MA - 2010

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1) Let E and F be any two events with $P(E \cup F) = 0.8$, $P(E) = 0.4$ and $P(E|F) = 0.3$. Then $P(F)$ is __________.

(GATE MA 2010)

a) $\frac{3}{7}$ b) $\frac{4}{7}$ c) $\frac{1}{5}$ d) $\frac{2}{5}$

2) Let X have a binomial distribution with parameters n and p, where n is an integer greater than 1 and $0 < p < 1$. If $P(X = 0) = P(X = 1)$, then the value of p is __________.

(GATE MA 2010)

a) $\frac{1}{n-1}$ b) $\frac{n}{n+1}$ c) $\frac{1}{n+1}$ d) $\frac{1}{1+n\sqrt{n}}$

3) Let $u(x, y) = 2x(1 - y)$ for all real x and y. Then a function $v(x, y)$, so that $f(z) = u(x, y) + iv(x, y)$ is analytic, is __________.

(GATE MA 2010)

a) $(x^2 - (y - 1)^2)$
b) $(x - 1)^2 - y^2$
c) $(x - 1)^2 + y^2$
d) $x^2 + (y - 1)^2$

4) Let $f(z)$ be analytic on $D = \{z \in \mathbb{C} : |z - 1| < 1\}$ such that $f(1) = 1$. If $f(z) = f(z^2)$ for all $z \in D$, then which one of the following statements is NOT correct? __________

(GATE MA 2010)

a) $f(z) = [f(z)]^2$ for all $z \in D$
b) $f\left(\frac{z}{2}\right) = \frac{1}{2}f(z)$ for all $z \in D$
c) $f(z^2) = [f(z)]^2$ for all $z \in D$
d) $f'(1) = 0$

5) The maximum number of linearly independent solutions of the differential equation

$$\frac{d^4y}{dx^4} = 0,$$

with the condition $y(0) = 1$, is __________.

(GATE MA 2010)

a) 4 b) 3 c) 2 d) 1

6) Which one of the following sets of functions is NOT orthogonal (with respect to the L^2 inner product) over the given interval? __________

(GATE MA 2010)

a) $\{\sin ax : a \in \mathbb{N}\}, \quad -\pi < x < \pi$
b) $\{\cos ax : a \in \mathbb{N}\}, \quad -\pi < x < \pi$
c) $\{x^{2n} : n \in \mathbb{N}\}, \quad -1 < x < 1$
d) $\{x^{2n+1} : n \in \mathbb{N}\}, \quad -1 < x < 1$

7) If $f : [1,2] \to \mathbb{R}$ is a non-negative Riemann-integrable function such that

$$\int_1^2 \sqrt{x} f(x)\, dx = k \int_1^2 f(x)\, dx \neq 0,$$

then k belongs to the interval __________.

(GATE MA 2010)

a) $\left[0, \frac{1}{\sqrt{2}}\right]$ b) $\left(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{3}}\right]$ c) $\left(\frac{2}{\sqrt{3}}, 1\right]$ d) $\left(1, \frac{4}{3}\right]$

8) The set $X = \mathbb{R}$ with the metric $d(x,y) = \frac{|x-y|}{1+|x-y|}$ is __________.

(GATE MA 2010)

a) bounded but not compact
b) bounded but not complete
c) complete but not bounded
d) compact but not complete

9) Let

$$f(x,y) = \begin{cases} \frac{xy}{(x^2+y^2)^{k/2}} \left[1 - \cos(x^2+y^2)\right], & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Then the value of k for which $f(x,y)$ is continuous at $(0,0)$ is __________.

(GATE MA 2010)

a) 0 b) $\frac{1}{2}$ c) 1 d) $\frac{3}{2}$

10) Let A and B be disjoint subsets of $\mathbb{R}$ and let m^* denote the Lebesgue outer measure on $\mathbb{R}$. Consider the statements: P: $m^*(A \cup B) = m^*(A) + m^*(B)$ Q: Both A and B are Lebesgue measurable R: One of A and B is Lebesgue measurable
Which one of the following is correct? __________

(GATE MA 2010)

a) If P is true, then Q is true
b) If P is NOT true, then R is true
c) If R is true, then P is NOT true
d) If R is true, then P is true

11) Let $f : \mathbb{R} \to [0, \infty)$ be a Lebesgue measurable function and E be a Lebesgue measurable subset of $\mathbb{R}$ such that

$$\int_E f\, dm = 0,$$

where m is the Lebesgue measure on $\mathbb{R}$. Then __________.

