep holding"
"sell or stay free"

Catalan Triangulation

	0	1	2	3	4	5	6
0	1	1	2				
1		1	2	2			
2			1	3	5	5	
3				1	4	14	14
4					1	5	28
5						1	42
6							42
7							132
8							132

16. Partition A Set into Two Subsets with minimum absolute sum difference (gfg)

[1, 2, 3, 4]

$$\{1, 2\} \quad \{3, 4\} = 1+2 - 3-4 = 4$$

$$\{1, 3\} \quad \{2, 4\} = 1+3 - 2-4 = 2$$

$$\{1, 4\} \quad \{2, 3\} = 1+4 - 2-3 = 0$$

curr subset + (sum = target(k))

f(ind, target)

{ if(target == 0) return true;

if(ind == 0) return (arr[0] == target);
 ("if it is last element next too should be 0.")

not_take = f(ind-1, target)

take = false

if(arr[ind] <= target)

take = f(ind-1, target - arr[ind])

} return take | not_take;

else,

Tabulation

dp[n][target]

for(ind=0 → n-1) {
 dp[ind][0] = true; }
 if(arr[0] <= target) dp[0][arr[0]] = true;

for(i=1 → n)

{ for(tar=1 → k)

{ notake = dp[i-1][target];

take = false;

if(arr[i] <= target)

{ take = dp[i-1][target - arr[i]]; }

dp[i][target] = take or nottake;

(s1, s2)

(s1+s2)

(s1-s2)

(s1+s2)

	0	1	2	3	4	5	6	7	-	-
0	T									
1	T	V								
2	T		V							
3	T			V						
4	T				V					

Subset ↘ yes/no
 ↘ Tabulation

If we check for a target(k)
 - we can determine if every possible target bw(1 & k) is ✓/✗.

targetsum {3, 2, 7}

	2	3	5	7	9	10	12
2	X	X	X	X	X	X	X
3		X	X	X	X	X	X
5			X	X	X	X	X
7				X	X	X	X
9					X	X	X
10						X	X
12							X

dp[3][2+1] ↗ 12-2=10
 ↗ target = s1+s2
 ↗ target - s1=s2
 ↗ abcdiff
 ↗ min = 2 ↗

$$\text{mini} = 1e9$$

```
for ( i=0 → totsum/2 )
  { if ( dp[n-1][i] == 7 )
    {
```

$$S_1 = i ;$$

$$S_2 = \text{totsum} - i ;$$

$$\text{mini} = \min(\text{mini}, \text{abs}(S_2 - S_1))$$

(44) Wildcard Matching

$$S_1 = "x?ay"$$

? → matching with single char

$$S_2 = "xyay" \quad * \rightarrow \text{Matches with sequence of length 0 or more}$$

$$S_1 = "ab*_cd" \quad \text{True}$$

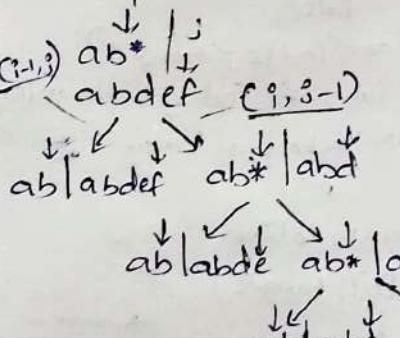
$$S_2 = "abdefcd"$$

$$S_1 = "ab*_cd"$$

$$S_2 = "abdefcd"$$

$$S_1[0:4]$$

$$S_2[0:6]$$



wildcardMatch(pattern, text)

```
(3) n, m → dp[m+1][n+1] → false
dp[0][0] = true;
for (int j=1; j<=m; j++)
  dp[0][j] = false;
```

```
for (int i=1, i<=n; i++)
  bool flag = true;
  for (int k=1; k<=i; k++)
    if (pattern[k-1] != '*')
      { flag = false;
        break;
      }
    dp[i][j] = flag; (T)
```

