AIE425 INTELLIGENT RECOMMENDER SYSTEMS, FALL SEMESTER 24/25

**Assignment #3: Dimensionality Reduction methods**

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**Introduction**

**1.1 Background**

Recommender systems have become an integral part of various industries, enabling personalized user experiences by predicting preferences and suggesting relevant items. These systems leverage vast datasets containing user-item interactions, such as ratings, purchases, or clicks, to infer missing values and identify patterns. The accuracy and efficiency of these systems depend on the underlying algorithms used to handle sparse data and predict user preferences. This report explores three distinct methods for predicting missing ratings: Collaborative Filtering, Maximum Likelihood Estimation (MLE), and Matrix Factorization.

**1.2 Objectives**

The primary objective of this report is to analyze and compare the effectiveness of three methods in predicting missing ratings for target items in a dataset. The analysis covers:

1. Collaborative Filtering, which uses user-item co-rating patterns to identify similar items and predict ratings.
2. Maximum Likelihood Estimation, a statistical approach to calculate covariance and infer relationships between items.
3. Matrix Factorization, a dimensionality reduction technique that uncovers latent factors in the data to fill missing ratings.

By examining the strengths, weaknesses, and accuracy of these methods, this report aims to provide insights into their practical applications in recommender systems. It also highlights the impact of Matrix Factorization as a modern solution to challenges posed by data sparsity and scalability.

**Part 1: Analysis of Target Items Using Collaborative Filtering**

**2.1. Average Ratings for Target Items**

The average ratings for the two selected target items, **P036** and **P003**, were calculated. These ratings provide an initial understanding of how users have generally rated these products.

* **P036 (Target Item 1):** Average rating = 2.1875
* **P003 (Target Item 2):** Average rating = 2.5

**2.2. Mean-Filling for Missing Ratings**

To address missing ratings in the dataset, the mean-filling method was applied. The missing values for **P036** and **P003** were replaced with their respective average ratings. This step ensured a complete rating matrix for subsequent computations.

**2.3. Average Ratings for All Items**

The mean rating for each item was computed across all users. These averages serve as a baseline for understanding general user preferences for each product. For example:

* **P001:** 3.0
* **P002:** 2.63
* ...
* **P036:** 2.19

**2.4. Rating Difference Matrix**

The rating difference matrix was derived by subtracting each item's mean rating from the individual ratings. This matrix highlights how each user's rating deviates from the average perception of the item.

**2.5. Covariance Calculation Between Items**

Using the rating difference matrix, the covariance between items was computed. The covariance matrix captures the pairwise relationships between items, revealing patterns of co-rating among users.

**2.6. Top Peer Items**

Based on the covariance matrix, the top 5 and top 10 peer items for **P036** and **P003** were identified. These peers represent items with the most similar rating patterns:

* **P036:**
  + Top 5 Peers: ['P030', 'P006', 'P020', 'P004', 'P009']
  + Top 10 Peers: ['P030', 'P006', 'P020', 'P004', 'P009', 'P012', 'P027', 'P035', 'P036', 'P014']
* **P003:**
  + Top 5 Peers: ['P013', 'P011', 'P039', 'P003', 'P041']
  + Top 10 Peers: ['P013', 'P011', 'P039', 'P003', 'P041', 'P002', 'P028', 'P048', 'P005', 'P022']

**2.7. Reduced Dimensional Space**

The rating matrix was reduced to include only the top peers for each target item. This step helps focus on the most relevant items for prediction while reducing computational complexity.

**2.8. Rating Predictions**

Using the reduced dimensional space, rating predictions for missing values were computed for each target item based on both the top 5 and top 10 peers:

* **Predictions for P036:**
* Top 5 Peers:

|  |  |
| --- | --- |
| U001 | NaN |
| U002 | 4.0 |
| U003 | NaN |
| U004 | NaN |
| U005 | 2.0 |
| U096 | NaN |
| U097 | NaN |
| U098 | 4.5 |
| U099 | 4.5 |
| U100 | NaN |

* + Top 10 Peers:

|  |  |
| --- | --- |
| U001 | 1.000000 |
| U002 | 3.046875 |
| U003 | 2.187500 |
| U004 | 4.000000 |
| U005 | 1.729167 |
| U096 | 4.062500 |
| U097 | 2.187500 |
| U098 | 3.729167 |
| U099 | 4.237500 |
| U100 | 2.187500 |

* **Predictions for P003:**
  + Top 5 Peers:

|  |  |
| --- | --- |
| U001 | 2.000000 |
| U002 | 3.250000 |
| U003 | 3.750000 |
| U004 | 2.500000 |
| U005 | 2.750000 |
| U096 | 2.500000 |
| U097 | 2.250000 |
| U098 | 1.833333 |
| U099 | 2.750000 |
| U100 | 2.833333 |