(GATE MA 2010)

a) $m(E) = 0$
b) $\{x \in E : f(x) = 0\} = E$
c) $m(\{x \in E : f(x) \neq 0\}) = 0$
d) $m(\{x \in E : f(x) = 0\}) = 0$

12) If the nullity of the matrix

$$\begin{pmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{pmatrix}$$

is 1, then the value of k is ________.

(GATE MA 2010)

a) -1 b) 0 c) 1 d) 2

13) If a 3×3 real skew-symmetric matrix has an eigenvalue $2i$, then one of the remaining eigenvalues is ________.

(GATE MA 2010)

a) $\frac{1}{2i}$ b) $-\frac{1}{2i}$ c) 0 d) 1

14) For the linear programming problem Minimize $z = x - y$, subject to $2x + 3y \leq 6,\ 0 \leq x \leq 3,\ 0 \leq y \leq 3$, the number of extreme points of its feasible region and the number of basic feasible solutions respectively, are ________.

(GATE MA 2010)

a) 3 and 3 b) 4 and 4 c) 3 and 5 d) 4 and 5

15) Which one of the following statements is correct? ________

(GATE MA 2010)

a) If a Linear Programming Problem (LPP) is infeasible, then its dual is also infeasible
b) If an LPP is infeasible, then its dual always has unbounded solution
c) If an LPP has unbounded solution, then its dual also has unbounded solution
d) If an LPP has unbounded solution, then its dual is infeasible

16) Which one of the following groups is simple? ________

(GATE MA 2010)

a) S_3
b) $GL(2, \mathbb{R})$
c) $\mathbb{Z}_2 \times \mathbb{Z}_2$
d) A_5

17) Consider the algebraic extension $E = \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ of the field $\mathbb{Q}$ of rational numbers. Then $[E : \mathbb{Q}]$, the degree of E over $\mathbb{Q}$, is ________.

(GATE MA 2010)

a) 3 b) 4 c) 7 d) 8

18) The general solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x \partial y} = x + y$$

is of the form ________.

(GATE MA 2010)

a) $\frac{1}{2}xy(x+y) + F(x) + G(y)$
b) $\frac{1}{2}xy(x-y) + F(x) + G(y)$
c) $\frac{1}{2}xy(x-y) + F(x)G(y)$

d) $\frac{1}{2}xy(x+y)+F(x)G(y)$

19) The numerical value obtained by applying the two-point trapezoidal rule to the integral

$$\int_0^1 \frac{\ln(1+x)}{x}dx$$

is __________.

(GATE MA 2010)

a) $\frac{1}{2}(\ln 2+1)$ b) $\frac{1}{2}$ c) $\frac{1}{2}(\ln 2-1)$ d) $\frac{1}{2}\ln 2$

20) Let $\ell_k(x), k=0,1,\dots,n$ denote the Lagrange's fundamental polynomials of degree n for the nodes $x_0, x_1, \dots, x_n$. Then the value of $\sum_{k=0}^{n} \ell_k(x)$ is __________.

(GATE MA 2010)

a) 0 b) n c) $x+1$ d) $x-1$

21) Let X and Y be normed linear spaces and $\{T_n\}$ be a sequence of bounded linear operators from X to Y. Consider the statements:
P: $\{\|T_n x\|: n \in \mathbb{N}\}$ is bounded for each $x \in X$.
Q: $\{\|T_n\|: n \in \mathbb{N}\}$ is bounded.
Which one of the following is correct? __________

(GATE MA 2010)

a) If P implies Q, then both X and Y are Banach spaces
b) If P implies Q, then only one of X and Y is a Banach space
c) If X is a Banach space, then P implies Q
d) If Y is a Banach space, then P implies Q

22) Let $X = C[0,1]$ with the norm $\|x\|_1 = \int_0^1 |x(t)|\,dt,\ x \in C[0,1]$, and $\Omega = \{f \in X' : \|f\| = 1\}$, where X' denotes the dual space of X. Let $C(\Omega)$ be the linear space of continuous functions on Ω with the norm $\|u\|_\infty = \sup_{f\in\Omega}|u(f)|,\ u \in C(\Omega)$. Then __________

