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Embedded System Design

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Multivariable Tank Level Control

Embedded Systems Project [ES-PRO]

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Table 1: Contribution List

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Multivariable Tank Level Control

Abstract

This paper focuses on studying the liquid level control method of the three-capacity water tank system, which is a simulation model for complex objects in multi-container process systems. The system has multiple inputs and outputs, is time-varying, and can experience leakage over time. The goal is to use state feedback control to enable the water tank to track a fixed liquid level, handle leakage, and meet control input constraints. The simulation model is built using Matlab/Simulink software environment, and the research is expected to have broad application prospects in various fields, such as agriculture, industry, chemical industry, and petroleum.

This study also includes the mathematical modeling of the three-capacity water tank system. Initially, we applied single-input single-output (SISO) control to the system and then progressed to multi-input multi-output (MIMO) control. Both simulation and testing of the control methods were conducted in the Simulink software environment, as well as on the real object itself. The results obtained from the simulations and tests are analyzed and compared, providing insights into the effectiveness and limitations of the proposed control methods. These findings contribute to the advancement of liquid level control in multi-container process systems and can be applied in various industries.

Part I

Introduction

The use of multi-container process systems is essential in many industries, such as agriculture, chemical, and petroleum industries, among others. These systems are often characterized by their complexity, such as multiple inputs, outputs, strong coupling, and time-varying behavior. One commonly used system is the three-capacity water tank system, which simulates such complexities. Control of the liquid level is crucial in these systems, and this study aims to present a comprehensive approach to liquid level control of the three-capacity water tank system. This includes mathematical modeling, application of SISO and MIMO control methods, and testing in both simulated and real-world environments. The study's findings [1] will provide insights into the effectiveness and limitations of the proposed control methods, which can be used to improve liquid level control in multi-container process systems.

Part II

3-Tank System

1 Structure

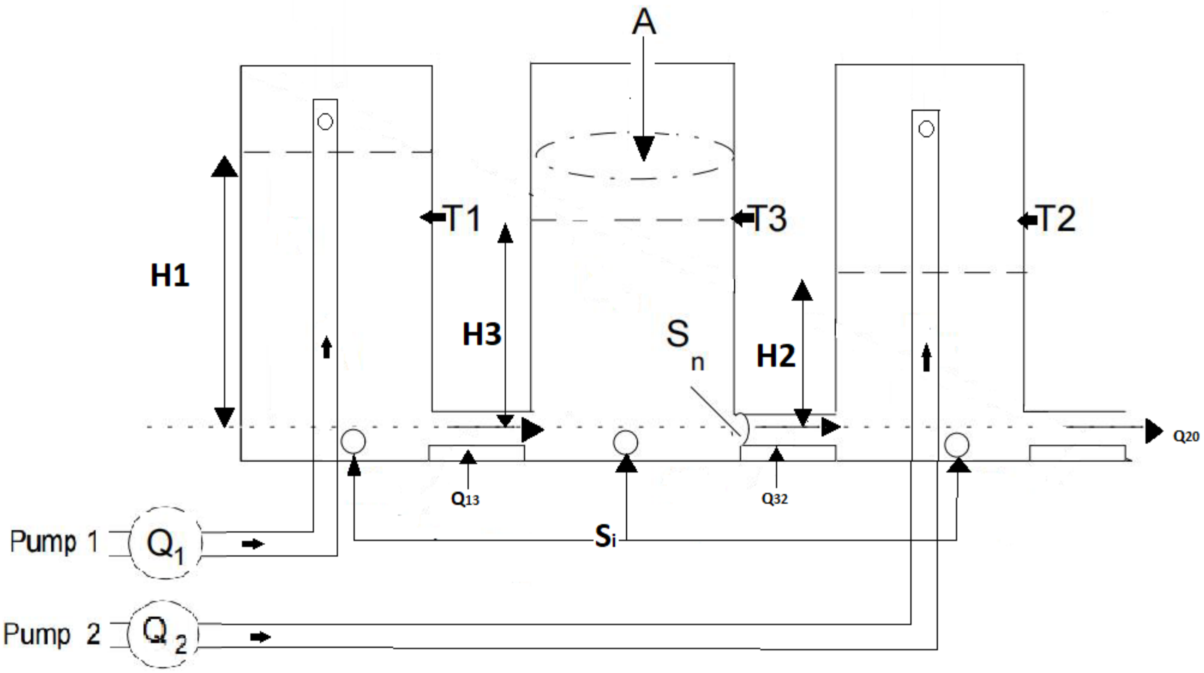


Figure 1: Three Tank System Structure

Figure 1 illustrates the configuration of a three-tank system comprising a water reservoir, two water pumps (pump1 and pump2), three water tanks (T1, T2, and T3), and three leakage valves (Si). The water tanks are interconnected by connecting pipes, which include a manually adjustable ball valve to mimic blockage or operational error. Each tank has a circular opening with a cross-section (Si) and a manual regulating ball valve to simulate leakage. When the system is in operation, the two water pumps draw water from the reservoir to supply water to tanks 1 and 2. The pressure of the water allows water to flow from tanks 1 and 2 to tank 3 via the connecting pipes. Some of the water in the three tanks flows back to the reservoir through the connecting pipe at the bottom of tank 2 represented by the flow Q20. The system is multi-input and multi-output, with the flow of water from the two pumps being the inputs and the liquid level height of tanks 1 and 2 being the usual outputs.

The symbols and abbreviations used in this text are shown in the following table:

Table 2: Abbreviations and symbols

Symbols	Description
azi	Outflow coefficients (i=1, = 2,3)
hi	Liquid levels (i = 1, 2, 3)
Qij	Flow rates (i=1, 2, 3)
Q1, Q2	Supplying flow rates
A	Section of cylinder
Si	Section of leak opening
Sn	Section of connection pipe
g	Gravity
sgn(z)	Sign of the argument Z
Δh	Liquid level difference between two tanks connected to each other
q	resulting flow rate in the connecting pipe
Tn	cylinder

The parameters used in this paper are shown in the following table:

Table 3: Parameters and Values

Parameter	Values & Units
An	0.0149 m^2
Sn	0.00005 m^2
g	9.81 m/s^2
Hmax	60 cm

2 Test Bench

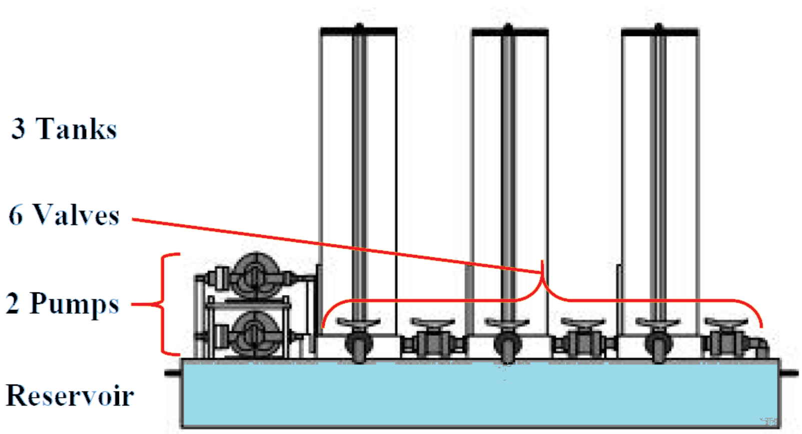


Figure 2: Test Bench of 3-Tank System

The test setup is primarily made up of three tanks, a reservoir, two pumps, and six ball valves. The tanks and reservoirs are interconnected using ball valves. The first and third tanks can be supplied with water by the two pumps. Additionally, each tank has a valve that can be used to release the water into the reservoir or another tank.

2.1 Controller

In this setup, a digital controller is implemented on a PC with an A/D-D/A plug-in card, and 12-bit converters are used to connect to the controlled plant. The A/D converter is responsible for reading the liquid levels in tanks T1, T2, and T3, and is initialized for a voltage range of -10V to 10V, with a resolution of 4.88mV. Two D/A converters are utilized to control the pumps, as illustrated in Figure 3, which shows the principal structure of the complete control loop.[1]

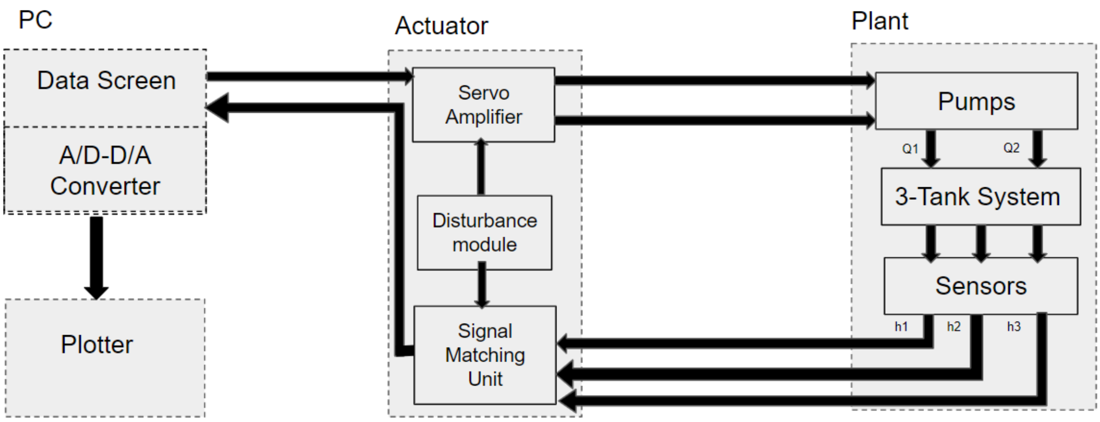


Figure 3: Control Loop Structure

2.2 Signal Adaption Unit

The purpose of the signal adaption unit is to adapt the voltage levels of the plant and the converter to each other. Here the output voltages of the sensors are adapted to the maximum resolution of the A/D- converters and on the other hand the output voltage range of the D/A-converters is adapted the servo amplifier of the corresponding pump.

2.3 Technical Data

The list of tables shown below represents the different parameters of the test bench, including the dimensions of the test bench itself, the reservoir, the cylindrical tank, and the valves. Each table corresponds to a specific parameter that needs to be tested and evaluated, providing a comprehensive assessment of the device under test. The dimensions of the test bench and the reservoir, for instance, can have a significant impact on the flow rate and pressure of the fluid being tested. Similarly, the dimensions of the cylindrical tank and the valves can affect the accuracy and consistency of the measurements taken. By including these dimensions in the list of parameters, the test bench provides a more complete picture of the performance of the device under test

- The table 4 contains the dimensions of the test bench.
- The table 5 contains the dimensions of the reservoir.
- The table 6 contains the dimensions of the cylindrical tank.

Table 4: Dimension Sizes and Weight of Test Bench

Dimension Sizes and Weight	Value	Unit
Length	1280	mm
Depth	350	mm
Height	890	mm
Weight	54	kg

Table 5: Dimension Sizes and Capacity of Reservoir

Reservoir	Value	Unit
Length	1200	mm
Depth	350	mm
Height	160	mm
Capacity ca.	43	l

Table 6: Dimensions of Cylindrical Tank

Cylinder tank	Value	Unit
Diameter outside	150	mm
Diameter inside	140	mm
Diameter inner flow pipe	25	mm
Height incl. socket and cover	720	mm

2.4 Callibration

The three Tank system calibration involves creating a Simulink interface that converts the control data from the control system derived to the corresponding values to the test bench to obtain the desired control of the system. The interface used to convert the data is called a look-up table. In Matlab, a look-up table (LUT) is a data structure that maps input values to corresponding output values. The components that need to be calibrated are:

- Calibration of pressure sensor from the tank which gives the amount of water in the tank.
- Calibration of the motor flow rate.

2.4.1 Tank Height Callibration

In this calibration, we collected the data from the pressure sensor as the water level in the tank rises from 0 to 574 mm. Due to the linear correlation between the information obtained from the pressure sensor and the real water level in the tank, we need two sets of data that consist of both the sensor readings and the corresponding height of the water in the tank. These two data sets can be conveniently obtained by employing the Simulink simulation 4 presented below: The two data sets obtained are :

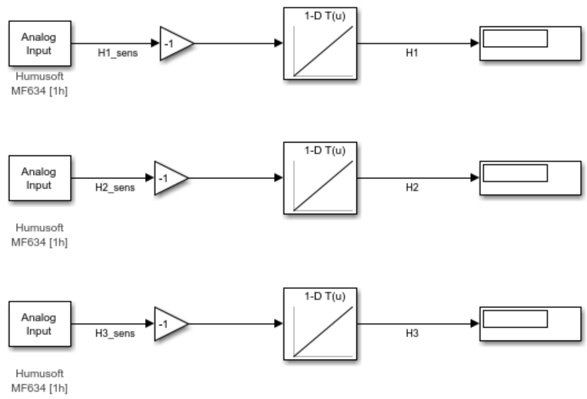


Figure 4: Simulink model for Height Calibration

- The sensor data is collected when there is no water present in the tank.
- then sensor data is collected when the water level of 0.575 m.

Once the data sets were gathered, the subsequent step involved creating the lookup tables. The area within the red box in the figures 31 6 7 indicates the location where the data sets were stored.

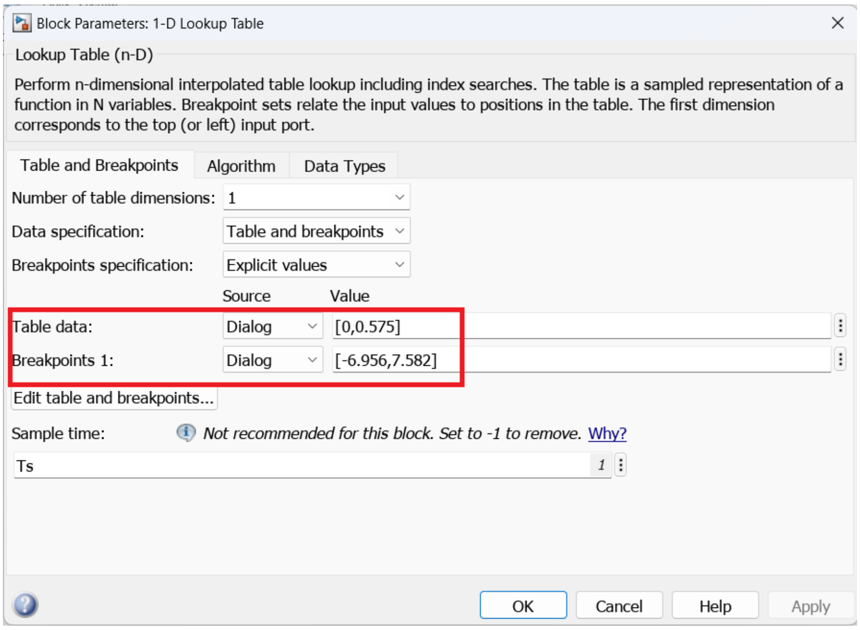


Figure 5: Look-up table for Tank 1

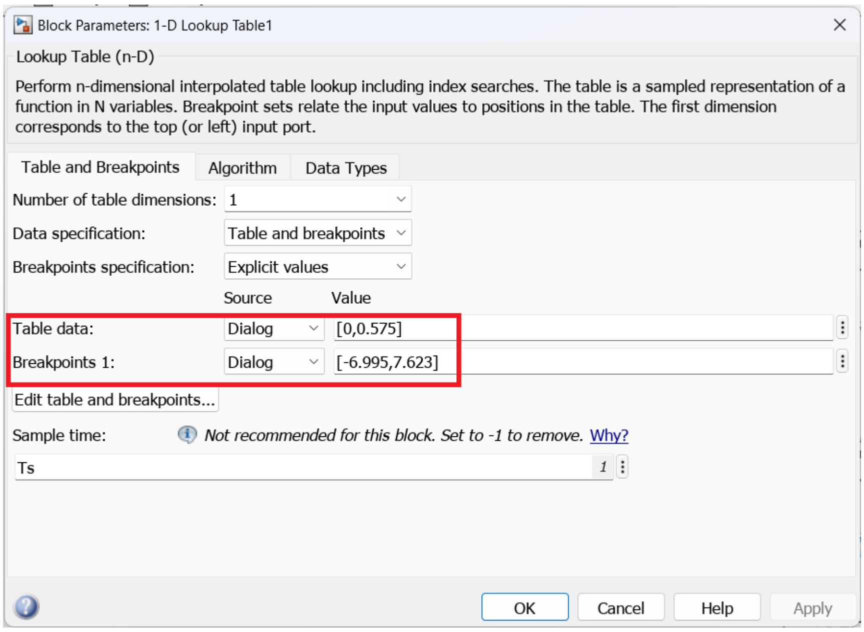


Figure 6: Look-up table for Tank 2

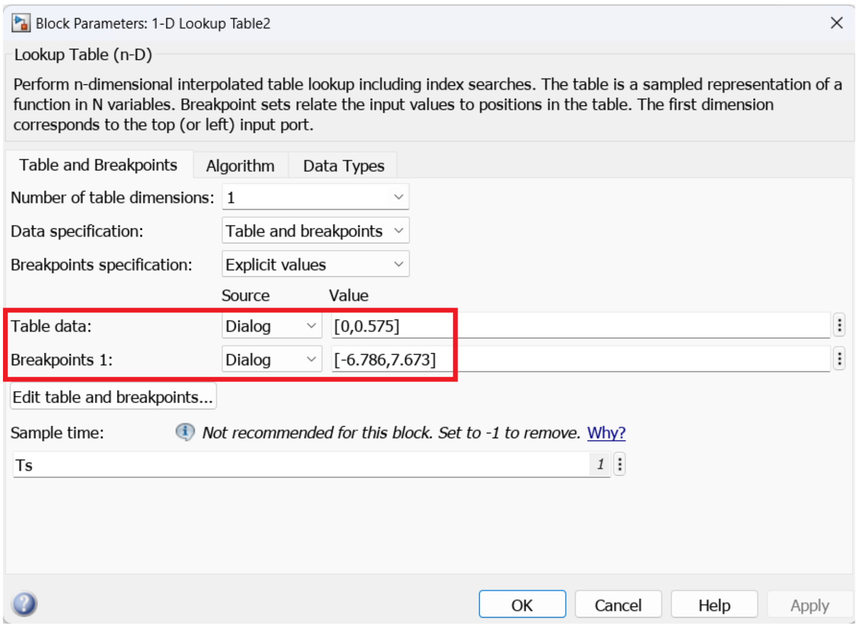


Figure 7: Look-up table for Tank 3

2.4.2 Motor Calibration

During the simulation, the lookup table will come into play, which establishes a linear correlation between the pump voltage and the flow rate. By regulating the voltage, it becomes possible to regulate the flow rate as well. The parameters for the look-up table are calculated using the following details in mind:

- The motor will cease functioning, or in other words, the flow rate will be zero, when a value of -10 V is applied to it.
- The motor will perform at its maximum capacity when a voltage of 10 V is applied.

Afterward, we administer a voltage of 10 volts and chart the variation of water level in tank 1 over time to estimate the motor's flow rate. The Simulink model 8 used to get the data is shown below:

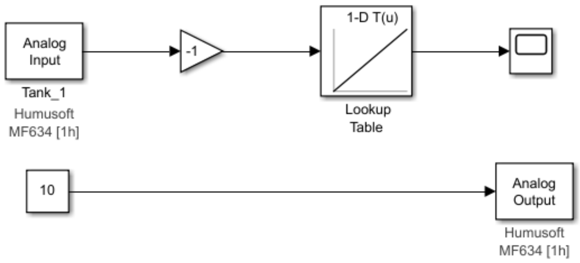


Figure 8: Simulink Model - Motor Rate Calculation

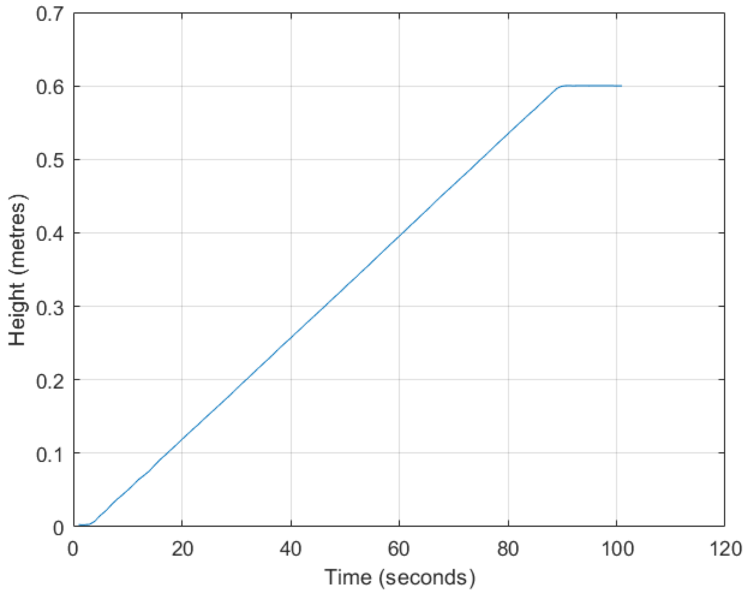


Figure 9: Variation of Height with time

The graph that represents the output is shown in the figure. From the figure, we can calculate the motor rate. The Equation is given by:

$$\text{Motor Rate} \;=\; \frac{\text{change in volume}}{\text{time taken}} \tag{1}$$

$$Q = \frac{An \times \Delta h}{\Delta t} \tag{2}$$

where:

Q = Water flow rate of motor 1.
An = Area of the cylinder subtracting the cross-section area of the motor inlet pipe.
Δh = Change in height during a particular time interval.
Δt = The time it takes for the water level to change in the water column.

On substituting the values we get the maximum motor flow rate as:

$$Q = 1.034 \times 10^{-4} \mathrm{m}^3\,\mathrm{s}^{-1} \tag{3}$$

Afterward, we insert these values into the look-up table, which yields a voltage output matching all the flow rate outputs of the controller we intend to create. The completed look-up table is shown by the figure 10 below:

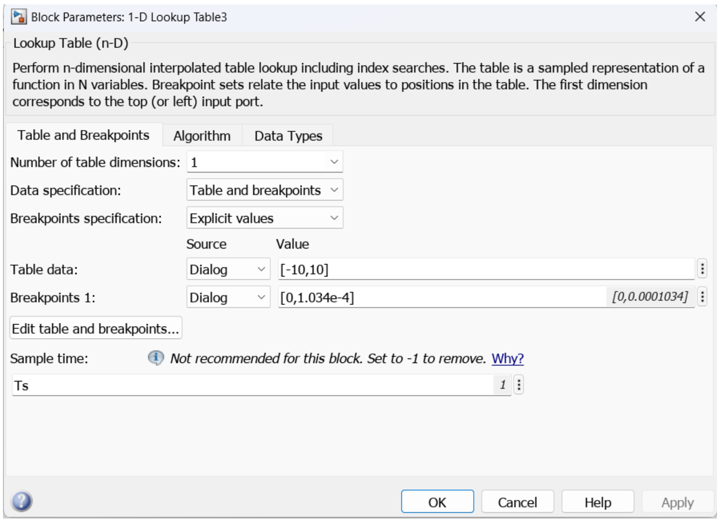


Figure 10: Look-up Table for Motor

3 Steady State Calculaiton

The three-tank system is a system that exhibits nonlinearity. To facilitate its control and analysis, it is imperative to linearize the system in the vicinity of its equilibrium point. To achieve this, the process of obtaining operating points will be elucidated. This involves determining the outflow coefficient, input flow rate of the pump, and the steady-state liquid level height of each tank.

3.1 Calculation of outflow coefficient

First, close all valves, and manually control the pump to adjust all tanks to reach the maximum initial liquid level. As shown in figure 11, The initial liquid levels of the three tanks are:

$$h1=0.5623(m),\ h2=0.5645(m),\ h3=0.5643(m)$$

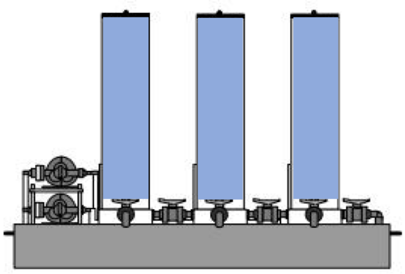


Figure 11: Tank Fully Filled

To begin, open the valves connecting the three tanks and tank 2 to the reservoir. Then, start the timer and allow 60 seconds to pass before measuring and noting down the liquid level height of every tank. After 60 seconds:

h1=0.3630(m), h2=0.1984(m), h3=0.3395(m)

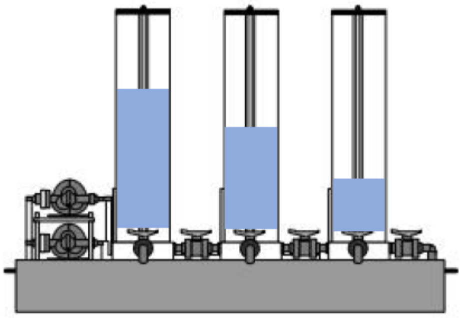


Figure 12: Tank Water level After 60s

After another 15 seconds, record the liquid level height of each tank as follows:

h1=0.3434(m), h2=0.1595(m), h3=0.3149(m)

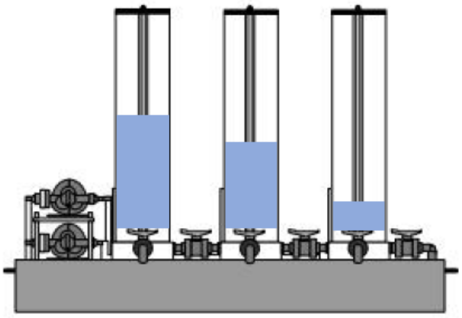


Figure 13: Tank Water Level After 75s

The recorded height measurements are then utilized to determine the change in height over a period of 15 seconds.

$$\Delta h1 = 0.3630 - 0.3434 = 0.0196\text{m} \tag{4}$$

$$\Delta h2 = 0.1984 - 0.1595 = 0.0389\text{m} \tag{5}$$

$$\Delta h3 = 0.3395 - 0.3149 = 0.0246\text{m} \tag{6}$$

The valve outflow coefficient is determined by applying the formulae that use the equations 4, 5, and 6 mentioned above. It is as follows:

$$az_1 = \frac{A\Delta h1}{Sn\Delta t\sqrt{2g(h_1 - h_2)}} \tag{7}$$

$$az_2 = \frac{A(\Delta h1 + \Delta h2 + \Delta h3)}{Sn\Delta t\sqrt{2gh_2)}} \tag{8}$$

$$az_3 = \frac{A(\Delta h1 + \Delta h3)}{Sn\Delta t\sqrt{2g(h_3 - h_2)}} \tag{9}$$

The area of the tank's cross-section with the area of the internal pipe deducted is denoted by 'A' and has a value of 0.0149 m^2.
From these equations, we can calculate outflow coefficient and is given by,

$$\text{az1} = 0.522,\ \text{az2} = 0.934,\ \text{az3} = 0.504$$

3.2 Calculation of steady state height

At a steady state, the water inflow and outflow of the entire system are balanced. Similarly, for each tank, the inflow of water is equivalent to the outflow of water. The level of liquid in the tanks remains constant, with varying heights corresponding to different rates of inflow and outflow of water. Therefore

$$Q_{01} = Q_{13} = Q_{32} = Q_{20} \tag{10}$$

where:

Q_{01} = Water flowing into Tank 1 from motor.
Q_{13} = Water flowing from tank 1 to tank 3.
Q_{32} = Water flowing from tank 3 to tank 2.
Q_{20} = Water flowing from tank 2 to the reservoir.

Using the equations mentioned below, we calculated the flow rate and water heights in other tanks based on a selected height of h02 = 0.0242 m.

$$Q_{20} = az_2 Sn\sqrt{2gh2} \tag{11}$$

$$Q_{13} = az_1 Sn(h1 - h3)\sqrt{2g|h1 - h3|} \tag{12}$$

$$Q_{32} = az_3 Sn(h3 - h2)\sqrt{2g|h3 - h2|} \tag{13}$$

Substituting the given data, we can determine the water inflow of the system and the height of each tank in the steady state, which are:

$$\begin{aligned} Q_{01} &= 3.102 \times 10^{-5}\,\mathrm{m}^3\,\mathrm{s}^{-1} \\ h01 &= 0.1750\mathrm{m} \\ h02 &= 0.0244\mathrm{m} \\ h03 &= 0.0946\mathrm{m} \end{aligned} \tag{14}$$

where,

Q_{01} = Motor flow rate at steady state condition.
$h01$ = Steady state height for tank 1.
$h02$ = Steady state height for tank 2.
$h03$ = Steady state height for tank 3.

Part III

SISO Model

To achieve control over a system, we typically begin by creating a mathematical model of the system, which is a crucial first step. For a single-input single-output (SISO) system, we can use a SISO model, such as a model of a three-tank system, to describe the behavior of the system where liquid flows between three interconnected tanks. In this model, the input is the flow rate of liquid into the first tank, and the output is the height of the liquid level in the tank 2. Once we have a SISO model, we can apply control logic to it to regulate the system. Subsequently, we can move towards modeling more complex multi-input multi-output (MIMO) systems.[2]

4 Mathematical model

A SISO system refers to a system that has only one input and one output. Assuming there is no leakage, the three-tank system can be represented by the following differential equation, based on the principle of material balance:

$$\begin{aligned} A\frac{dh_1}{dt} &= Q_1 - Q_{13} \\ A\frac{dh_2}{dt} &= Q_{32} - Q_{20} \\ A\frac{dh_1}{dt} &= Q_{13} - Q_{32} \end{aligned} \tag{15}$$

where,

$\frac{dh_i}{dt}$ = change in height of the ith$tank$
A = Area of cross section of tank
Q_{ij} = Flow rate from tank i to j
Q_1 = Flow rate of the water entering tank1 from motor

4.1 Toricelli's Law

Torricelli's law, also known as Torricelli's theorem, is a principle in fluid mechanics that relates the flow rate of a fluid through an orifice to the height of the fluid above the orifice. The law states that the velocity of a fluid flowing out of an orifice under gravity is equal to the velocity that an object would attain if it fell from a height equal to the height of the fluid level above the orifice, neglecting air resistance [3]. This can be expressed mathematically as:

$$v = \sqrt{2gh} \tag{16}$$

Where:

v = velocity of the fluid flowing out of the orifice
g = acceleration due to gravity
h = the height of the fluid level above the orifice.

The flow rate of the fluid, can be calculated by multiplying the velocity by the area of the orifice:

$$Q = A * v \tag{17}$$

Where:

Q = Flow rate of the liquid
A = Area of the orifice.

Therefore, Torricelli's law provides a way to estimate the flow rate of a fluid through an orifice based on the height of the fluid level above the orifice.By using this equation we can calculate the values of Flow rates in equation 15:

$$\begin{aligned} Q_{13} &= az_1 * S_n * sgn(h_1 - h_3)(\sqrt{2g|h_1 - h_3|}) \\ Q_{32} &= az_3 * S_n * sgn(h_3 - h_2)(\sqrt{2g|h_3 - h_2|}) \\ Q_{20} &= az_2 * S_n(\sqrt{2g|h_1 - h_3|}) \end{aligned} \tag{18}$$

Then, the differential equation of the three-tank system can be expressed as:

$$\begin{aligned} A\frac{dh_1}{dt} &= Q_1 - az_1 * S_n * sgn(h_1 - h_3)(\sqrt{2g|h_1 - h_3|}) \\ A\frac{dh_2}{dt} &= az_3 * S_n * sgn(h_3 - h_2)(\sqrt{2g|h_3 - h_2|}) - az_2 * S_n(\sqrt{2g|h_1 - h_3|}) \\ A\frac{dh_3}{dt} &= az_1 * S_n * sgn(h_1 - h_3)(\sqrt{2g|h_1 - h_3|}) - az_3 * S_n * sgn(h_3 - h_2)(\sqrt{2g|h_3 - h_2|}) \end{aligned} \tag{19}$$

from the equation 19 we can create a non linear model, which will be the representation of the SISO system. The created simulation model is shown in the appendix figure 78.

```
An = 0.0154-pi*0.0125*0.0125; % m^2 The cross-section of the tank
Sn = 0.00005; % 5*10e-5 m^2 The cross section of the pipes
g = 9.8;

%------------SISO Non-linear system--------------------------------------
az1 = 0.522; % valve outflow coefficient
az2 = 0.934;
az3 = 0.504;
```

5 Steady-State Validation

The steady state height of each tank at a particular flow rate has been calculated through mathematical modeling and is represented by equation 14. In this section, the validity of this calculation will be confirmed using the real test stand. The Simulink model used for this validation is shown in figure 14.