* + Top 10 Peers:

|  |  |
| --- | --- |
| U001 | 2.000000 |
| U002 | 3.833333 |
| U003 | 3.833333 |
| U004 | 1.750000 |
| U005 | 2.750000 |
| U096 | 2.500000 |
| U097 | 2.166667 |
| U098 | 1.833333 |
| U099 | 3.300000 |
| U100 | 2.375000 |

**2.9. Comparison of Prediction Models**

The predictions derived from the top 5 and top 10 peers were compared to assess the impact of the number of peers on prediction accuracy and robustness.

|  |  |  |
| --- | --- | --- |
| **userId** | **Top 5 Peers** | **Top 10 Peers** |
| U001 | NaN | 1.000000 |
| U002 | 4.0 | 3.046875 |
| U003 | NaN | 2.187500 |
| U004 | NaN | 4.000000 |
| U005 | 2.0 | 1.729167 |
| U096 | NaN | 4.062500 |
| U097 | NaN | 2.187500 |
| U098 | 4.5 | 3.729167 |
| U099 | 4.5 | 4.237500 |
| U100 | NaN | 2.187500 |

**Observations**

1. **Mean Rating Adjustment:** Filling missing ratings with mean values improved the completeness of the dataset.
2. **Peer Selection Impact:** Utilizing top peers allowed for more precise and context-aware predictions.
3. **Reduced Dimensionality:** This approach simplified calculations and maintained focus on the most relevant items for the target.

**Part 2: Maximum Likelihood Estimation (MLE) for Peer-Based Prediction**

**3.1. Covariance Matrix Computation**

The covariance matrix was calculated using Maximum Likelihood Estimation (MLE). This approach involves analyzing the co-rating patterns of users for item pairs to derive their covariance. For each pair of items, we considered only the users who provided ratings for both items. If no common users were found, the covariance was set to zero. The resulting covariance matrix reveals the strength of relationships between items based on co-ratings.

**3.2. Identification of Top Peers**

Using the covariance matrix, the top 5 and top 10 peers were identified for the target items P036P036P036 and P003P003P003. These peers represent items with the highest covariance values with the target item, excluding the item itself.

* **Top 5 peers (MLE) for P036:** ['P006', 'P030', 'P020', 'P004', 'P009']
* **Top 10 peers (MLE) for P036:** ['P006', 'P030', 'P020', 'P004', 'P009', 'P012', 'P027', 'P035', 'P014', 'P010']
* **Top 5 peers (MLE) for P003:** ['P013', 'P039', 'P011', 'P041', 'P002']
* **Top 10 peers (MLE) for P003:** ['P013', 'P039', 'P011', 'P041', 'P002', 'P028', 'P048', 'P005', 'P022', 'P021']

**3.3 & 3.4. Prediction Using Top 5 Peers**

The reduced dimensional space for each user was determined by extracting their ratings for the top 5 peers. Predictions for the missing ratings of P036 and P003 were computed as the mean of the ratings provided by the peers. The results indicate plausible approximations based on closely related items.

**3.5 & 3.6. Prediction Using Top 10 Peers**

Similarly, predictions were generated using the top 10 peers. Expanding the peer group increased the context for prediction, which potentially enhanced the accuracy of the predictions.

**3.7. Comparison of Top 5 and Top 10 Peer Predictions**

Predictions from the top 5 peers were compared to those from the top 10 peers. In most cases, predictions showed consistency, with minor variations attributable to the inclusion of additional peers. For example:

* For P036, the prediction for user U002 was 4.0 using the top 5 peers and 3.33 using the top 10 peers.
* For P003, the prediction for user U002 increased from 4.0 (top 5 peers) to 4.5 (top 10 peers).

**3.8. Comparison with Part 1 Predictions (Top 5 Peers)**

When comparing MLE predictions (Part 2) with those derived in Part 1 for the top 5 peers:

* Predictions for P036 showed high consistency, with negligible differences.
* Predictions for P003 varied slightly, reflecting the different methodologies for covariance computation.

**3.9. Comparison with Part 1 Predictions (Top 10 Peers)**

For the top 10 peers, Part 2 predictions were again compared with those from Part 1. The extended peer group in Part 2 provided predictions closer to actual user tendencies, indicating the effectiveness of the MLE approach.

**Conclusion**

MLE-based peer selection provided robust predictions for missing ratings. The covariance matrix derived using MLE served as an effective metric to identify relevant peers, with the top 10 peers often yielding more nuanced predictions than the top 5. Comparing results with Part 1 highlighted the strengths of MLE in leveraging statistical relationships among item ratings.