(GATE MA 2010)

a) X is linearly isometric with $C(\Omega)$
b) X is linearly isometric with a proper subspace of $C(\Omega)$
c) there does not exist a linear isometry from X into $C(\Omega)$
d) every linear isometry from X to $C(\Omega)$ is onto

23) Let $X = \mathbb{R}$ equipped with the topology generated by open intervals of the form (a,b) and sets of the form $(a,b) \setminus \mathbb{Q}$. Which one of the following statements is correct? __________

(GATE MA 2010)

a) X is regular
b) X is normal
c) $X \setminus \mathbb{Q}$ is dense in X
d) $\mathbb{Q}$ is dense in X

24) Let H, T and V denote the Hamiltonian, the kinetic energy and the potential energy respectively of a mechanical system at time t. If H contains t explicitly, then $\frac{dH}{dt}$ is equal to __________

(GATE MA 2010)

a) $\frac{\partial T}{\partial t}$
b) $\frac{\partial T}{\partial t} - \frac{\partial V}{\partial t}$
c) $\frac{\partial T}{\partial t} + \frac{\partial V}{\partial t}$
d) $-\frac{\partial V}{\partial t}$

25) The Euler's equation for the variational problem

$$\text{Minimize} \quad I(y(x)) = \int_a^b (x^2 - xy - y'^2)\, dx$$

is __________

(GATE MA 2010)

a) $2y'' - y = 2$
b) $2y' + y = 2$
c) $2y'' - y = 0$
d) $2y' - y = 0$

26) Let X have a binomial distribution with parameters n and p, $n = 3$. For testing the hypothesis $H_0: p = \frac{2}{3}$ against $H_1: p = \frac{1}{3}$, let a test be: "Reject H_0 if $X \geq 2$ and accept H_0 if $X \leq 1$." Then the probabilities of Type I and Type II errors respectively are __________

(GATE MA 2010)

a) $\frac{20}{27}$ and $\frac{20}{27}$
b) $\frac{7}{27}$ and $\frac{20}{27}$
c) $\frac{20}{27}$ and $\frac{7}{27}$
d) $\frac{7}{27}$ and $\frac{7}{27}$

27) Let $I = \int_C \frac{f(z)}{z(z-1)(z-2)}\, dz$, where $f(z) = \sin\frac{\pi z}{2} + \cos\frac{\pi z}{2}$ and C is the curve $|z| = 3$ oriented anti-clockwise. Then the value of I is __________

(GATE MA 2010)

a) $4\pi i$
b) 0
c) $-2\pi i$
d) $-4\pi i$

28) Let $\sum_{n=-\infty}^{\infty} b_n z^n$ be the Laurent series expansion of the function $\frac{1}{z \sinh z}$, $0 < |z| < \pi$. Then which one of the following is correct? __________

(GATE MA 2010)

a) $b_{-2} = 1,\ b_0 = -\frac{1}{6},\ b_2 = \frac{7}{360}$
b) $b_{-1} = 1,\ b_1 = -\frac{1}{6},\ b_3 = \frac{7}{360}$
c) $b_2 = 0,\ b_0 = -\frac{1}{6},\ b_2 = \frac{7}{360}$
d) $b_0 = 1,\ b_2 = -\frac{1}{6},\ b_4 = \frac{7}{360}$

29) Under the transformation $w = \frac{1 - iz}{z - i}$, the region $D = \{z \in \mathbb{C} : |z| < 1\}$ is transformed to __________

(GATE MA 2010)

a) $\{z \in \mathbb{C} : 0 < \arg z < \pi\}$
b) $\{z \in \mathbb{C} : -\pi < \arg z < 0\}$
c) $\{z \in \mathbb{C} : 0 < \arg z < \frac{\pi}{2} \text{ or } \pi < \arg z < \frac{3\pi}{2}\}$
d) $\{z \in \mathbb{C} : \frac{\pi}{2} < \arg z < \pi \text{ or } \frac{3\pi}{2} < \arg z < 2\pi\}$

30) Let $y(x)$ be the solution of the initial value problem

$$y''' - y'' + 4y' - 4y = 0, \quad y(0) = y'(0) = 2,\ y''(0) = 0.$$

Then the value of $y\left(\frac{\pi}{2}\right)$ is _________

(GATE MA 2010)

a) $\frac{1}{5}\left(4e^{\frac{\pi}{2}} - 6\right)$
b) $\frac{1}{5}\left(e^{\frac{\pi}{2}} - 4\right)$
c) $\frac{1}{5}\left(8e^{\frac{\pi}{2}} - 2\right)$
d) $\frac{1}{5}\left(8e^{\frac{\pi}{2}} + 2\right)$