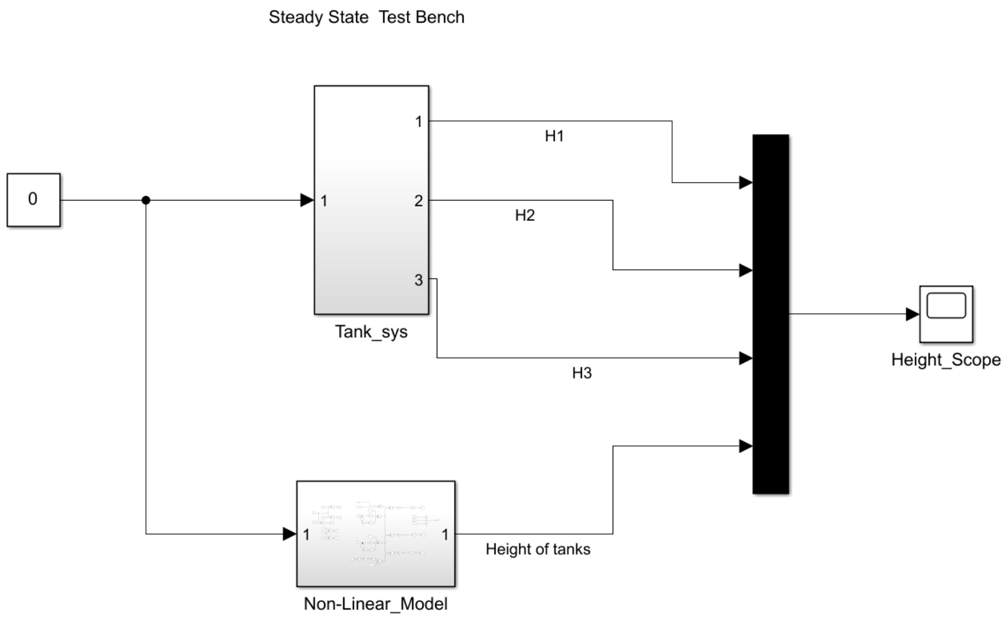


Figure 14: Simulink Model for Steady state Validation

- As the basic flow rate needed to achieve steady state remains constant and does not require alteration, it was incorporated within the models. This can be observed in the appendix figures (78, 77) representing the tank system and non-linear system, where a constant model denoted by Q01 was used.
- In figure 14 above, a constant with a value of 0 was included to represent the anticipated future flow rate values from the control unit.

5.1 Matlab Code for Non-Linear Validation

```
An = 0.0154-pi*0.0125*0.0125; % m^2 The cross section we used of the tank
Sn = 0.00005; % 5*10e-5 m^2The cross section of the pipes
g = 9.8; % acceleration due to gravity

%-------------SISO Non-linear system------------------------------------
az1 = 0.495;  % valve outflow coefficients
az2 = 0.88;
az3 = 0.529;
h02 = 0.0242; % steady state height of tank h2
```

```
h03 = 0.0943; % steady state height of tank h3
h01 = 0.1746; % steady state height of tank h1
Q01= 3.1020e-5; % Steady state flow rate
Ts = 1; % sampling time
```

- The values of outflow coefficients that we obtained mathematically is calibrated or modified to match with the values obtained from testing in the test stand.
- The modified values obtained are shown in the Matlab code above.

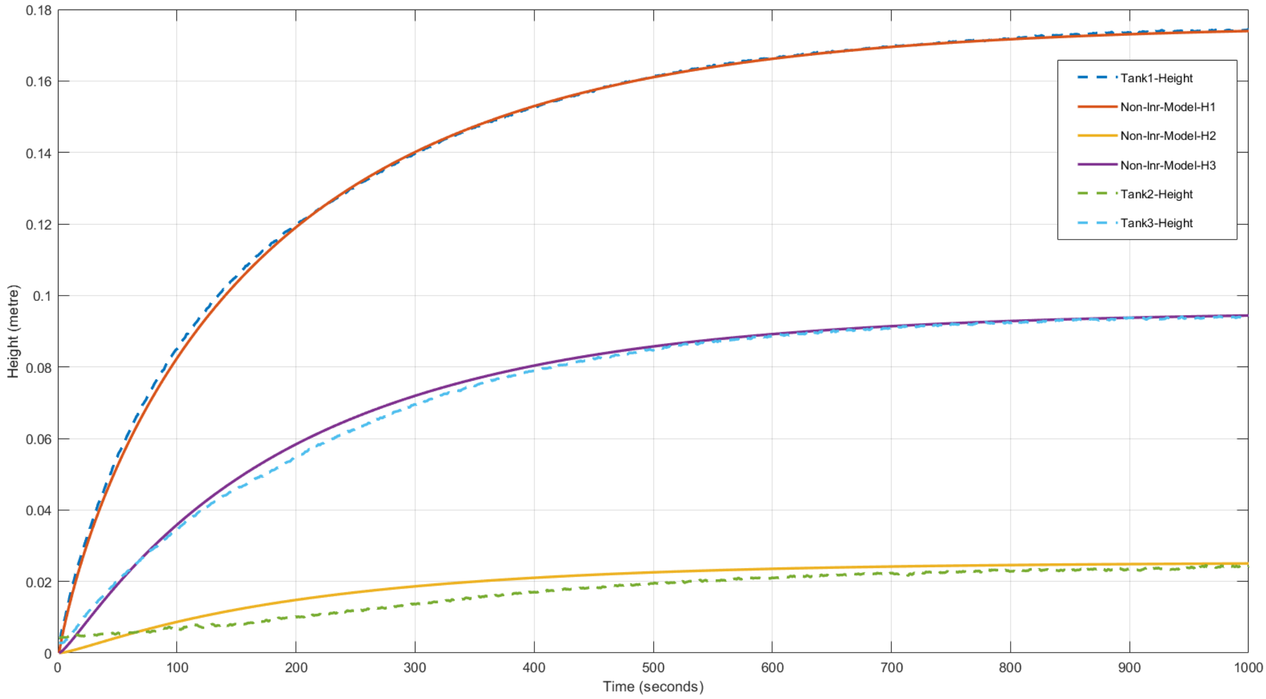


Figure 15: validation output graph for steady state

From the graph 15 above following things can be understood:

- The graph illustrates a comparison between the height levels of three water tanks in both the simulated and real test stands.
- In the graph, the output from the testing stand is represented by dotted lines, while the simulated non-linear system is represented by bold lines.
- The image clearly demonstrates the close resemblance between the system and the simulated system, achieved through the calibration of the valve outflow coefficients.

This steady state point is then used as working point of the SISO System and linearised around these points.

6 Linearization

Linearization in control system engineering is the process of approximating a non-linear system with a linear system in a specific operating region. The linearized model provides a simplified representation of the original non-linear system and is often used in control system analysis and design. It is important to note that the validity of the linearized model depends on the accuracy of the operating point and the degree of non-linearity of the system. Linearization is a common technique used in control system engineering, particularly in the design of feedback controllers, which are used to regulate the behavior of non-linear systems [2].

6.1 State-Space Representation

For the three tank system, where all differential equations have square-root characteristics, linearization near the origin is either not possible or produces poor results. To overcome this, a steady-state representation is required. In classical control theory, a linear steady-state system can be described using ordinary differential equations or transfer functions, with a single output variable directly related to the input. However, the system also contains other independent variables, and the differential equation or transfer function is not suitable for describing the intermediate variables. As a result, these methods cannot fully capture all the information of the system and may not reveal the complete motion state of the system. Therefore, using differential equations or transfer functions to describe a linear steady system has limitations.[4]
The state space method describes the dynamic characteristics of a system using a first-order differential equation system composed of state variables. This method captures changes in all independent variables of the system, allowing for the determination of all internal motion states of the system simultaneously and easy handling of initial conditions. This approach offers a powerful tool for improving system performance during control system design, as it is not limited to input, output, and error quantities.[5]
Generally, the state-space representation is as follows:

$$\begin{aligned} \dot{X} &= Ax + Bu \\ Y &= Cx + Du \end{aligned} \tag{20}$$

where,

A = System matrix
B = Input matrix
C = Output matrix
D = Feedthrough (or feedforward) matrix (direct matrix)

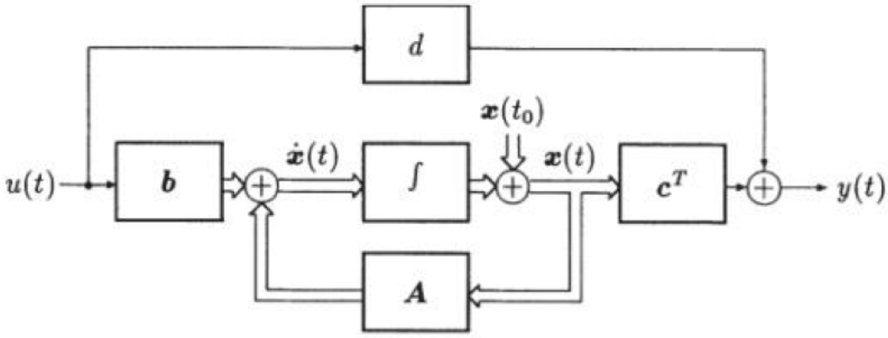


Figure 16: State-Space Representation of SISO system

The linearized state-space model parameters are given by:

$$
\begin{aligned}
A &= \begin{bmatrix} -a_1 & a_1 & 0 \\ a_1 & (-a_1 - a_3) & a_3 \\ 0 & a_3 & (-a_3 - a_2) \end{bmatrix} \\
B &= \begin{bmatrix} \frac{1}{An} \\ 0 \\ 0 \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
D &= 0
\end{aligned}
\tag{21}
$$

where,

$$
\begin{aligned}
a_1 &= \mathrm{az1} \times Sn \times g/(An \times \sqrt{2 \times g \times (h01 - h03)}) \\
a_2 &= \mathrm{az2} \times Sn \times g/(An \times \sqrt{2 \times g \times h02}) \\
a_3 &= \mathrm{az3} \times Sn \times g/(An \times \sqrt{2 \times g \times (h03 - h02)})
\end{aligned}
$$

The figure 17 below shows the simulated model of the linearized system:

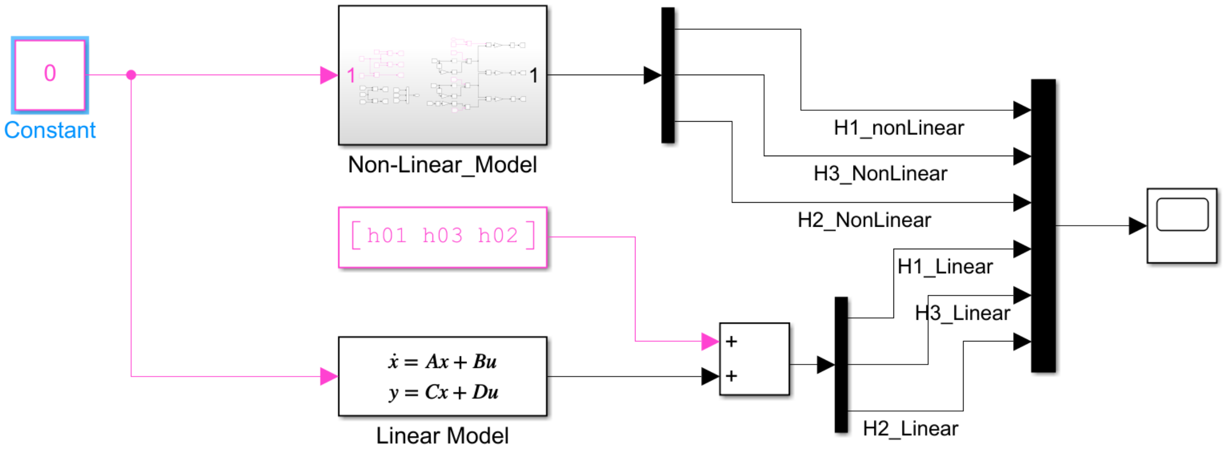


Figure 17: Linearized model

- The Simulink model depicted in Figure 17 is utilized to contrast the linearized and non-linear models.
- The fixed values h01, h02, and h03 denote the steady state points or working points about which the system is linearized.

The matlab code which contains the parameters of the linear model block (System Matrices) are shown below:

```
%---------------------SISO Linear system---------------------------------
a1=az1*Sn*g/(An*sqrt(2*g*(h01-h03)));
a2=az2*Sn*g/(An*sqrt(2*g*h02));
a3=az3*Sn*g/(An*sqrt(2*g*(h03-h02)));
A = [-a1 a1 0;a1 (-a1-a3) a3; 0 a3 (-a3-a2)];
B = [1/An; 0; 0];
C = eye(3);
D = [0;0;0];
```

The result of the comparison is shown in the figure below:

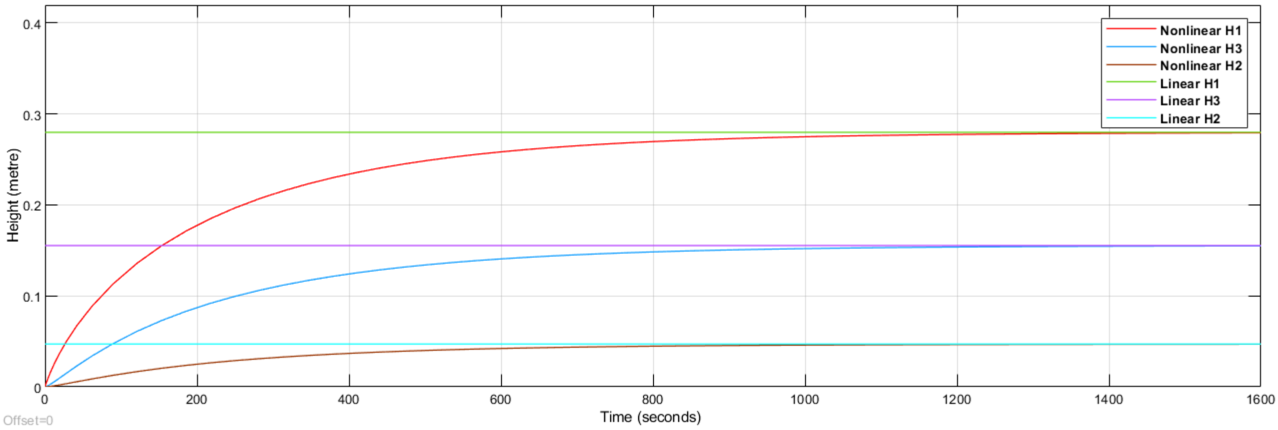


Figure 18: Linear and nonlinear model comparison - steady state

Based on the graph 18, it is evident that even though there is a lag the simulated non linear system attains the steady state point. Evaluating the linear model's performance necessitates altering the height from the steady-state state. This can be achieved by employing a proportional control (p-control) with feedback, and subsequently, we can compare the behavior of the non-linear and linear systems.

7 Sate Feedback

Feedback control is a control system technique that uses information from the output of a process or system to adjust the input to achieve the desired output. In feedback control, the output is measured and compared with the desired or setpoint value, and the resulting error signal is used to adjust the input.

State feedback control is a type of feedback control that uses the state variables of a system to calculate the control input(motor flow rate). State variables are variables that describe the internal state of a system, in this case the heights of the tank.In state feedback control, a mathematical model of the system is used to calculate the control input based on the current state of the system. The control input is designed to drive the system to a desired state or trajectory. State feedback control can provide better control performance than traditional feedback control methods, but it requires a good understanding of the system dynamics and the ability to measure or estimate the state variables.

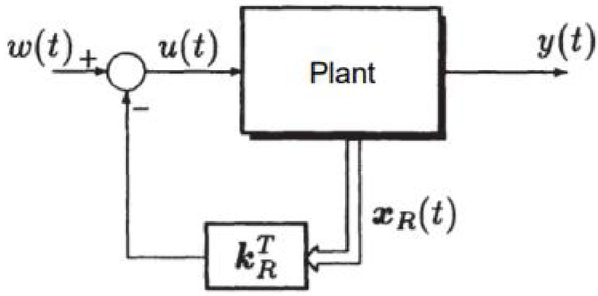


Figure 19: State Feedback Block Diagram

Instead of using the output signal as the feedback, the state-feedback is used here to make the SISO system to be a closed-loop system. The control law for the state feedback is as follows:

$$u(t) = w(t) - k^t x(t) \tag{22}$$

Where,

$w(t)$ = reference input
kT = state-feedback vector

Therefore,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(w(t) - k^t x(t)) \\ &= (A - Bk^T)x(t) + Bw(t) \\ &= A_{cl}x(t) + Bw(t) \end{aligned} \tag{23}$$

Where,

A_{cl} = A - Bk^t

In this chapter a general state feedback control system is designed by using Ackermann's formula. This approach eliminates the need to represent the system in a controllable canonical form.This formula provides a direct solution for calculating the state feedback gain matrix that will place the closed-loop poles of the system at desired locations.
The Ackermann's equation generates a state feedback vector . It is given as :

$$k^T = q^T P_\alpha(A) \tag{24}$$

where,

q^T = last row of the inverse state controllability matrix
$P_\alpha(A)$ = characteristic polynomial

To achieve a steady-state accuracy in control, the reference value undergoes a pre-amplification or pre-filtering by a factor known as P. This factor can be designed as a low-pass filter, hence the name pre-amplifier or pre-filter.The pre-factor P is calculated using state feedback vector and is described as following :

$$P = \frac{1}{\underline{c}^t(\underline{Bk^t} - \underline{A})^{-1}\underline{B}} \tag{25}$$

8 Height Control

We use the principles of state feed back to implement a p-State feedback control on the simulation model of the three tank siso system. With this control we should be able to increase and decrease the height of the water level in tank 2.
The poles of the controller required to calculate the state-feedback vector using Ackermann's formula is:
p = $\begin{bmatrix} -0.06 & -0.1 & -0.01 \end{bmatrix}$
A function is called "acker" in Matlab ,which can be directly used to implement the pole placement by using Ackermann's formula.The Matlab code used is shown below:

```
%--------State Feedback Control-------------------------%
Ct = C(3,:);
p = [-0.06,-0.1,-0.01]; % pole placement
K = acker(A,B,p); % state feedback vector using Ackermann
P = 1/(Ct*(((B*K-A)^-1)*B)); %Calculation of pre-amplifier
%------------------------------------------------------%
```

8.1 Simulation Model and Inference

The Simulation Model is shown in appendix under figure 79.

- We apply a proportional control and state feedback to both the nonlinear and linear systems in our simulation model.
- Once the system reaches a steady-state height, we introduce a height change to both the linear and nonlinear models.
- The refh2 block refers to the desired input height. Through the step signal (see figure 20) we can introduce the change in height at a particular time instance.
- For the test results shown below the height of the tank h2 is increased by 0.02 m when time is 1500 s.
- The simulation model's scope, as depicted in Appendix under Figure 79, illustrates the comparison results of the nonlinear and linear models, along with the desired height input reference signal.

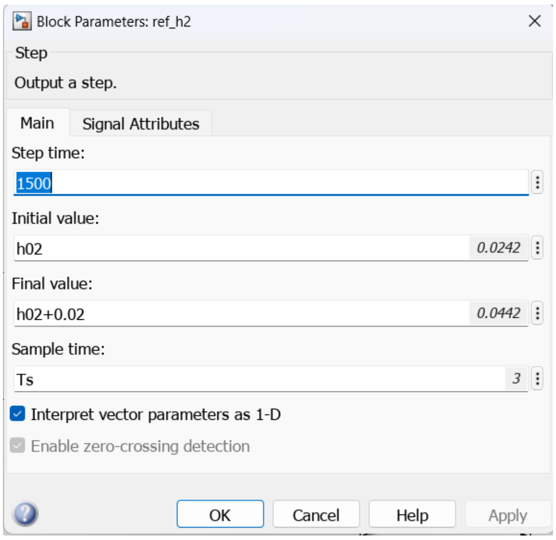


Figure 20: Input Reference height Details

8.1.1 Simulation Results

The graph shows how the water level changes in each tank when a change in height is introduced, while a proportional state feedback control is applied to both the nonlinear and linear models. It also illustrates the relationship between the nonlinear and linear models.

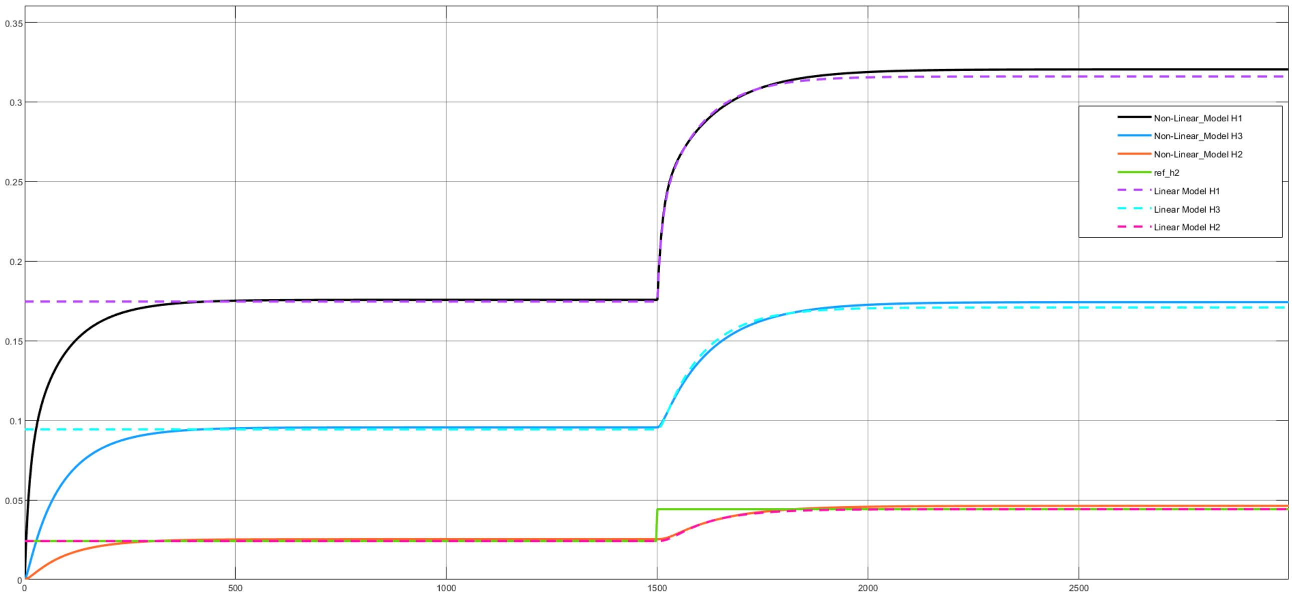


Figure 21: Simulation Results of P-state feedback Control

- In Figure 21, the dashed lines represent the output of the linear system, while the solid lines (except the green line) represent the output of the nonlinear system.

- The green line corresponds to the desired height input reference signal, labeled as "ref_h2".
- The graph in Figure 21 demonstrates that applying feedback control results in quicker attainment of steady state within 500 seconds, compared to the roughly 1000 seconds depicted in Figure 18.
- At 1500 seconds, the required height of the system was increased by 0.02 m. The figure shows that the height of Tank 2 in both the linear and nonlinear system models gradually increases and eventually reaches the desired height.
- As the height of Tank 2 increased, the corresponding heights in Tank 1 and Tank 3 also increased to maintain the desired height in Tank 2.
- Both the linear and nonlinear system models produce similar outputs and reach the desired height of Tank 2. Therefore, this control will be applied to the real test stand to verify its efficacy.

8.2 Test-Stand Validation

The real test stand was subjected to p-state feedback control, and the control application can be observed in the model presented in figure 80 located in the appendix. To confirm the effectiveness of the control, it was compared in real time with a non-linear model on the actual test stand. This validation process was carried out to ensure the reliability of the control system on the real-world test stand.

8.2.1 Inference

During the application of control to the test stand, several observations were made:

- When a sampling time of 0.1 seconds was utilized, it was not possible to obtain a real-time graph representing the height over a long period, even up to 1000 seconds. The graph repeatedly reset around the 100-second mark.
- A proper graphical representation of the control over a period of 3000 seconds was obtained only by increasing the sampling time to 3 seconds.
- The systems's I-O card (Humusoft MF634) can store only 1000 rows of data points, and beyond this limit, the data is reset, and previous data is lost.
- However, increasing the sampling time also resulted in a delay in the control system's response to changes in the system being controlled. This delay caused poor performance of the control system, as there was a lag and difference in height when compared to the non-linear system model.

As a result of the aforementioned issues, it was not possible to obtain accurate testing results for the SISO model.

9 Leakage Handling

Simulating leakages on a tank system enables engineers and designers to comprehend the system's behavior under different conditions and develop more efficient and effective designs. With a better understanding of the leaks, informed decisions can be made on preventing and managing them, thus enhancing the system's safety and reliability.

Before implementing control, simulating the leaks in the system is necessary. This can be achieved by introducing step signal blocks in the non-linear system to produce an additional water flow, subtracted from the original input flow into the tank. These blocks are labeled des_q1, des_q2, and des_q3 on tanks 1, 2, and 3, respectively, as shown in Figure 81 in the appendix.
The value of the step signal for each of the leakage step signal is shown in the figures below:

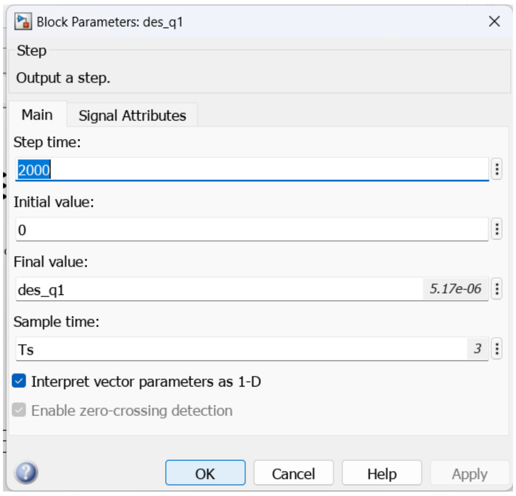


Figure 22: Leakage in tank 1 simulation

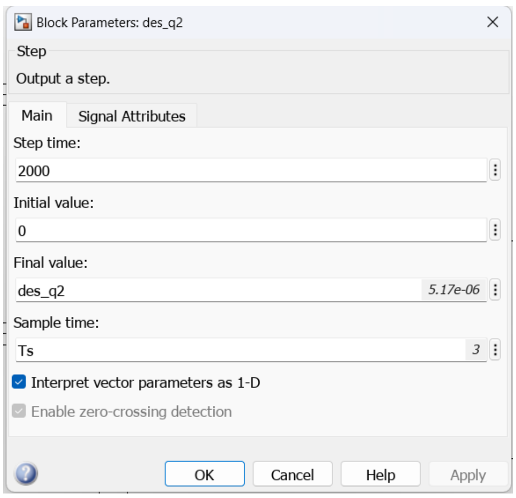


Figure 23: Leakage in tank 2 simulation

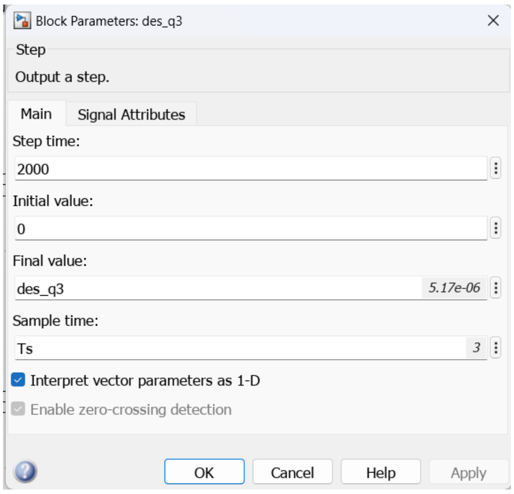


Figure 24: Leakage in tank 3 simulation

To handle leakage in the system or design a robust control that can handle real-world scenarios, such as leakages, PI state feedback control is implemented over the system. This control strategy utilizes a feedback loop to adjust the input to the system based on its output. In the case of a tank system with leakage, the PI state feedback control regulates the fluid level in the tank by adjusting the inflow rate to compensate for the leakage. With PI state feedback control, the tank system can maintain a consistent fluid level despite the presence of leakage, thereby increasing efficiency and reliability while reducing the risk of overflow or underflow. Furthermore, feedback control can provide early warnings of potential leaks, enabling prompt maintenance and repair, and minimizing the impact of the leak on the system.

9.1 PI Controller Design

Before building the simulation in Matlab, the control loop and structure of PI output feedback control needs to be constructed. The figure below gives the structure of a pi control applied to a system. The order of the closed-loop transfer function is equal to the system transfer function n plus the order of the controller. The PI controller of SISO system has an integrator which means the order of the closed loop will increase by 1.

As the structure of the system changed, the state-space representation needs to be

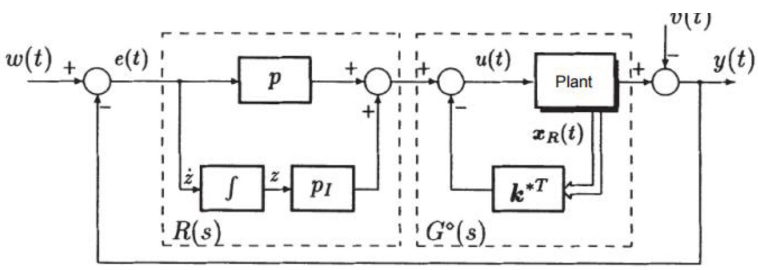


Figure 25: PI Control Applied to a plant

redefined. The state vector of the closed-loop system can be defined:

$$\begin{aligned}\dot{z} &= -y(t) + w(t)\\ y &= c^T x(t)\\ \dot{z} &= -c^t x(t) + w(t)\end{aligned} \tag{26}$$

This leads to the following system description of the augmented system in state-space:

$$\begin{bmatrix}\dot{x}(t)\\ \dot{z}(t)\end{bmatrix} = \begin{bmatrix}\underline{A} & 0\\ -c^T & 0\end{bmatrix}\begin{bmatrix}x(t)\\ z(t)\end{bmatrix} + \begin{bmatrix}b\\ 0\end{bmatrix}u(t) \tag{27}$$

$$y(t) = \begin{bmatrix}c^T & 0\end{bmatrix}\begin{bmatrix}x(t)\\ z(t)\end{bmatrix} \tag{28}$$

9.2 Matlab Code

The code shown below is a continuation of the code used for P state feedback control.

```
A_pi = [A,zeros(3,1);-C(3,:),0]; % Augmented system matrix
B_pi = [B;0];
C_pi = eye(4);
D_pi = [0;0;0;0];
Eig = eig(A_pi);
p_pi = [-0.008,-0.01,-0.05,-0.05];% controller pole placement
k_pi = acker(A_pi,B_pi,p_pi);
pi =  -k_pi(end);
kt_pi = k_pi(1:3) - P*Ct;
P_pi= 1/(C_pi(4,:)*(((B_pi*k_pi-A_pi)^-1)*B_pi));
des_q1 =  Q01/2; % leakage constant for tank 1
des_q2 =  Q01/2; % leakage constant for tank 2
des_q3 =  Q01/2; % leakage constant for tank 3
```

To use Ackermann's formula for pole placement [6] in the augmented system, full controllability of the system with respect to u(t) is a requirement. Placement of poles in the closed-loop transfer function can have negative impacts on step responses, particularly if the placement results in large p values. When using PI state-feedback control, it is essential to place poles near the zero point to keep the p value smaller than that of feedback control.