$S_1 = "ab*_cd"$	$S_2 = "abdefcd"$
$f(i, j)$	$f(i-1, j-1)$

Recursion $T.C. \rightarrow O(n \cdot m) \rightarrow O(n^2)$
 (both are exhausted)

if ($i < 0 \& j > 0$) return true;
 if ($i < 0 \& j >= 0$) return false;
 if ($j < 0 \& i >= 0$) return false;
 for ($k=0$ to i)
 if ($S_1[k] \neq '*' \& S_1[k] \neq S_2[j]$) return false;
 else return true;

If ($S_1[i] = S_2[j] \& S_1[i] \neq '?'$)
 return $f(i-1, j-1)$;

if ($S_1[i] = '*' \& S_1[i] \neq '?'$)
 return $f(i-1, j) \text{ or } f(i, j-1)$;

$T.C. \rightarrow \text{exponential}$
 $S.C. \rightarrow O(n+m)$
 overlapping subproblem
 Recursion

memorization
 $T.C. \rightarrow O(n \times m)$
 $S.C. \rightarrow O(n \times m + O(n+m))$
 Aux. Space

Tabulation
 $S.C. \rightarrow O(n \times m)$

```

for (int p=1; i <= n; i++)
{ for (j=1; j <= m; j++)
    { if (pattern[i-1] == text[j-1] || pattern[p-1] == '?')
        { dp[p][j] = dp[p-1][j-1];
        }
        else if (pattern[p-1] == '*')
        { dp[p][j] = dp[p-1][j] | dp[p][j-1];
        }
        else dp[p][j] = false;
    }
}

```

Space Optimization

```

vector<bool> prev(m+1, false);
prev[0] = true;

```

```

for (int j=1; j <= m; j++)
{ prev[j] = false;
for (int i=1; i <= n; i++)
{ bool flag = true;
    for (int k=1; k <= i; k++)
    { if (pattern[k-1] != '*')
        { flag = false
        .. break;
    }
}

```

3 3

// for every row - you are assigning the 0th col's value

if (flag) curr[0] = flag

```

for (int p=1; j <= m; j++)
{ if (pattern[p-1] == text[j-1] || pattern[p-1] == '?')
    { curr[j] = prev[j-1];
    }
    else if (pattern[p-1] == '*')
    { curr[j] = prev[j] | curr[j-1];
    }
    else curr[j] = false;
}

```

3 prev = curr;

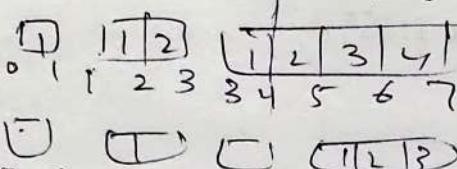
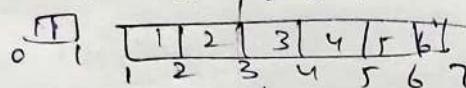
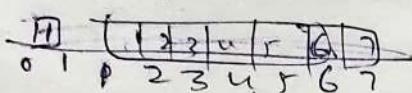
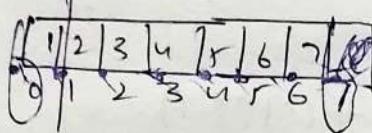
return prev[m];

	*	a	b	*	c	d
*	T	F	F	F	F	F
a	F	T	F	R	F	F
ab	F	F	T	F	F	F
abd	F	F	F	T	F	F
abde	F	F	F	T	F	F
abdef	F	F	F	T	F	F
abcdef	F	F	F	T	T	F
abcdefd	F	F	F	T	F	T

