Statistics

Assignment -1

Q3) Three coins are tossed, find the probability that two heads and one tail are obtained.

Sol:- Let's take number of coins = 3 coins

Now, we have to write the sample spaces of those three coins.

=> [H, H, H]   [T, T, T]   [H, T, H]   [T, H, T]   [H, T, T]   [T, T, H]
[H, H, T]   [T, H, H]

Now, we have total sample spaces = 8

After finding the sample spaces, we have to find the probability that two heads and one tail are obtained.

Formula of probability => $P(A) = \dfrac{No. of\ favourable\ outcomes}{Total\ no. of\ events\ in\ sample\ space}$

Now, find the no. of favourable outcomes from the sample spaces.

No. of favourable outcomes = [H, H, T]   [T, H, H]   [H, T, H]

So, the number of favourable outcomes will be 3 from the 8 sample spaces.

Now, Apply the probability formula to get the result.

$$P(A) = \dfrac{No. of\ favourable\ outcomes}{Total\ no. of\ events\ in\ sample\ space} = \dfrac{3}{8}$$

Q4) Two dice are rolled, find the probability that sum is

ⓐ Equal to 1

ⓑ Equal to 4

ⓒ Sum is divisible by 2 and 3.

Sol:- Let's take number of dice = 2 Dice

Now, we have to write the sample space when the two dice are rolled.

=> (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)

(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

So, when 2 dice are rolled total possible outcomes are 36.

Then we have total sample space = 36.

ⓐ Now, we have to find the probability that sum is equal to 1.

Formula of probability => $P(S) = \dfrac{No. of\ favourable\ outcomes}{Total\ no. of\ events\ in\ sample\ space}$

Here, Favourable outcomes will be zero (0). Because, in sum equal to 1, it is not possible that sum always exceed to 1.

So, the required probability will be:

$$P(\text{sum}) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of events in sample space}} = \frac{0}{36} = 0$$

(b) Now, we have to find the probability that sum is equal to 4.

Formula of probability $\Rightarrow P(s) = \dfrac{\text{No. of favourable outcomes}}{\text{Total no. of events in sample space}}$

Now, we are finding the favourable outcomes from the given sample space.

No. of favourable outcomes = $(1,3), (2,2), (3,1)$

So, the no. of favourable outcomes will be 3 from the 36 sample spaces.

Now, apply the probability formula to get the result.

$$P(s) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of events in sample space}} = \frac{3}{36} = \frac{1}{12}$$

(c) Now, we have to find the probability that sum is divisible by 2 and 3.

Formula of probability $\Rightarrow P(s) = \dfrac{\text{No. of favourable outcomes}}{\text{Total no. of events in sample space}}$

Now, we are finding the favourable outcomes from the given sample space.

No. of favourable outcomes =

→ 18 outcomes are divisible by 2 = 18/36

→ 12 outcomes are divisible by 3 = 12/36

→ 6 outcomes are divisible by 2 and 3 = 6/36

So, the no. of favourable outcomes will be 6 from the 36 sample spaces.

Now, apply the probability formula to get the result

$$P(sum) = \frac{No.\ of\ favourable\ outcomes}{Total\ no.\ of\ events\ in\ sample\ space} = \frac{6}{36} = \frac{1}{6}$$

Q5) Calculate the

Q5) A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

Sol:- Let's take total number of balls = (2+3+2) = 7

Now, let's find the sample space.

Sample space = No. of ways of drawing 2 balls out of 7 = $7C_2$

$$= \frac{(7 \times 6)}{(2 \times 1)} = \frac{42}{2} = 21$$

Now, we have total sample spaces = 21.

Now, find the probability that none of the balls drawn is blue.

formula of probability $\Rightarrow$ $P(B) = \dfrac{No. of\ favourable\ outcomes}{Total\ no. of\ events\ in\ sample\ space}$

Here, favourable outcome of drawing 2 balls, none of which is blue. So, number of ways of drawing 2 balls out of (2+3) balls.

$= 5C_2$

$= \dfrac{(5 \times 4)}{(2 \times 1)} = \dfrac{20}{2} = 10$

So, the no. of favourable outcomes will be 10 from the 21 sample spaces.

Now, find the probability to get the result.

$P(Balls) = \dfrac{No. of\ favourable\ outcomes}{Total\ no. of\ events\ in\ sample\ space} = \dfrac{10}{21}.$

Q8) Calculate Expected Value for the problem below.