9.3 Simulink Model

The Simulink model (shown in Appendix Figure 82) features a PI feedback control. As depicted in the figure, the PID block comprises the values for the proportional and integral parts of the controller, while the differential part is maintained at 0. The parameters for the PID block can be found in Figure 26. The variables P_pi and pi are retrieved from the Matlab workspace, which is updated by the code displayed above.

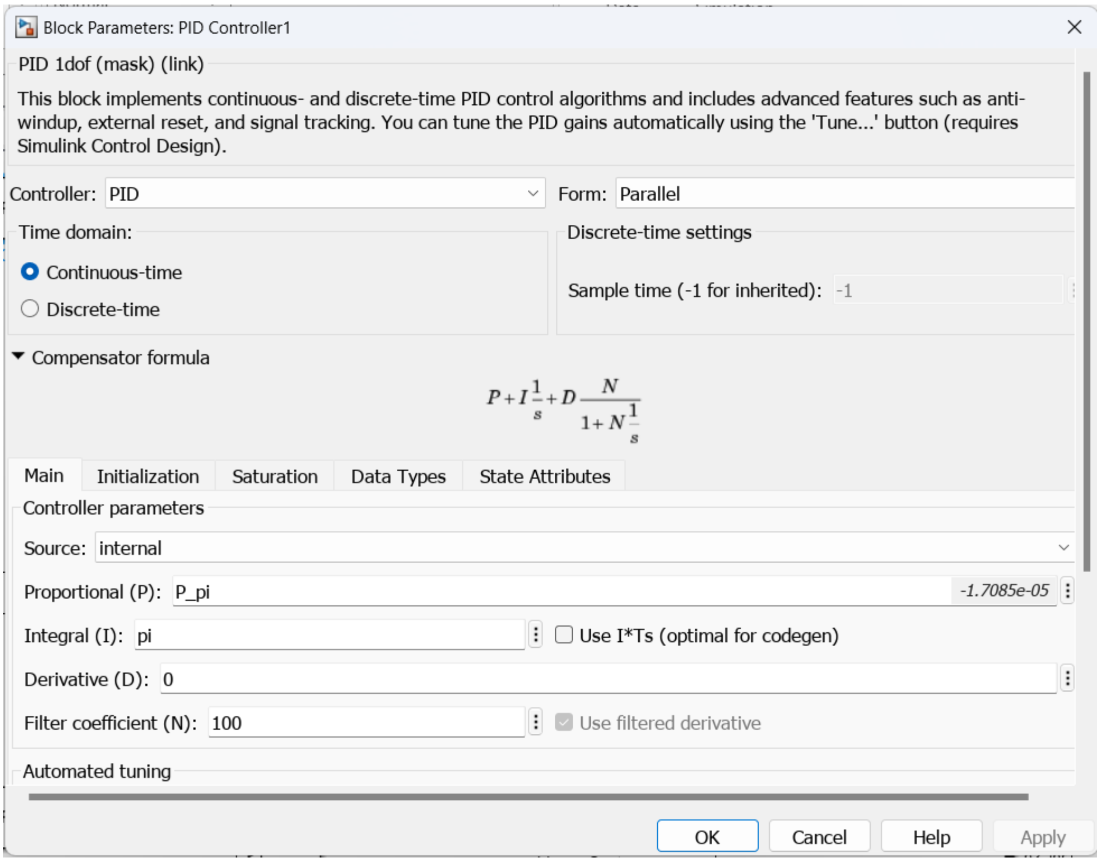


Figure 26: PID parametrs

9.4 Simulation Leakage Test

In the case of a non-linear system, leakage tests can be conducted on each tank individually, as illustrated in Figures 22, 23, and 24. However, for a linear system, testing is performed by introducing a disturbance in the input flow from the motor. The graphs below provide a comparison of the response of the tanks in a non-linear system under leakage conditions versus a linear system under disturbance conditions. The following combinations can be used for conducting leakage tests in non-linear systems:

- When Leakage is applied only on either of the tanks
- When leakage is applied to all of the tanks simultaneously.

9.4.1 Leakage test on Tank H1

The output graph when a leakage of Q01/2 is applied to tank H1 is shown in figure 27below. The leakage was applied by setting the value of des_q1 in Matlab code as Q01/2 and the rest of the leakage step signals as 0.

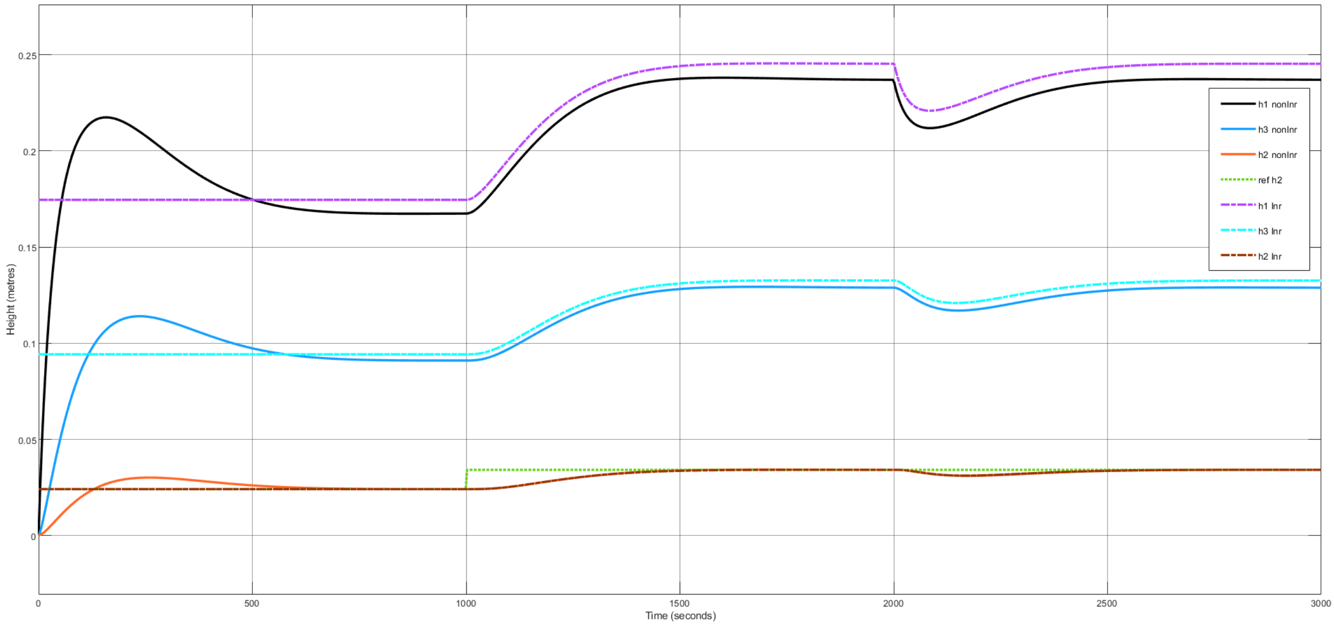


Figure 27: Output Graph when Tank H1 Leaks

- The system is observed to attain a steady state condition around 700 seconds, as illustrated in figure 27.
- A height increment of 0.01 m is introduced at 1000 seconds, and the new height is reached by each tank at approximately 900 seconds.
- At 2000 seconds, a leakage of Q01/2 is introduced in Tank 1.
- As a result of the leakage, the water level in Tank 1 decreases, but the controller compensates for this by increasing the Motor input.
- The decrease in water level in Tank 1 causes a corresponding decrease in the water level of Tanks 2 and 3. However, they eventually return to their original height as the water level in Tank H1 rises back to its pre-leakage height.

9.4.2 Leakage test on Tank H3

The output graph when a leakage of Q01/2 is applied to tank H3 is shown in figure 28below. The leakage was applied by setting the value of des_q3 in Matlab code as Q01/2 and the rest of the leakage step signals as 0.

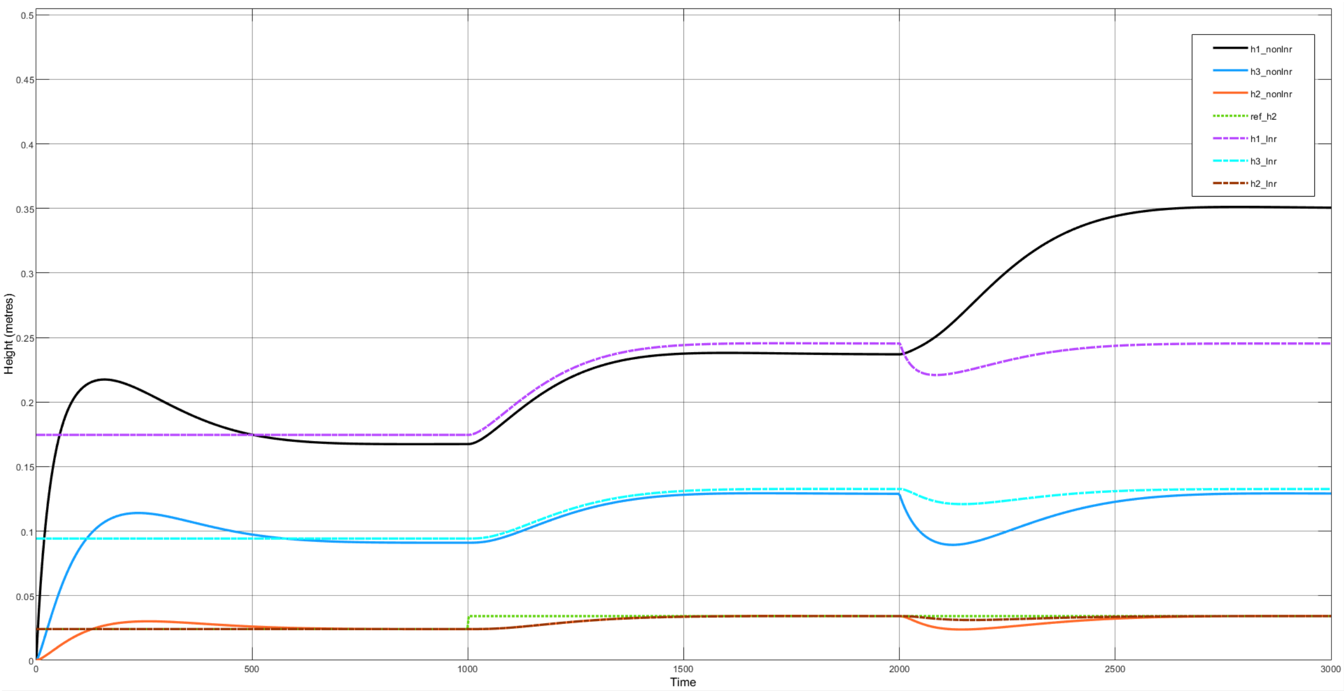


Figure 28: Output Graph when Tank H3 Leaks

- The system is observed to attain a steady state condition around 700 seconds, as illustrated in figure 28.
- A height increment of 0.01 m is introduced at 1000 seconds, and the new height is reached by each tank at approximately 900 seconds.
- At 2000 seconds, a leakage of Q01/2 is introduced in Tank 3.
- As a result of the leakage, the water level in Tank 3 decreases, but the controller compensates for this by increasing the Motor input.
- The decrease in the water level in Tank 3 causes a corresponding decrease in the water level of Tank 2 and an increase in the water level in Tank 1.
- The water level in Tank 1 increases to supply the additional pressure required to compensate the leakage flow in tank 3.

9.4.3 Leakage test on Tank H2

The output graph when a leakage of Q01/2 is applied to tank H2 is shown in figure 29below. The leakage was applied by setting the value of des_q2 in Matlab code as Q01/2 and the rest of the leakage step signals as 0.

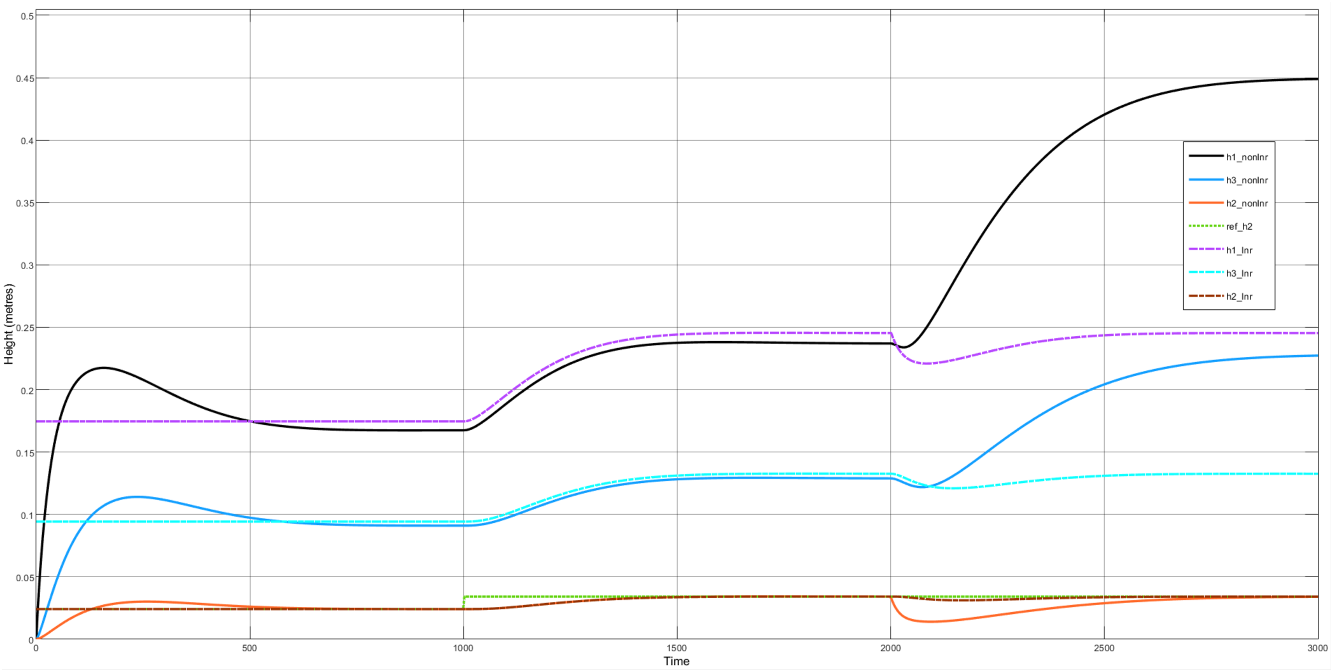


Figure 29: Output Graph when Tank H2 Leaks

- The system is observed to attain a steady state condition around 700 seconds, as illustrated in figure 29.
- A height increment of 0.01 m is introduced at 1000 seconds, and the new height is reached by each tank at approximately 900 seconds.
- At 2000 seconds, a leakage of Q01/2 is introduced in Tank 3.
- As a result of the leakage, the water level in Tank 2 decreases, but the controller compensates for this by increasing the Motor input.
- The decrease in the water level in Tank 2 causes a corresponding increase in the water level in Tank 1 and Tank 3.
- The water level in Tank 1 and Tank 3 increases to supply the additional pressure required to compensate the leakage flow in tank 2.

9.4.4 Leakage test on All Tanks Simultaneously

The output graph when a leakage of Q01/6 is applied to all the Tanks simultaneously is shown in figure 30below. The leakage was applied by setting the value of des_q1, des_q2 and des_q3 in Matlab code as Q01/6.

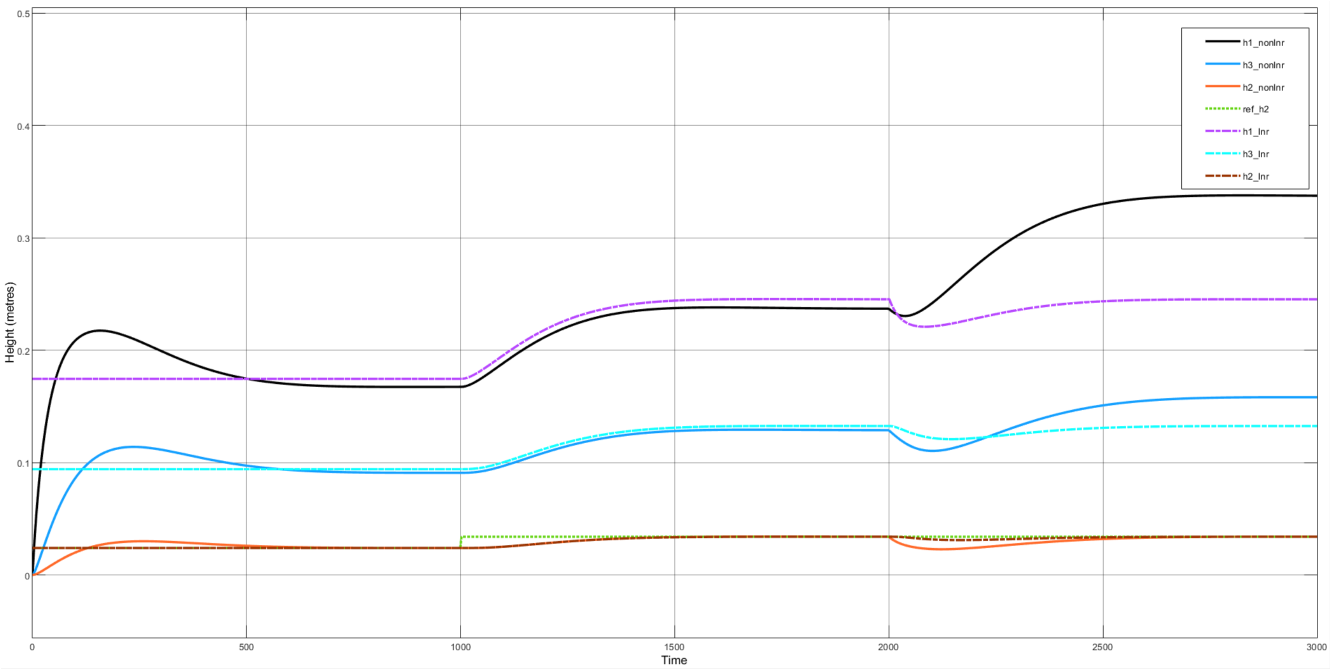


Figure 30: Output Graph when all Tanks Leak

- The system is observed to attain a steady state condition around 700 seconds, as illustrated in figure 29.
- A height increment of 0.01 m is introduced at 1000 seconds, and the new height is reached by each tank at approximately 900 seconds.
- At 2000 seconds, a leakage of Q01/2 is introduced in Tank 3.
- As a result of the leakage, the water level in Tank 1,2 and 3 decreases, but the controller compensates for this by increasing the Motor input.
- To compensate for the pressure loss due to the leakages in Tanks 2 and 3, the water level in Tank 1 increases.
- In order to compensate for the flow of leakage in Tank 2, the water level in Tank 3 rises to provide the necessary additional pressure.

SISO systems can be analyzed and designed using various mathematical tools and techniques, such as transfer functions, block diagrams, and feedback control. They offer several advantages, including simplicity, ease of implementation, and better performance in certain applications. However, SISO systems may not be suitable for more complex control problems that require multiple inputs and outputs, in which case multi-input multi-output (MIMO) systems would be more appropriate. The next section deals with the MIMO systems and the control applied to them.

Part IV

MIMO System

10 Establishment Of MIMO System

Process control in modern industries revolves around several key variables, including the crucial element of liquid level. Ensuring accurate management of this factor is essential to maintain the desired product quality and maximize financial gains. One approach used is manipulating input or output flow rates through designed systems that regulate specific ranges.

The effective management of liquid levels in industrial applications often depends on sophisticated control mechanisms like the three-tank setup comprising interconnected tanks regulated via flow valves that modulate inputs/outputs for desired level adjustments. Yet such arrangements pose considerable difficulties due to their strong coupling and nonlinear nature that complicate tank interaction dynamics making them multi-input multi-output (MIMO) schemes where any adjustment made on one side impacts all facets of a given scenario simultaneously.

The three-tank system's strong coupling and nonlinearity make it an ideal system for studying and developing liquid-level control strategies. To manage the system's complexity and achieve accurate control, researchers have developed various control methods such as proportional-integral-derivative (PID) control, fuzzy logic control, and neural network control.

Improving the three-tank system's control accuracy can improve product quality and increase economic benefits by reducing waste, increasing throughput, and minimizing downtime. As a result, the investigation of liquid level control in three-tank systems has significant research value and practical applications in industrial processes.

Real industrial processes often involve multiple input variables and multiple output variables. This means that any change or disturbance in one loop can have an impact on the other loops through the interactions between the different inputs and outputs. These interactions are known as cross-couplings and are one of the most important features of a MIMO system.

In contrast to SISO (single-input, single-output) systems, where the control engineer can design each loop independently, in a MIMO system, the engineer must consider the interactions between the loops when designing the control system. The engineer must take into account the cross-couplings between the different inputs and outputs and ensure that the control system is designed to handle these interactions.

Designing a control system for a MIMO system can be more difficult than designing one for a SISO system. The engineer must consider the interactions between the loops and ensure that the system remains stable even when disturbances occur. To handle the interactions between the different inputs and outputs, the engineer may need to use advanced control techniques such as model predictive control, adaptive control, or multivariable control.

Overall, cross-couplings between loops are a critical feature of MIMO systems that must be carefully considered by control engineers when designing control systems for industrial processes.

As a result, high-performance requirements in MIMO control are known to be much more difficult than in SISO control. The cross-coupling and loop interactions among the variables cause this complication. The requirements for high performance in a MIMO control system are known to be much more difficult than in SISO control due to the cross-coupling and loop interactions among the various variables. The presence of multiple time delays and inverse responses can complicate and limit the performance of an MI-MO system.

Several control techniques have been designed to cancel the interactions between the various inputs and outputs in order to achieve effective control of an MI-MO system. Among these methods are:

Decoupling: This technique involves designing the control system to eliminate or reduce the cross-coupling between the different loops. This can be achieved through the use of decoupling matrices or decoupling algorithms.

Model Predictive Control (MPC): MPC is a control technique that uses a mathematical model of the system to predict future behavior and optimize the control inputs. MPC can handle the interactions between the different inputs and outputs and is particularly useful for systems with multiple time delays.

Adaptive Control: Adaptive control is a technique that adjusts the control inputs in real-time based on changes in the system. This technique can handle the inverse responses and non-linearities that are often present in MIMO systems.

Multi variable Control: Multivariable control is a technique that considers the interactions between the different inputs and outputs when designing the control system. This technique can handle the cross-coupling and loop interactions and optimize the control inputs for the entire system.

Understanding the interactions between the various inputs and outputs and designing a control system that can handle these interactions is critical to effective MI-MO control. Engineers can effectively control complex MI-MO systems and achieve high performance by employing decoupling, MPC, adaptive control, and multivariable control. The basic requirements that needed to be accomplished were:

- The mixing was to be carried out between Tank 1 and Tank 3, and the final product would be present in Tank 2, which would be drained.
- In order to do this, the mixing ratio was to be fixed, which was fixed to 1:0.5 between Tank 1 and Tank 3.

Inlets and Outlets List		
Tanks	Inlets	Outlets
Tank1	1	1
Tank2	2	1
Tank3	1	1

Table 7: Inlets and Outlets List

- The ratio and level of all tanks had to be maintained at all times.

The schematic of the process is shown in Fig 1. and Table 1. shows the inlets and outlets as adjusted.

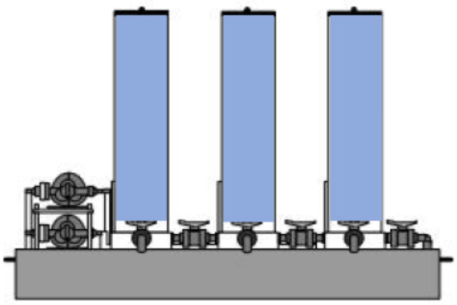


Figure 31: Look-up table for Tank 1

11 MIMO nonlinear mathematical model

The three-tank system you mentioned is a complicated system with numerous inputs and outputs. The system is based on Bernoulli's law of liquids, which states that as a fluid's velocity increases, its pressure decreases.

Three tanks, two pumps, and three valves make up the system. At the bottom of each tank is a flow valve, and Tank 1 has one inlet and one outlet. Tank number two has two inlets and one outlet. Tank 3 has a single inlet and outlet.

The complexity of the system arises from the multiple inputs and outputs, which result in strong interactions between the tanks. Any change in one tank's input or output will affect the inputs and outputs of the other tanks.

For example, increasing the flow rate of the inlet to Tank 1 raises the level of water in Tank 1, which affects the flow rates of the other tanks. This interdependence of inputs and outputs makes the system difficult to control and optimize.

To manage the system's interactions, it is necessary to think of the system as a Multi-input Multi-output (MI-MO) process. In other words, the system has multiple inputs and multiple outputs that interact with one another. Understanding the system's MI-MO nature can aid in the development of control strategies that take into account the system's interdependencies and optimize its performance

The liquid entering the reservoir from the tanks is pumped back into the tanks via the pumps, making it a closed system. Because the liquid that enters the reservoir from the tanks is pumped back into the tanks via the pumps, the system described here is a closed system. This means the system has no external input or output of liquid.

These pumps, on the other hand, will be turned off automatically. To prevent overfilling of tanks T1 and T2, the pumps are set to turn off automatically when the liquid level in these tanks exceeds a predetermined limit. This keeps the system from overflowing and causing damage or spills.

When the liquid in T1 or T2 exceeds a certain limit. The system contains five additional valves in addition to T2's outflow valve. Two of them are used to connect every two consecutive tanks (one for the T1-T3 connection and another for the T3-T2 connection). These valves can be manually adjusted to close the connection between two consecutive tanks, which is useful for isolating one tank from the rest of the system if necessary.

The other three valves are located at the bottom of each tank and are known as leak valves. Manually draining a tank is possible with these valves. This is useful for maintenance purposes or in the event of a leak or spill. Overall, the system is designed to be flexible and versatile, with several control mechanisms that allow for precise control over the flow of liquid and the ability to isolate and drain individual tanks if necessary.

From 1 In a three-tank system, the liquid levels in Tank 1 and Tank 2 can be controlled independently by using the inlet flows Q1 and Q2 through the inlet pumps. This means that the inlet flow rates in each tank can be adjusted to maintain the desired liquid levels.

The liquid level in the middle tank, on the other hand, may not be directly affected by the inlet flows and thus may be more difficult to control. This is due to the fact that the liquid level in the middle tank is affected by the liquid levels in tanks 1 and 2, as well as any other external disturbances to the system.

To control the liquid level in the middle tank, additional control strategies, such as feedback control or model predictive control, may be required. Feedback control entails continuously measuring the liquid level in the middle tank and adjusting the inlet flow rates to maintain the desired level. Model predictive control entails using a mathematical model of the system to predict future behavior and optimize control inputs.

The pumps are controlled by analog signals, which means that the control signals for the pumps range from -10V to +10V. Pressure sensors are used to measure the height of the water level.

Two digital signals operate the system's valves. The first signal is used to begin closing the valve, while the second signal is used to open the valve. If none of the signals are activated, the valve will remain in its current position. Each valve also has three output signals. The first is an analog voltage signal corresponding to the current position of the valve. The other two are informative logical signals that indicate whether the valve is fully opened or fully closed.

Based on the water level measurements from the pressure sensors, this system is designed to control the flow of water by adjusting the pump speed and opening and closing valves as needed. Water flow can be controlled precisely and efficiently by using analog signals for pump control and digital signals for valve control. The valve output signals can also be used for monitoring and troubleshooting because they provide useful information about the current state of the system.

As, controlling a system with three tanks can be difficult, especially if the liquid levels are not directly related to the inlet flows. However, with the proper control strategies in place, it is possible to maintain the desired liquid levels in each of the tanks while also achieving stable and efficient system operation.

In a global system, where the pump flow rates are the input signals and the levels of T1 and T2 are the output signals, all of them can be used for control. This means that the pump flow rates can be changed to maintain the desired levels in T1 and T2.

It is important to note, however, that better controllability can be obtained when the number of outputs does not exceed the number of control inputs. Each output in this case can be effectively controlled by a corresponding input signal. When the number of outputs exceeds the number of control inputs, achieving effective control can be more difficult.

For example, in a system with two control inputs (pump flow rates) and three outputs (liquid levels in T1, T2, and the middle tank T3), it may be more difficult to achieve effective control of all three outputs at the same time. In this case, it may be necessary to prioritize the control of certain outputs over others or to employ more advanced control strategies such as model predictive control or multivariable control.

The overall number of inputs to the model:

- 2 analog signals controlling the pumps,
- 12 digital signals (2 for each of the 6 valves) for
  The plant provides 21 measurable outputs which can be used as a control feedback or for measurements of plant characteristics:
- 3 analog signals representing level heights in the three tanks, opening/closing of the valves.
- 6 analog signals representing the position of the valves,

The material balance principle states that for any system, the total amount of material that enters the system must equal the total amount of material that leaves the system, plus any accumulation of material within the system. However, the system has two inputs and one output; the input quantities are Q1 and Q2, and the output quantity is Q20.

According to the material balance principle, without taking into account leakage. The three-capacity water tank system can be expressed by the following differential equation

$$A\frac{dh_1}{dt} = Q_1 - Q_{13} \tag{29}$$

$$A\frac{dh_2}{dt} = Q_2 + Q_{32} - Q_{20} \tag{30}$$

$$A\frac{dh_3}{dt} = Q_{13} - Q_{32} \tag{31}$$

where h1, h2, and h3 represent the liquid levels in each tank, The cross-sectional areas of all three tanks are the same and are symbolized by A. Q1, and Q2 denote the flow rates of pumps 1 and 2, and The plus sign represents the inflow of liquid to the tank while minus sign stands forwhere h1, h2, and h3 represent the liquid levels in each tank The cross-sectional areas of all three tanks are the same and are represented by A. Q1 and Q2 denote the flow rates of pumps 1 and 2, respectively. The plus sign represents the inflow of liquid into the tank, while the minus sign represents the outflow of liquid from the tank. These three balance equations state that the change in volume in each tank is equal to the sum of the flow rates that enter and exit the tank. However, flow Q13, Q32, and Q20 remain unknown in equations (29), (30), and (31). Torricelli's Law is applied to obtain them: Law is used:

Q13, Q23, and Q20 are unknown and can be represented by the Torricelli rule:

$$Q_{ij} = \; az_i S_n sgn(h_i - h_j)(2g|h_i - h_j|)^{1/2} \tag{32}$$

where az is the outflow coefficient, sgn is the sign of the argument z, and g is the gravitation constant.

Because the flow through a valve depends only on the level difference, the valve position, and constants representing pipes and cylindrical tanks, the resulting equations to calculate the partial flows of the entire model can be written as follows: Then, Q13, Q32, Q20 can be expressed as:

$$Q_{13} = \; az_1 S_n sgn(h_1 - h_3)(2g|h_1 - h_3|)^{1/2} \tag{33}$$

$$Q_{32} = \; az_3 S_n sgn(h_3 - h_2)(2g|h_3 - h_2|)^{1/2} \tag{34}$$

$$Q_{20} = \; az_2 S_n (2gh_2)^{1/2} \tag{35}$$

Then, the differential equation of the three-tank system can be expressed as:

$$A\frac{dh_1}{dt} = Q_1 - az_1 S_n sgn(h_1 - h_3)(2g|h_1 - h_3|)^{1/2} \tag{36}$$

$$A\frac{dh_2}{dt} = Q_2 + az_3 S_n sgn(h_3 - h_2)(2g|h_3 - h_2|)^{1/2} - az_2 S_n (2gh_2)^{1/2} \tag{37}$$

$$A\frac{dh_3}{dt} = az_1 S_n sgn(h_1 - h_3)(2g|h_1 - h_3|)^{1/2} - az_3 S_n sgn(h_3 - h_2)(2g|h_3 - h_2|)^{1/2} \tag{38}$$

The above formula is the MI-MO system's nonlinear mathematical equation. The nonlinear model of the SI-SO system is established in MATLAB using the formula. Nonlinear models are mathematical models that describe systems in which the relationship between the input and output variables is not linear.