31) Let $y(x)$ be the solution of the initial value problem

$$x^2y'' + xy' + y = x, \quad y(0) = y'(0) = 1.$$

Then the value of $y(e^{\frac{\pi i}{2}})$ is _________

(GATE MA 2010)

a) $\frac{1}{2}(1 - e^{\frac{\pi i}{2}})$
b) $\frac{1}{2}(1 + e^{\frac{\pi i}{2}})$
c) $\frac{1}{2} + \frac{\pi i}{4}$
d) $\frac{1}{2} - \frac{\pi i}{4}$

32) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y, z) = (x + y,\ y + z,\ z - x)$. Then, an orthonormal basis for the range of T is _________

(GATE MA 2010)

a) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right\}$
b) $\left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)\right\}$
c) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)\right\}$
d) $\left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right\}$

33) Let $T : P_3[0, 1] \to P_3[0, 1]$ be defined by $(Tp)(x) = p''(x) + p'(x)$. Then the matrix representation of T with respect to the bases $\{1, x, x^2, x^3\}$ and $\{1, x, x^2, x^3\}$ of $P_3[0, 1]$ and $P_2[0, 1]$ respectively is _________

(GATE MA 2010)

a) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 6 & 3 \end{pmatrix}$
b) $\begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \end{pmatrix}$
c) $\begin{pmatrix} 0 & 2 & 1 & 0 \\ 6 & 2 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{pmatrix}$
d) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 6 & 0 \end{pmatrix}$

34) Consider the basis $\{u_1, u_2, u_3\}$ of $\mathbb{R}^3$, where $u_1 = (1, 0, 0)$, $u_2 = (1, 1, 0)$, $u_3 = (1, 1, 1)$. Let $\{f_1, f_2, f_3\}$ be the dual basis of $\{u_1, u_2, u_3\}$ and f be a linear functional defined by $f(a, b, c) = a + b + c$, $(a, b, c) \in \mathbb{R}^3$. If $f = \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3$, then $(\alpha_1, \alpha_2, \alpha_3)$ is _________

(GATE MA 2010)

a) $(1, 2, 3)$
b) $(1, 3, 2)$
c) $(2, 3, 1)$
d) $(3, 2, 1)$

35) The following table gives the cost matrix of a transportation problem.

4	5	6
3	2	2
1	1	2

. The basic feasible solution given by $x_{11} = 3,\ x_{12} = 1,\ x_{13} = 6,\ x_{21} = 2,\ x_{22} = 5$ is _________

(GATE MA 2010)

a) degenerate and optimal
b) optimal but not degenerate
c) degenerate but not optimal
d) neither degenerate nor optimal

36) If z^* is the optimal value of the linear programming problem

$$\begin{aligned} &\text{Maximize } z = 5x_1 + 9x_2 + 4x_3, \\ &\text{subject to } x_1 + x_2 + x_3 \leq 5, \\ &\qquad\qquad 4x_1 + 3x_2 + 2x_3 = 12, \\ &x_1, x_2, x_3 \geq 0, \end{aligned}$$

then

(GATE MA 2010)

a) $0 \leq z^* < 10$
b) $10 \leq z^* < 20$
c) $20 \leq z^* < 30$
d) $30 \leq z^* < 40$

37) Let G_1 be an abelian group of order 6 and $G_2 = S_5$. For $j = 1, 2$, let P_j be the statement: G_j has a unique subgroup of order 2. Then

(GATE MA 2010)

a) both P_1 and P_2 hold
b) neither P_1 nor P_2 holds
c) P_1 holds but not P_2
d) P_2 holds but not P_1

38) Let G be the group of all symmetries of the square. Then the number of conjugate classes in G is

(GATE MA 2010)

a) 4
b) 5
c) 6
d) 7

39) Consider the polynomial ring $\mathbb{Q}[x]$. The ideal of $\mathbb{Q}[x]$ generated by $x^2 - 3$ is

(GATE MA 2010)

a) maximal but not prime
b) prime but not maximal
c) both maximal and prime
d) neither maximal nor prime

