

MTL107: NUMERICAL METHODS AND COMPUTATION  
MINOR EXAMINATION

Total Marks: 40

Time: Two Hour

1. (10 Marks) Consider the following matrix:

$$\begin{bmatrix} 2 & 4 & -2 & -2 \\ 1 & 2 & 4 & -3 \\ -3 & -3 & 8 & -2 \\ -1 & 1 & 6 & -3 \end{bmatrix}$$

Using the partial pivoting find the  $LU$  factorization of the matrix in the following form:

$$L_3 P_3 L_2 P_2 L_1 P_1 A = U$$

where  $P_1, P_2$  and  $P_3$  are the permutation matrices and  $L_1, L_2$  and  $L_3$  are the lower triangular matrices representing the row operations. Then show that,  $P_3 P_2 L_1 P_2^{-1} P_3^{-1}$  and  $P_3 L_2 P_3^{-1}$  are lower triangular matrices. Finally find  $L$  using permutation matrices  $P_1, P_2$  and  $P_3$  and  $L_1, L_2$  and  $L_3$  such that  $PA = LU$  with  $P = P_3 P_2 P_1$ . Explain all the steps clearly and write all the steps with complete details.

2. (5 Marks) Show that a symmetric strictly positive definite matrix do not need any pivoting for  $LU$  decomposition.
3. (5 Marks) Consider the following matrix,

$$\begin{bmatrix} 1 & -1 & 2 \\ -1 & 3 & 6 \\ 2 & 6 & 4 \end{bmatrix}$$

**CANCELLED QUESTION**

Prove that the matrix is symmetric positive definite. First find the  $LDL^T$  decomposition of the matrix. Using  $LDL^T$  decomposition find the Cholesky decomposition of the matrix.

—————Please Turn Over—————

Handwritten calculations and diagrams at the bottom right of the page, including a circle containing  $\lambda = \frac{1}{\sqrt{18}}$  and other mathematical expressions.

✓ 4. (5 Marks) Consider the function defined by  $f(x) = \sin(\pi x)$ . Show that when  $-1 < a < 0$  and  $2 < b < 3$ , Bisection method in interval  $[a, b]$  converges to

a) 0, if  $a + b < 2$ ,

b) 2, if  $a + b > 2$ ,

c) 1, if  $a + b = 2$ ,

No graphical proof will be accepted. Write complete details of your arguments. In each case, determine the number of iteration needed to achieve  $10^{-3}$  accuracy. Final value should be a positive integer.

5/ (5 Marks) Derive Secant method as inverse interpolation method to find root of a function. Consider  $f(x) = \sin(\pi x)$  and initial guesses 0.1 and 0.12. Compute first two iterations using Secant Method.

✓ 6. (5 Marks) Consider the fixed point problem  $x = g(x)$  with  $g(x) = x^2 \sin(\pi x)$ . Find the appropriate interval, so that the fixed point iteration converge (You have to prove it).

✓ 7. (5 Marks) State the sufficient conditions for the convergence of the Newton's method. Under those conditions prove that Newton's method convergence and convergence is quadratic.

————— Good Luck —————