The parameters An, Sn, and the gravitational acceleration g are set in MATLAB to create the model. These parameters describe the physical properties of the system, such as the cross-sectional area of the pipes and the acceleration due to gravity. The flow coefficient is assumed to have random values, which simplifies the model. The valves of the pipe connections, pumps, and leaks are also modeled using standard relationships. These relationships describe how the various system components influence the flow rate of the fluid. For example, the flow rate through a valve can be modeled as a function of the valve opening, whereas the flow rate through a pump can be modeled as a function of the pump speed.

The system model is depicted. This appendix includes a diagram or schematic of the MI-MO system that depicts the various components and their connections. The model itself is most likely written in MATLAB code, which is used to simulate the system's behavior under various conditions.

Matlab code

```
%------------MIMO Non-linear system---------------------
ts=1;
An = 0.0154-pi*0.0125*0.0125; % m^2 The cross-section we used of the tank
Sn = 0.00005; % 5*10e-5 m^2The cross-section of the pipes
g = 9.8;      % gravity
az1 = 0.54;
az2 = 0.928;
az3 = 0.5432;
```

12 Validate steady-state working point on MIMO system

12.1 Flow control

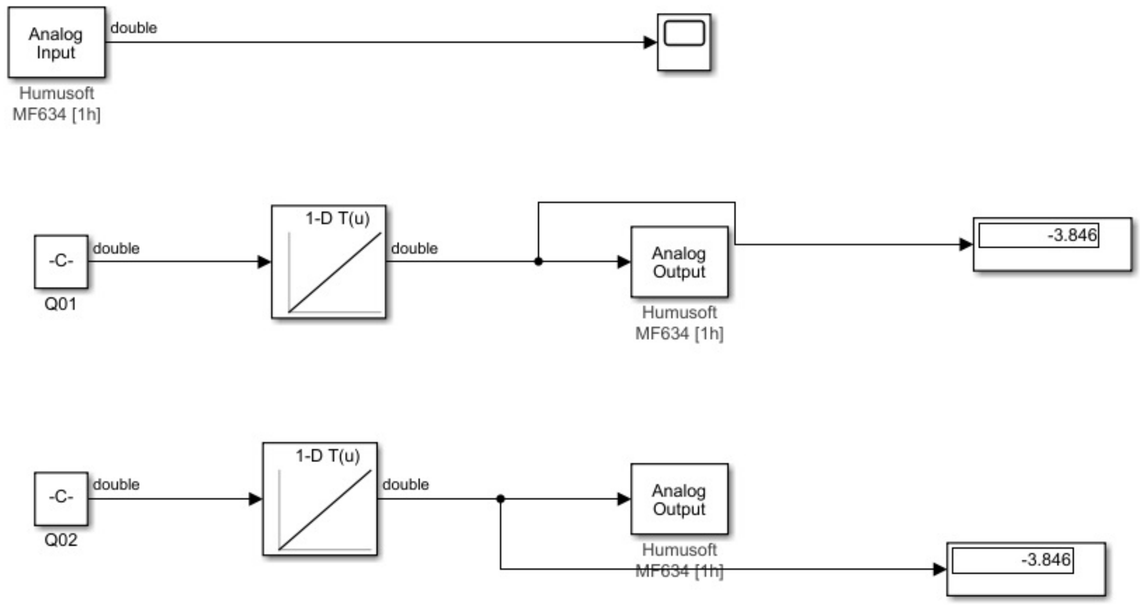


Figure 32: Flow control

The final equilibrium height of tank 2, denoted as h02, has been set to 0.102m. This means that when the system reaches equilibrium, the water level in tank 2 will settle at this level.

In addition, the flow coefficients of the three water tanks are all treated as random values. The flow coefficient is a measurement of the resistance of a fluid to flow through a system. In this case, assuming that the flow coefficients of all three tanks are different, the resistance to flow varies for each tank.

The equilibrium heights of the three water tanks are h01=0.272m, h03=0.188m, and h02=0.102m under these conditions. This means that the water level in Tank 1 will be 0.272m, the water level in Tank 3 will be 0.188m, and the water level in Tank 2 will be 0.102m.

Furthermore, the flow rates of pumps 1 and 2 should be $3.3809 * 10 - 5m3/s$ and $3.3385 * 10 - 5m3/s$, respectively, to achieve this equilibrium height. This means that the pump should be putting water into the system at this rate in order to keep the water levels at the specified levels.

Result, the model describes the behavior of a system model with three water tanks and two pumps, and how the system can reach a stable equilibrium state with specified water levels in the tanks and a specific flow rate from the pump.

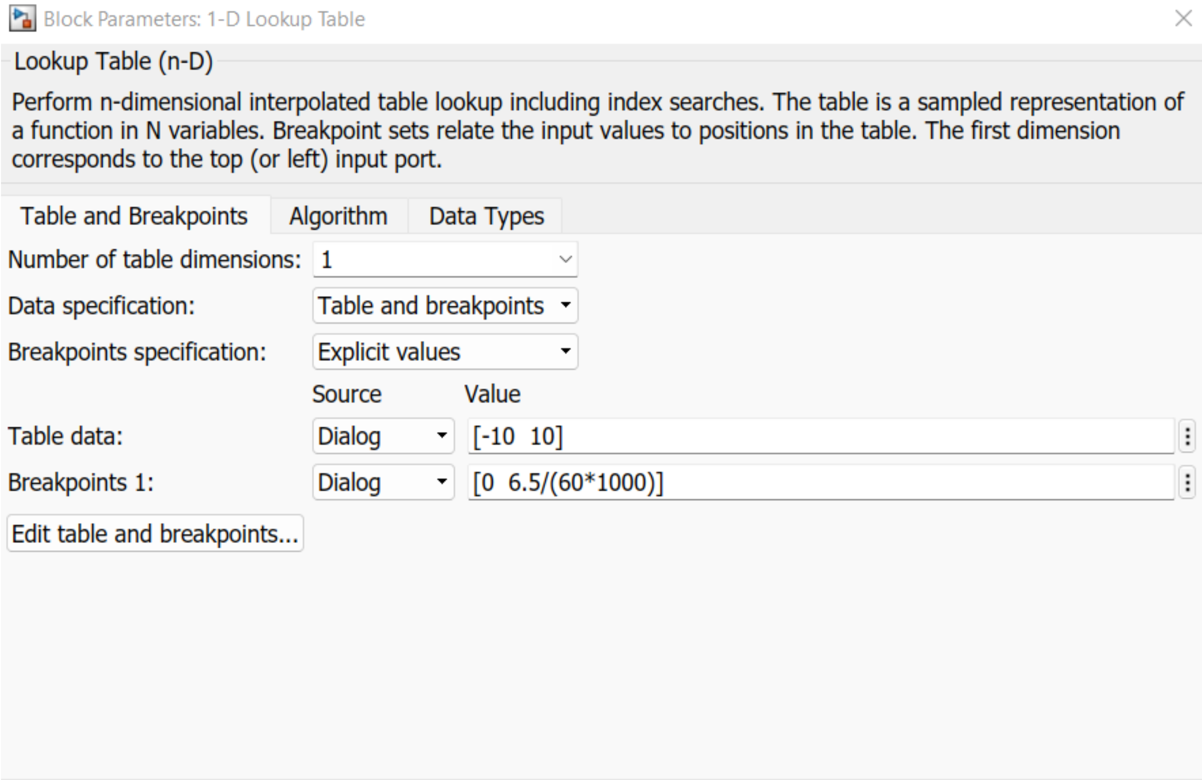


Figure 33: Lookup table flowrate inside

A lookup table that maps the voltage of the pump to the corresponding flow rate. In this context, a lookup table is a data structure that contains a set of values that can be searched and retrieved based on the input value. Because the pump's voltage has a linear relationship with the flow rate, the flow rate corresponding to the pump voltage can be filled in the position in the lookup table to output the flow rate.

The pump's voltage is proportional to its flow rate. This means that as the voltage of the pump increases or decreases, the flow rate of the pump will also increase or decrease linearly.

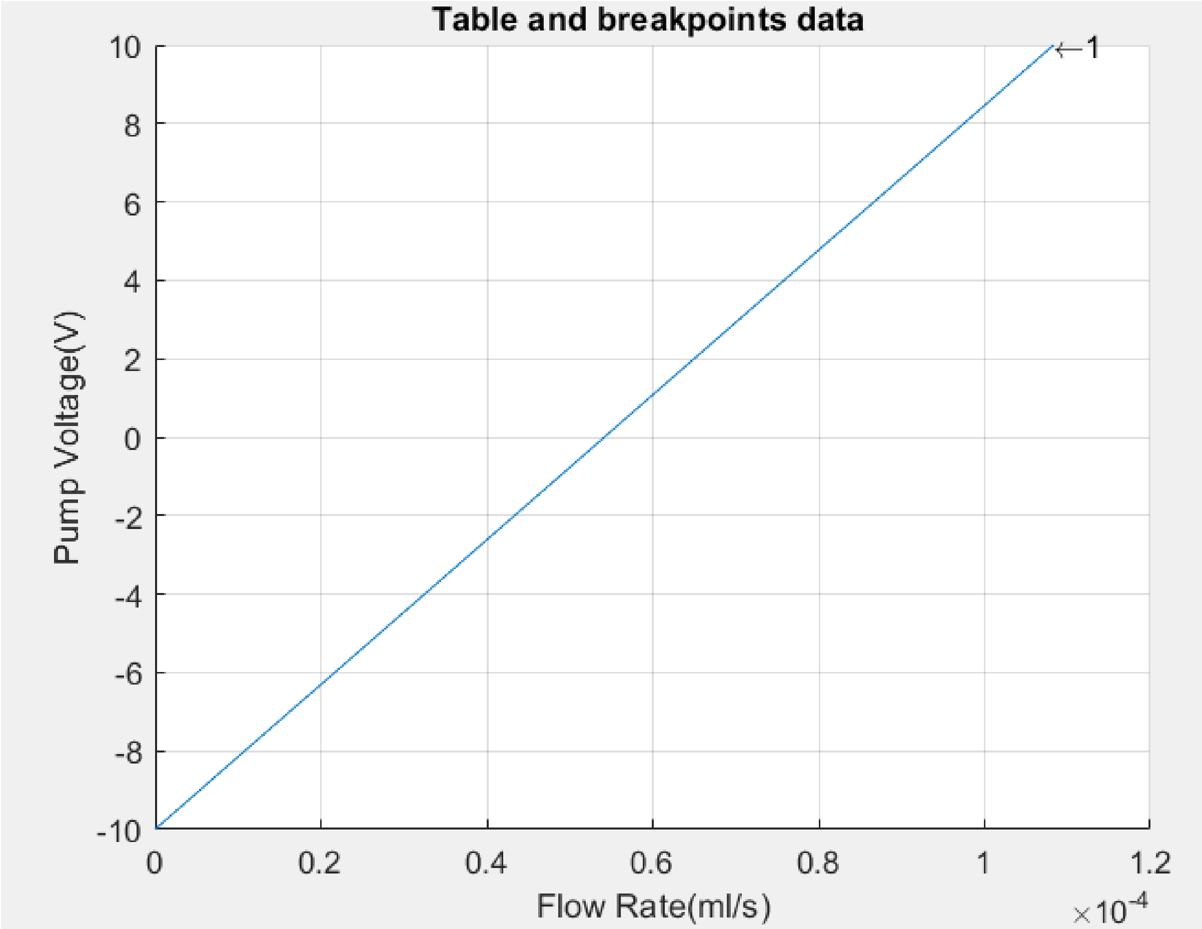


Figure 34: Flowrate graph

It implies that by adjusting the voltage of the pump, the flow rate of the fluid can be changed and that this change can be used to achieve a specific goal or purpose.

This means that if the flow rate of a fluid needs to be increased or decreased, the voltage supplied to the pump can be adjusted accordingly. For example, if the flow rate needs to be reduced, the voltage supplied to the pump can be reduced, which reduces the flow rate of the fluid. If the flow rate needs to be increased, the voltage supplied to the pump can be increased, which will increase the flow rate of the fluid.
In this way, the flow rate can be controlled by controlling the voltage of the pump in the subsequent experiment, the flow can also be controlled. The purpose of control can be achieved.

12.2 Height level control

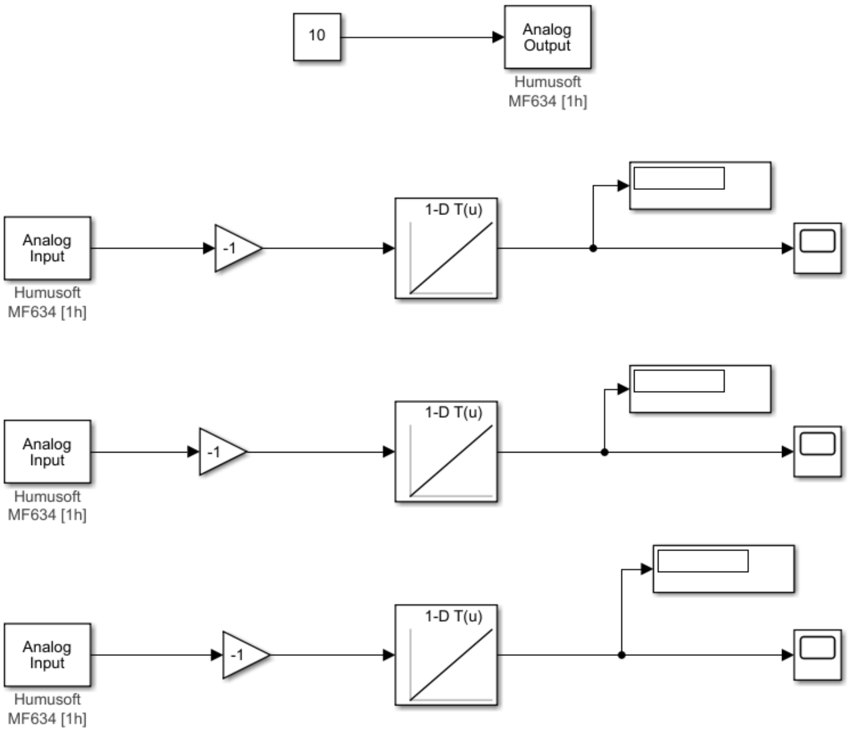


Figure 35: Height level control

The model describes how a sensor measures the height of liquid in a tank by producing different voltages for different liquid heights.

When the liquid level in the tank is 0 cm, the sensor generates a voltage of -9.612V. This means that when the liquid level in the tank is at the bottom, the sensor produces a negative voltage, indicating that there is no liquid in the tank. When the tank is empty or at its lowest level, the voltage reading of -9V can be considered the reference voltage or baseline for the sensor.

In contrast, when the liquid level in the tank is 400cm, the sensor produces a voltage of +2.445V. The same results are obtained for each tank, and the corresponding sensor readings are shown in Figs 8 and 9. This means that when the liquid level reaches the top of the tank, the sensor produces a positive voltage, indicating that the tank is full. When the tank is full or at its maximum level, the voltage reading of +9V can be considered the maximum voltage for the sensor.

Based on these voltage readings, we can conclude that the sensor has a linear relationship between the voltage output and the height of the liquid in the tank. The voltage output varies linearly with the level of liquid in the tank.

Based on the voltage reading of the sensor, we can use this linear relationship to determine the height of the liquid in the tank.

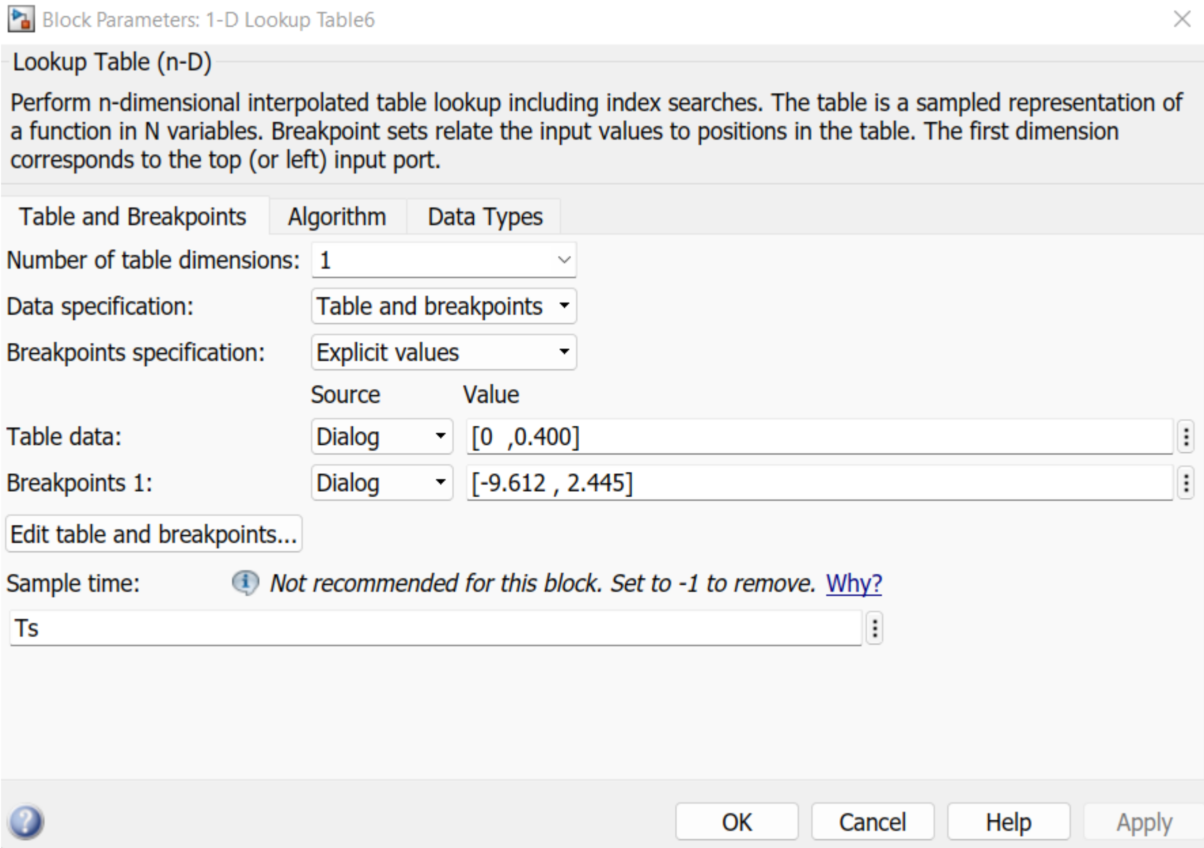


Figure 36: Lookup table height tank 1

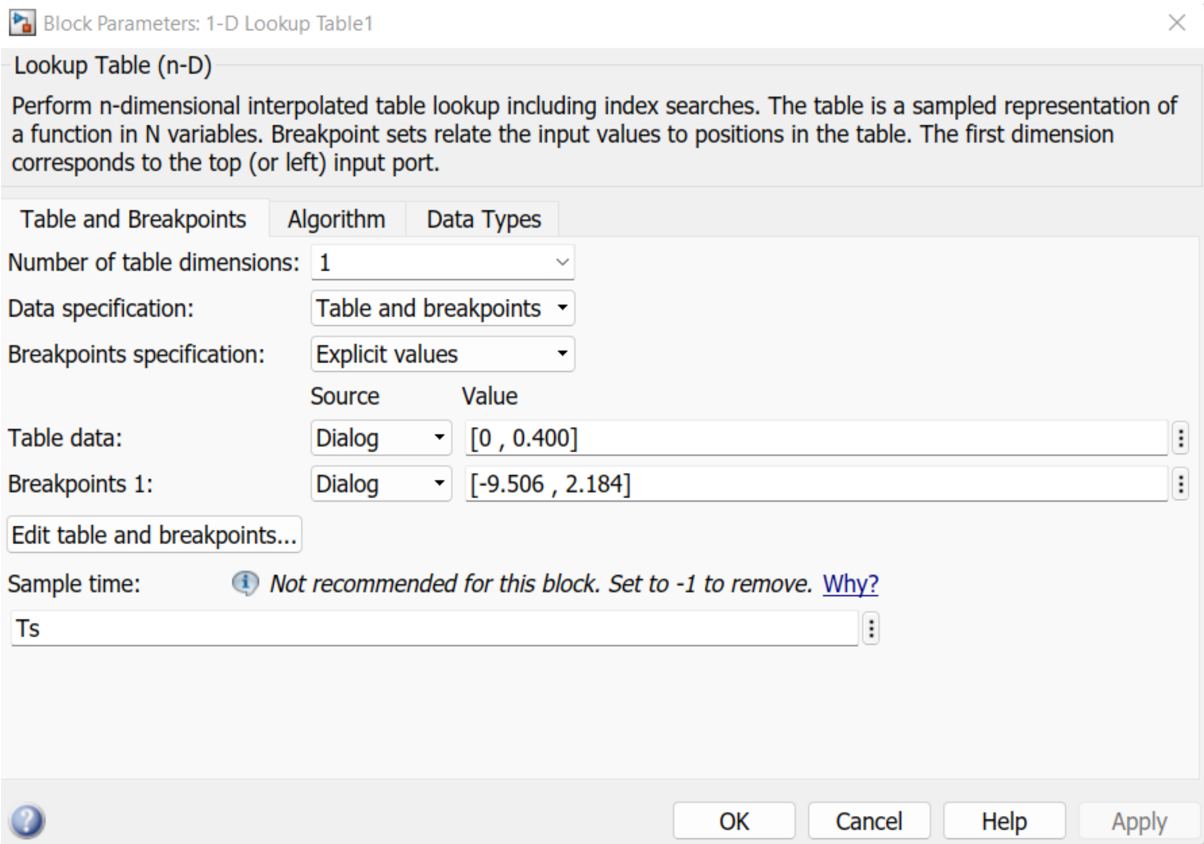


Figure 37: Lookup table height tank 2

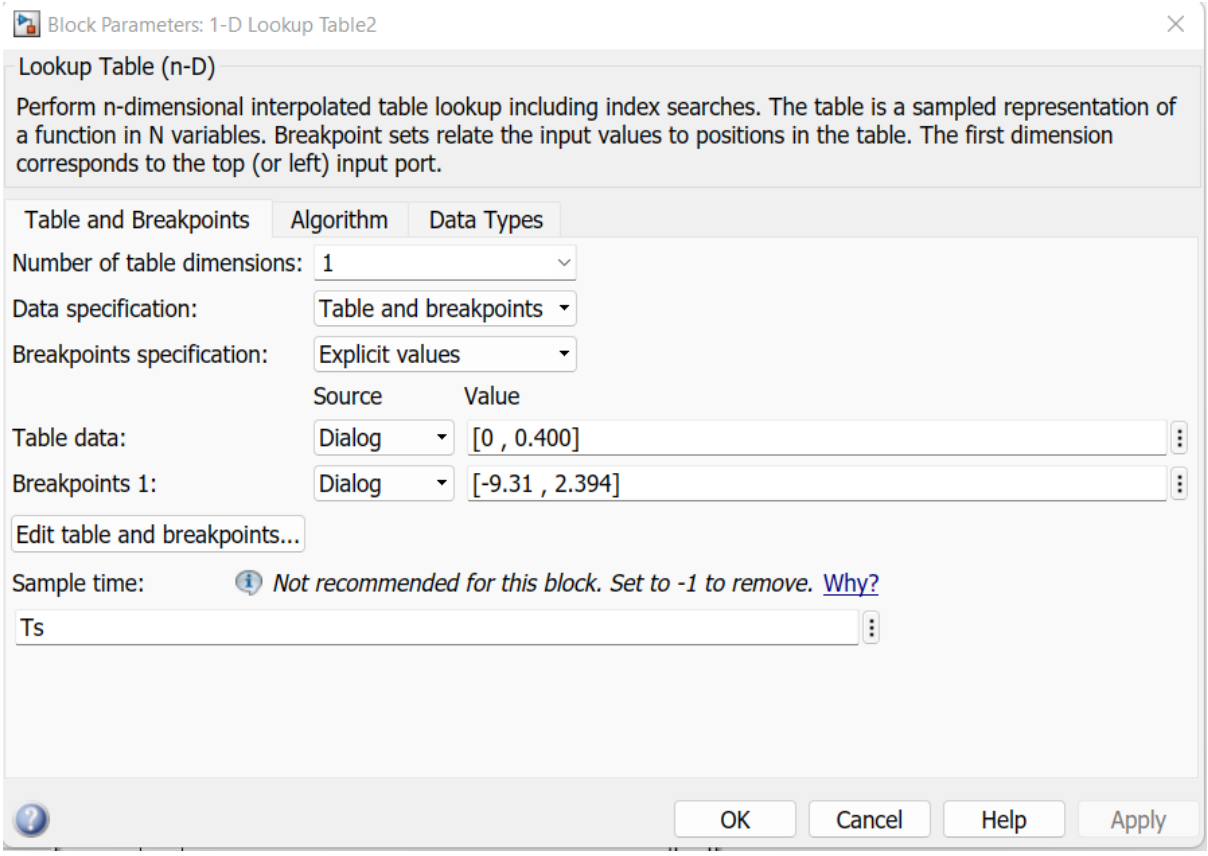


Figure 38: Lookup table height tank 3

Similarly, the voltage measured by the sensor can be used to determine the liquid level height of the current three tanks .
The sensor can produce a voltage output corresponding to the height of the liquid in each tank if it is placed at the appropriate height in each tank. As previously stated, the voltage output of the sensor changes linearly with the height of the liquid, so we can determine the height of the liquid in the tank by measuring the voltage output of the sensor.

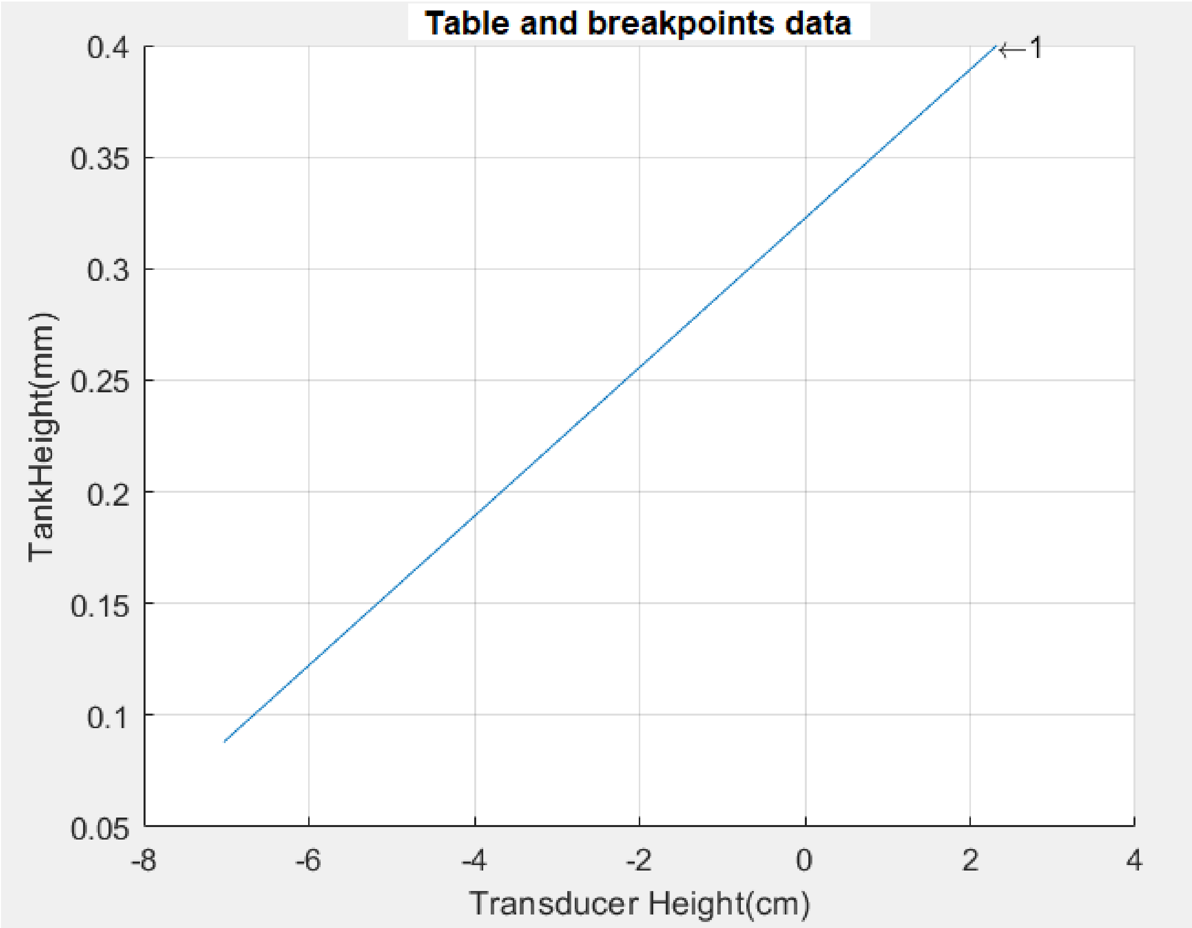


Figure 39: Height graph

The process of connecting a test bench to a computer and using MATLAB to simulate the behavior of a three-tank system.

First, the test bench is linked to the computer, most likely via some sort of interface such as a data acquisition card or USB device. When the test bench is connected, MATLAB can be used to simulate the system's behavior.

By simulating the system in MATLAB, we can obtain data about the system's behavior, such as flow rates, liquid levels, and other relevant variables. This information can be used to analyze and improve the system's behavior.

We can also control the system using the computer by simulating its behavior with MATLAB. For example, we can adjust the flow rates of the pumps or other system parameters in real-time to achieve the desired behavior of the system.

12.3 Validated Non-linear model

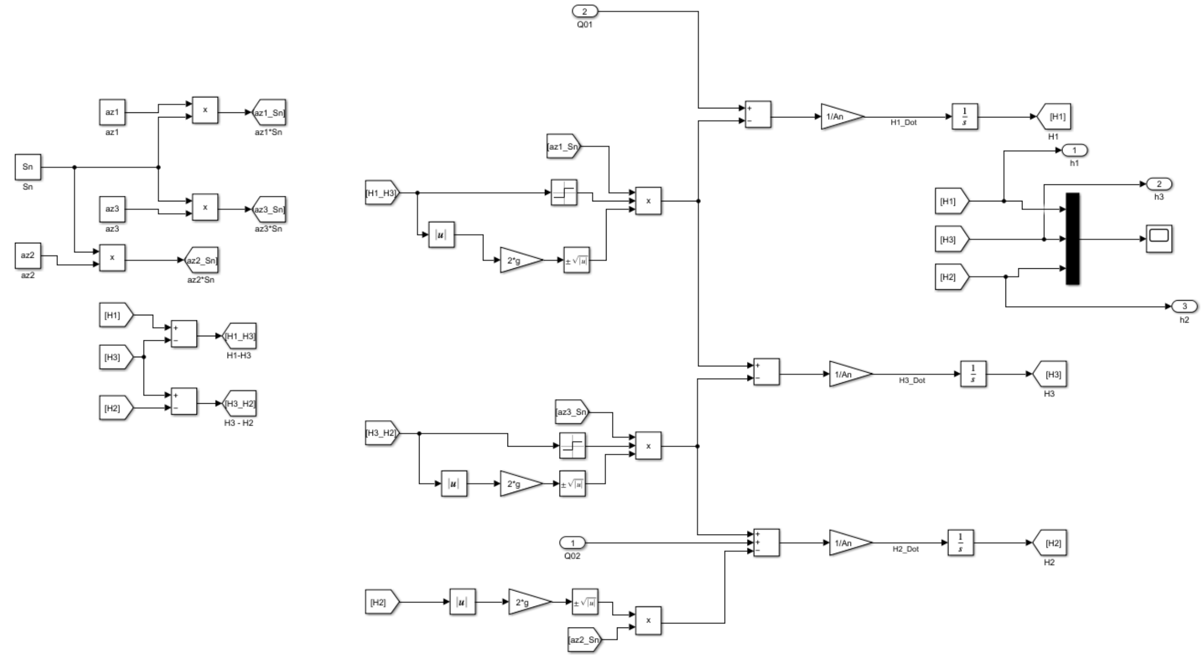


Figure 40: Nonlinear Model

The rate at which water enters the tank from the top is proportional to the voltage, V, applied to the pump. The water exits the tank through an opening in the tank base at a rate proportional to the square root of the tank's water height, H. A nonlinear plant is produced when the square root of the water flow rate is present. It describes the system's dynamics and allows us to simulate the system's behavior under various conditions. The model accounts for the system's non-linearities, such as the non-linear behavior of the valves and the non-linear dynamics of the water flow.