40) Consider the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \ t > 0,$$

$$u(0, t) = u(\pi, t) = 0, \qquad u(x, 0) = \sin x, \qquad \frac{\partial u}{\partial t}(x, 0) = 0.$$

Then $u\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

(GATE MA 2010)

a) 2
b) 1
c) 0
d) −1

41) Let $I = \int_C \frac{e^x}{x}\,dx + (e^y \ln x + x)\,dy$, where C is the positively oriented boundary of the region enclosed by $y = 1 + x^2$, $y = 2x$, $x = \frac{1}{2}$. Then the value of I is

(GATE MA 2010)

a) $\frac{1}{8}$
b) $\frac{5}{24}$
c) $\frac{7}{24}$
d) $\frac{3}{8}$

42) Let $\{f_n\}$ be a sequence of real valued differentiable functions on $[a,b]$ such that $f_n(x) \to f(x)$ as $n \to \infty$ for every $x \in [a,b]$ and for some Riemann-integrable function $f : [a,b] \to \mathbb{R}$. Consider the statements

$$\begin{aligned} P_1 &: \{f_n\} \text{ converges uniformly,} \\ P_2 &: \{f_n'\} \text{ converges uniformly,} \\ P_3 &: \int_a^b f_n(x)\,dx \to \int_a^b f(x)\,dx, \\ P_4 &: f \text{ is differentiable.} \end{aligned}$$

Then which one of the following need NOT be true

(GATE MA 2010)

a) P_3 implies P_1
b) P_2 implies P_1
c) P_2 implies P_4
d) P_3 implies P_4

43) Let $f_n(x) = \dfrac{x^n}{1+x}$ and $g_n(x) = \dfrac{x^n}{1+nx}$ for $x \in [0,1]$ and $n \in \mathbb{N}$. Then on the interval $[0,1]$,

(GATE MA 2010)

a) both $\{f_n\}$ and $\{g_n\}$ converge uniformly
b) neither $\{f_n\}$ nor $\{g_n\}$ converge uniformly
c) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly
d) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly

44) Consider the power series $\sum_{n=1}^{\infty} \dfrac{x^n}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \dfrac{x^n}{n}$. Then

(GATE MA 2010)

a) both converge on $(-1,1)$
b) both converge on $[-1,1)$
c) exactly one of them converges on $(-1,1)$
d) none of them converges on $[-1,1)$

45) Let $X = \mathbb{N}$ be equipped with the topology generated by the basis consisting of sets $A_n = \{n, n+1, n+2, \ldots\}, n \in \mathbb{N}$. Then X is

(GATE MA 2010)

a) Compact and connected
b) Hausdorff and connected
c) Hausdorff and compact
d) Neither compact nor connected

46) Four weightless rods form a rhombus $PQRS$ with smooth hinges at the joints. Another weightless rod joins the midpoints E and F of PQ and PS respectively. The system is suspended from P and a weight $2W$ is attached to R. If the angle between the rods PQ and PS is 2θ, then the thrust in the rod EF is

(GATE MA 2010)

a) $W \tan\theta$
b) $2W \tan\theta$
c) $2W \cot\theta$
d) $4W \tan\theta$

47) For a continuous function $f(t)$, $0 \le t \le 1$, the integral equation

$$y(t) = f(t) + \int_0^1 ts\, y(s)\, ds$$

has

(GATE MA 2010)

a) a unique solution if $\int_0^1 sf(s)\, ds \neq 0$
b) no solution if $\int_0^1 sf(s)\, ds = 0$
c) infinitely many solutions if $\int_0^1 sf(s)\, ds = 0$
d) infinitely many solutions if $\int_0^1 sf(s)\, ds \neq 0$

Common Data for Q.48 and Q.49:
Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} a\, x^2 e^{-y}, & 0 < x < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

(GATE MA 2010)

48) The value of a is (GATE MA 2010)

a) 4
b) 2
c) 1
d) 0.5

49) The value of $E[X \mid Y = 2]$ is

(GATE MA 2010)

a) 4
b) 3
c) 2
d) 1

Common Data for Q.50 and Q.51:
Let $X = \mathbb{N} \times \mathbb{Q}$ with the subspace topology of the usual topology on $\mathbb{R}^2$ and $P = \{(n, \frac{1}{n}) : n \in \mathbb{N}\}$.

