

GATE MA 2017

EE25BTECH11030-AVANEESH

- 1) Consider the vector space $V = \{a_0 + a_1x + a_2x^2 : a_i \in \mathbb{R} \text{ for } i = 0, 1, 2\}$ of polynomials of degree at most 2. Let $f : V \rightarrow \mathbb{R}$ be a linear functional such that $f(1+x) = 0$, $f(1-x^2) = 0$ and $f(x^2-x) = 2$. Then $f(1+x+x^2)$ equals _____. (GATE MA 2017)
- 2) Let A be a 7×7 matrix such that $2A^2 - A^4 = I$, where I is the identity matrix. If A has two distinct eigenvalues and each eigenvalue has geometric multiplicity 3, then the total number of nonzero entries in the Jordan canonical form of A equals _____. (GATE MA 2017)
- 3) Let $f(z) = (x^2 + y^2) + i2xy$ and $g(z) = 2xy + i(y^2 - x^2)$ for $z = x + iy \in \mathbb{C}$. Then, in the complex plane \mathbb{C} .
 - a) f is analytic and g is NOT analytic
 - b) f is NOT analytic and g is analytic
 - c) neither f nor g is analytic
 - d) both f and g are analytic
 (GATE MA 2017)
- 4) If $\sum_{n=-\infty}^{\infty} a_n(z-2)^n$ is the Laurent series of the function $f(z) = \frac{z^4+z^3+z^2}{(z-2)^3}$ for $z \in \mathbb{C} \setminus 2$, then a_{-2} equals _____. (GATE MA 2017)
- 5) Let $f_n : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f_n(x) = \frac{2x^2}{x^2+(1-2nx)^2}$, $n = 1, 2, \dots$. Then the sequence (f_n)
 - a) converges uniformly on
 - b) does NOT converge uniformly on but has a subsequence that converges uniformly on
 - c) does NOT converge pointwise on
 - d) converges pointwise on but does NOT have a subsequence that converges uniformly on
 (GATE MA 2017)
- 6) Let $C : x^2 + y^2 = 9$ be the circle in \mathbb{R}^2 oriented positively. Then $\frac{1}{\pi} \oint_C (3y - e^{\cos x^2})dx + (7x + \sqrt{y^4 + 11})dy$ equals _____. (GATE MA 2017)
- 7) Consider the following statements:

(P): There exists an unbounded subset of \mathbb{R} whose Lebesgue measure is equal to 5.

(Q): If $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and $g : \mathbb{R} \rightarrow \mathbb{R}$ is such that $f = g$ almost everywhere on \mathbb{R} , then g must be continuous almost everywhere on \mathbb{R} .

Which of the above statements hold TRUE?

 - a) Both P and Q
 - b) Only P
 - c) Only Q
 - d) Neither P nor Q
 (GATE MA 2017)
- 8) If x^3y^2 is an integrating factor of $(6y^2 + axy)dx + (6xy + bx^2)dy = 0$ where $a, b \in \mathbb{R}$, then
 - a) $3a - 5b = 0$
 - b) $2a - b = 0$
 - c) $3a + 5b = 0$
 - d) $2a + b = 0$
 (GATE MA 2017)
- 9) If $x(t)$ and $y(t)$ are the solutions of the system $\frac{dx}{dt} = y$ and $\frac{dy}{dt} = -x$ with the initial conditions $x(0) = 1$ and $y(0) = 1$, then $x(\pi/2) + y(\pi/2)$ equals _____. (GATE MA 2017)
- 10) If $y = 3e^{2x} + e^{-2x} - \alpha x$ is the solution of the initial value problem $\frac{d^2y}{dx^2} + \beta y = 4\alpha x$, $y(0) = 4$ and $\frac{dy}{dx}(0) = 1$, where $\alpha, \beta \in \mathbb{R}$, then

(GATE MA 2017)

- 31) Let (x_n) and (y_n) be two sequences in a complete metric space (X, d) such that $d(x_n, x_{n+1}) \leq \frac{1}{n^2}$ and $d(y_n, y_{n+1}) \leq \frac{1}{n}$ for all $n \in \mathbb{N}$. Then
- both (x_n) and (y_n) converge
 - (x_n) converges but (y_n) need NOT converge
 - (y_n) converges but (x_n) need NOT converge
 - neither (x_n) nor (y_n) converges

(GATE MA 2017)

- 32) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = 0$ if x is rational, and if x is irrational then $f(x) = 9^n$, where n is the number of zeroes immediately after the decimal point in the decimal representation of x . Then the Lebesgue integral $\int_0^1 f(x)dx$ equals _____. (GATE MA 2017)