Furthermore, by simulating the system's behavior with MATLAB, we can control the system through the computer.

The non-linear MATLAB Simulink model is typically composed of a set of differential equations that describe the system's dynamics. These equations take into account the mass balance of the water in each tank as well as the behavior of the valves that control the flow of water between the tanks. The model is made up of a series of blocks, each representing a different component of the system, such as tanks, valves, and pumps. Lines connect the blocks.

The model is intended to simulate the behavior of the system under various conditions, such as changes in the set points of the water levels in the tanks, disturbances in the water flow, and changes in the valve positions.

The model can be used to create and test control algorithms for regulating water levels in tanks. Typically, feedback control is used in the design of the control algorithms, where the control algorithm measures the water levels in the tanks and adjusts the valve positions to maintain the desired water levels.

12.4 Validated non-linear model with teststand

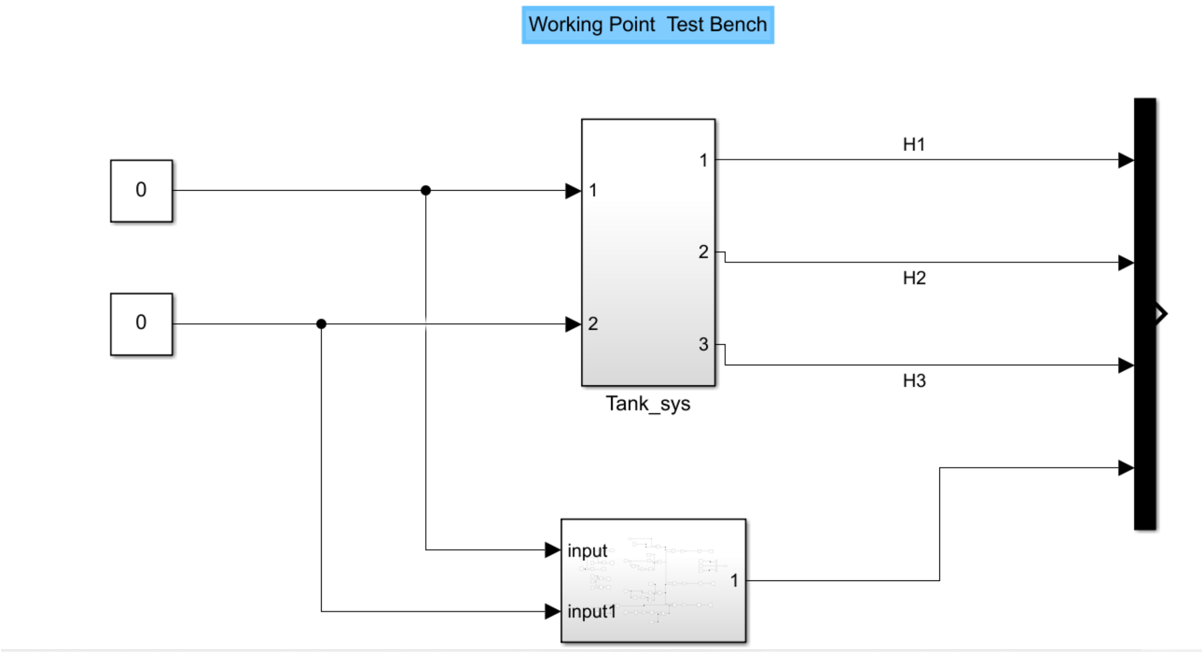


Figure 41: Nonlinear Model With Teststand

The Three Tank System Control is a system that consists of three interconnected water tanks, with the main goal of controlling the water levels in these tanks. Adjusting the flow rates of the pumps and the valve positions allows you to control the heights of the water levels in the tanks.

We set the final equilibrium height of tank 2, h02, to 0.102m and the height of tank 1, h01, to 0.272m in the given scenario. When the flow coefficients of the three water tanks are assumed to be random values, the equilibrium heights of the three tanks are h01=0.272m, h02=0.102m, and h03=0.188m, respectively.

To achieve this equilibrium height, set the flow rate of pump 1 to $3.3809(-5)m3/s$ and the flow rate of pump 2 to $3.3385(-5)m3/s$. These flow rates can be adjusted by varying the speed of the pumps or the positions of the valves.

Once the flow rates of the pumps are set to the required values, the system will eventually reach the desired equilibrium heights of the water levels in the tanks. It is important to note that the equilibrium heights will only be maintained as long as the pump flow rates and valve positions remain constant. Any changes to these parameters will cause the water levels in the tanks to fluctuate.

Matlab code

```matlab
%--------------------- MIMO Non-linear system ------------------------

An = 0.0154-pi*0.0125*0.0125; % m^2 The cross-section we used of the tank
Sn = 0.00005; % 5*10e-5 m^2The cross-section of the pipes
g = 9.8;      % gravity
az1 = 0.525;
az2 = 0.949;
az3 = 0.52;
h02 = 0.102; %0.155
h03 = 0.188;
h01=  0.272;
% h01 = ((az2/az3)^2+(az2/az1)^2+1)*h02;
Q01 = az1*Sn*(sqrt(2*g*(h01-h03)));%0.35
Q02 = az2*Sn*(sqrt(2*g*(h02)))-Q01;
% Q01=3.3809e-05;
% Q02=3.3385e-05;
```

Experimental results TThe scenario in which there are two equilibrium heights, h01 and h02. An equilibrium height is the height at which an object remains stable in the absence of any external force acting on it.

These equilibrium heights are used as a reference point for tracking the height change of h01 and h02 over time. This means that any change in height is measured in relation to these two points. Using the equilibrium heights as a reference point can be useful in situations where the absolute height values are unimportant and the change in height over time is what is important. Any change in height can be expressed consistently and meaningfully by using the equilibrium heights as a reference.

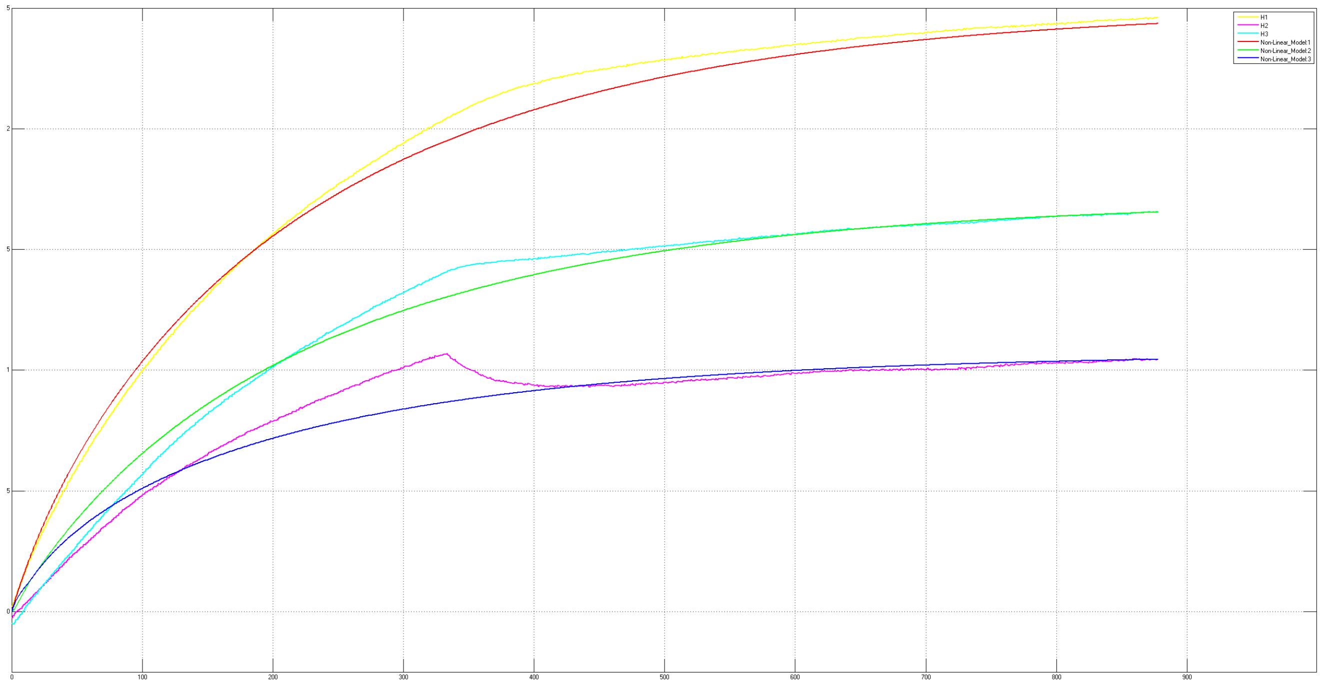


Figure 42: Teststand Nonlinear Graph

The three-tank system control entails using pumps to control the liquid levels in three interconnected tanks. The system began with a single pump but was later upgraded to a double pump. According to the statement, the liquid level takes longer to reach equilibrium as a result of this change.

The graph compares the height levels of three water tanks in both the simulated and actual test stands.

The output of the testing stand is represented by yellow, pink, and blue color lines in Figure 13, while the output of the simulated non-linear system is represented by red, green, and dark blue color lines.

The image clearly shows the close resemblance between the system and the simulated system, which was achieved through valve calibration of outflow coefficients.

Despite this, the testing stand and simulated non-linear system are balanced near 600s, and the error between the actual height changes and the simulated height changes is within the allowable range. This indicates that the MIMO system is capable of achieving the desired control objectives.

Consequently, the height at which the liquid level is balanced and the flow rates of the two pumps can be used as working points for linearizing the MIMO system. Linearization is the process of approximating a nonlinear system with a linear model around a specific operating point. The MIMO system can be approximated as a linear system by using balanced height and flow rates as the operating point. Which can simplify the design of a controller that can regulate the system around this operating point.

13 Linearized MIMO system

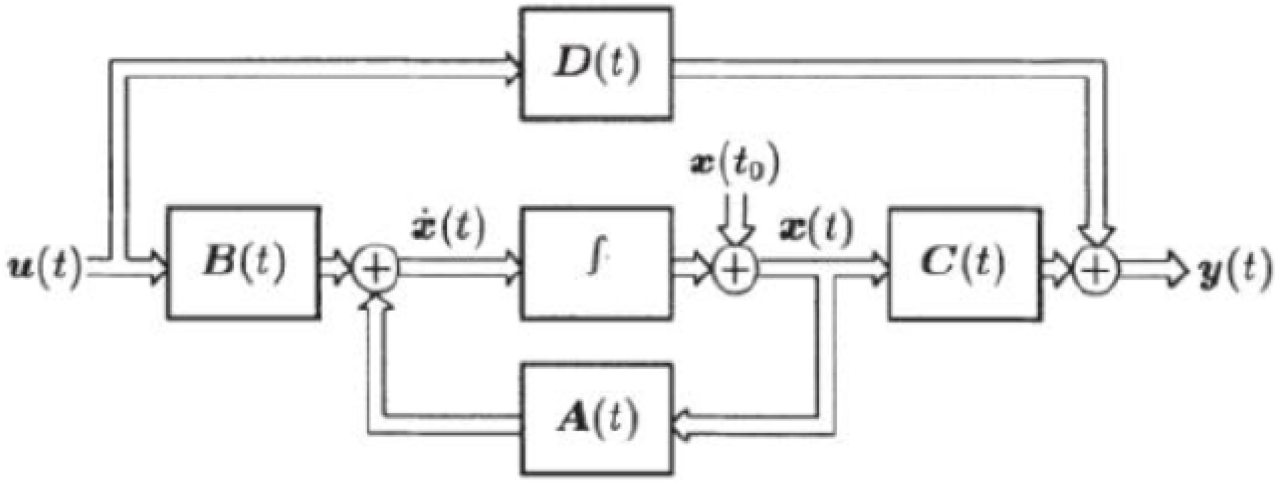


Figure 43: State-Space Representation of time continuous linear Systems for the MIMO

The linearized state-space model parameters are given by:

$$A = \begin{bmatrix} -a_1 & a_1 & 0 \\ a_1 & (-a_1 - a_3) & a_3 \\ 0 & a_3 & (-a_3 - a_2) \end{bmatrix} \tag{39}$$

$$B = \begin{bmatrix} \frac{1}{A_n} & 0 \\ 0 & 0 \\ 0 & \frac{1}{A_n} \end{bmatrix} \tag{40}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{41}$$

$$D = 0 \tag{42}$$

Where,

$$a1 = \ az1 * Sn * g/(An * \sqrt{2 * g * (h01 - h03)}) \tag{43}$$

$$a2 = \ az2 * Sn * g/(An * \sqrt{2 * g * h02}) \tag{44}$$

$$a3 = \ az3 * Sn * g/(An * \sqrt{2 * g * (h03 - h02)}) \tag{45}$$

where A is the state matrix, B is the input matrix, C is the output matrix, D is the matrix that describes which inputs affect the outputs directly, and h01, h02, and h03 are the operating points of the three levels, respectively.

13.1 Comparison of MIMO system non-linearization and Linearization

The established model is shown in the figure below:

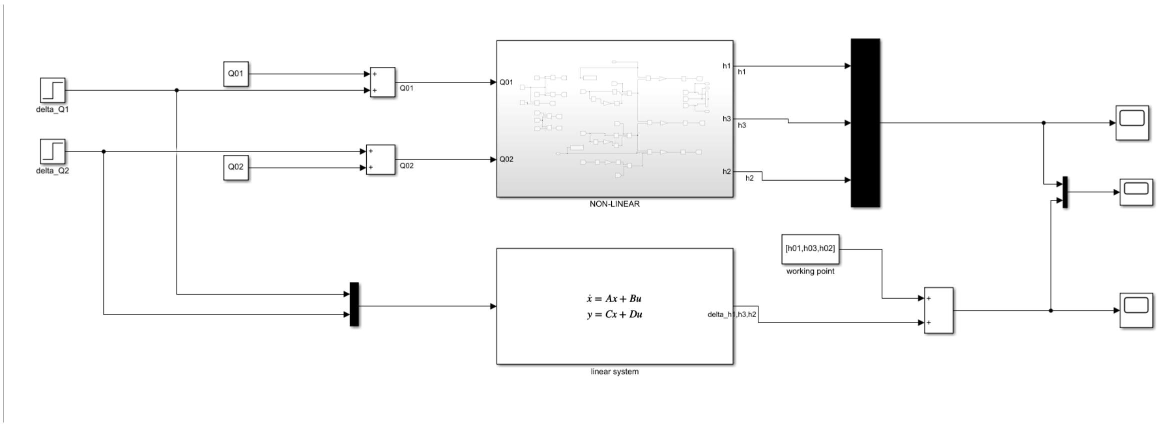


Figure 44: Linearization model

After finishing the model-building process, the linearization parameters must be determined. These parameters include the operating point, the linearization method, and the order of the linearization. Figure 15 depicts the fixed values h01, h02, and h03, which denote the steady state points or working points around which the system is linearized.

The operating point is the point at which the non-linear model will be linearized. It is usually selected as the system's steady-state operating point. The steady-state operating point is the point at which the system has achieved a stable equilibrium.

The linearization method used is determined by the type of nonlinear model being analyzed. Taylor series approximation, Jacobian linearization, and numerical linearization are all common methods. The Taylor series approximation method is used to linearize a nonlinear function around a point. The Jacobian linearization method is used to linearize a system of nonlinear equations around an operating point. The numerical linearization method is used to linearize a nonlinear function using numerical methods.

The level of accuracy of the linearized model is determined by the order of linearization. Higher orders of linearization result in more accurate models, but they also require more computation time and resources. The order of linearization is usually determined by the desired level of accuracy and the available computational resources.

Matlab code

```
%------------------------- MIMO linear system ------------------------

a1=az1*Sn*g/(An*sqrt(2*g*(h01-h03)));
a2=az2*Sn*g/(An*sqrt(2*g*h02));
a3=az3*Sn*g/(An*sqrt(2*g*(h03-h02)));
A = [-a1 a1 0;a1 (-a1-a3) a3; 0 a3 (-a3-a2)];
B = [1/An 0; 0 0; 0 1/An];
C = eye(3);
D = [0 0; 0 0; 0 0];
```

The equilibrium working point of a system is the state in which all of the forces and moments are balanced and the system is stable.

When the new equilibrium point is reached, the simulated height of the system is calculated for both the nonlinear and linearized models. The nonlinear model is a mathematical model that takes into account the system's nonlinear behavior, whereas the linearized model is a simplified version of the nonlinear model that assumes the system behaves linearly around the equilibrium point.

In comparison to the linearized model, the simulated height of the nonlinear model will exhibit more complex behavior. This is because the nonlinear model takes into account the system's nonlinear behavior. The linearized model, on the other hand, assumes the system behaves linearly around the equilibrium point. The linearized model, on the other hand, is easier to analyze and can be used to make approximations about the behavior of the system around the equilibrium point.

The figure depicting the simulated height of the nonlinear and linearized models can be used to compare the behavior of the two models and analyze the system's behavior around the equilibrium point. The differences between the two models can also provide insights into the system's nonlinear behavior and help to identify areas where the linearized model may be inaccurate. Also, the figure shows here the dotted line represents the non-linear heights of the system and the normal line indicates the linear height of the system. Even though there is a small error in linearizing both systems get normalized which has shown in the graph. From this, we can compare the behavior of the non-linear and linear systems.

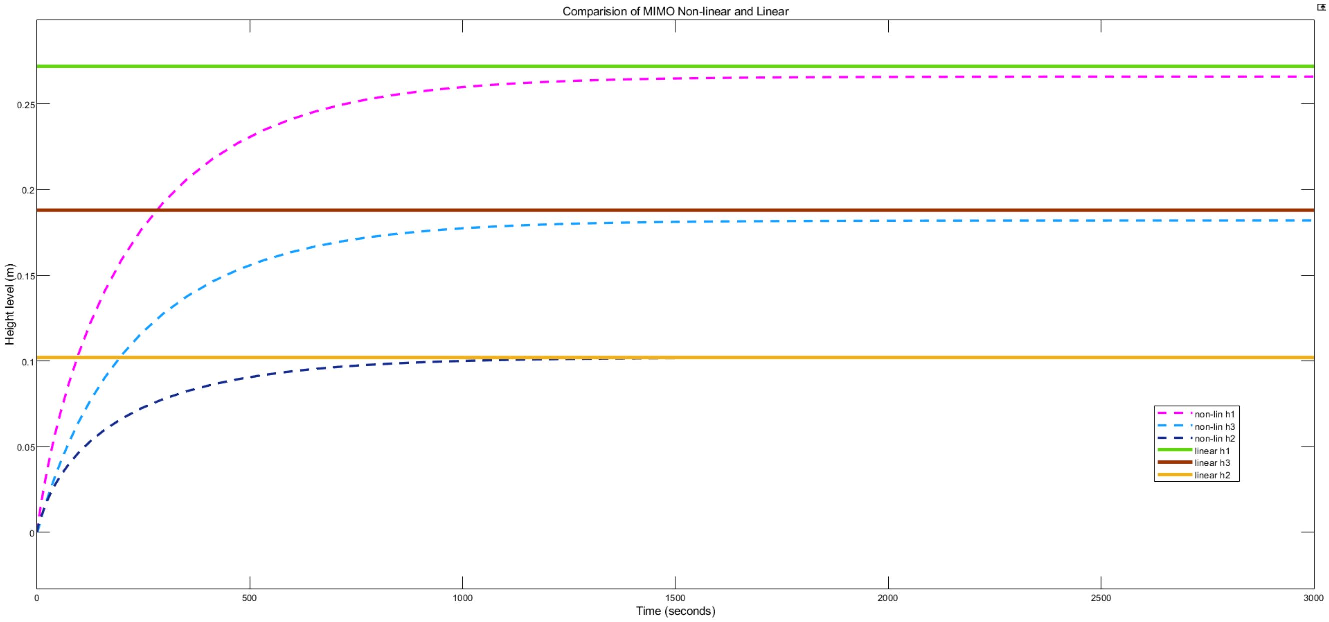


Figure 45: Comparison of Nonlinearization and Linearization of MIMO System

14 State Feedback control

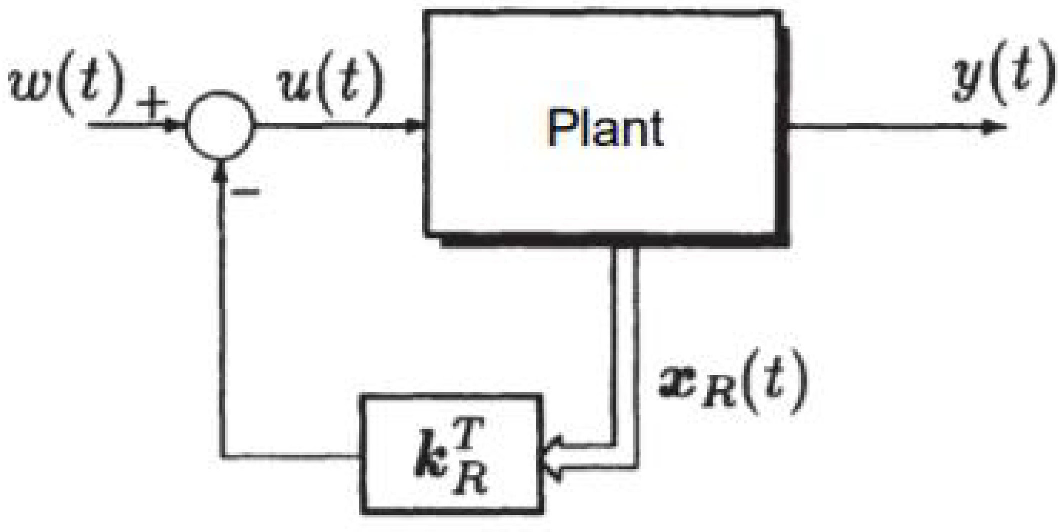


Figure 46: State feedback

Full-state feedback control is a common technique used in control systems engineering to control a dynamic system using feedback information from the system's full state. This technique is based on the idea of measuring the entire state of the system, including all relevant variables that describe its behavior, and then using this information to design a feedback controller that can manipulate the system to achieve the desired behavior.

There are two approaches to designing a full-state feedback controller: pole placement and linear quadratic regulator (LQR). Both approaches involve developing a control law that maps the current state of the system to a control signal that can be applied to the system to achieve the desired behavior. The control law is typically represented as a matrix equation of the form u(t) = Kx(t), where u(t) is the control signal, x(t) is the current state of the system, and K is a matrix of control gains that determine how the state variables are mapped to the control signal.

Pole placement and LQR are similar in that they both involve designing a control law based on the state feedback information. However, they differ in the specific design criteria used to determine the control gains.

Pole placement entails positioning the system's closed-loop poles at desired locations in the complex plane by selecting appropriate control gain values. The stability and performance of the system are determined by the closed-loop poles, so selecting appropriate pole locations is critical for achieving desired system behavior.

Alternatively, LQR entails minimizing a cost function that describes the desired trade-off between control effort and system performance. Typically, the cost function contains terms that penalize both the system's deviation from its desired behavior and the magnitude of the control signal. LQR generates control gains by minimizing this cost function, achieving the desired trade-off between control effort and system performance.

Despite differences in design criteria, pole placement and LQR can be used to generate control laws that achieve similar system behavior and can often be implemented using the same control structure. In practice, the choice between pole placement and LQR is determined by the system's specific requirements and the design constraints imposed.

A Single-Input-Single-Output (SISO) system is a type of control system with only one input and one output. In SISO system design, it is frequently necessary to use a feedback controller to achieve the desired system behavior. The pole placement method is a common method for designing a feedback controller.

So, in the Single-Input-Single-Output (SISO) system, we used the pole placement method. The pole placement method entails selecting the desired locations of the closed-loop poles in the complex plane. The poles of a system are the values of s that make the denominator of the transfer function equal to zero. The location of the poles determines the system's stability and response. If the poles are in the left half of the complex plane, the system is stable, whereas poles in the right half of the complex plane indicate instability.

The feedback controller is designed to place the closed-loop poles at the desired pole locations once the desired pole locations have been selected. The feedback controller can be designed with either state or output feedback. The controller in state feedback is designed based on the state of the system, whereas the controller in output feedback is designed based on the output of the system.

Typically, the feedback controller is built by solving the algebraic Riccati equation or by employing the pole placement method. The controller gain matrix K is calculated using the pole placement method by selecting the closed-loop system poles and using the Ackermann formula to determine the control gain matrix. The control law is then given by u = -Kx, where u is the control input, x is the state vector, and K is the control gain matrix.

The pole placement method has the advantage of allowing the closed-loop poles to be placed at any desired location in the complex plane, assuming the system is controllable and observable. This allows for greater freedom in designing a feedback controller that meets specific performance requirements. However, the method requires accurate knowledge of the system model and may be sensitive to modeling errors. As a result, it is critical to validate the controller design through simulation and testing before implementing it in a real system.

The pole placement method is a well-known technique for creating feedback controllers. One of the main limitations of the pole placement method is that it requires the designer to specify the desired closed-loop pole locations. This can be difficult for complex systems or higher-order systems with many poles to control.

In contrast, the Linear Quadratic Regulator (LQR) method is a widely used technique for designing optimal controllers for linear and nonlinear systems. The LQR method is a feedback control strategy that attempts to minimize a quadratic cost function that represents the control system's performance criteria. The LQR method determines the optimal state feedback control law that minimizes the cost function, and the resulting controller is known as the Linear Quadratic Regulator.

The LQR method is particularly useful for systems in which the cost function can be defined in terms of the system's state and control input. This cost function typically includes terms that penalize deviations from desired state values as well as control effort. The LQR method generates a state feedback control law that meets the desired performance specifications while requiring the least amount of control effort by minimizing this cost function.

Both the Linear Quadratic Regulator (LQR) method and the pole placement method are used to create feedback controllers for Multi-Input Multi-Output (MIMO) systems. However, for MIMO systems, the LQR method has several advantages over the pole placement method.

Simultaneous control of all states:
The LQR method allows you to create a controller that controls all of the MIMO system's states at the same time. The pole placement method, on the other hand, requires the designer to specify the desired closed-loop pole locations for each individual state. To achieve the desired performance specifications, the LQR method ensures that all states are controlled in a coordinated manner.

Optimization of a cost function:
The LQR method optimizes a cost function that reflects the desired performance specifications of the MIMO system. The cost function is defined in terms of system states and

control inputs, and the LQR method finds the optimal state feedback control law that minimizes the cost function. The pole placement method, on the other hand, does not directly optimize a cost function.

Robustness to modeling uncertainties:
The LQR method outperforms the pole placement method when it comes to modeling uncertainties and disturbances. By minimizing the cost function, the LQR method explicitly takes the effects of uncertainties and disturbances into account. The final controller is intended to be resistant to these effects.

Handling of non-square systems:
The LQR method can handle non-square MIMO systems in which the number of inputs does not equal the number of outputs. The pole placement method, on the other hand, necessitates that the MIMO system be square.

Consequently, the LQR method is a powerful technique for designing feedback controllers that can meet desired performance specifications while requiring minimal control effort. It allows for simultaneous control of all states, optimizes a cost function, is resistant to modeling uncertainties, and can handle non-square systems. While the pole placement method is a useful technique, its limitation of requiring the designer to specify the desired closed-loop pole locations can be addressed by using the LQR method.

The optimal gain matrix K in the Linear Quadratic Regulator (LQR) method is found by selecting closed-loop characteristics that balance performance and control effort. The LQR method, in particular, seeks to minimize a quadratic cost function that reflects the system's desired performance specifications while also taking into account the effort required to achieve that performance.

The cost function used in the LQR method has two terms: a quadratic term that penalizes deviations of system states from their desired values and a quadratic term that penalizes the control effort required to achieve the desired performance. By adjusting the relative weights of these terms, we can balance the desired performance against the control effort.

After that, the optimal gain matrix K is computed by solving the associated algebraic Riccati equation, which includes the system dynamics and the cost function. The optimal state feedback control law that minimizes the cost function is provided by the resulting gain matrix.

The LQR method is a feedback control strategy that aims to minimize a quadratic cost function over a finite time horizon. The cost function considers the deviations of the system state from the desired values as well as the control effort required to achieve the desired performance.

The LQR controller solves the following optimization problem:

$$\begin{aligned} J(u) = \int_0^T x(t)^T Q x(t) + u(t)^T R u(t) dt \\ dx/dt = Ax + Bu \end{aligned} \tag{46}$$

Where,
x is the state of the system,
u is the control input,
Q is a positive semidefinite weighting matrix on the state,
R is a positive definite weighting matrix on the control,
A and B are the system matrices,
and T is the final time.

The optimal feedback control law that can be used to control the system is provided by the solution to the LQR problem. The LQR problem is solved to obtain the gain matrix K with the lowest cost. By solving the associated algebraic Riccati equation, which includes the system dynamics and the cost function, the optimal gain matrix K is calculated.

The optimal control input u is then given by:

$$u(t) = -Kx(t) \tag{47}$$

where K denotes the optimal gain matrix and x(t) denotes the system's state at time t.

The weighting matrices Q and R are used in the preceding expression to balance the trade-off between system performance and control effort. The matrix Q represents a positive semidefinite weighting matrix on the state, while the matrix R represents a positive definite weighting matrix on the control input.

The diagonal elements of the matrix Q represent the relative importance of each state variable in the control problem. We put more emphasis on reducing error in the corresponding state variable by assigning a larger value to a diagonal element of Q. Assigning a lower value to a diagonal element of Q, on the other hand, places less emphasis on reducing the error in that state variable. As a result, by appropriately selecting the diagonal elements of Q, we can prioritize the state variables and ensure that the control effort is directed toward the most critical variables.

Similarly, the matrix R's diagonal elements represent the relative importance of each control input in the control problem. We make it more expensive to apply a large control effort in the corresponding input by assigning a larger value to a diagonal element of R. Assigning a lower value to a diagonal element of R, on the other hand, makes it less expensive to apply a large control effort in that input. As a result, by carefully selecting the diagonal elements of R, we can limit the magnitude of the control effort and keep the control input from becoming saturated.

Therefore, the diagonal elements of the weighting matrices Q and R in the LQR problem are used to assign relative importance to the state variables and control inputs, respectively. By appropriately selecting the values of Q and R, we can prioritize the state variables and limit the control effort, achieving the desired system performance with minimal control effort.

As previously stated, the matrix R is a positive definite weighting matrix on the control input. It is also a diagonal matrix that assigns a weight to each control variable in the cost function. The diagonal elements of R represent the relative importance of each control variable in the control problem. We make it more expensive to apply a large control effort in the corresponding control variable by assigning a larger value to a diagonal element of R. Assigning a lower value to a diagonal element of R, on the other hand, makes it less expensive to apply a large control effort in that control variable.

The LQR method prioritizes state variables while also attempting to minimize the control effort required to achieve the desired system performance. This is accomplished by penalizing the control inputs via matrix R. We can obtain an optimal feedback control law that minimizes the control effort required to achieve the desired system performance by minimizing the cost function J(u).

Accordingly, the LQR method balances the trade-off between system performance and control effort by adjusting the weighting matrices Q and R. The matrix Q is used to penalize poor performance in the state vector, while the matrix R is used to penalize excessive actuator effort in the control variables. We can prioritize the state variables and limit the control effort by adjusting Q and R, resulting in the desired system performance with minimal control effort.

The Linear Quadratic Regulator (LQR) is a powerful control technique used to design feedback controllers for linear time-invariant systems. It is based on finding a control law that minimizes a quadratic cost function that captures the trade-off between control effort and tracking error.

During the LQR design process, we select the weighting matrices Q and R to define the cost function. The tracking error is weighed using the matrix Q, and the control effort is weighed using the matrix R. The designer can choose between tracking performance and controlling effort by adjusting the values of Q and R.