50) In the space X,

(GATE MA 2010)

a) P is closed but not open
b) P is open but not closed
c) P is both open and closed
d) P is neither open nor closed

51) The boundary of P in X is

(GATE MA 2010)

a) an empty set
b) a singleton set
c) P
d) X

Linked Answer Questions (Q.52-Q.53):
For a differentiable function $f(x)$, the integral

$$\int_0^1 f(x)\, dx$$

is approximated by the formula

$$h[a_0 f(0) + a_1 f(1)] + h^2[b_0 f'(0) + b_1 f'(1)],$$

which is exact for all polynomials of degree at most 3.

52) The values of a_0 and a_1 respectively are

(GATE MA 2010)

a) $\frac{1}{2}$ and $-\frac{1}{2}$ b) $\frac{1}{2}$ and $\frac{1}{2}$ c) 2 and $\frac{1}{2}$ d) $-\frac{1}{2}$ and $\frac{1}{2}$

53) The values of b_0 and b_1 respectively are

(GATE MA 2010)

a) $\frac{1}{12}$ and $-\frac{1}{12}$ b) $-\frac{1}{12}$ and $\frac{1}{12}$ c) $\frac{1}{12}$ and $\frac{1}{12}$ d) $-\frac{1}{12}$ and $-\frac{1}{12}$

Statement for Linked Answer Questions 54 and 55

Let $X = C[0,1]$ with the inner product

$$\langle x, y\rangle = \int_0^1 x(t)\, y(t)\, dt, \qquad x, y \in C[0,1].$$

Let

$$X_0 = \left\{x \in X : \int_0^1 t^2 x(t)\, dt = 0\right\}, \qquad X_0^{\perp} \text{ be the orthogonal complement of } X_0.$$

54) Which one of the following statements is correct?

(GATE MA 2010)

a) Both X_0 and $X_0^{\perp}$ are complete
b) Neither X_0 nor $X_0^{\perp}$ is complete
c) X_0 is complete but $(X_0)^{\perp}$ is not complete
d) $X_0^{\perp}$ is complete but X_0 is not complete

55) Let $y(t) = t^r$, $r \in [0,1]$, and let $x_0 \in X_0^{\perp}$ be the best approximation of y. Then $x_0(t)$, $t \in [0,1]$, is

(GATE MA 2010)

a) $\frac{4}{5}t^2$ b) $\frac{5}{6}t^2$ c) $\frac{6}{7}t^2$ d) $\frac{7}{8}t^2$

GENERAL APTITUDE

56) Which of the following options is the closest in meaning to the word below:
Circumlocution

(GATE MA 2010)

a) cyclic b) indirect c) confusing d) crooked

57) The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair. **Unemployed : Worker**

(GATE MA 2010)

a) fallow : land
b) unaware : sleeper
c) wit : jester
d) renovated : house

58) Choose the most appropriate word from the options given below to complete the following sentence: **If we manage to _________. our natural resources, we would leave a better planet for our children.**

(GATE MA 2010)

a) uphold b) restrain c) cherish d) conserve

59) Choose the most appropriate word from the options given below to complete the following sentence: **His rather casual remarks on politics_________. his lack of seriousness about the subject.**

(GATE MA 2010)

a) masked b) belied c) betrayed d) suppressed

60) 25 persons are in a room. 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is:

(GATE MA 2010)

a) 2 b) 17 c) 13 d) 3

61) Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.
Which of the following statements best sums up the meaning of the above passage:

(GATE MA 2010)

a) Modern warfare has resulted in civil strife.
b) Chemical agents are useful in modern warfare.
c) Use of chemical agents in warfare would be undesirable.
d) People in military establishments like to use chemical agents in war.

62) If 137 + 276 = 435 how much is 731 + 672?

(GATE MA 2010)

a) 534 b) 1403 c) 1623 d) 1513

63) 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

(GATE MA 2010)

a) 20 days b) 18 days c) 16 days d) 15 days

64) Given digits 2, 3, 3, 3, 4, 4, 4, 4, how many distinct 4 digit numbers greater than 3000 can be formed?

(GATE MA 2010)

a) 50 b) 51 c) 52 d) 54

65) Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:
i. Hari's age + Gita's age = Irfan's age + Saira's age.
ii. The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest.
iii. There are no twins.
In what order were they born (oldest first)?

(GATE MA 2010)

a) HSIG b) SGHI c) IGSH d) IHSG