18. Minimum cost to cut the stick (15u7)

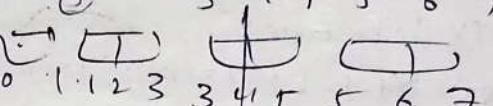
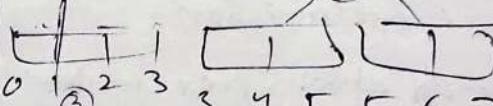
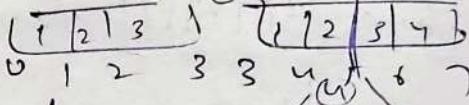
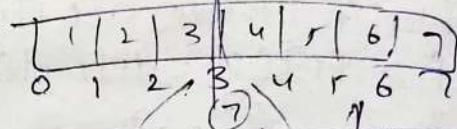
Cost \rightarrow length of the stick you cutting

$$\text{cuts}[] = [1, 3, 4, 5] \quad n=7$$



$$7 + 6 + 4 + 3 \\ = (20)$$

$(3, 5, 1, 4)$ another case



.. (2)

$$7 + 4 + 3 + 2 \\ = (16) \min$$

$(1, 3, 4, 5)$

$(1, 3, 4, 5, 2)$

Solve

Solve

array (Sorted array)

$(1, 3, 5, 7)$ 2 ps not
dependent

i 2 3 4

$\text{dp}[1, 2, 3, 5, 7, 6, 12]$

$f(i, j)$ append

$$(Cuts[3+1] - cuts[i-1]) + f(i, ind-1) + f(ind+1, j)$$

$f(i, j)$ cuts.insert(cuts.begin(), i);
cuts.pushback(n);

$\text{dp}[1, 2, 3, 5, 7, 6, 12]$

$\{$ if ($i > j$) return 0;
min = INT_MAX
for (ind i \rightarrow j)

$f(dp[i][j] != -1) \rightarrow$ return dp[i][j];

{ cost = (cuts[3+1] - cuts[i-1]) +
 $f(i, ind-1) + f(ind+1, j)$

$\}$ } min = min (min, cost)
return dp[i][j] = min;