The weights (x) of patients at a clinic (in pounds), are

108, 110, 123, 134, 135, 145, 167, 187, 199

Assume one of the patients is chosen at random. What is the Expected value of the weight of that patient?

Sol:- Let's write the given data of the weights of patients

$$x = 108, 110, 123, 134, 135, 145, 167, 187, 199$$

Note: The weights are in pounds

We have to assume one of the patients is chosen at random.

We will use probability to find the expected value of the weight of that patient.

Here, to find the expected value $E(x)$ or mean μ of a discrete random variable X, simply multiply each value of the random variable by its probability and add the products.

The formula is given as $E(x) = \mu = \sum x \cdot P(x)$.

By apply this formula we will get,

$$E(x) = \frac{1}{9} \times 108 + \frac{1}{9} \times 110 + \frac{1}{9} \times 123 + \frac{1}{9} \times 134 + \frac{1}{9} \times 135 + \frac{1}{9} \times 145 +$$

$$\frac{1}{9} \times 167 + \frac{1}{9} \times 187 + \frac{1}{9} \times 199$$

We will get the Expected value of the weight of that patient
is 145.333 pounds.

Q11) Suppose we want to estimate the average weight of an adult male in mexico. We draw a random sample of 2,000 men from a population of 3,000,000 men and weigh them. We find that the avg person in our sample weighs 200 pounds, and the standard deviation of the sample is 30 pounds. Calculate 94%, 98%, 96% confidence interval?

Sol:- Let's write the formula for confidence interval.

$$C = x \pm z(1-\alpha) \frac{\sigma}{\sqrt{n}}$$

Here,

x = sample average or mean

$z(1-alpha) \frac{\sigma}{\sqrt{n}}$ = Margin of error

Now,

$\frac{\sigma}{\sqrt{n}} = \frac{30}{\sqrt{2000}} = 0.6708$

<u>Z values</u>

$94 = 1.882$

$98 = 2.326$

$96 = 2.053$

So, the confidence interval for 94% is

$\rightarrow 200 \pm 1.882 * 0.678$

$= 200 \pm 1.262$

$\Rightarrow [200+1.262] \ [200-1.262]$

The confidence interval for 98% is

$\rightarrow 200 \pm 2.326 * 0.678$

$= 200 \pm 1.560$

$\Rightarrow [200+1.560] \ [200-1.560]$

The confidence interval for 96% is

$\rightarrow 200 \pm 2.053 * 0.678$

$= 200 \pm 1.377$

$\Rightarrow [200 +1.377] \ [200-1.377]$

Q12) Below are the scores obtained by a student in tests

34, 36, 36, 38, 38, 39, 39, 40, 40, 41, 41, 41, 41, 42, 42, 45, 49, 56

1) Find mean, median, variance, standard deviation.

2) What can we say about the student marks?

Sol:- Let's write the given data

34, 36, 36, 38, 38, 39, 39, 40, 40, 41, 4.1, 41, 41, 42, 42, 4.5, 49, 56

1) We have to find the mean, median, variance and standard deviation of the given data.

Formula of mean = $\dfrac{\text{Sum of observations}}{\text{Total no. of observations}}$

Mean = $\dfrac{34+36+36+38+38+39+39+40+40+41+41+41+41+42+42+45+49+56}{18}$

$$\boxed{\text{Mean} = \dfrac{738}{18} = 41}$$

Formula of median = $\left[\dfrac{n+1}{2}\right]^{th}$

Here, the given data & median is in even.

So, the median will be

$$\boxed{\text{Median} = \dfrac{40+41}{2} = \dfrac{81}{2} = 40.5}$$

Now, we have to find the variance from the given data.

Step 1: Compute the mean of the 18 values given.

$$Mean = \frac{34+36+36+38+38+39+39+40+40+41+41+41+41+42+42+45+49+56}{18}$$

$$Mean = \frac{738}{18} = 41$$

Step 2: Make a table with three columns, one for the x values, the second for the deviations and third for squared deviations. As the data is not given as sample data so we use the formula for population variance. Thus, the mean is denoted by μ.

Value X	$x - \mu$	$(x-\mu)^2$
34	16	256
36	18	324
36	18	324
38	20	400
38	20	400
39	21	441
39	21	441
40	22	484
40	22	484
41	23	529
41	23	529
41	23	529
41	23	529
42	24	576
42	24	576
45	27	729
49	31	961
56	38	1444

2) This class contains students that are actually mediocre. Most of the students in the class are having an average percentage of 65 and there are only a few students securing value above 90%