- 33) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $f(x, y) = \begin{cases} \sin(\frac{y^2}{x}) \sqrt{x^2 + y^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then, at $(0, 0)$,
- f is continuous and the directional derivative of f does NOT exist in some direction
 - f is NOT continuous and the directional derivatives of f exist in all directions
 - f is NOT differentiable and the directional derivatives of f exist in all directions
 - f is differentiable

(GATE MA 2017)

- 34) Let D be the region in \mathbb{R}^2 bounded by the parabola $y^2 = 2x$ and the line $y = x$. Then $\iint_D 3xy, dx, dy$ equals _____. (GATE MA 2017)

- 35) Let $y_1(x) = x^3$ and $y_2(x) = x^2|x|$ for $x \in \mathbb{R}$. Consider the following statements:

(P): $y_1(x)$ and $y_2(x)$ are linearly independent solutions of $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 0$ on \mathbb{R} .

(Q): The Wronskian $W(y_1, y_2)(x) = y_1(x) \frac{dy_2}{dx}(x) - y_2(x) \frac{dy_1}{dx}(x) = 0$ for all $x \in \mathbb{R}$.

Which of the above statements hold TRUE?

- Both P and Q
- Only P
- Only Q
- Neither P nor Q

(GATE MA 2017)

- 36) Let α and β with $\alpha > \beta$ be the roots of the indicial equation of $x^2 \frac{d^2y}{dx^2} - (x+1) \frac{dy}{dx} + y = 0$ at $x = 0$. Then $\alpha - 4\beta$ equals _____. (GATE MA 2017)

- 37) Let S_9 be the group of all permutations of the set $1, 2, 3, 4, 5, 6, 7, 8, 9$. Then the total number of elements of S_9 that commute with $\tau = (1\ 2\ 3)(4\ 5\ 6\ 7)$ in S_9 equals _____. (GATE MA 2017)

- 38) Let $\mathbb{Q}[x]$ be the ring of polynomials over \mathbb{Q} . Then the total number of maximal ideals in the quotient ring $\mathbb{Q}[x]/\langle x^4 - 1 \rangle$ equals _____. (GATE MA 2017)

- 39) Let $e_n : n \in \mathbb{N}$ be an orthonormal basis of a Hilbert space H . Let $T : H \rightarrow H$ be given by $Tx = \sum_{n=1}^{\infty} \frac{1}{n} \langle x, e_n \rangle e_n$. For each $n \in \mathbb{N}$, define $T_n : H \rightarrow H$ by $T_n x = \sum_{k=1}^n \frac{1}{k} \langle x, e_k \rangle e_k$. Then

- $|T_n - T| \rightarrow 0$ as $n \rightarrow \infty$
- $|T_n - T| \rightarrow 0$ as $n \rightarrow \infty$ but for each $x \in H$, $|T_n x - Tx| \rightarrow 0$ as $n \rightarrow \infty$
- for each $x \in H$, $|T_n x - Tx| \rightarrow 0$ as $n \rightarrow \infty$ but the sequence $(|T_n|)$ is unbounded
- there exist $x, y \in H$ such that $\langle T_n x, y \rangle \rightarrow \langle Tx, y \rangle$ as $n \rightarrow \infty$

(GATE MA 2017)

- 40) Consider the subspace $V = \{x_n : n \in \mathbb{N} : \sum_{n=1}^{\infty} |x_n| < \infty\}$ of the Hilbert space ℓ^2 of all square summable real sequences. For $n \in \mathbb{N}$, define $T_n : V \rightarrow \mathbb{R}$ by $T_n((x_k)) = \sum_{i=1}^n x_i$. Consider the following statements:

(P): $T_n : n \in \mathbb{N}$ is pointwise bounded on V .

(Q): $T_n : n \in \mathbb{N}$ is uniformly bounded on $x \in V : |x|_2 = 1$.

Which of the above statements hold TRUE?

- a) Both P and Q
b) Only P

- c) Only Q
d) Neither P nor Q

(GATE MA 2017)

- 41) Let $p(x)$ be the polynomial of degree at most 2 that interpolates the data $(-1, 2)$, $(0, 1)$ and $(1, 2)$. If $q(x)$ is a polynomial of degree at most 3 such that $p(x) + q(x)$ interpolates the data $(-1, 2)$, $(0, 1)$, $(1, 2)$ and $(2, 11)$, then $q(3)$ equals _____.

(GATE MA 2017)

- 42) Let J be the Jacobi iteration matrix of the linear system $\begin{pmatrix} 1 & 2 & 1 & 2 & 1 & 2 & -4 & 2 & 1 \end{pmatrix} \begin{pmatrix} x & y & z \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$. Consider the following statements:

(P): One of the eigenvalues of J lies in the interval .

