MTL107: NUMERICAL METHODS AND COMPUTATION MINOR EXAMINATION

Total Marks: 40

Time: Two Hour

1. (10 Marks) Consider the following matrix:

$$\begin{bmatrix} 2 & 4 & -2 & -2 \\ 1 & 2 & 4 & -3 \\ -3 & -3 & 8 & -2 \\ -1 & 1 & 6 & -3 \end{bmatrix}$$

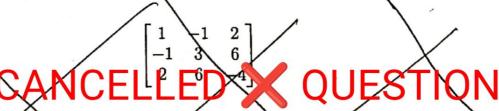
Using the partial pivoting find the LU factorization of the matrix in the following form:

$$L_3P_3L_2P_2L_1P_1A = U$$

where P_1 , P_2 and P_3 are the permutation matrices and L_1 , L_2 and L_3 are the lower triangular matrices representing the row operations. Then show that, $P_3P_2L_1P_2^{-1}P_3^{-1}$ and $P_3L_2P_3^{-1}$ are lower triangular matrices. Finally find L using permutation matrices P_1 , P_2 and P_3 and L_1 , L_2 and L_3 such that PA = LU with $P = P_3P_2P_1$. Explain all the steps clearly and write all the steps with complete details.

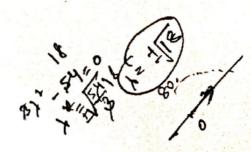
2. (5 Marks) Show that a symmetric strictly positive definite matrix do not need any pivoting for LU decomposition.

3 (5 Marks) Consider the following matrix,



Prove that the matrix is symmetric positive definite. First find the $L\Omega L^{\mathsf{T}}$ decomposition of the matrix. Using LDL^{T} decomposition find the Cholesky decomposition of the matrix.

Please Turn Over



(5 Marks) Consider the function defined by $f(x) = \sin(\pi x)$. Show that when -1 < a < 0 and 2 < b < 3, Bisection method in interval [a, b] converges to

a) 0, if a+b < 2,

b) 2, if a+b>2,

c) 1, if a+b=2,

No graphical proof will be accepted. Write complete details of your arguments. In each case, determine the number of iteration needed to achieve 10^{-3} accuracy. Final value should be a positive integer.

- 5/(5 Marks) Derive Secant method as inverse interpolation method to find root of a function. Consider $f(x) = \sin(\pi x)$ and initial guesses 0.1 and 0.12. Compute first two iterations using Secant Method.
- 6. (5 Marks) Consider the fixed point problem x = g(x) with $g(x) = x^2 \sin(\pi x)$. Find the appropriate interval, so that the fixed point iteration converge (You have to prove it).
 - (5 Marks) State the sufficient conditions for the convergence of the Newton's method. Under those conditions prove that Newton's method convergence and convergence is quadratic.

----Good Luck-