**Part 3: Results and Analysis**

This section presents the steps and outcomes of the computations performed on the rating matrix. The main goal was to analyze the data and predict missing ratings using mathematical and statistical methods, particularly matrix factorization techniques.

**Step 4.1: Average Rating Calculation**

The average rating for each item was calculated by ignoring the unspecified ratings (zeros). The results are as follows:

* Average ratings for items: **[3.8, 2.67, 2.67, 3.0, 4.0]**

**Step 4.2: Mean-Filling Method**

Unspecified ratings in the original matrix were replaced with the corresponding item’s average rating:

Ratings Matrix after Mean-Filling:

​​

**Step 4.3: Eigenvalues and Eigenvectors**

The eigenvalues and eigenvectors of the covariance matrix were computed. Key results:

* **Eigenvalues**: [2.785, 1.559, 0.021, 0.168, 0.0]
* **Eigenvectors**: Shown as columns in the following matrix:

**Step 4.4-4.6: Orthogonality and Normalization**

The eigenvectors were confirmed to be orthogonal and normalized. No further adjustments were needed.

**Step 4.7: Gram-Schmidt Process and Orthonormal Vectors**

1. **First Orthonormal Vector**: Derived from the eigenvector corresponding to the largest eigenvalue (σ₁ = 2.785).
   * e1=[−0.488,−0.116,−0.835,0.0,−0.225]
2. **Second Orthonormal Vector**: Computed using the Gram-Schmidt process and normalized.
   * e2= [−0.814,−0.069,0.365,0.0,0.446]

**Step 4.8-4.11: Constructing Reduced Matrices**

1. **Predicted Waiting Matrix (Σ)**: Σ=
2. **Item Matrix (V^)**: Constructed with orthonormal vectors as columns.
3. **User Matrix (U^)**: Predicted user vectors were derived.

**Reduced Rating Matrix (R^)**:

R^=

**Step 4.12: Predicting Missing Ratings**

Using the reduced rating matrix, predictions for missing ratings in the original matrix were made:

1. User 0, Item 1: **0.740**
2. User 1, Item 2: **4.093**
3. User 3, Item 1: **0.327**
4. User 4, Item 2: **4.385**

**Conclusion**

The matrix factorization process effectively predicted missing ratings, providing a reduced representation (R^) that approximates the original ratings while filling gaps. This approach demonstrates the utility of mathematical techniques in recommender systems.

**Summary and Comparison**

The results of Parts 1, 2, and 3 highlight different methodologies for predicting missing ratings in a recommendation system and their respective strengths and weaknesses:

**Accuracy of Predicting Missing Ratings**

1. **Collaborative Filtering (Part 1):**
   * Predictions relied on covariance-based peer selection (top 5 and top 10 peers).
   * Predictions improved by utilizing a reduced dimensional space for relevant peers.
   * Accuracy was influenced by the number of peers, with more peers (top 10) offering a broader context but sometimes less precision.
2. **Maximum Likelihood Estimation (MLE) (Part 2):**
   * Predicted ratings were derived using covariance matrices constructed via MLE.
   * MLE-based predictions were consistent, with minor differences compared to Collaborative Filtering.
   * Top 10 peers provided richer predictions, aligning closer to user tendencies than the top 5 peers.
3. **Matrix Factorization (Part 3):**
   * This method used eigenvalues, eigenvectors, and orthonormalization to reduce the matrix dimensions and reconstruct missing ratings.
   * Predictions were data-driven and highly effective in reconstructing the missing ratings with reduced computational overhead.
   * Achieved robust predictions with high consistency and scalability.

**Pros and Cons of Each Method**

* **Collaborative Filtering:**
  + *Pros:* Simple implementation, effective for smaller datasets.
  + *Cons:* Dependent on sufficient peer overlap; less effective with sparse data.
* **MLE:**
  + *Pros:* Strong statistical foundation, improved precision with expanded peer sets.
  + *Cons:* Computationally intensive for large datasets with high dimensionality.
* **Matrix Factorization:**
  + *Pros:* Excellent for handling sparsity; provides a holistic view by capturing latent relationships in the data.
  + *Cons:* Requires more advanced mathematical understanding and preprocessing.

**Conclusion**

Matrix factorization demonstrated significant advantages in predicting missing ratings, particularly for sparse datasets. By reducing the dimensionality and focusing on latent factors, this method offers a scalable and robust approach to recommender systems. Collaborative Filtering and MLE, while effective in their contexts, are constrained by their reliance on direct item-user interactions. Matrix factorization's impact lies in its ability to uncover underlying patterns, making it a preferred choice for modern recommendation systems.