(Q): The Jacobi iteration converges for the above system.

Which of the above statements hold TRUE?

- a) Both P and Q
b) Only P

- c) Only Q
d) Neither P nor Q

(GATE MA 2017)

- 43) Let $u(x, y)$ be the solution of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 4u$ satisfying the condition $u(x, y) = 1$ on the circle $x^2 + y^2 = 1$. Then $u(2, 2)$ equals _____.

(GATE MA 2017)

- 44) Let $u(r, \theta)$ be the bounded solution of the following boundary value problem in polar coordinates:

$$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0, \quad 0 < r < 2, \quad 0 \leq \theta \leq 2\pi,$$

$$u(2, \theta) = \cos^2 \theta, \quad 0 \leq \theta \leq 2\pi.$$

Then $u(1, \pi/2) + u(1, \pi/4)$ equals

a) 1

b) $\frac{9}{8}$

c) $\frac{7}{8}$

d) $\frac{3}{8}$

(GATE MA 2017)

- 45) Let T_u and T_d denote the usual topology and the discrete topology on \mathbb{R} , respectively. Consider the following three topologies:

$$T_1 = \text{usual topology on } \mathbb{R}^2 = \mathbb{R} \times \mathbb{R},$$

$$T_2 = \text{topology generated by the basis } \{U \times V : U \in T_u, V \in T_d\} \text{ on } \mathbb{R} \times \mathbb{R},$$

$$T_3 = \text{dictionary order topology on } \mathbb{R} \times \mathbb{R}.$$

Then

a) $T_3 \subseteq T_1 \subseteq T_2$

b) $T_1 \subseteq T_2 \subseteq T_3$

c) $T_3 \subseteq T_2 \subseteq T_1$

d) $T_1 \subseteq T_2 = T_3$

(GATE MA 2017)

- 46) Let X be a random variable with probability mass function $p(n) = \left(\frac{3}{4}\right)^{n-1} \left(\frac{1}{4}\right)$ for $n = 1, 2, \dots$. Then $E(X - 3 \mid X > 3)$ equals _____.

(GATE MA 2017)

- 47) Let X and Y be independent and identically distributed random variables with probability mass function $p(n) = 2^{-n}$, $n = 1, 2, \dots$. Then $P(X \geq 2Y)$ equals (rounded to 2 decimal places) _____.

(GATE MA 2017)

- 48) Let X_1, X_2, \dots be a sequence of independent and identically distributed Poisson random variables with mean 4. Then

$$\lim_{n \rightarrow \infty} P\left(4 - \frac{2}{\sqrt{n}} < \frac{1}{n} \sum_{i=1}^n X_i < 4 + \frac{2}{\sqrt{n}}\right)$$

equals _____. (GATE MA 2017)

- 49) Let X and Y be independent and identically distributed exponential random variables with probability density function $f(x) = e^{-x}$, $x > 0$. Then $P(\max(X, Y) < 2)$ equals (rounded to 2 decimal places) _____. (GATE MA 2017)

- 50) Let E and F be any two events with $P(E) = 0.4$, $P(F) = 0.3$, and $P(F|E) = 3P(F|E^c)$. Then $P(E|F)$ equals (rounded to 2 decimal places) _____. (GATE MA 2017)

- 51) Let X_1, X_2, \dots, X_m ($m \geq 2$) be a random sample from a binomial distribution with parameters $n = 1$ and p , $p \in (0, 1)$, and let $\bar{X} = \frac{1}{m} \sum_{i=1}^m X_i$. Then a uniformly minimum variance unbiased estimator for $p(1 - p)$ is

- a) $\frac{m}{m-1} \bar{X} (1 - \bar{X})$ c) $\frac{m-1}{m} \bar{X} (1 - \bar{X})$
b) $\bar{X} (1 - \bar{X})$ d) $\frac{1}{m} (1 - \bar{X})$

(GATE MA 2017)

- 52) Let X_1, X_2, \dots, X_9 be a random sample from a $N(0, \sigma^2)$ population. For testing $H_0 : \sigma^2 = 2$ against $H_1 : \sigma^2 = 1$, the most powerful test rejects H_0 if $\sum_{i=1}^9 X_i^2 < c$, where c is to be chosen such that the level of significance is 0.1. Then the power of this test equals _____. (GATE MA 2017)

- 53) Let X_1, X_2, \dots, X_n ($n \geq 2$) be a random sample from $N(0, \theta)$ population, where $\theta > 0$, and let $W = \sum_{i=1}^n X_i^2$. Then the maximum likelihood estimator of θ is

