MTL103: Practice Sheet 5
Integer Programming

1. Convert the following constraints into linear by using integer variables:
   (a) either $x = 0$ or $x \in [1, 2]$.
   (b) x takes only one of the value in $\{0, 1, 2, 3, 4, 5\}$.
   (c) (either $2x_1 + 9x_2 \leq 18$ or $x_1 + x_2 \leq 6$), and $x_1,\ x_2 \geq 0$.
   (d) (either $2x_1 - 3x_2 = 0$ or $3x_1 + x_2 \geq 1$), and $x_1,\ x_2 \geq 0$.
   (e) at least one of the two but not necessarily both $(x_1 + x_2 \leq 1,\ 4x_1 + x_2 \leq 2)$ and $x_1,\ x_2 \geq 0$.
   (f) at least two of the following three $(6x_1 + 5x_2 \leq 60,\ 4x_1 + 5x_2 \leq 50\ 3x_1 + 7x_2 \leq 80)$.
   (g) If $6x_1 + 5x_2 > 30$ then $4x_1 + 7x_2 \leq 28$.

2. Consider the example of a manufacturer of animal feed who is producing feed mix for dairy cattle. In our simple example the feed mix contains two active ingredients and a filler to provide bulk. One kg of feed mix must contain a minimum quantity of each of four nutrients as below:

Nutrient	A	B	C	D
gram	90	50	20	2

The ingredients have the following nutrient values and cost

	A	B	C	D	Cost/Kg
Ingredient 1 (gm/Kg)	100	80	40	10	40
Ingredient 2 (gm/Kg)	200	150	20	-	60

If we use any of ingredient 2 we incur a fixed cost of Rs 15. Also, we need not satisfy all four nutrient constraints but need only satisfy three of them (i.e., now the optimal solution could only have three (any three) of these nutrient constraints satisfied and the fourth violated). Give the complete MIP formulation of the problem with these two new conditions added.

3. Suppose a bakery sells eight varieties of doughnuts. The preparation of varieties 1, 2, and 3 involves a rather complicated process, and so the bakery has decided that it would rather not bake these varieties unless it can bake and sell at least 10 dozen doughnuts of varieties 1, 2, and 3 combined. Suppose also that the capacity of the bakery prohibits the total number of doughnuts baked from exceeding 30 dozens, and that the per unit profit f or a variety j doughnut is Rs P_j. Formulate the problem as mixed integer programming problem.

4. Suppose a Research and Development company has a sum of money, Rs M , available for investment. The company has determined that there are N projects suitable for investment and at least Rs p_j must be invested in project j if it is decided that project j is worthy of investment. The company net profit that can be made by an investment in project j is Rs P_j. The company's dilemma is that it cannot invest in all N projects. Formulate the optimization model so to advise in which projects company should invest to maximize the profit. (This problem is *resource allocation problem*).

5. A thief enters in a Mall with a knapsack (shoulder bag) of carrying capacity V Kg of weight. There are n number of distinct items available for him to carry in the bag. Each item i with a size s_i and a worth value v_i. The objective is to maximize the total value of the items in the knapsack. Advise (!) a thief as to what items he should pick by constructing a model for him. (This problem is called *knapsack problem* and undoubtedly the most studied one in the MIP literature).

6. A company has selected m possible sites for distribution of its products in a certain area. There are n customers in the area and the transport cost of supplying all the customer j requirements over the given planning period from site i is c_{ij}. If the site i is constructed and developed for distribution purposes then it incurs a cost f_i to the company. Which sites should be selected by the company to minimize the total construction plus transport cost? Formulate it as an integer programming problem. (This problem is known as *facility location problem*).

7. Consider maximization of a piecewise linear function

$$f(x) = \begin{cases} 0 & x = 0 \\ 0.5x + 1 & 0 < x \leq 10 \\ x - 4 & 10 \leq x \leq 15 \\ 1.6x - 8 & 15 \leq x \leq 25 \\ -2x + 82 & 25 \leq x \leq 30 \end{cases}$$

over the interval $[0, 30]$. Convert this optimization problem into a mixed integer programming problem.

8. Use the Gomory cut technique to solve the following problems:

   (i). $\max\ v = 4x_1 + 3x_2$
   subject to $3x_1 + 5x_2 \leq 11,\ 4x_1 + x_2 \leq 8,\ x_1, x_2 \geq 0,\ x_1$ is integer

   (ii). $\max\ v = x_1 + x_2$
   subject to $2x_1 + 5x_2 \leq 16,\ 6x_1 + 5x_2 \leq 30\ x_1, x_2 \geq 0,$ are integer

   (iii). $\min\ v = 3x_1 - x_2$
   subject to $-10x_1 + 6x_2 \leq 15,\ 14x_1 + 18x_2 \leq 63,\ x_1, x_2 \geq 0,\ x_1$ is integer

   (iv). $\max\ v = 2x_1 + 20x_2 - 10x_3$
   subject to $2x_1 + 20x_2 + 4x_3 \leq 15,\ 6x_1 + 20x_2 + 4x_3 = 20,\ x_1, x_2, x_3 \geq 0,\ x_1, x_2$ are integer

9. Consider the integer program:

$$\begin{aligned} \max\ v = 15x_1 + 32x_2 & \\ \text{subject to } 7x_1 + 16x_2 &\leq 52 \\ 3x_1 - 2x_2 &\leq 9 \\ x_1, x_2 &\geq 0 \\ x_1, x_2 \text{ are integer} & \end{aligned}$$

   Show that the optimal solution of relaxed problem is $\left(4, \dfrac{3}{2}\right)$. Rounding the optimal solution will yield two infeasible points. Use Gomory cut technique to obtain the optimal solution.

10. Use Branch and Bound technique to solve:

   (i.) $\max\ v = x_1 - 3x_2$
   subject to $x_1 + x_2 \leq 5,\ -x_1 + 2x_2 \leq \dfrac{11}{2},\ x_1, x_2 \geq 0,\ x_2$ is integer

   (ii.) $\max\ v = 9x_1 + 10x_2$
   subject to $2x_1 + 5x_2 \leq 15,\ x_1 \leq 3,\ x_1, x_2 \geq 0,$ are integer