The selection of Q and R is critical because it has a direct impact on the performance and stability of the LQR controller. If Q is too large, the control law will prioritize tracking performance over control effort, which can result in excessive control effort and instability. If Q is too small, the control law will prioritize control effort over tracking performance, which can lead to poor performance tracking.

Similarly, if R is set too high, the control law will prioritize minimizing control effort over tracking performance, resulting in a slow and sluggish response. If R is too small, the control law will favor tracking performance over control effort, resulting in excessive control effort and instability.

Choosing the values of Q and R is frequently based on trial and error, with our experience and intuition serving as guides. In most cases, we began with an educated guess for Q and R and then simulated the closed-loop system to assess performance. We changed the values of Q and R based on the simulation results to improve the system's performance and stability.

There are also some methods for selecting Q and R based on optimization criteria, such as the H-infinity optimization or the pole placement method, but these methods are more advanced and require a deeper understanding of control theory.

Accordingly, determining the values of Q and R is a critical step in the LQR design process. It necessitates careful consideration of the trade-off between tracking performance and control effort, and it frequently relies on our own experience and intuition to determine the best values.

The control system, which controls two different input flow rates. The maximum flow rate represents the maximum amount of flow that can be achieved, and the two input flow rates are expected to be less than this maximum.

If the input flow rates are less than the maximum, the transducer's effort (i.e., the control action taken by the transducer) will be penalized. This means that the control system is programmed to prevent the transducer from operating at full capacity when it is not required.

This penalty can be implemented by increasing the corresponding value in the R matrix. The R matrix is a component of the control system that specifies the cost of the control effort. By increasing the value for the corresponding input, the control system prioritizes minimizing the control effort for that input.

Furthermore, it is assumed that the input pump voltage remains within the range of -10v to 10v. The Q matrix can be penalized in order to bring the system to a steady state as quickly as possible. The Q matrix is a control system component that specifies the cost of error in system output.The control system will prioritize reducing error and reaching a steady state as soon as possible by penalizing the Q matrix.

15 Optimal control

15.1 Linear Quadratic Regulator (LQR)/Linear Quadratic Gaussian (LQG) control

The Linear Quadratic Regulator (LQR) controller is a popular control design method that is used in a wide variety of control applications. The LQR controller is based on the optimal control theory, which is concerned with running a dynamic system at the lowest possible cost.

The system dynamics are described by a set of linear differential equations in the linear quadratic control problem, and the cost is described by a quadratic function. The LQR controller's goal is to minimize the cost function while taking into account system dynamics and any control constraints.

The Linear Quadratic Regulator (LQR) is a full-state feedback optimal control law, $u(t) = -Kx(t)$, that regulates the control system by minimizing a quadratic cost function.

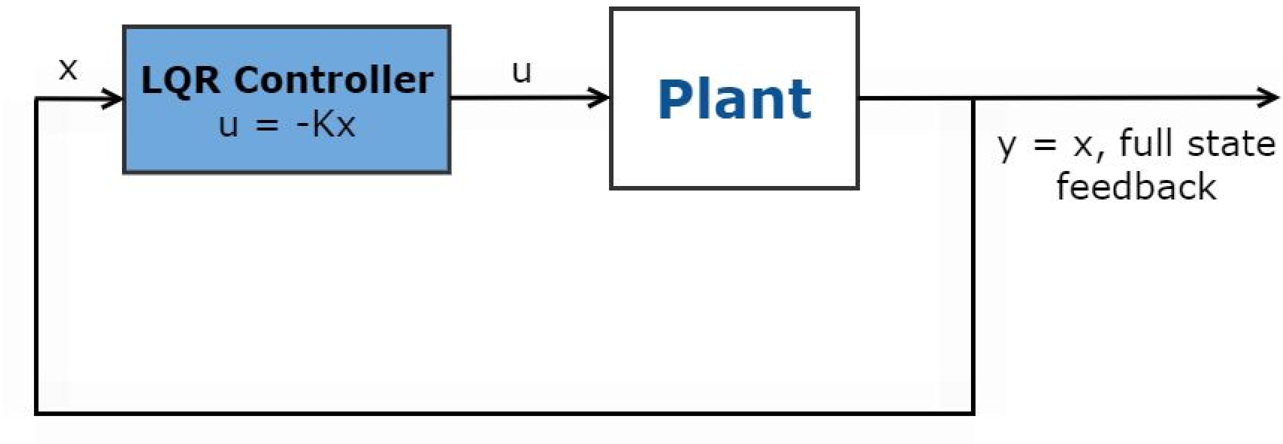


Figure 47: Linear Quadratic Regulator controller

In the context of the Linear Quadratic Regulator (LQR) controller, the performance index (PI) is a way of measuring the system's performance in relation to a quadratic cost function. This cost function is affected by system states (x) and control inputs (u).

As shown below.

$$J(u) = \int_0^\infty (x^T Q x + u^T R u) dt \tag{48}$$

Q is a positive semi-definite matrix that ensures that the scalar quantity $XTQX$ is always positive for all X(t) values and brings all states to equilibrium. R is a positive-definite matrix that ensures that the scalar quantity $uTQu$ is always positive for all values of U(t), penalizing the control input. The goal is to find the best state feedback controller gains. The designer makes the choice of matrix Q and R. The closed-loop system's set-point tracking responses will vary depending on which of these matrices is used. Selecting 'Q' large keeps 'J' small, resulting in smaller states. Small values of 'R' make control effortless, resulting in a small performance index 'J' in the equation. Larger values of 'Q' and smaller values of 'R' result in the location of closed loop poles far from the origin, ensuring relative stability. In other words, the system will remain stable even in the presence of disturbances or modeling errors.

Consider a system described by the state space equation

$$\begin{aligned} \dot{x} &= [A][X] + [B]u \\ y &= [C][X] + [d]u \end{aligned} \tag{49}$$

The optimal control, minimizing 'U' is given by the linear feedback law

$$u(t) = -Kx(t) \tag{50}$$

with $K = R^{-1}B^T P$, where 'P' is the unique positive definite solution to the Continuous Algebraic Ricattic Equation (CARE) given by

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{51}$$

Let the scalar function be, $V(x) = X^T PX$ with $V(x) > 0$

The time derivative of V(x) is, $\dot{V}(x) = \dot{X}^T PX + X^T P\dot{X}$

Now,

$$\dot{V}(x) = (AX + Bu)^T PX + X^T P(AX + Bu) \tag{52}$$

$\dot{V}(x) = X^T(A^T P + PA)X + u^T B^T PX + X^T PBu$ From CARE we have,

$(A^T P + PA) = -Q + PBR^{-1}B^T P$

Now,

$$\dot{V}(x) = (B^T PX + Ru)^T R^{-1}(B^T PX + Ru) - (XTQX + u^T Ru) \tag{53}$$

Integrating $\dot{V}(x)$ we get,

$$\begin{aligned} \int_0^\infty \dot{V}(x)dt &= -J + \int_0^\infty (B^T PX + Ru)^T R^{-1}(B^T PX + Ru)dt \\ J = XT(0)PX(0) &+ \int_0^\infty (B^T PX + Ru)^T R^{-1}(B^T PX + Ru)dt \end{aligned} \tag{54}$$

The minimum value of 'J' is achieved when $U = -R^{-1}B^T PX = -KX$

The design procedure is described by the following steps:

- The weighting matrices Q and R are selected.
- The Continuous Algebraic Ricatic Equation (CARE) is solved to get the P matrix.
- The linear quadratic controller gain (K) is computed.
- The time response of the system is simulated.
- If the transient specifications are not met, then the weight matrices are tuned.

LQR (Linear Quadratic Regulator) control is a type of optimal control that is based on the state-space representation of a system. It is a popular control strategy in control engineering because of its ability to provide an optimal control solution for a wide range of systems.

The LQR control strategy entails creating a state feedback controller that minimizes a cost function that is a combination of the quadratic forms of the state and control inputs. The cost function represents the system's performance, and the LQR controller seeks to minimize it by selecting the best control inputs.

The LQR controller is created by solving two coupled algebraic Riccati equations that provide the optimal feedback gains for the controller. These gains are then used to compute the optimal control input for the system.

The LQR control method, as opposed to the pole placement method, provides a more systematic approach to optimal control design. The LQR control considers the entire system dynamics, including its state-space representation and cost function, whereas the pole placement method only takes the system's poles into account.

Furthermore, because it can handle systems with time-varying parameters or uncertainties, the LQR control is more flexible than the pole placement method. LQR control can also be used to develop robust control strategies that are less susceptible to disturbances or modeling errors.

Simulink Model of LQR Control :

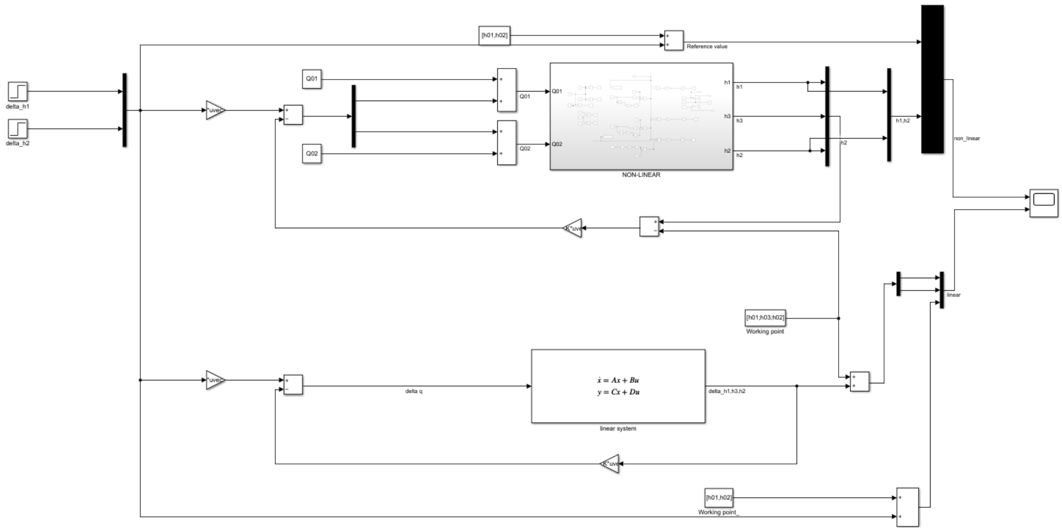


Figure 48: Simulink model of LQR Control

Here, the values are adjusted as follows in Matlab:

MATLAB Code for LQR State-Feedback Control

Matlab code

```
%---------------------Control law -----------------------
Q = [100 0 0;
    0 1000 0;
    0 0 100];
R = [100000000000 0;
    0 10000000000];
Kop = lqr(A,B,Q,R);
Pop = ([1 0 0;0 0 1]*(B*Kop-A)^-1*B)^-1;
%------------------------------------------------------------
```

Simulation result:

Non-Linear System	Linear System
For Reference Height:	h1= 0.27m, h2=0.102m

Table 8: Height Parameter

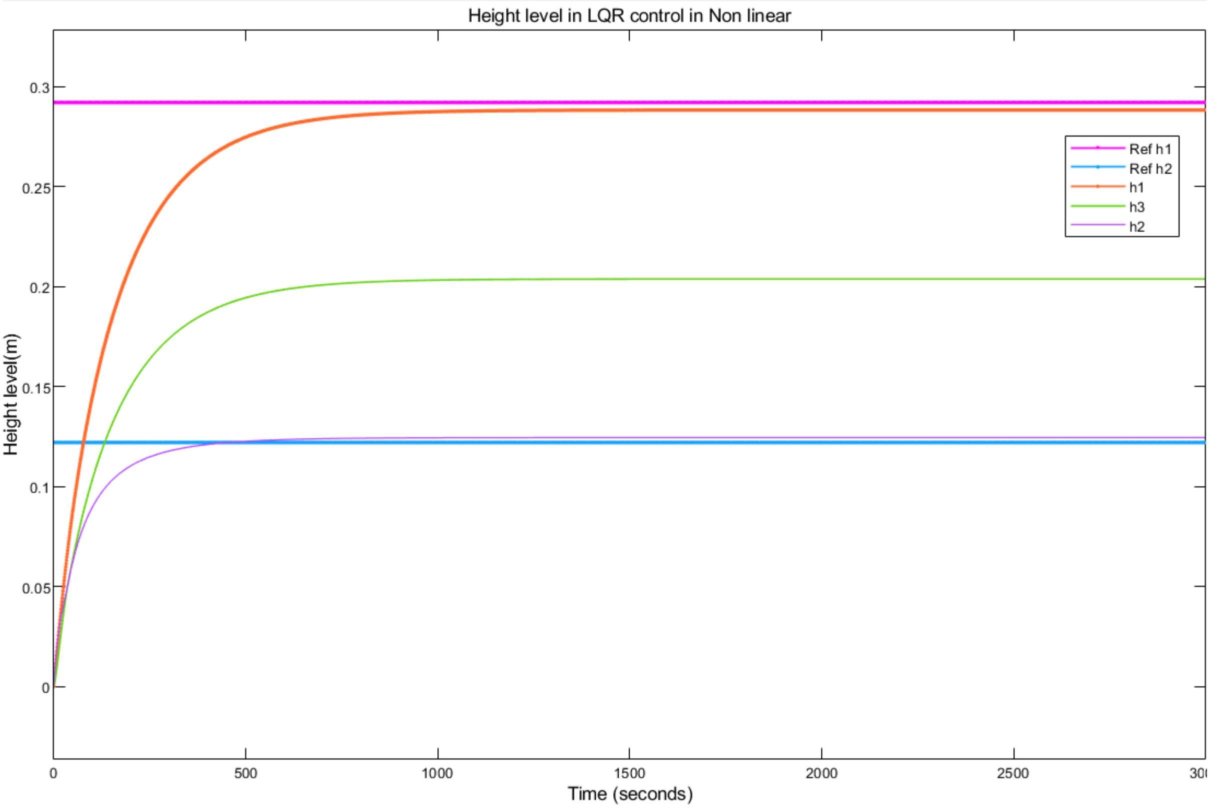


Figure 49: LQR Non-linear Graph

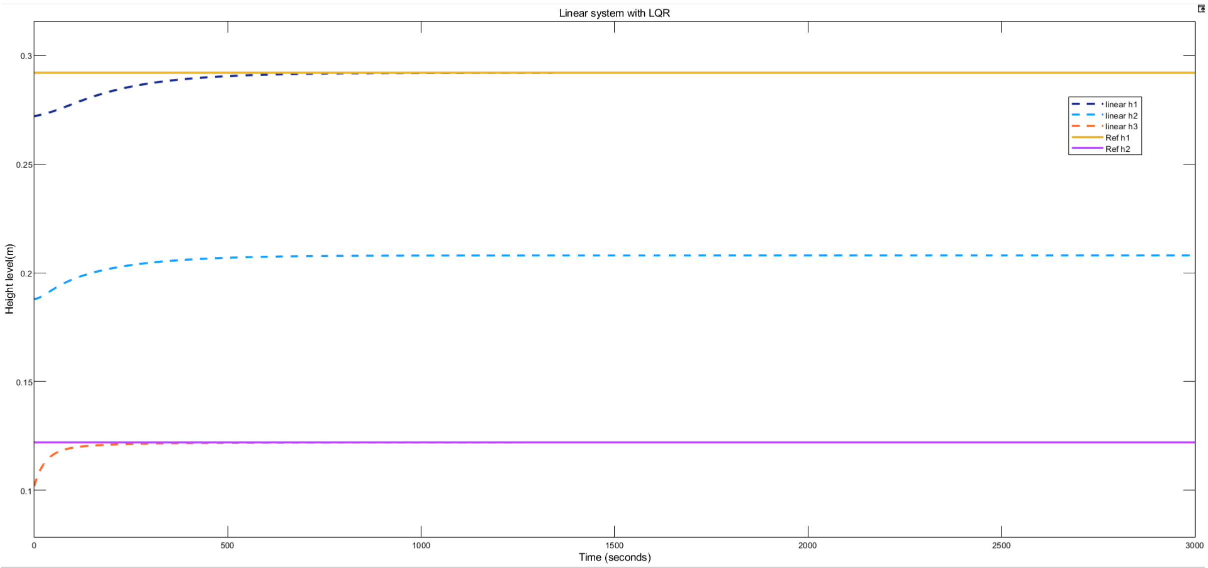


Figure 50: LQR Linear Graph

To begin, it states that an adjusted LQR (Linear Quadratic Regulator) control has been applied to both linear and non-linear systems. LQR is a control strategy used in control theory to regulate the behavior of a dynamic system by minimizing the sum of the quadratic cost functions of the control input and the state variables. The adjusted LQR indicates that some modifications or adaptations to the LQR control have been made to suit the system under consideration.

Along with this, the adjusted LQR control performed well in both linear and nonlinear systems. This indicates that the control strategy was effective in regulating the system's behavior and achieving the desired objectives, which could be stability, tracking, or optimization depending on the specific problem.

According to the figures above, the adjusted LQR control works well in both linear and nonlinear systems. The system's reference height and height h1=0.27m and h2=0.102m are balanced. Changing the Q and R values through trial and error. Finally, we have a precise height with a low margin of error.

15.2 LQR testing

The three-tank system is made up of three tanks that are linked in series. The first and second tanks are supplied with the input flow rate, and the output flow rate from each tank is controlled by two pumps. A level sensor measures the height of each tank, and the control goal is to keep the level of each tank at a preset value. A state space representation can be used to model the system, with the state variables being the heights of the three tanks and the input variables being the voltages applied to the two pumps.

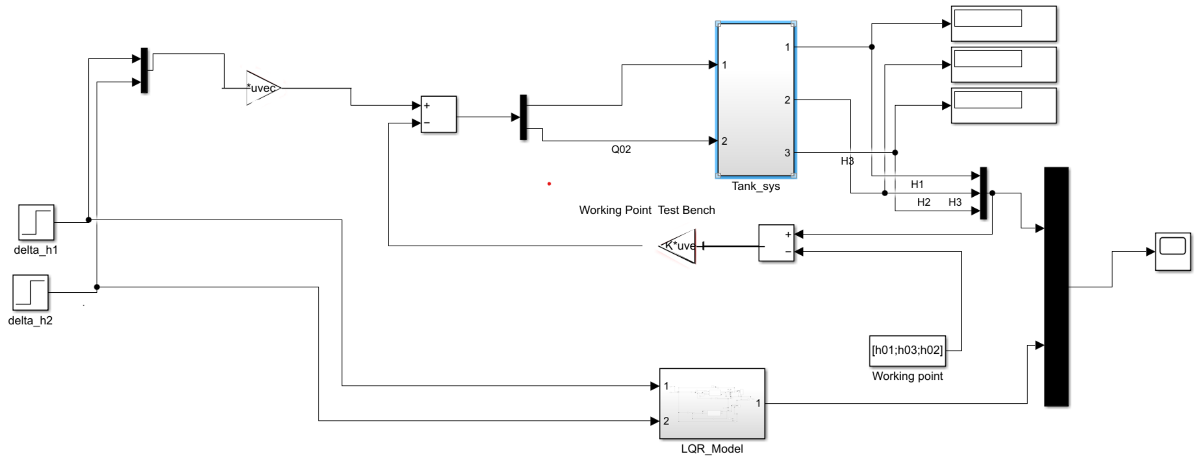


Figure 51: Simulink model of LQR Control on MIMO System

We used the LQR method to develop an optimal control strategy for the three-tank system. The LQR approach entails defining a cost function that takes into account the desired control objective as well as the control effort required to achieve it. The cost function is defined as a quadratic function of the state and input variables, and the optimal control law is designed by minimizing this cost function.

The three tank system has two pumps as actuators, and the pumps are controlled by input voltages ranging from -10 to 10, which correspond to the minimum and maximum flow rates. However, in the previous control design, the pump would occasionally stop working for a few seconds because the input voltage generated by the controller exceeded the pump voltage range. To avoid this happening again, the system must be able to quickly reach the desired height level while ensuring that the flow rates Q1 and Q2 do not exceed the maximum flow. finally, it is critical to design a controller that can achieve better performance under these conditions.

In a control system, the state vector x may have negative values, resulting in a negative overall cost. To avoid this, the value is squared to ensure positivity, yielding a quadratic cost function with a specified minimum value. To obtain the gain matrix with the lowest cost, the Linear Quadratic Regulator (LQR) problem can be solved.

We employ a diagonal matrix with positive coefficients to define the weighting matrices Q and R. These matrices are used to penalize poor performance as well as actuator effort. By setting corresponding values to large values, the Q matrix can be used to target specific states with a low error, whereas the R matrix can be adjusted to penalize actuator effort.

It is critical to ensure that the flow rate inputs do not exceed the maximum flow rate in the case of the three-tank system. To accomplish this, increase the corresponding values in the R matrix to penalize actuator effort. Furthermore, the Q matrix can be penalized to allow the system to reach a steady state as soon as possible while keeping the input pump voltage within the -10V to 10V range.

Overall, the LQR problem offers a method for optimizing system control by minimizing

Measurement jack	value	unit
Tank 1... Tank 3 (Outputs of sig. adapt. unit or disturbance unit)		
-Range of liquid levels	+10..-10	v
-Nominal value for Ocm	+9	v
-Nominal value for 60cm	-9	v
Q1 and Q2 (=Outputs of PC- controller)		
-Flow rate 0 ml/s	-10	v
-Flow rate 100 ml/s	+10	v

Table 9: Data sheet of test stand

a quadratic cost function. Q and R weighting matrices can be tweaked to target specific states and actuator effort, respectively. As a result, a gain matrix is produced that provides an optimal solution for the constraints specified.

Here, the values are adjusted as follows in Matlab:

```
 %------------------LQR------------------------ %
Control law
 Q = [1000 0 0;
       0 100 0;
       0 0 100];
R = [100000000 0; 0 100000000];

Kop = lqr(A,B,Q,R);
Pop = ([1 0 0;0 0 1]*(B*Kop-A)^-1*B)^-1;
%----------------------------------------------%
```

Result :

The LQR controller was built in MATLAB Simulink and tested on a three-tank system. The simulation results demonstrated that the controller was able to quickly reach the desired levels while keeping the flow rate within the maximum limit. When the performance of the LQR controller was compared to that of a standard proportional-integral (PI) controller, it was discovered that the LQR controller performed better in terms of settling time and overshoot.

Based on the figures, it is clear that the optimized LQR controller performs well in both linear and nonlinear systems. While the simulation results showed that the flow rate Q1 exceeded the maximum flow in the non-linear system, the controller did not exceed the maximum value during actual system testing. It is important to remember that real-world systems are influenced by a variety of factors, which may result in different simulation results. As a result, the control parameters must be tested and adjusted multiple times on the test bench to determine the optimal settings for the controller.

Test result on test stand:

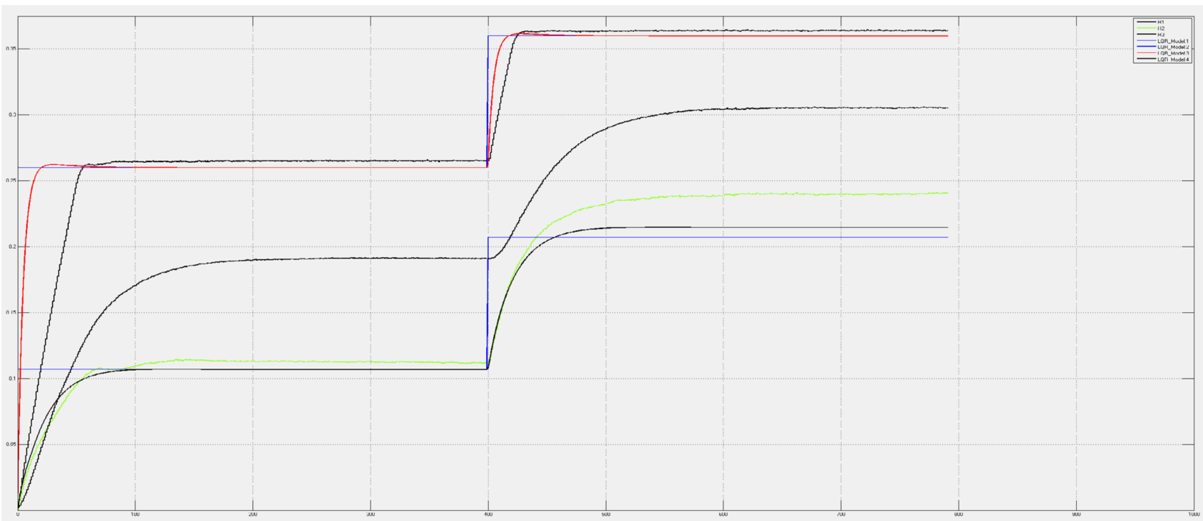


Figure 52: LQR control of MIMO system on test stand

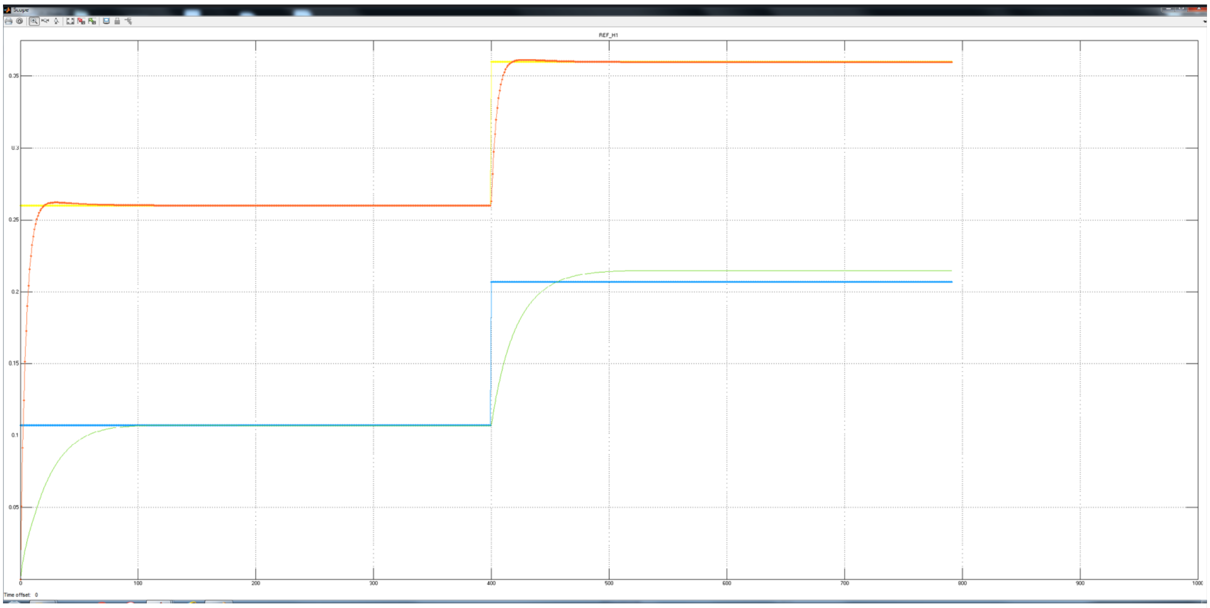
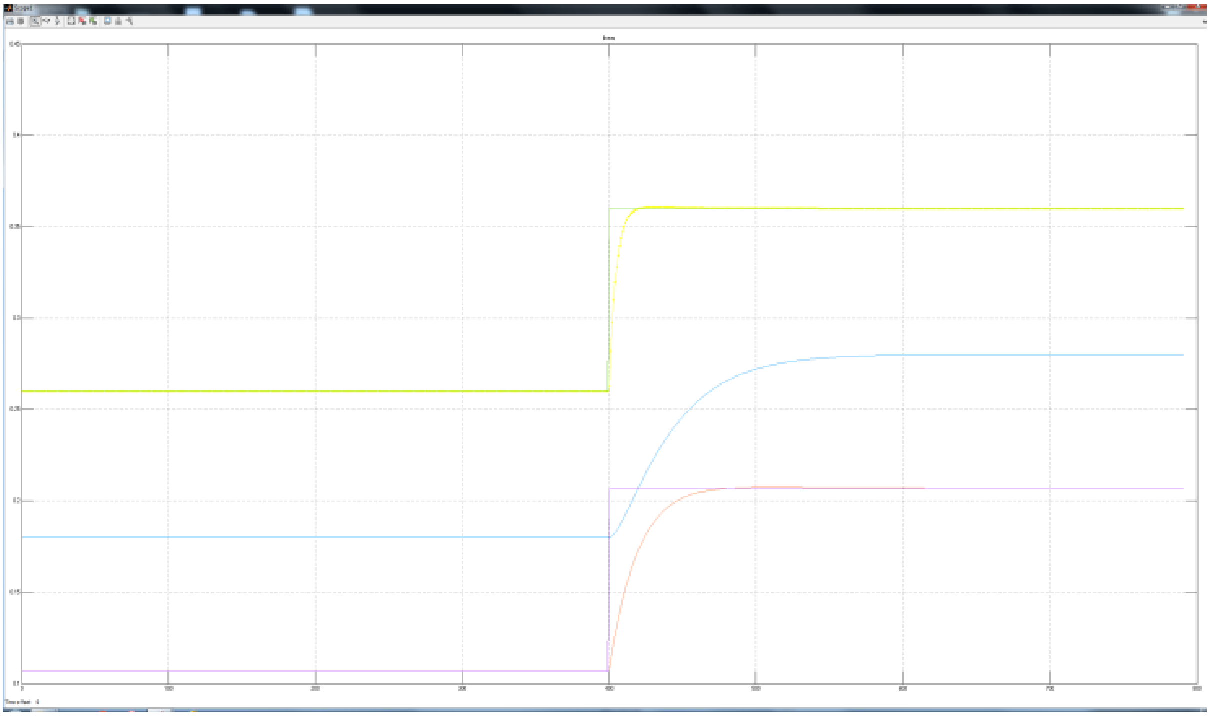
For Reference Height: h1 = 0.1m, h2 = 0.1m
Non-Linear System 
Linear System 

Table 10: Simulation results of LQR control of MIMO system on test stand

The blue line in this figure is the reference line, read, and the last black is a nonlinear system, as well as the upper side black, is tank system height 1, and the green line is tank system height 2. Because there are multiple inputs and outputs that are interconnected in a MIMO (multi-input multi-output) system, the control problem can be even more complex than in a SISO (single-input single-output) system. A simple state feedback control in a MIMO system may not achieve accurate control performance, and the system's output response may still contain steady-state errors, as in the SISO case.

The figure obtained from the MIMO tank system's LQR control shows that there is a difference between the actual output response and the desired reference input value, indicating the presence of a steady-state error. This error can introduce an offset in the feedback loop, causing the system's operating point to shift by a constant value and preventing the output control variables from reaching the desired reference input values.

If the values of h01, h02, and h03 in the figure of the 3-tank system are slightly different from the reference value, this indicates that the system has a steady state error. The steady-state error occurs when the system's output response does not converge to the desired reference value even when the input to the system is held constant. In the case of the 3-tank system, the reference value for the liquid height in each tank is not accurately reached, resulting in a steady state error. This steady-state error can occur for a variety of reasons, including modeling errors, sensor noise, or imperfect actuator response.

To address this issue, a control system can be designed to provide a corrective action to reduce steady-state error and ensure that output variables accurately reach their desired reference values. In the case of the three-tank system, PI output feedback can be used to eliminate steady-state error and improve control system accuracy. This method works by providing a correction high pass filter response to negate the output deviation. The integral action in the PI controller allows the controller to adjust the output response in order to eliminate the steady-state error. The output feedback controller is designed in such a way that it can provide correction for all output variables at the same time.

It is important to note that designing a MIMO LQR control system can be difficult because it involves designing a controller that can effectively control multiple inputs and outputs in a coordinated manner. The controller design should take into account the dynamic interrelationship between the various inputs and outputs, and the weighting matrices used in the design should be carefully chosen to achieve optimal performance. Furthermore, the measurement sensors used to provide feedback to the controller can have an impact on the performance of the control system.

16 Basics of P, PI, PID Controller

Different control strategies are now used in process industries based on their needs and convenience. The feedback control strategy is the most widely used control strategy in industry. It is the control mechanism that manipulates a variable based on information from measurements of the controlled variables to achieve the desired result. The error between the actual process output and the setpoint is 'driven' by the feedback controller. Feedback controllers are classified into several types.