- a) $\sqrt{1 - 4W}/2$ c) $-\sqrt{1 - 4W}/2$
b) $\sqrt{1 + 4W}/2$ d) $-\sqrt{1 + 4W}/2$

(GATE MA 2017)

- 54) Consider the following transportation problem (entries inside cells denote per unit cost of transportation):

	Destination 1	Destination 2	Destination 3	Supply
Origin 1	4	3	6	20
Origin 2	7	10	5	30
Origin 3	8	9	7	50
Demand	10	30	60	

With demands: 10, 30, 60 units respectively. The optimal cost of transportation equals

_____. (GATE MA 2017)

- 55) Consider the linear programming problem (LPP): Maximize $kx_1 + 5x_2$ subject to $x_1 + x_2 \leq 1$, $2x_1 + 3x_2 \leq 1$, $x_1, x_2 \geq 0$. If $x = (x_1, x_2)$ is an optimal solution of the above LPP with $k = 2$, then the largest value of k (rounded to 2 decimal places) for which x remains optimal equals _____. (GATE MA 2017)

- 56) The ninth and the tenth of this month are Monday and Tuesday

- a) figuratively c) respectively
b) retrospectively d) rightfully

(GATE MA 2017)

- 57) It is _____ to read this year's textbook _____ the last year's.

- a) easier, than c) easier, from
b) most easy, than d) easiest, from

(GATE MA 2017)

- 58) A rule states that in order to drink beer, one must be over 18 years old. In a bar, there are 4 people: P (16 years old), Q (25 years old), R (drinking milkshake), and S (drinking a beer). What must be checked to ensure that the rule is being followed?

- a) Only P's drink
- b) Only P's drink and S's age
- c) Only S's age
- d) Only P's drink, Q's drink and S's age

(GATE MA 2017)

59) Fatima starts from point P, goes North for 3 km, and then East for 4 km to reach point Q. She then turns to face point P and goes 15 km in that direction. She then goes North for 6 km. How far is she from point P, and in which direction should she go to reach point P?

- a) 8 km, East
- b) 12 km, North
- c) 6 km, East
- d) 10 km, North

(GATE MA 2017)

60) 500 students are taking one or more courses out of Chemistry, Physics, and Mathematics. Registration records indicate course enrolment as follows: Chemistry (329), Physics (186), Mathematics (295), Chemistry and Physics (83), Chemistry and Mathematics (217), Physics and Mathematics (63). How many students are taking all 3 subjects?

- a) 37
- b) 43
- c) 47
- d) 53

(GATE MA 2017)

61) "If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country, I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters." Which of the following statements best reflects the author's opinion?

- a) An intimate association does not allow for the necessary perspective.
- b) Matters are recorded with an impartial perspective.
- c) An intimate association offers an impartial perspective.
- d) Actors are typically associated with the impartial recording of matters.

(GATE MA 2017)

62) Each of P, Q, R, S, W, X, Y and Z has been married at most once. X and Y are married and have two children P and Q. Z is the grandfather of the daughter S of P. Further, Z and W are married and are parents of R. Which one of the following must necessarily be FALSE?

- a) X is the mother-in-law of R
- b) P and R are not married to each other
- c) P is a son of X and Y
- d) Q cannot be married to R

(GATE MA 2017)

63) 1200 men and 500 women can build a bridge in 2 weeks. 900 men and 250 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week?

- a) 3000
- b) 3300
- c) 3600
- d) 3900

(GATE MA 2017)

64) The number of 3-digit numbers such that the digit 1 is never to the immediate right of 2 is

- a) 781
b) 791
- c) 881
d) 891

(GATE MA 2017)

- 65) A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot. (Listed locations P, Q, R, S, T with heights; diagram referenced.)

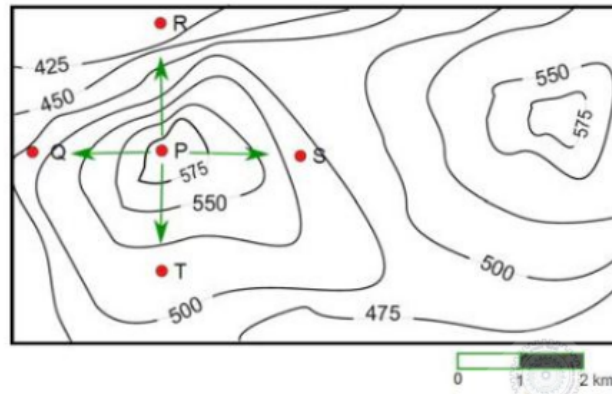


Fig. 1. Q.65

Which of the following is the steepest path leaving from P?

- a) P to Q
b) P to R
- c) P to S
d) P to T

(GATE MA 2017)