$\text{dp} \rightarrow [c+1][c+1]$

T.C $\rightarrow (m^2 \times m) \quad m3$

S. $\rightarrow O(m^2)$ aux. stack space

```

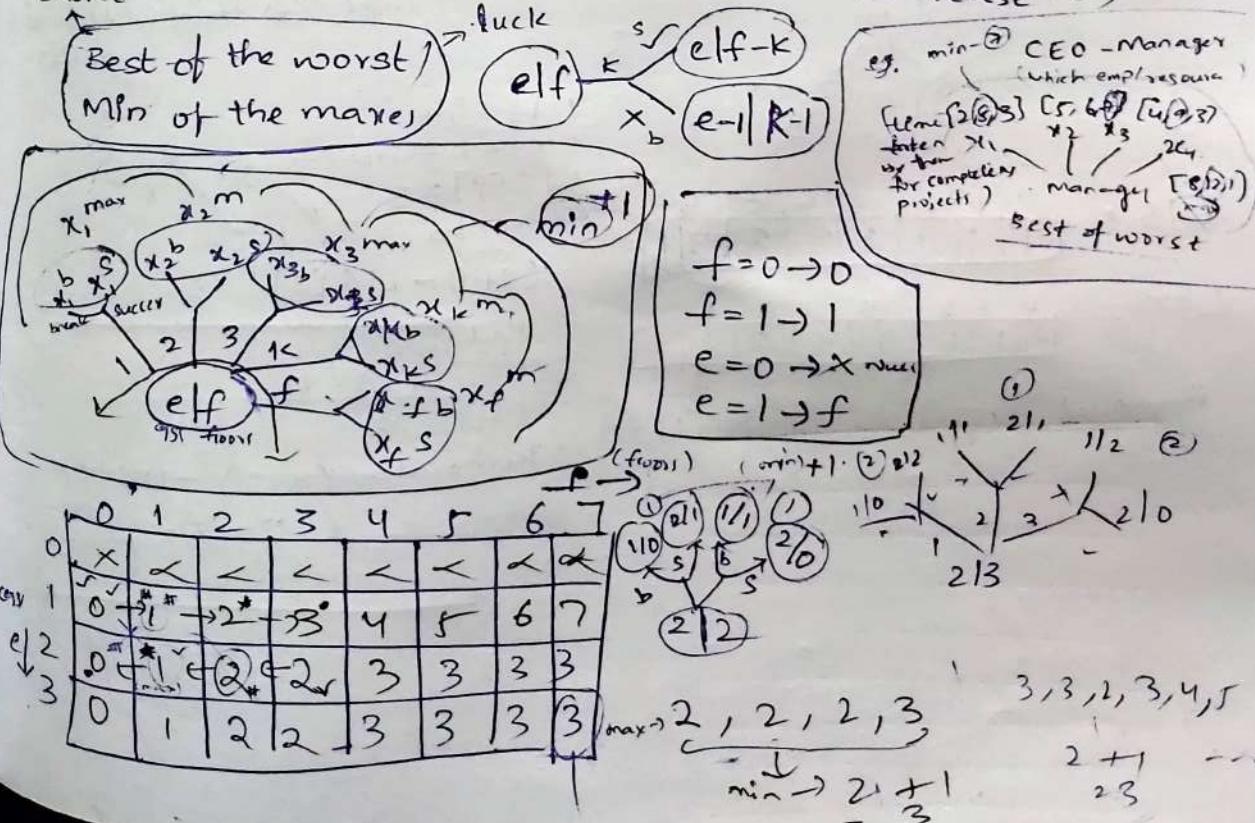
12)
not
int cost (int n, int c, vector<int> &cuts)
{
    cuts.push_back(n);
    cuts.insert(cuts.begin(), 0);
    sort(cuts.begin(), cuts.end());
    vector<vector<int>> dp(c+2, vector<int>(c+2));
    for (int i=c; i>=1; i--)
    {
        for (int j=1; j<=c; j++)
        {
            if (j>i) continue;
            int mini = INT_MAX;
            for (int ind=i; ind<=j; ind++)
            {
                int cost = cuts[j+1] - cuts[i-1] + dp[i][ind-1] +
                           dp[ind+1][j];
                mini = min(mini, cost);
            }
            dp[i][j] = mini;
        }
    }
    return dp[1][c];
}

```

19. Super Egg Drop (887) {
 (where egg breaks) >= c - breaks
 (critical floor) < c - survives }
 IP: $k=1, n=2$ eggs floors
 OIP: 2 no. of eggs no. of floors

 → Don't DP on floors → DP on moves
 → With 'm' moves & 'k' eggs,
 how many floors can we
 guarantee.

7) (min no. of attempts/moves that you need to determine with certainty what val off is)
choice (if egg breaks no longer use, else reuse it)



int eggDrop(int n, int k)

```
f int [][] dp = new int [n+1] [k+1];
for (int i=1; i<=n; i++)
    for (int j=1; j<=k; j++)
        { if (i == 1)
            { dp[i][j] = j;
            }
            else if (j == 1)
                { dp[i][j] = 1;
                }
            else
                { int min = Integer.MAX_VALUE;
                    for (int m=1; m<=j-1; m++)
                        if (dp[i-1][m] >= 0; m-->0; m++)
                            v1 = dp[i][m]; // egg survive
                            v2 = dp[i-1][m]; // egg breaks
                            val = Math.max (v1, v2);
                            min = Math.min (val, min);
                    dp[i][j] = min+1;
                }
        }
    return dp[n][k];
}
```

(cont)

20. Min no. of removals to make Mountain array

INPUT = $\begin{bmatrix} 2 & 1 & 1 & 5 & 6 & 2 & 3 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix}$

i \nearrow left side strictly increasing pattern
consec
 \searrow right side strictly decreasing pattern

6
1
5
2 or 3
1
n = 8

LIS = $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 2 & 3 & 2 & 3 & 1 \end{bmatrix}$

left side :
elements = $(i+1) \times 1$
 $LIS[i] = 3$
Removal = $\sum_{i=1}^n (i+1) - LIS[i]$; // 2

