MTL103: Tutorial Sheet-2

1. Graph the convex hull of points $(0,5)$, $(3,5)$, $(6,3)$, $(5,0)$, $(3,3)$, $(2.5,2.5)$. Which of these points are extreme points of the hull? Express the non extreme point (among given points), if any, as a convex combination of the extreme points.

2. Find the extreme points of the set $\{(x_1,x_2,x_3) : x_1 - x_2 + x_3 \le 1,\ -x_1 + 2x_2 \le 4,\ x_1,x_2,x_3 \ge 0\}$. Does this set have any recession direction?

3. Express the point $x = (0,1)$ as a convex combination of the extreme points of the set $\{(x_1,x_2)^T : x_1 - x_2 \le 3,\ 2x_1 + x_2 \le 4,\ x_1 \ge -3\}$.

4. Find the extreme directions (if any) and extreme points of the set described by

$$\{\,(x_1,x_2)^T \mid 5x_1 + 3x_2 \ge 15,\quad -x_1 + x_2 \le 1,\quad x_1 \ge 0,\quad x_2 \ge (3/2)\,\}.$$

5. Plot the feasible region

$$S = \{(x_1,x_2) : -x_1 + x_2 \le 1,\ \ x_1 + x_2 \le 5,\ \ 4x_1 - 3x_2 \le 6,\ \ x_1 - 2x_2 \le 1,\ \ x_1,x_2 \ge 0\}.$$

Find all the basic feasible solutions of the problem. If we move from vertex $(2,3)$ to vertex $(3,2)$, then determine the entering and leaving variables.

6. Find the number of degenerate and non-degenerate basic feasible solutions for the system graphically $2x + 3y \le 21,\ \ 3x - y \le 15,\ \ x + y \ge 5,\ \ y \le 5,\ \ x,\ y \ge 0.$

7. Why the variable x_1 is present in all the basic solutions of the following system ?

$$\begin{aligned} x_1 + x_2 - 2x_3 + x_4 &= 1 \\ 2x_1 - x_2 + 2x_3 + 2x_4 &= 2 \\ 3x_1 + 2x_2 - 4x_3 - 3x_4 &= 3 \end{aligned}$$

8. Solve the problem graphically:

$$\begin{array}{rcl} \min\ \ 4x_1 - 3x_2 & & \\ \text{s.t}\ \ 2\mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 & = & 10 \\ x_1 + x_2 + x_4 & = & 7 \\ 2x_1 + 3x_2 - x_5 & = & 12 \\ x_i & \ge & 0,\ i = 1,\dots,5 \end{array}$$

9. Find all basic solutions of the following systems and classify them as degenerate/non-degerate

$$(a)\ \begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 7 \\ 2x_1 + x_2 + x_3 + 2x_4 &= 3 \end{aligned} \qquad (b)\ \begin{aligned} 8x_1 + 6x_2 + 12x_3 + x_4 + x_5 &= 6 \\ 9x_1 + x_2 + 2x_3 + 6x_4 + 10x_5 &= 11 \end{aligned}$$

10. Consider the following linear programming problem, where b and a_i are 3×1 column vectors for $i = 1,2,3,4$.

$$\min\ \ c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4,\ \ \text{subject to}\ \ a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = b,\ \ x_1,x_2,x_3,x_4 \ge 0.$$

Suppose $x^* = (x_1^*, 0, x_3^*, x_4^*)$ is a basic feasible solution, where B be the corresponding basis matrix. Let $d = (d_1, 5, d_3, d_4)^T$ be such that $x^* + d$ is a feasible solution of given LP. Prove that $\begin{pmatrix} d_1 \\ d_3 \\ d_4 \end{pmatrix} = -5B^{-1}a_2$.

11. Solve the following LPP without using algorithm

$$\max Z = 4x_1 + 5x_2 + 11x_3 + 2x_4,\ \ \text{subject to}\ \ 21x_1 + 7x_2 - 3x_3 + 10x_4 = 210,\ \ x_1,x_2,x_3,x_4 \ge 0.$$

12. Reduce the solution $(2,4,1)^T$ to a basic feasible solution of $Ax = b,\ x \ge 0$ if where $A = [a_1,a_2,a_3]$,

$$a_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix},\ a_2 = \begin{pmatrix} -1 \\ 4 \end{pmatrix},\ a_3 = \begin{pmatrix} 2 \\ 0 \end{pmatrix},\ b = \begin{pmatrix} 0 \\ 18 \end{pmatrix}.$$