16.1 Proportional Controller (P) controller

The most basic type of controller is a proportional controller, also known as a P controller. It computes the difference between the desired setpoint and the measured process variable and then applies a proportional correction to the control output. The correction is directly proportional to the error, which means that as the error increases, so does the controller's output. The P controller is useful for processes with relatively stable dynamics and no significant oscillations or disturbances.

The proportional gain is defined mathematically as the ratio of the output response to the error signal. In general, increasing the value of a proportional constant accelerates the response time of the control system. When the gain of the controller is increased above a certain threshold, the process response begins to oscillate. As the gain is increased further, the system becomes unstable. Only the proportional constant needs to be observed in the P controller; the other two values, the integral constant and the derivative constant are set to zero.

16.2 Proportional-Integral Controller (PI)

A proportional-integral controller, or PI controller, adds an integral term to the output of a P controller. The integral term accumulates the error over time, allowing the controller to eliminate steady-state errors in the process variable. The PI controller is commonly used for processes that have steady-state errors due to factors such as friction or external disturbances.

The PI controller is a combination of proportional and integral terms that is important in increasing response speed and eliminating steady-state error. It adds the error and increases the integral constant until the error is zero. In the case of the PI controller, the steady-state error is thus zero. This also increases. Feedback is the system's time constant, and it pushes it toward instability. Its step response is oscillating in nature, so its settling time is large.

16.3 Proportional-Integral-Derivative Controller (PID)

A proportional-integral-derivative controller, or PID controller, adds a derivative term to the output of a PI controller. The derivative term computes the rate of change of the error, allowing the controller to respond quickly to changes in the process variable. The PID controller is the most common type of controller and is used in a wide range of applications, including temperature control, speed control, and position control. A PID controller is an appropriate combination of proportional, integral, and derivative terms that provides all of the desired performances of a closed-loop system. It also has no steady-state error and very little overshoot. PID controllers are recommended for use in slow processes that are free of noise.

$$Gc(S) = Kc(1 + \frac{1}{\tau_{is}} + \tau_d S) \tag{55}$$

The tuning of a process refers to the adjustment of control parameters to achieve the desired control response. The tuning of the PID controller refers to the determination of proportional gain (Kc), integral time (i), and derivative time (d). Because the PID controller has three parameters that can be adjusted, many different methods have been developed. Many researchers have proposed different tuning methods for PID controllers. Some of the most important tuning methods are described below.

16.4 PI-Output with LQR Control:

A controller's goal in control theory is to regulate a system's output by adjusting its inputs in response to sensor feedback. In some cases, the system may experience a steady-state error, which means the output does not reach the desired setpoint even when the controller is fully engaged.

The addition of an integrator to the controller is a common method for eliminating steady-state errors. Over time, an integrator accumulates errors and produces an output that continuously adjusts the input until the error reaches zero.

The letters "P" and "I" in a PI controller stand for proportional and integral, respectively. As in a P controller, the controller applies a proportional correction to the input based on the error, but it also includes an integral term that accumulates the error over time. The controller can eliminate steady-state errors by using this integrator term.

Adding an integrator to the controller, on the other hand, raises the order of the closed-loop system. When there are multiple control objectives in a MIMO (multiple-input, multiple-output) system, adding an integrator to each controller increases the order of the system by 2. This can make the system more complicated and difficult to control.

In conclusion, while a PI controller with an integrator is effective at eliminating steady-state errors, the increased order of the closed-loop system can make the system more complex and difficult to control in MIMO systems.

The system structure can be illustrated using an additional integrator as follows:

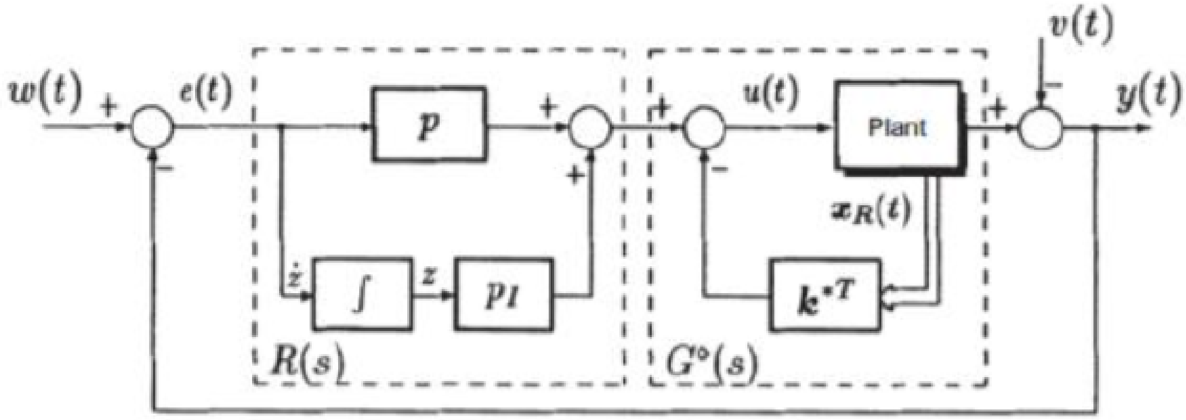


Figure 53: Block diagram of LQR with PI-output feedback control

To design a PI controller, appropriate values for the proportional and integral gains must be chosen. In practice, however, it is frequently difficult to choose these gains solely on intuition, especially for complex systems with many inputs and outputs.

One approach to overcoming this difficulty is to tune the gains using an optimization-based approach. In this approach, a cost function that measures the controller's performance is defined, and the gains are chosen to minimize this cost function subject to constraints.

The cost function is typically a quadratic function of the error and control signals, with weighting matrices Q and R specifying the relative importance of each term. In particular, the cost function, The cost function is defined as:

$$J(u) = \int x(t)^T Q x(t) + u(t)^T R u(t) dt \tag{56}$$

where e(t)' denotes the transpose of the error signal e(t), and the integral is computed over a finite time horizon.

The weighting matrix Q is a positive semi-definite matrix that penalizes the error signal, while the weighting matrix R is a positive definite matrix that penalizes the control signal. The selection of these matrices determines the relative importance of regulating the output while minimizing control effort.

For example, if the desired output is a temperature setpoint and the measured output is the actual temperature, Q could be chosen to penalize deviations from the setpoint, while R could be chosen to penalize rapid changes in the heater or cooler input.

The weighting matrices Q and R can be chosen using various methods, such as trial-and-error, heuristics, or optimization algorithms. The optimal choice depends on the specific system being controlled and the performance criteria of interest.

MATLAB Code for PI LQR Control

```
%% ----------------------PI LQR------------------------

%Qpi = [100 0 0 0 0; 0 100 0 0 0; 0 0 10 0 0; 0 0 0 10 0; 0 0 0 0 10 ];
%Rpi = [100000000000 0;0 1000000000000];
Qpi = [1000 0 0 0 0;0 100 0 0 0;0 0 10000 0 0;0 0 0 10000 0;0 0 0 0 1000 ];
Rpi = [10000000000000000 0;0 1000000000000000];%linear ALSO works fine on nonlinear
lqrKpi = lqr(Api,Bpi,Qpi,Rpi);
lqrKtpi = lqrKpi(:,1:3);
K2_auglqr = lqrKtpi-Pop*Ct;
pi_gainlqr =-lqrKpi(:,4:5);
%-------------------------------------------------------------
```

Simulink Model of PI LQR Control

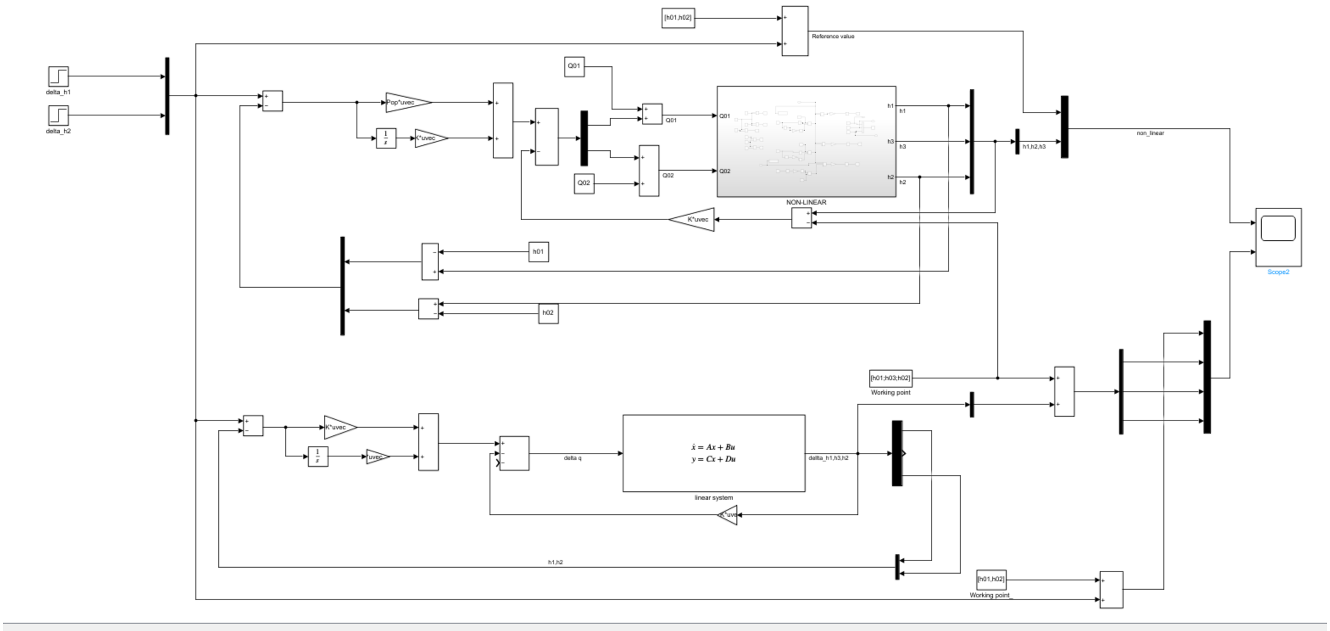


Figure 54: Simulink Model of PI LQR Control

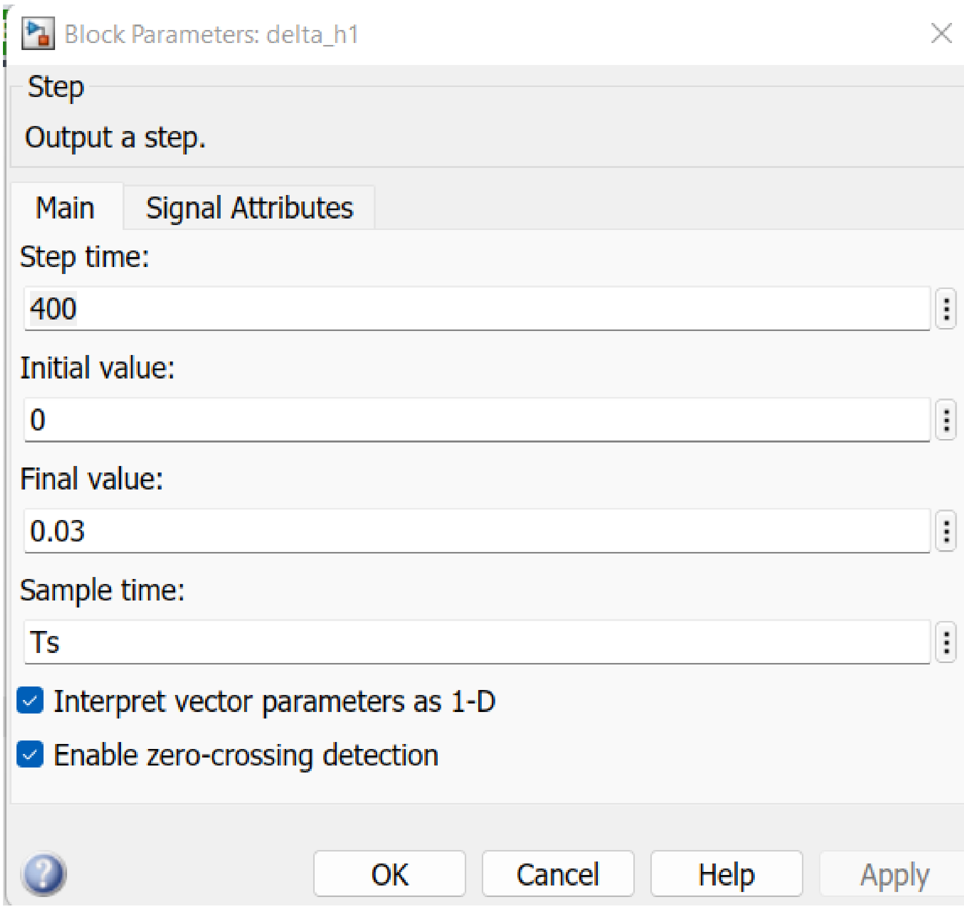


Figure 55: Input Reference Height 1

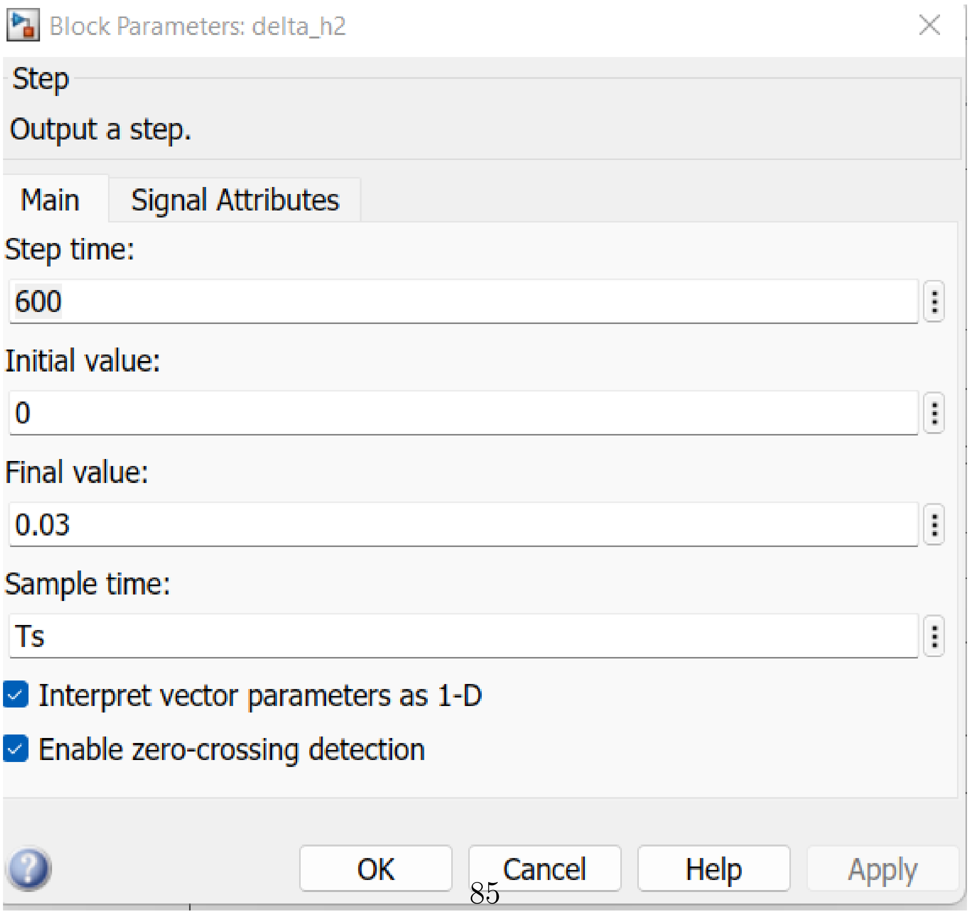


Figure 56: Input Reference Height 2

Non-Linear System	Linear System
For Reference Height:	h1= 0.27m, h2=0.102m

Table 11: Height Parameter

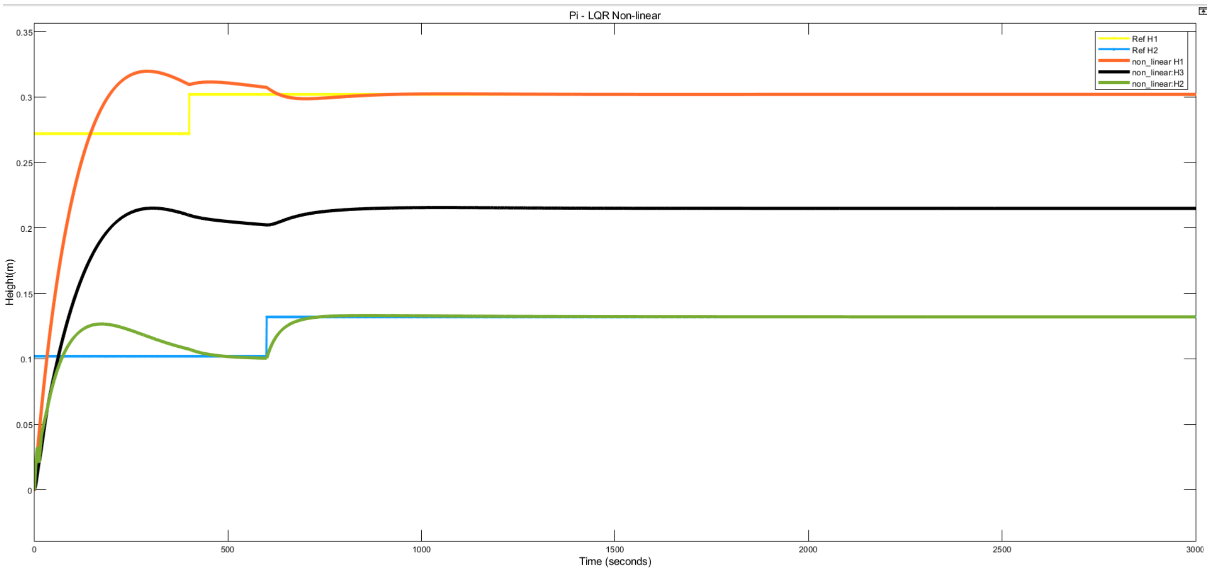


Figure 57: PI LQR Non-linear Graph

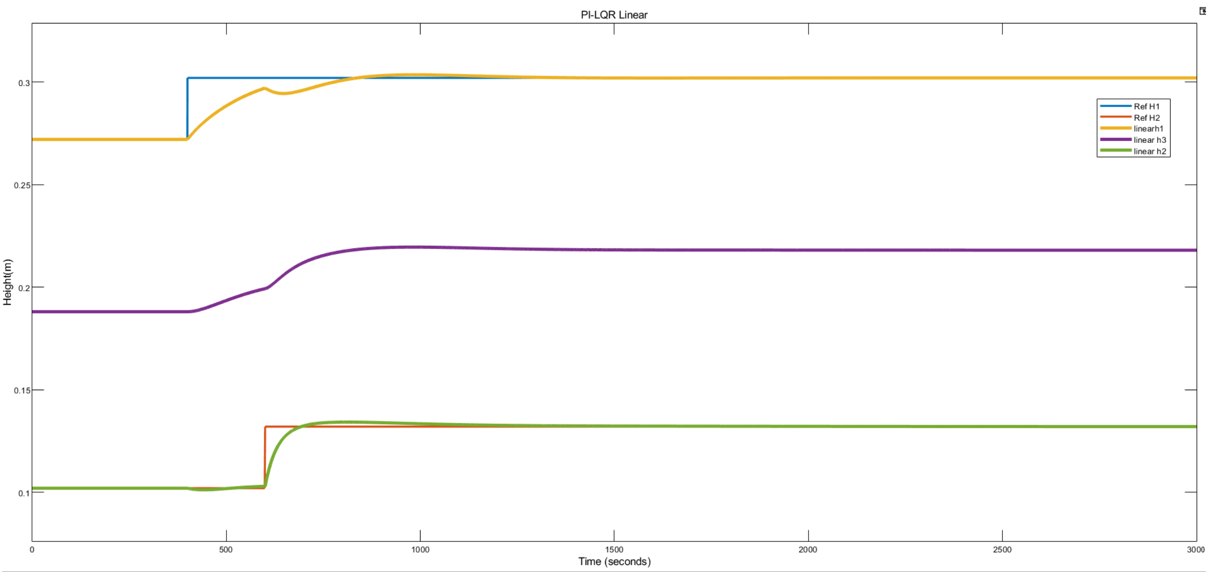


Figure 58: PI LQR Linear Graph

In order to obtain an accurate representation of this behavior. This means that the simulation ran for 3000 seconds in order to collect data on the systems being studied.

Tanks 1 and 2. Each tank has a reference height value of 0.27m for Tank 1 and 0.102m for Tank 2. This means that these tanks have a specific liquid (Water) height that is used as a reference point in the simulation.

The figure depicts the system's output; we changed the height in tank 1 and tank 2 as a step response, as shown in Figs 24 and 25, and it balances well to the reference height (0.27m and 0.10m) in both linear and non-linear systems. This can be achieved by adjusting matrix Q and R matrixes.

In brief, the flow rates Q1 and Q2 in the simulation of the nonlinear system have some troubleshooting. This implies that the nonlinear system was pushed beyond its operating limits or constraints, and we attempted to adjust the control strategy to prevent this. The implications of this are dependent on the specific system and the consequences of exceeding the maximum flow rate. It could result in system failure, decreased performance, or safety risks, among other things.

The absence of significant oscillations or divergent behavior in the system's output indicates the stability of the system. Furthermore, the flow rates of both pumps in the system have been regulated so that they do not exceed the system's maximum flow rate.

This is accomplished by imposing a penalty on the flow rate value in order to keep it within the allowable limits of the actuator matrix.

However, it is important to note that even with a state feedback control, non-linear systems can be sensitive to changes in initial conditions. This means that small variations in the initial state of the system can lead to significant changes in the output behavior, which can potentially destabilize the system.

The difference between simulation and real-world test stand results

As previously stated, the flow rate of the controller exceeded the maximum value in the simulation. However, it goes on to say that the flow rate of the controller did not exceed the maximum value on the test stand (presumably a physical setup for testing the system). This indicates that there is a mismatch between the simulation and the real-world results.

There are numerous factors that can affect the performance of a real system that may not be accounted for in the simulation. Environmental conditions, component tolerances, and other unforeseen issues that may arise during operation are examples of these factors. Therefore, it is common practice to test and adjust the control parameters of a system on a test bench multiple times in order to optimize the system's performance and ensure it meets the required specifications.

16.5 Steady-State validation of PI-LQR Control System

The three-tank system is a process control system comprised of three tanks connected in series. The system's goal is to keep the level of each tank at a preset value. The system is modeled using a state space representation, with the state variables being the tank heights and the input variables being the voltages applied to the pumps. PI control is an effective method for controlling the flow rate from each tank while maintaining the desired level set points.

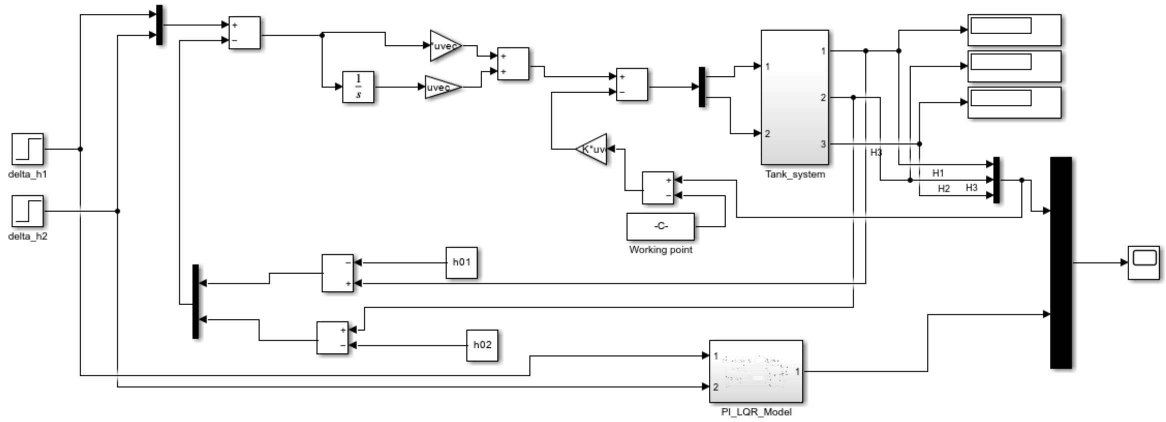


Figure 59: Simulink Model for PI-LQR Control System

• The PI-LQR Tank control system is made up of two major components: the proportional-integral (PI) controller and the linear quadratic regulator (LQR). The PI controller is used to adjust the system input based on the difference between the desired and actual output. The LQR is used to create the optimal feedback gain matrix to minimize a specific cost function.

• The system's state-space representation can be described by the matrices A, B, C, and D. In this case, you designed the LQR gain matrix lqrKpi and the PI gain matrix pi-gainer using the LQR and PI controller design methods, respectively.

• The LQR gain matrix lqrKpi determines how much the control input should be adjusted based on the current state of the system, whereas the PI gain matrix pi-gainer determines how much the control input should be adjusted based on the integral of the error between the desired output and the actual output.

• The weighting matrices used in the LQR design are matrices Qpi and Rpi. The matrix Qpi is used to weigh the importance of each state variable in the cost function, whereas the matrix Rpi is used to weigh the importance of each control input in the cost function.

• The matrices Q and R are also weighting matrices used in the design of the • The matrices Q and R are also weighting matrices that are used in the design of the optimal feedback gain matrix Kop. The matrix Q is used to weigh the importance of each state variable in the cost function, while the matrix R is used to weigh the importance of each control input in the cost function.

• The matrix Kop is the optimal feedback gain matrix designed using the LQR method. It calculates how much the control input should be adjusted based on the current state of the system in order to minimize the cost function.

• the matrix Pop is the pre-compensator matrix that is intended to eliminate the system's steady-state error. It is used to alter the system's input based on the desired output in order to compensate for any error in the system.

• The PI controller: This controller computes a control input that drives the tank to a desired velocity and heading based on measurements of the tank's velocity and heading. To achieve the desired performance, the PI gains pi-gainlqr are selected.

• The LQR controller: This controller computes a control input that drives the tank to a desired position and orientation based on measurements of the tank's position and orientation. The LQR gain matrix K2-auglqr is chosen to achieve the desired performance while accounting for the effect of the PI controller on the system. Overall, the pi-lqr tank control system employs a combination of the PI controller and the LQR to create an optimal feedback gain matrix, which is used to adjust the control input to the system based on the current state of the system and the desired output, in order to minimize the cost function and achieve optimal performance.

MATLAB Code for PI LQR Control Validation

```
Qpi = [10 0 0 0 0 ;0 10 0 0 0;0 0 100 0 0;0 0 0 1000 0;0 0 0 0 1000 ];

Rpi = [100000000000 0;0 100000000000];

lqrKpi = lqr(Api,Bpi,Qpi,Rpi);

lqrKtpi = lqrKpi(:,1:3);

K2-auglqr = lqrKtpi-Pop*Ct;

pi-gainlqr = -lqrKpi(:,4:5);
```

16.5.1 Test result on test stand

Based on a simulation time of 1000s and reference height values of 0.1m and 0.1m for tank 1 and tank 2, respectively, the output of the Non-Linear System with the reference inputs was found to be stable. It is also worth noting that Non-Linear systems are stable for these input reference heights.

When presenting the results of the PI-LQR control, you can mention that the simulation time of 1000s was sufficient to obtain an accurate representation of the behavior of the systems under consideration. Furthermore, the stability of Non-Linear systems for reference heights of 0.1m and 0.1m can be highlighted as a positive result of the control design.

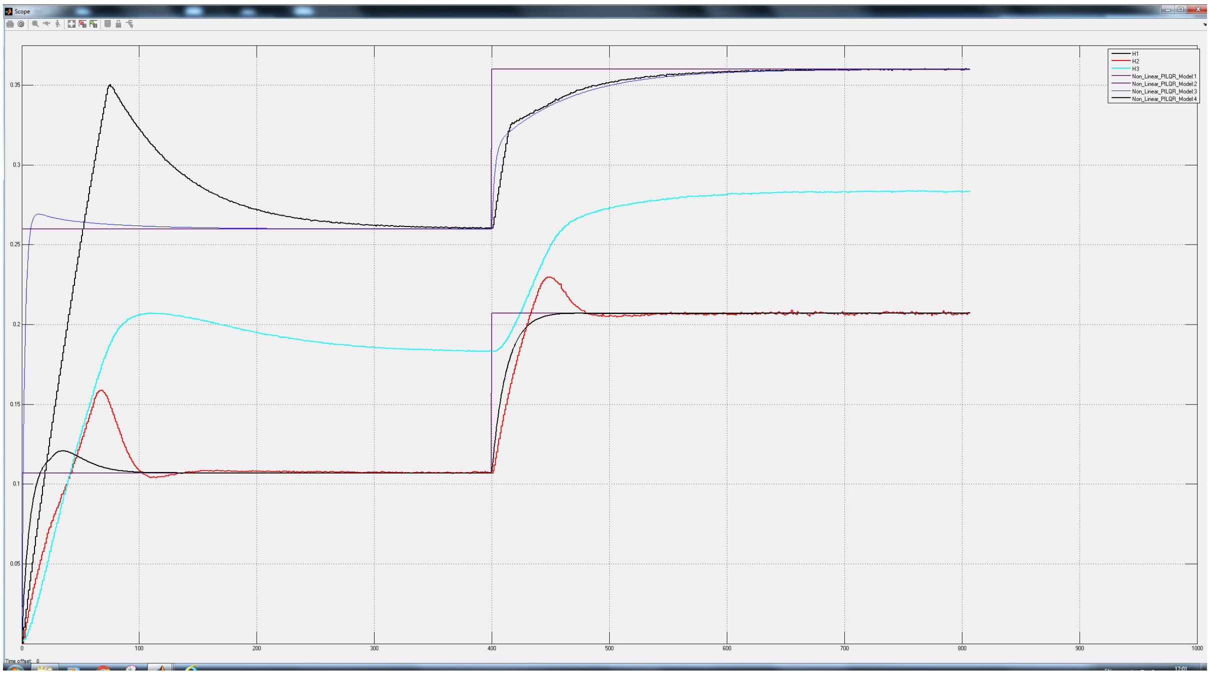


Figure 60: Validation PI-LQR Control System Geaph

Based on the simulation results, it can be seen that the PI control logic was successful in stabilizing both systems and eliminating the steady-state error. The system was able to reach the reference height level in a short period of time without exceeding the maximum flow rate. This indicates that the PI-LQR control was effective in regulating system dynamics while keeping the system within safe operating limits.

Furthermore, the LQR control approach used in this study has several advantages over other control methods. One of the primary advantages of LQR control is that it can achieve optimal control performance while meeting certain constraints or limitations. This is especially useful for systems with multiple inputs and outputs, where determining the best control parameters can be difficult. LQR control allows for parameter adjustment based on specific control requirements, which can assist in achieving the desired control goal.

Overall, the PI-LQR control system results show that it is effective at regulating the behavior of the system under consideration. These findings can be applied to the development of control strategies for similar systems as well as the optimization of existing control systems.

17 The Leakage test performed on a MIMO system

The three-tank system is a process control system that consists of three series-connected tanks. The system's goal is to keep each tank's level at a predetermined level. The system is represented using a state space representation, with the state variables being the tank heights and the input variables being the voltages applied to the pumps. PI control is a reliable method for regulating the flow rate from each tank while maintaining the desired level set points.

Leakage can be applied in the form of flow rate changes. It will have an effect on the water level in each of the tank heights H1, H2, and H3. We attach a PI-LQR controller, which will control the model and keep the liquid level steady and stable.

The design of the nonlinear system for the Leakage:

In a system of interconnected tanks, the Leakage is applied to each tank in the system. When there is a leak in the system, it is critical to account for the effect of the leak on fluid flow into and out of the tanks.

To accomplish this, Leakage is added as an input to each tank. This means that the flow of fluid through the inlet is treated as an additional input to the tank, alongside the regular input flow.