LDS = $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 1 & 3 & 3 & 2 & 2 & 1 \end{bmatrix}$

right side :
ele = $n - i = 4$
 $LDS[i] = 3$
Removal = $(n - i) - LDS[i]$

Removal = $n - LDS[i] - LIS[i] + 1$

4 - 3 = 1

21. CGFG) Matrix chain multiplication

- ↳ An optimization problem that finds the most efficient order to multiply a sequence of matrices.
- ↳ Goal - minimize the no. of scalar multiplications by choosing - the best parenthesization.
- ↳ used in - compilers, db query optimization, scientific computing, deep learning graph execution.

→ New pattern - [partition DP] (Tough pattern)

(solve a problem in partition)

$$(1+2+3) \times 5$$

$$(1+2)(3 \times 5)$$

→ MCM

$$ABC \quad \begin{matrix} A = 10 \times 30 \\ B = 30 \times 5 \\ C = 5 \times 60 \end{matrix} \quad \begin{matrix} (A)(BC) \\ (AB)(C) \end{matrix}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} \quad \begin{bmatrix} 10 \times 30 \\ 30 \times 5 \\ 5 \times 60 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\text{Operations} = 10 \times 30 \times 5 \quad \text{not } C = R$$

$$= 1500 \times 5 = 7500$$

$$(AB)(C) = (10 \times 30 \times 5) + (10 \times 5 \times 60) = 1500 + 3000 = 4500 \text{ operations}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{matrix} 2 \times (2) \\ 2 \times 1 \end{matrix} \quad \begin{bmatrix} 1 \times 2 + 2 \times 3 \\ 3 \times 2 + 1 \times 3 \end{bmatrix} \quad \begin{matrix} 2 \times 1 \\ 2 \times 1 \end{matrix}$$

$$2 \times 2 \times 1 = 4 \text{ multiplications}$$

$$A(Bc) = \begin{matrix} 30 \times 5 \\ 10 \times 30 \end{matrix} \quad \begin{matrix} 5 \times 60 \\ 30 \times 60 \end{matrix} = (30 \times 5 \times 60) + (10 \times 30 \times 60) = 9000 + 18000 = 27000$$

→ ABCD

$$A(B(CD)) \quad (AB)(CD) \quad (A(BC))D$$

$$\text{Given, arr[]} \rightarrow \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

↓
dimension of N-1 matrix $\Rightarrow (N=5)$

$$\begin{matrix} 1^{\text{st}} \rightarrow A[0] \times A[1] \\ 2^{\text{nd}} \rightarrow A[1] \times A[2] \\ 3^{\text{rd}} \rightarrow A[2] \times A[3] \\ \vdots \\ i^{\text{th}} \rightarrow A[i-1] \times A[i] \end{matrix}$$

→ Partition DP \rightarrow Rules:

1. Start with entire blockarray $f(i, j)$ \uparrow start end

2. Try all partitions \rightarrow [Run a loop to try all partition]

3. Return \leftarrow the best possible 2 partitions

→ $f(i, j)$

$$\begin{matrix} (AB)(CD) & A(BCD) & (ABC)(CD) \\ \downarrow & \downarrow & \downarrow \\ f(i, j) & f(i, j) & f(i, j) \end{matrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 10, 20, 30, 40, 50 \end{bmatrix}$$

$$\begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

$f(i, j) \rightarrow$ ref min multiplication to
matrix (matrix \rightarrow mat_{i,j})

$f(i, j)$ single matrix \rightarrow $f(i, j) = \min_{k=i}^{j-1} (f(i, k) + f(k+1, j))$

if $f(i, j) = 0$ then $dP(i, j) = 1$

for $k \leftarrow i$ to $j-1$

steps = $(A[i-1] \times arr[k] \times arr[j])$
 $+ f(i, k) + f(k+1, j)$

$\min = \min(\min, steps)$

return \min (min of all steps)

