MTL103: Minor Exam on 18/03/2021 starting at 3:55 pm

Max Marks 30 Max Time: 95 minutes till 5:30 pm including the uploading time

Start answering each question on a new page.
Write your name and entry number on top of each page.
For each question, do all parts of the question at one go, clearly mentioning the part number in your answer - no recheck/regrade request will be entertained on mixing the parts and/or not correctly mentioning the part number in the answer.
Upload your PDF file on Gradescope ONLY with clear and correct mapping of answers.
Submission of answer sheet on any other platform or emailing will NOT be considered for any reason.

No re-minor for any reason. No marks will be awarded if answer is not supported with correct justification.

1. While solving some maximization LPP, we arrive at the following tableau:

v_B	x_B	y_1	y_2	y_3	y_4	y_5
x_3	4	-1	η	1	0	0
x_4	1	α	-4	0	1	0
x_5	β	γ	3	0	0	1
	$z_j - c_j \rightarrow$	δ	-2	0	0	0

where α, β, γ, η, δ are unknown real parameters.
For each of the following statements, determine the range sets of the parameters that will make that statement true.
(a) The current solution is an optimal solution and that the problem has multiple optimal solutions.
(b) The problem is unbounded.
(c) The current solution is degenerate basic feasible solution and the solution in the next iteration is non-degenerate non- optimal.
(d) The current solution is feasible but not optimal and the solution in the next iteration is optimal. [8]

2. Consider the problem of maximizing $c^T x$ over a polyhedron $P \subset \mathbb{R}^n$, where $c \in \mathbb{R}^n$ is a fixed vector and $x \in P$. Prove that a feasible solution $\hat{x}$ of the problem is its unique optimal solution if and only if $c^T d < 0$ for every non-zero feasible direction d at $\hat{x}$. [4]

3. The simplex method is applied to a maximization LPP in the standard form. In this framework, for each of the following statements, state which one os TRUE or FALSE. If TRUE, prove; and if FALSE give counterexample to justify. NO marks will be awarded merely for writing TRUE/FALSE unless it is justified with correct mathematical reasoning.
(a) A variable that has just left the basis at an iteration can not reenter in the basis in the immediate next iteration.
(b) A variable that has just entered in the basis at an iteration can not leave the basis in the immediate next iteration.
(c) If an artificial variable left the basis at an iteration then it can not reenter into the basis at any of the subsequent iteration. [6]

4. Let $A = conv\{\,(1,\,1),\ (-2,\,-3),\ (3,\,-1)\,\}$ and $B = conv\{\,(0,\,3),\ (1,\,-1),\ (4,\,2)\,\}$, where $conv$ denotes the convex hull. Let $S = conv(A \cup B)$. Write the point $\left(2,\,\frac{1}{3}\right)$ as a convex combination of the extreme points of the set S. [6]

5. Without computing the basic feasible solutions and without using the simplex method, estimate the least upper bound (if exists) for the following LPP

$$\begin{aligned}
\max \quad & z = 10x_1 + 24x_2 + 20x_3 + 20x_4 + 25x_5 \\
\text{subject to} \quad & \\
& x_1 + x_2 + 2x_3 + 3x_4 + 5x_5 \le 19 \\
& 2x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 \le 57 \\
& x_i \ge 0, \quad \forall\, i = 1, \ldots, 5.
\end{aligned}$$

[6]