**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with ** = 45 minutes and ** = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Solution:- Given mean = 45, standard deviation = 8

As per given condition the work will start after 10 min so mean will be 45+10 = 55

We have asked customer the car will be ready after 1 hour(x) = 60

Hence,

Z = (60 – 55)/8 = 0.625

From z table

Z value will be 0.73237

the probability that the service manager cannot meet his commitment =

1 – 0.73237 = 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean ** = 38 and Standard deviation ** =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Solution:- A. First find out the probability for employees older than 44:

X = 44, mean = 38, std = 6

1 – Stats.norm.cdf(44,38,6) = 1 – 0.8413 = 0.1587

Now find the probability of employees between 38 and 44:-

Stats.norm.cdf(44,38,6) – stats.norm.cdf(38,38,6) = 0.8413 – 0.5 = 0.3413

As we can clearly see that the probability of employees age between 38-44 is more than employees age more than 44. So, the given statement is False.

B. Lets calculate the probability of employees age under 30:

X = 30, mean = 38, std = 6

Stats.norm.cdf(30,38,6) = 0.0912

So the total number of employees age under 30 is 0.0912\*400 = 36.48

So, we can say that the statement is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Solution:- As we know that

if X ~ N (µ1, σ1^2), and Y ~ N (µ2, σ2^2) are two independent random variables then,

X + Y ~ N (µ1 + µ2, σ1^2 + σ2^2), and

X − Y ~ N (µ1 − µ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY, where X and Y are as defined above, I

.e., Z isl inear combination of X and Y then,

Z ~ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore, I

n the question2X1~ N (2µ,4 σ^2) and

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)2X1-(X1+X2) = N (4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Solution:- Mean = 100 and std = 20

Probability of the random variable taking a value between them is 0.99

hence,

Z value at 99% = Stats.norm.ppf(0.995) = 2.5758

1st value will be 2.5758\*20 + 100 = 151.51

2nd value will be (-2.5758)\*20 + 100 = 48.484

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Solution:- Total profit = profit 1 + profit 2

Mean = profit 1(mean) + profit 2(mean)

= 5 + 7 = 12

Std = sqrt(9+16)

= sprt(25)

= 5

Mean in rs = 12\*45 = 540

Std in rs = 5\*45 = 225

A) Range for 95% :- Stats.norm.interval(0.95,540,225)

Range is rs(99.008, 980.991) in millions

B) the 5th percentile:-

From z score we need to find the value of 0.5000 – 0.050 = 0.4500

We are getting the value of -1.645

the 5th percentile of profit = mean + (-1.645)\*std

= 540 – (1.645\*225)

= 540 – 370.125

= 169.87 = 170 in million

C) Probability of 1st division making loss = stats.norm.cdf(0,5,3) = 0.0479

Probability of 2nd division making loss = stats.norm.cdf(0,7,4) = 0.04005

We can see that 1st division can make more loss compared to 1st division.