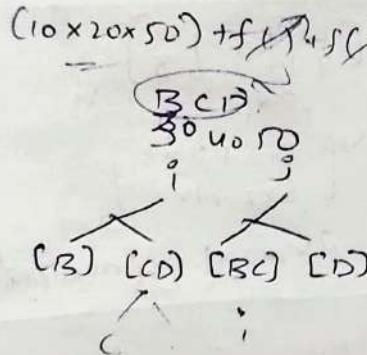
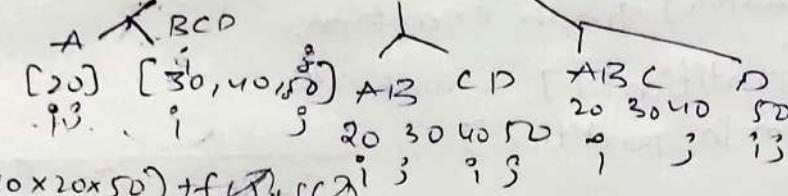
$$\rightarrow k = (i \rightarrow j \rightarrow l)$$

$$k=1 \cdot f(i, k) f(k+1, j) \quad | \quad k = (i+1 \rightarrow j)$$

$$k=2 \cdot f(1, 2), f(3, 4) \quad | \quad f(i, k-1), f(k, j)$$

$$k=3 \cdot f(1, 3), f(4, 4) \quad | \quad k = 2 \cdot f(1, 2), f(3, 4)$$

$$\rightarrow f(10, 20, 30, 40, 50) \quad | \quad k = 1 \cdot f(1, 3) - f(4, 4)$$



(T.C) Recursion $\rightarrow TC$: exponential x
 ↓
 overlapping subproblems
 memorization
 (partially T.C.E)
 $f(i, j)$ changing variable,
 $dp[i][j]$
 $\rightarrow TC \rightarrow O(N^3) \times n \rightarrow O(n^3)$
 $SC \rightarrow O(n^2) + O(n)$

Bottom-up (Tabulation)

Rules:

- copy the base case
- write down changing states
- $i = (n-1 \text{ to } 0)$
 $j = (i+1 \text{ to } n-1)$
- copy the recurrence

$dp[N][N]$

smaller prob to bigger prob \rightarrow

1st step
 \rightarrow

$f(i, j)$
 min cost + to
 $min(dp[i-1][j])$

$T.C \rightarrow N^3$

$S.C \rightarrow O(n^2)$

int matrixMultiplication (vector<int>&arr, int n) {
 $\begin{array}{c} i \\ | \\ A \end{array}$
 $\begin{array}{c} j \\ | \\ B \end{array}$
 $\begin{array}{c} k \\ | \\ C \end{array}$
 $\begin{array}{c} l \\ | \\ D \end{array}$

0	$A \cdot B$ 6000	$A \cdot BC$ 18000	$A \cdot BCD$ 180000
0	0	$B \cdot C$ 24000	$B \cdot C \cdot D$ 60000
0	0	0	$C \cdot D$ 60000
0	0	0	0

$\bullet A \cdot B \rightarrow 10 \times 20 \times 30 = 6000$
 $\bullet A \cdot (B \cdot C) \rightarrow$ best way to multiply $A \cdot B$, $C \rightarrow 18000$
 $\bullet (A \cdot B) \cdot C \rightarrow$ good order $= 360000$

```

    & int dp[N][N];
    & for (int p=1; i<N; i++) {
        &     dp[i][i] = 0; // B.C
    & }
    & for (int p=N-1; i=1; i--) {
        &     for (int j=i+1; j<N; j++) {
            &         int mini = INT_MAX;
            &         for (int k=j; k<i; k++) {
                &             int steps = (arr[p-1] * arr[k] * arr[3]);
                &             if (steps < mini) mini = steps;
            &         }
            &         dp[i][j] = mini + dp[i][k] + dp[k+1][j];
        &     }
    & }
    & return dp[1][N-1];
    
```

