Prof: Vivek Mukundan Uploaded By-Rahul & Saurath

## INDIAN INSTITUTE OF TECHNOLOGY DELHI DEPARTMENT OF MATHEMATICS MTL104 (LINEAR ALGEBRA AND APPLICATIONS) Minor II

Time: 1 hour

Maximum Marks: 35

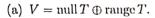
1. (1 points) Suppose  $S, T \in \mathcal{L}(V)$  and S is invertible. Suppose  $p \in \mathcal{P}(\mathbb{F})$  is a polynomial. Prove that

$$p\left(STS^{-1}\right) = Sp(T)S^{-1}.$$

(b) (4 points) Apply LU decomposition to solve the system Ax = b, where

$$A = \begin{pmatrix} 2 & 2 & 2 \\ 4 & 7 & 7 \\ 6 & 18 & 22 \end{pmatrix} \text{ and } b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

- 2. (a) (2 points) Suppose  $T, S \in \mathcal{L}(V)$  be linear operators on finite dimensional vector spaces V over  $\mathbb{C}$ . Suppose that TS = ST, then show that there is a eigenvector w that is common to both T and S.
  - (b) (3 points) Let V be the vector space of  $n \times n$  matrices with entries in  $\mathbb{C}$ . For a matrix  $A \in V$  define a linear operator  $T_A : V \to V$  such that  $T_A(B) = AB$ . If A is diagonalizable, show that  $T_A$  is diagonalizable.
- 3. (5 points) Suppose V is finite-dimensional and  $T \in \mathcal{L}(V)$ . Prove that the following are equivalent:



(b) 
$$V = \text{null } T + \text{range } T$$
.

(c) 
$$\operatorname{null} T \cap \operatorname{range} T = \{0\}$$



$$\langle p, q \rangle = \int_0^1 p(x)q(x)dx.$$

- (b) (2 points) What happens if the Gram-Schmidt Procedure is applied to a list of vectors that is not linearly independent?
- 5. (2 points) Find the eigenvalues of the linear operator T on  $\mathbb{R}^2$  which takes the circle  $\{(x_1,x_2)|x_1^2+x_2^2=1\}$  to the ellipse  $\{(x_1,x_2)|x_1^2/a^2+x_2^2/b^2=1\}$ .
  - (3 points) Let T be the linear operator on  $\mathcal{P}_2(\mathbb{R})$  defined by  $T(f(x)) = f(1) + f'(0)x + (f'(0) + f''(0))x^2$ . Check diagonalizability of this operator.
  - (a) (3 points) Suppose  $\{v_1, \ldots, v_n\}$  form a linearly independent set of vectors. Show that there exists  $w \in V$  such that  $\langle w, v \rangle > 0$  for all  $1 \le j \le m$ .
  - (b) (2 points) Show that the function that takes  $((x_1, x_2), (y_1, y_2)) \in \mathbb{R}^2 \times \mathbb{R}^2$  to  $|x_1y_1| + |x_2y_2|$  is not an inner product on  $\mathbb{R}^2$ .
- 7. (a) (3 points) Suppose  $T \in \mathcal{L}(V)$  and U is a subspace of V. Prove that U is invariant under T if and only if  $U^{\perp}$  is invariant under the adjoint  $T^*$ .
  - (b) (2 points) Show that dim null  $T^* = \dim \text{null } T + \dim W \dim V$  for every  $T \in \mathcal{L}(V, W)$ .



