MTL103: Quiz 1 on 08/03/2021 at 7:00 pm

Max Marks 10 | Max Time: 35 minutes

Time to upload answers: 10 minutes

Start the answer of each question on the new page. Write your name and entry number on the top of each page. Upload your PDF file on Gradescope with clear and correct mapping.

No re-quiz for any reason. No marks will be awarded if answer is not supported with step-wise working.

1. Consider the following LPP

$$\begin{aligned}
&\max z = 4x_1 + x_2 + 3x_3 + 5x_4 \\
&\text{ s.t.} \\
&4x_1 - 6x_2 - 5x_3 - 4x_4 \geq -20 \\
&3x_1 - 2x_2 + 4x_3 + x_4 \leq 10 \\
&8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20 \\
&x_1,\ x_2,\ x_3,\ x_4 \geq 0
\end{aligned}$$

Fill all the missing entries in the following simplex table

c_B	v_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7
5	x_4	$--$	0	$--$	13/10	1	$--$	0	$--$
0	x_6	$--$	0	$--$	$--$	0	$--$	1	$-2/5$
4	x_1	$--$	1	$-3/5$	$--$	0	$--$	0	$--$
		$z_j - c_j \rightarrow$	0	$--$	$--$	0	4/5	0	$--$

where $x_5,\ x_6,\ x_7 \geq 0$ are the slack variables in the first, second and third constraints (read in order), respectively. Is this table yield an optimal solution or indicate unbounded LPP? Show your work clearly. [3]

2. For any non-zero cost vector $c = (c_1,\ c_2)^T$, can $x^* = (1,\ 3)^T$ be an optimal solution of the problem given below. Explain your answer with clear justification.

$$\begin{aligned}
\max \quad & c_1x_1 + c_2x_2 \\
& 2x_1 + 3x_2 \leq 11 \\
& 3x_1 - 2x_2 \leq 9 \\
& x_1,\ x_2 \geq 0.
\end{aligned}$$

Solve the LPP graphically when $c = (1,\ 5)^T$, and find the optimal value of the LPP. [3]

3. Consider the following LPP

$$\begin{aligned}
&\max z = x_1 - 4x_2 \\
&\text{ s.t.} \\
&x_1 - x_2 \geq -4 \\
&4x_1 + 5x_2 \leq 45 \\
&5x_1 - 2x_2 \leq 20 \\
&x_1 \geq 0.
\end{aligned}$$

Describe the range set of x_2 for which the LPP
(a) becomes unbounded; (b) infeasible. Justify your answer. [2]

4. Consider the polyhedral set described by the following inequalities:

$$\begin{aligned}
-3x_1 + x_2 &\leq -2 \\
-x_1 + x_2 &\leq 2 \\
-x_1 + 2x_2 &\leq 8 \\
x_1 \geq 0,\ x_2 &\geq 2
\end{aligned}$$

Find the extreme directions of the set. Explain and show the steps clearly. [2]