(312) Burst Balloons

→ $\text{arr}[] = [3, 1, 5, 8]$ each i^{th} balloon has $\text{arr}[i]^{\text{th}}$ coin on it
 calculate max coins u can collect by bursting balloons in increasing order

$$\begin{array}{ll} [3, 1, 5, 8] & 1 \times 3 \times 1 = 3 \\ [1, 5, 8] & 1 \times 1 \times 5 = 5 \\ [5, 8] & 1 \times 5 \times 8 = 40 \\ [8] & 1 \times 8 \times 1 = 8 \end{array}$$

$$\rightarrow \text{nums}[i-1] \times \text{nums}[i] \times \text{nums}[i+1]$$

$$\begin{array}{ll} [3, 1, 5, 8] & 3 \times 1 \times 5 = 15 \\ [3, 5, 8] & 3 \times 5 \times 8 = 120 \\ [3, 8] & 1 \times 3 \times 8 = 24 \\ [8] & 1 \times 8 \times 1 = 8 \end{array}$$

→ mcm*

$$\{b_1, b_2, b_3, \boxed{b_4, b_5, b_6}\}$$

$$b_3 \times b_4 \times b_5 + b_1, b_2, b_3 + b_5, b_6$$

Can I say this? (No)

$$\{b_1, b_2, b_3, \boxed{b_4, b_5, b_6}\}$$

$$\{b_1, b_2, b_3, b_5, b_6\}$$

(last 2 are right)
not independent

$$\rightarrow 1 \left[\begin{array}{|c|c|c|} \hline b_1 & b_2 & b_3 \\ \hline \end{array} \right] \left[\begin{array}{|c|c|c|} \hline b_4 & b_5 & b_6 \\ \hline \end{array} \right] + f(1, 5)$$

$$\text{arr}[i-1] \times \text{arr}[ind] \times \text{arr}[i+1]) + f(i, ind-1) + f(ind+1, i)$$

$\{b_1, b_4\}$ or $\{b_2, b_4\}$ or $\{b_3, b_4\}$ or $\{b_4, b_5\}$ or $\{b_5, b_6\}$

(b_4) -

$$[b_4] = 1 \times b_4 \times 1$$

→ $f(i, j)$

{ if ($i > j$) return 0;

$$- \text{if } d[i][j] != -1:$$

mani = INT_MAX

ret = dp[i][j]

for (ind = $i \rightarrow j$)

$$\{ \text{cost} = \text{arr}[i-1] \times \text{arr}[ind] \times \text{arr}[i+1]$$

$$+ f(i, ind-1) + f(ind+1, j)$$

$$\} \quad \text{mani} = \max(\text{mani}, \text{cost})$$

$$\} \quad \text{ret} = \min(\text{ret}, \text{mani})$$

T.C - $O(n^3)$

T.C $\rightarrow n \times n \times n \approx n^3$, SC $\rightarrow O(n^2) + \text{arr}(dn)$

Bottom-up

$f(1, n)$

↑ ↑

↓ ↓

Recursion \rightarrow Top Down

↓ ↓ Tabulation \rightarrow Bottom up

$dP[n+1][n+1] \rightarrow 0$

1. write B.C.

2. $i = n-1$
 $j = 1 \rightarrow n$

3. copy the recurrence in the recursion

int maxCoins(vector<int> &a)

```

{ int n = a.size();
  a.push_back(1);
  a.insert(a.begin(), 1);
  vector<vector<int>> dP(n+2, vector<int>(n+2, 0));
  for (int i=n; i>=1; i--)
  {
    for (int j=1; j<=n; j++)
      if (i>j) continue;
      int maxP = INT_MIN;
      for (int pnd = i; pnd<=j; pnd++)
      {
        int cost = a[i-1] * a[pnd] * a[j+1]
                  + dP[i][pnd-1] + dP[pnd+1][j];
        maxP = max(maxP, cost);
      }
      dP[i][j] = maxP;
    }
  return dP[1][n];
}
  
```