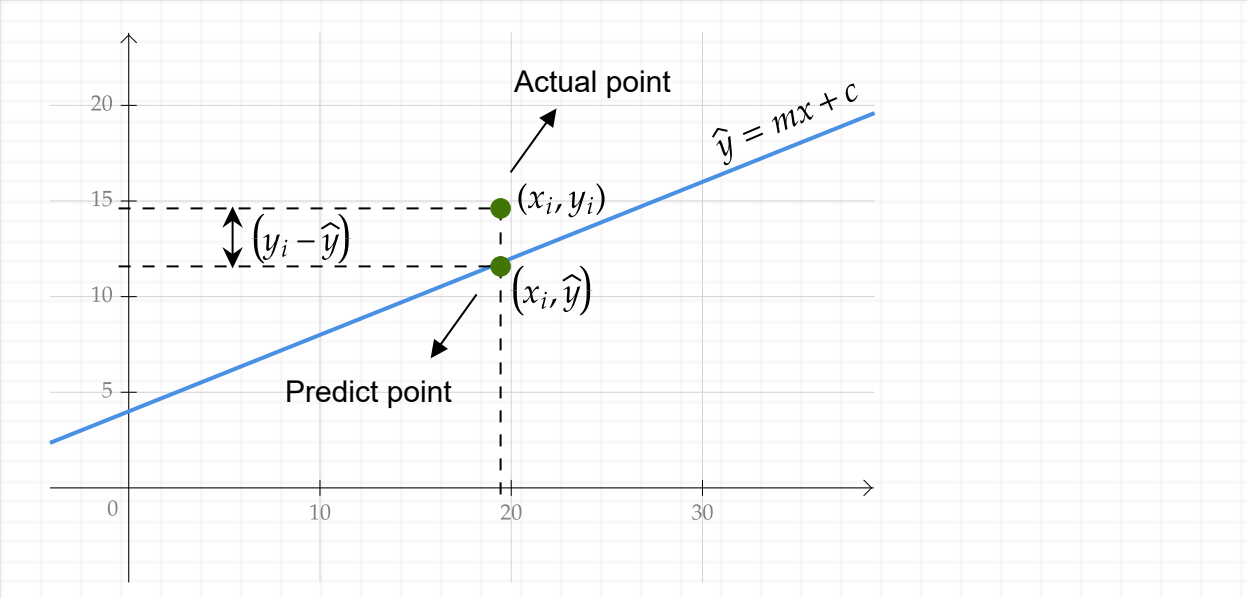
Linear regression slope (m) formula derivation



$$S = \sum \left(y_i - \hat{y}\right)^2$$

$$\hat{y} = mx + c$$

$$S = \sum (y_i - (mx_i + c))^2$$

Apply partial differential with respective c

$$\frac{\partial S}{\partial c} = \frac{\partial}{\partial c} \sum (y_i - (mx_i + c))^2$$

$$= 2 \times \sum \left((y_i - (mx_i + c)) \times \frac{\partial}{\partial c} (y_i - (mx + c)) \right)$$

$$= 2 \times \sum (y_i - (mx_i + c)) \times (-1) \qquad \therefore \ \frac{\partial}{\partial c} (y_i - (mx_i + c) = -1$$

$$= -2 \times \sum (y_i - (mx_i + c))$$

Apply partial differential with respective m

$$\frac{\partial S}{\partial m} = \frac{\partial}{\partial m} \sum (y_i - (mx_i + c))^2$$

$$= 2 \times \sum \left((y_i - (mx_i + c)) \times \frac{\partial}{\partial m} (y_i - (mx_i + c)) \right)$$

$$= 2 \times \sum (y_i - (mx_i + c)) \times (-x_i) \qquad \therefore \ \frac{\partial}{\partial m} (y_i - (mx_i + c) = -x_i$$

$$= -2 \times \sum x_i (y_i - (mx_i + c))$$

Partial derivatives equal to 0

$$\frac{\partial S}{\partial c} = -2 \times \sum (y_i - (mx_i + c)) = 0$$

$$\frac{\partial S}{\partial m} = -2 \times \sum x_i (y_i - (mx_i + c)) = 0$$

Find c (intercept)

$$\frac{\partial S}{\partial c} = 0 = \sum (y_i - (mx_i + c))$$

$$= \sum y_i - \sum mx_i - \sum c$$

$$= \sum y_i - \sum mx_i - nc$$

$$nc = \sum y_i - \sum mx_i$$

$$c = \frac{\sum y_i}{n} - m\frac{\sum x_i}{n}$$

$$c = \bar{y} - m\bar{x} \qquad \therefore \quad \frac{\sum y_i}{n} = \bar{y} \;\; ; \;\; \frac{\sum x_i}{n} = \bar{x}$$

Find m (slope)

$$\frac{\partial S}{\partial m} = 0 = \sum x_i(y_i - (mx_i + c))$$

$$= \sum x_i\left(y_i - \left(mx_i + \left(\bar{y} - m\bar{x}\right)\right)\right.$$

$$= \sum x_i\left(y_i - \bar{y} - mx_i + m\bar{x}\right)$$

$$= \sum x_i\left(y_i - \bar{y}\right) - m\sum x_i\left(x_i + \bar{x}\right)$$

$$m\sum x_i\left(x_i + \bar{x}\right) = \sum x_i\left(y_i - \bar{y}\right)$$

$$m = \frac{\sum x_i\left(y_i - \bar{y}\right)}{\sum x_i\left(x_i + \bar{x}\right)}$$

$$m = \frac{\sum x_i\left(y_i - \bar{y}\right) + \sum \bar{x}\left(y_i - \bar{y}\right)}{\sum x_i\left(x_i + \bar{x}\right) + \sum \bar{x}\left(x_i - \bar{x}\right)} \qquad \therefore \sum \bar{x}\left(y_i - \bar{y}\right) = 0 \;\; ; \;\; \sum \bar{x}\left(x_i - \bar{x}\right) = 0$$

$$m = \frac{\sum \left(x_i y_i - x_i\bar{y} + \bar{x}y_i - \overline{xy}\right)}{\sum \left(x_i^2 + 2x_i\bar{x} - \bar{x}^2\right)}$$

$$m = \frac{\sum \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sum \left(x_i - \bar{x}\right)}$$