Assume there is a tank with an input flow of 10 liters per minute and leakage that causes a reduction of 2 liters per minute of fluid. To account for this leak in the model, the 2 liters per minute of fluid will affect the liquid height of each tank, and the original input flow of 10 liters per minute would be subtracted by 2 liters per minute, resulting in a net input flow of 8 liters per minute to the tank.

17.1 MIMO Leakage test with simulation model

Any signal or noise that affects the system is referred to as leakage. It can have a negative impact on system performance, so it is critical to test the system's ability to suppress these leakages.

To see if the MIMO system under the control of the LQR controller can effectively suppress leakages, we must simulate the system in a controlled environment. This can be accomplished by introducing Leakage into the system and measuring its response. Leakage can be introduced in a variety of ways, including adding noise to the inputs, changing the system's parameters, or introducing external signals that affect the system.

Once the Leakage is added, the system's response can be measured using performance metrics such as settling time, rise time, overshoot, and steady-state error. These metrics can be used to assess the effectiveness of the LQR controller in preventing leakage. Assume the performance metrics are within an acceptable range. In that case, we can conclude that the MIMO system is under the control of the LQR controller and can well suppress the Leakages that may occur in the operation process.

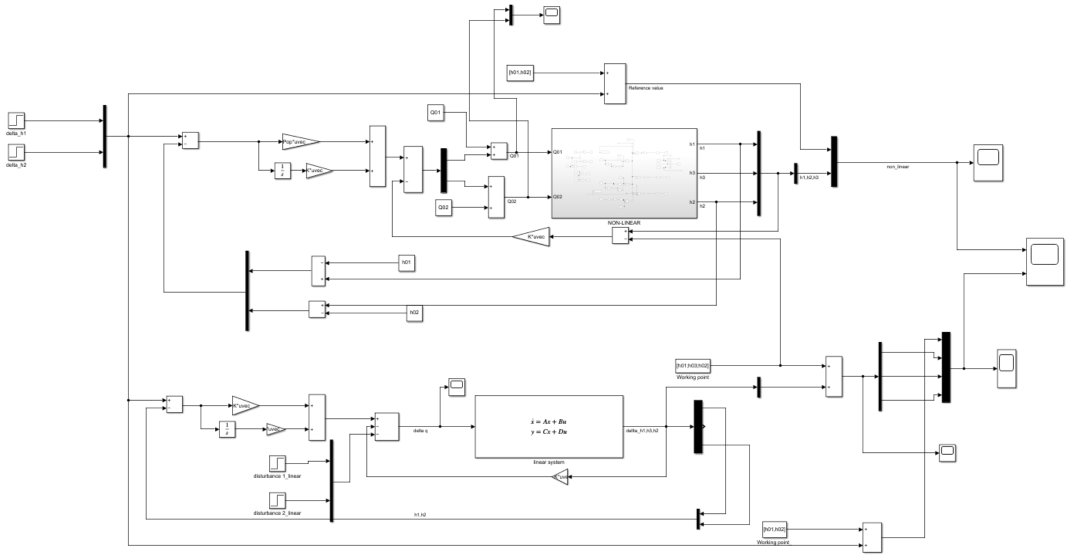


Figure 61: Simulation model of Leakage applied on the MIMO system

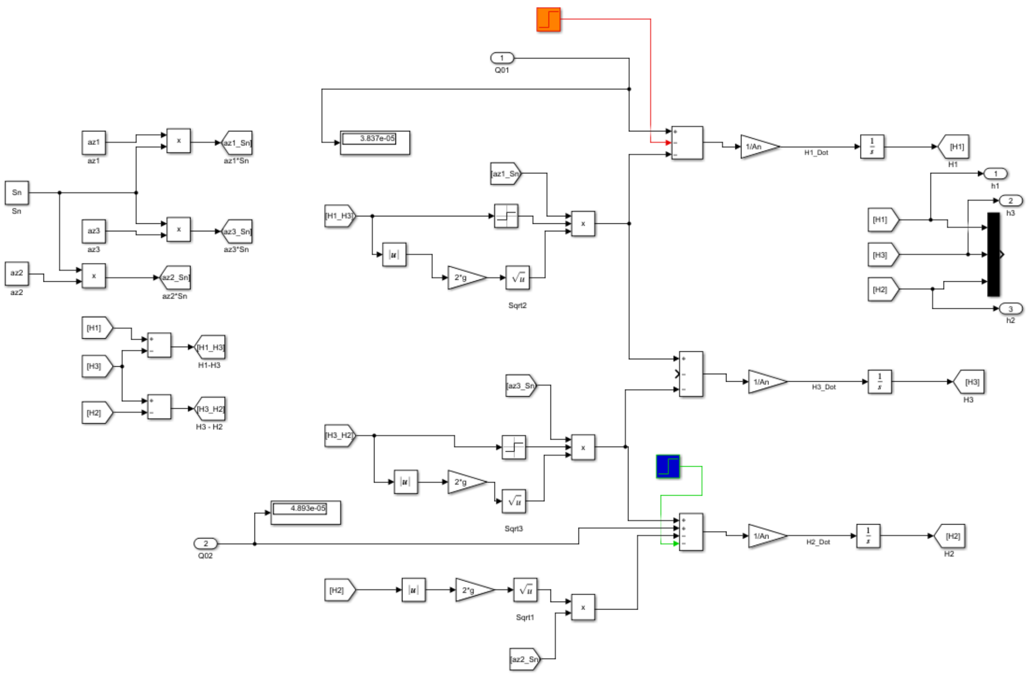


Figure 62: Leakage of Non-linear Model

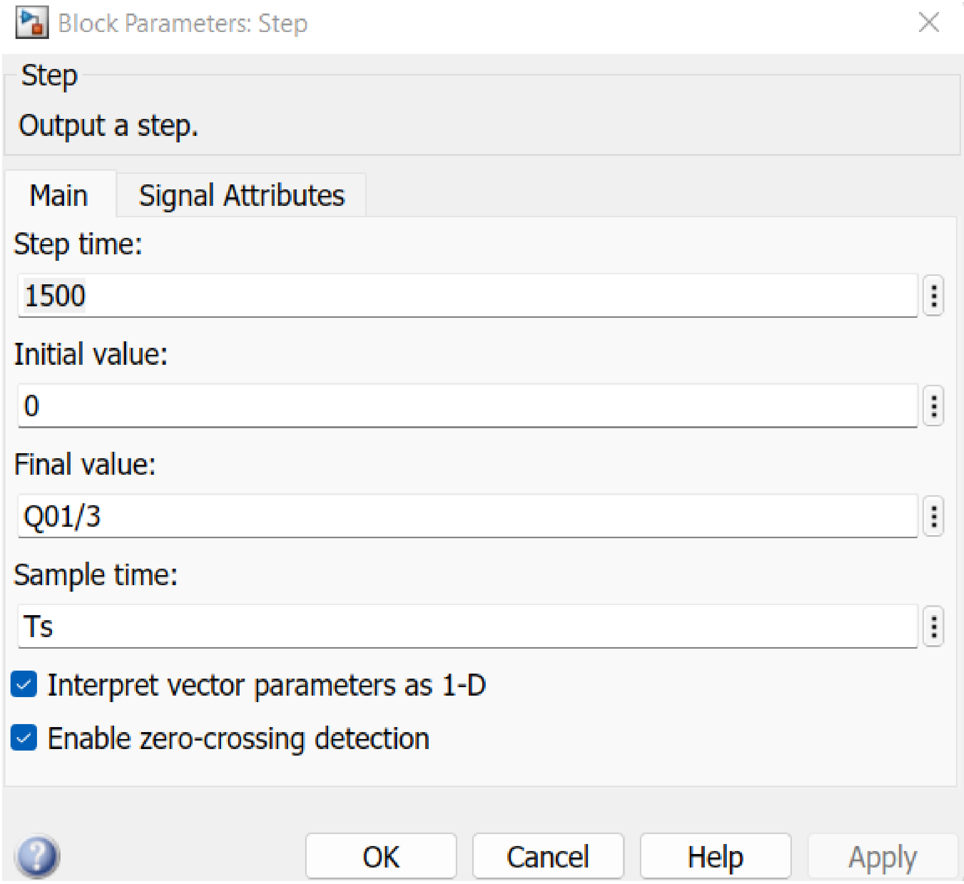


Figure 63: Leakage-step Response Tank 1

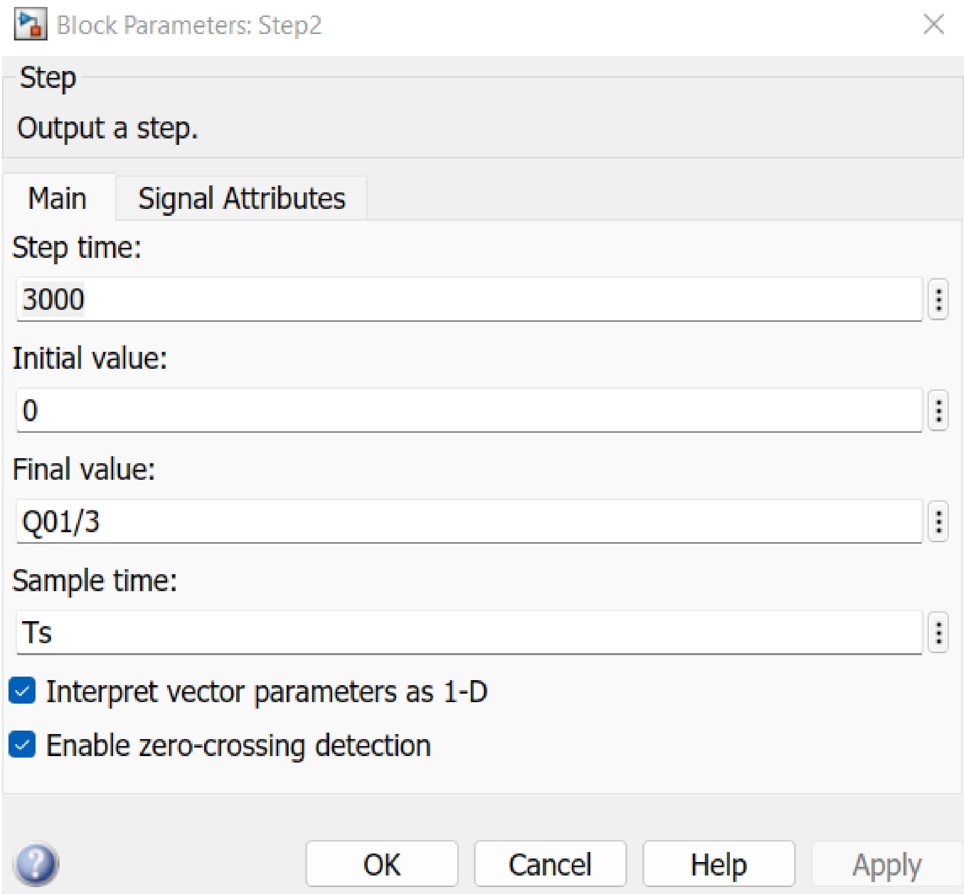


Figure 64: Leakage-step Response Tank 2

A system consists of tanks, each affected by a different type of interference.

The step reference in the figure represents the input interference of both the linear and nonlinear systems. Input interference is defined as any external signal that affects the system's inputs. The step reference appears to represent a change in the system's input values.

The interference of the orange font is applied as a Leakage of $\frac{Q01}{3}$ to Tank 1, and the interference of the blue font is applied as a Leakage of $\frac{Q01}{3}$ to Tank 2. It is likely that the interference of orange and blue fonts represents different types of signals affecting each tank. For example, the interference in Tank 1 could be a change in the flow rate of the liquid entering the tank, whereas the interference in Tank 2 could be a change in the flow rate of the liquid entering the tank.

The fact that each tank experiences different types of interference suggests that the system is a MIMO (Multiple Input Multiple Output) systems, which means that there are multiple inputs and multiple outputs. The inputs in this case are the signals affecting each tank, and the outputs could be various system measurements, such as the level of liquid in each tank or the flow rate of liquid leaving each tank.

The purpose of the testing procedure is to see how the LQR controller reacts to a system leakage, specifically a change in the flow input to Tanks 1 and 2.

The testing procedure entails adjusting the Leakage plate to reduce the flow input to tank 1 by frac $Q - 013$ 1500 seconds after the system reaches the steady-state working point and adjusting the Leakage plate to reduce the flow input to tank 2 by frac $Q - 013$ 3000 seconds after the system reaches the steady-state working point. The steady-state working point is the system's desired operating point when the liquid levels in both tanks are at their desired levels.

By introducing a leakage into the system, we can see how the LQR controller reacts and whether it is able to keep the liquid levels in both tanks at their steady-state working point. The LQR controller is intended to adjust the system's inputs in response to changes in the system's state in order to minimize a cost function.

The testing procedure will most likely involve monitoring the liquid levels in both tanks over time and measuring the performance of the LQR controller in terms of how quickly it can respond to the disturbance and restore the liquid levels to their steady-state working point. The performance of the LQR controller can be measured using various metrics such as settling time, rise time, overshoot, and steady-state error.

Overall, the described testing procedure is a critical step in evaluating the performance of the LQR controller in a real-world application. We can ensure that the controller can maintain the desired operating point of the system even in the presence of external factors that may affect the system by testing the system's response to leakage.

The test results are shown as follows:

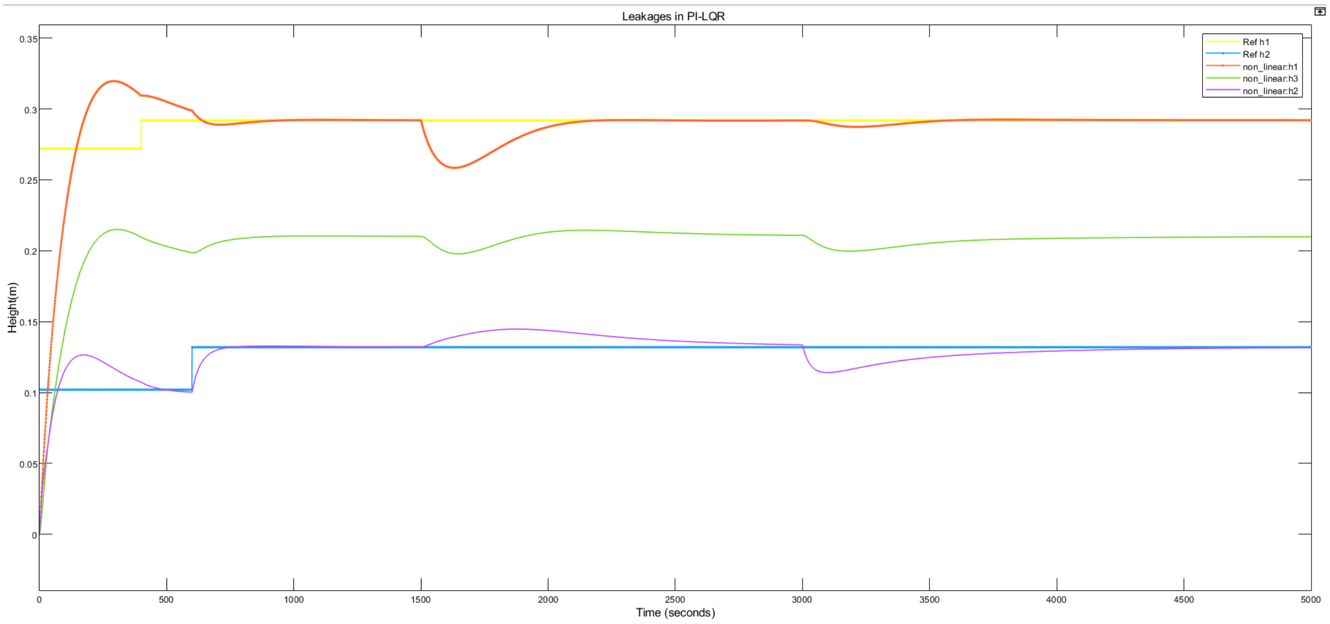


Figure 65: Simulation Leakage Graph

In response to external flow leakage, the test results of a system with two tanks that are controlled by a PI (Proportional-Integral) controller.

The figure above depicts the leakage applied in a system, where the settings of these leakage plates are mentioned above as at 1500 seconds the leakage of Q01/3 is applied for tank 1 and is represented as an orange line in the graph after the system reaches the steady-state working point. The water level in Tank 1 falls as a result of the leak, but the controller compensates by increasing the Motor input.

Where the settings of these Leakage plates are mentioned above, the leakage of Q01/3 is applied for tank 2 at 3000 seconds and is represented as a violet line in the graph after the system reaches the steady-state working point. The water level in Tank 2 drops as a result of the leak, but the controller compensates by increasing the Motor input, and by doing so, it affects Tank 3 as well as the height. Finally, notice how the LQR controller keeps the liquid levels in Tanks 1 and 2 at the steady-state working point.

The results of the tests show that under the control of the PI controller, both linear and nonlinear systems can effectively suppress external flow leakages. This means that the PI controller can change the system's inputs in response to changes in the system's state in order to keep the liquid levels in both tanks at the desired levels.

The term "linear system" most likely refers to a simplified mathematical model of the system in which the inputs and outputs are assumed to have linear relationships. The term "nonlinear system" most likely refers to a more complex mathematical model of the system that takes nonlinear relationships between inputs and outputs into account.

The results of the tests show that both the linear and nonlinear models can effectively suppress external leakage, indicating that the PI controller is resistant to different modeling assumptions. This is an important result because it implies that the controller can keep the system operating at the desired operating point even in the presence of uncertainties and modeling errors.

Therefore, the test results show that after applying Leakages, the liquid levels in both tanks are maintained at their desired levels, indicating that the PI controller can effectively regulate the system's behavior. This is an important goal in many engineering applications where it is necessary to maintain certain operating conditions to ensure the system's safety and efficiency.

Accordingly, the test results indicate that the PI controller is an effective control strategy for regulating the behavior of a system with two tanks in the presence of external flow leakage. The findings also suggest that both linear and nonlinear modeling assumptions can be used to effectively represent system dynamics.

A critical testing procedure is required to evaluate the performance of the LQR controller in a real-world application. This testing procedure involves examining the system's response to leakages, which ensures that the controller can keep the system operating at the desired operating point despite external factors that may affect it. This step contributes to the system's stability and reliability under varying conditions.

17.1.1 MIMO Leakage validation:

Leakage refers to any signal or noise that negatively affects a system's performance. To test the ability of a MIMO system with an LQR controller to suppress leakages, simulations can be performed with added leakages and the system's response can be measured using performance metrics such as settling time and steady-state error. If the performance metrics are within acceptable ranges, it can be concluded that the MIMO system can effectively suppress the leakages that may occur during operation. The figure describes a system consisting of two tanks, each affected by a different type of interference. The step reference in the figure represents the input interference of tank nonlinear systems, which refers to any external signal that is affecting the inputs of the system. The both blue box represents interference applied as a Leakage of Q01 to Tank 1 and Tank 2, These different types of interference suggest that the system is a MIMO (Multiple Input Multiple Output) system with multiple inputs and multiple outputs.

The goal of the testing procedure is to observe how the LQR (Linear Quadratic Regulator) controller responds to a Leakage in the system, specifically a change in the flow input to Tank 1 and Tank 2. The LQR controller is designed to adjust the inputs to the system in response to changes in the system's state, in order to minimize a cost function. The testing procedure involves setting the Leakage plate to reduce the flow input to Tank 1 by Q01, 500 seconds after the system reaches the steady-state working point, and the same for Tank 2 by Q01, 500 seconds after the system reaches the steady-state working point.

The steady-state working point is the desired operating point of the system, where the liquid levels in both tanks are at their desired values. By introducing a Leakage to the

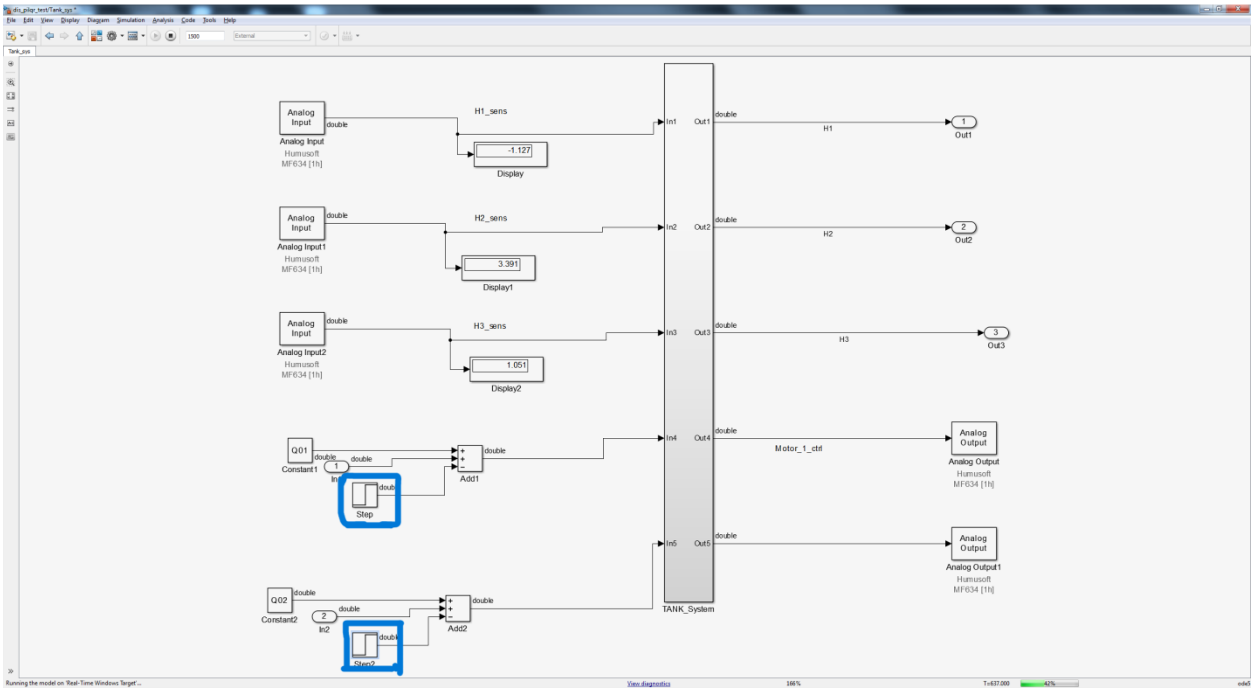


Figure 66: Disturbances applied on the MIMO system

system, the response of the LQR controller can be observed, and whether it is able to maintain the liquid levels in both tanks at their steady-state working point.

The testing procedure will likely involve monitoring the liquid levels in both tanks over time and measuring the performance of the LQR controller in terms of how quickly it can respond to the disturbance and bring the liquid levels back to their steady-state working point. The performance of the LQR controller can be measured using various metrics, such as settling time, rise time, overshoot, and steady-state error.

Overall, the testing procedure described is an important step in evaluating the performance of the LQR controller in a real-world application. By testing the system's response to Leakage, it can be ensured that the controller can maintain the desired operating point of the system even in the presence of external factors that may affect the system.

The objective of the test was to evaluate the effectiveness of a PI controller in regulating the behavior of a system with two tanks in the presence of external flow Leakage. The PI controller is a type of feedback control system that adjusts the input to the motor based on the difference between the desired water level and the actual water level in the tanks.

The test involved applying Leakage to Tank 1 and Tank 2 at the same time using a valve. The change in water level in each tank over time was represented on a graph as a black line and a red line, respectively. The results showed that the water level in both tanks decreased due to the Leakage, but the PI controller compensated for this by increasing the input to the motor. This allowed the controller to maintain the desired water level in both tanks by adjusting the inputs to the system in response to changes in the system's state.

The results demonstrated that the Tank nonlinear systems were effective in suppressing

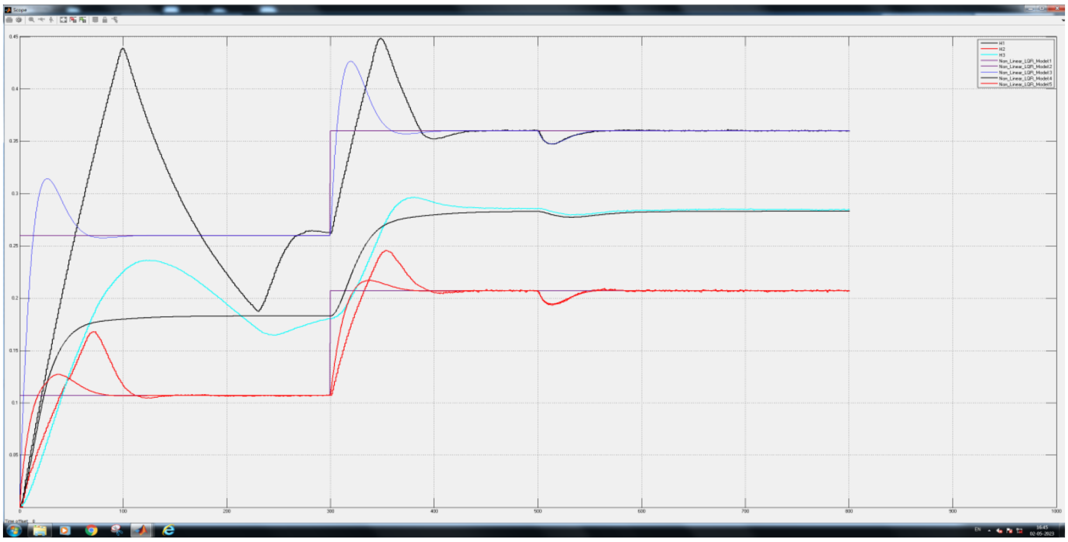


Figure 67: MIMO system PI LQR control disturbances test

external flow Leakages under the regulation of the PI controller. This implies that the PI controller can adjust the inputs to the system in response to changes in the system's state, maintaining the liquid levels in both tanks at their desired values. The findings indicate that the PI controller is robust to different modeling assumptions, as it was able to maintain the desired operating point of the system in the presence of uncertainties and modeling errors.

It is crucial to ensure the stability and reliability of the system under different conditions. Therefore, evaluating the performance of the LQR (Linear Quadratic Regulator) controller in real-world applications is essential. This involves examining the system's response to Leakages, which helps to guarantee that the controller can maintain the desired operating point of the system despite external factors that may affect it. The LQR controller is a feedback control system that minimizes the difference between the actual and desired states of a system by adjusting the input to the system. By evaluating the performance of the LQR controller, one can determine its effectiveness in regulating the behavior of the system under different conditions.

18 GUI Model Simulation

18.1 Assumption

User should understand the dynamics of a tank system with a simple input and outflow proportional to depth.

18.2 Aim

The purpose of this GUI is to explore the impact of uncertainty on our ability to control the depth in the tank.

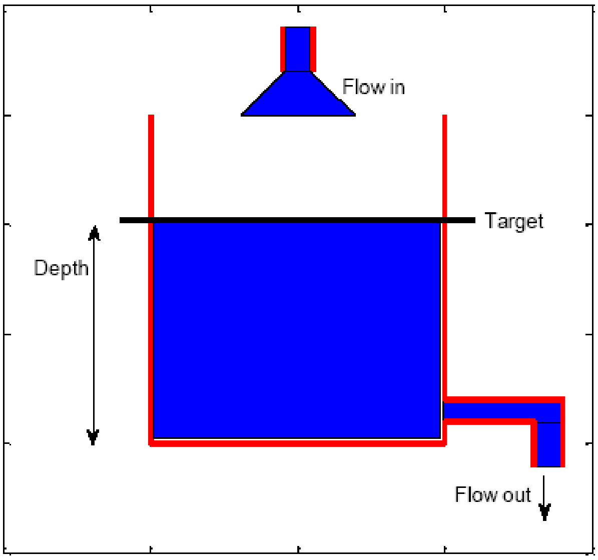


Figure 68: The general diagram of tank with water level

18.3 Model

For h the depth, A the cross-sectional area, R a constant linked to the outflow pipe, and f_{in} = flow in, then a simplified linear model is:

$$A\frac{dh}{dt} + Rh == f_{in} \tag{57}$$

18.4 Add Uncertainty

1. The flow in varies from the measured/planned flow by an unknown, but small amount f_{dist}.
2. The resistance R of the outflow pipe is different to the expected value, and thus the outflow f_{out} and system parameter R are different from that expected.

$$A\frac{dh}{dt} + Rh + R_u h == f_{in} + f_{dist} \tag{58}$$

18.4.1 Control of Level

Four alternatives are possible which the user selects with a drop down menu.

1. Manual control (estimate fin required) and enter desired flow through a slider.
2. User designed Proportional control.
3. User designed PI control.
4. Auto-tuned PI (based on A and R and tuned to give a time constant equivalent to open-loop dynamics).

18.4.2 Control Law Structure

$$f_{in} = \left(K_p + \frac{K_i}{s}\right)(r - h) \tag{59}$$

18.4.3 System Parameters

Users are also able to change the default A, R parameters to investigate tanks with different time constants and gains.

The inflow keeps changing (this is set to fixed values that change randomly every 300 seconds so user can see the impact of uncertainty in the inflow). The time constant and gain change as the R value changes. The true R (that is $R+R_u$) is unknown because, due to stiction and other uncertainty, the outlet value aperture is never known precisely. Each disturbance can be switched off individually to observe each effect separately.

18.4.4 PI Design

A typical feedback loop has compensator M(s) that can be either proportional or PI. Assume the system G(s) is 1st order so that:

$$G(s) = \frac{k}{T_s + 1} \tag{60}$$

$$M(s) = \frac{K_p s + K_i}{s} \tag{61}$$

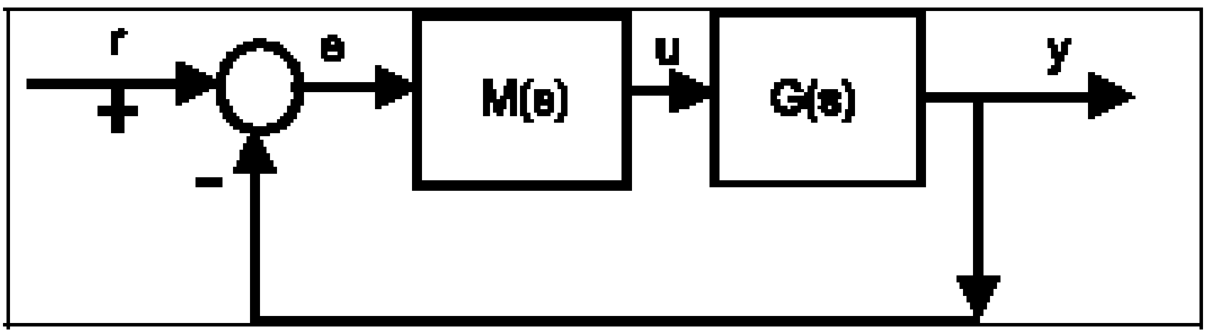


Figure 69: Control loop flow

18.4.5 Modelling

Switch on the inlet flow disturbance which means that the actual flow (green) will differ from the expected flow. The red line shows a scaled version of the disturbance flow only.

The GUI is slow to detect changes in the target depth and some other parameters. Wait a few cycles (say 600 sec) to be sure the effect is in place. The uncertain value of R must be entered manually (within limits of 0.01<R<0.1, 0.008<R+Ru<0.12). This allows the user to investigate more precisely the impact of such changes. With small R, the system time constant may be larger than 500 and level control will become difficult. With large R, you may need to exceed flow limits of 0.1 and thus not obtain target.

$$A\frac{dh}{dt} + [R\rho g]h = f_{in} \tag{62}$$

$$f_{in} == K_p\left(r - h\right) + K_i \int_0^t \left(r - h\right)\, dt \tag{63}$$

$$A\frac{dh}{dt} + [R\rho g]h = K_p\left(r - h\right) + K_i \int_0^t \left(r - h\right)\, dt \tag{64}$$

$$A\frac{d^2h}{dt^2} + [R\rho g]\frac{dh}{dt} = K_p\left(\frac{dr}{dt} - \frac{dh}{dt}\right) + K_i\left(r - h\right) \tag{65}$$

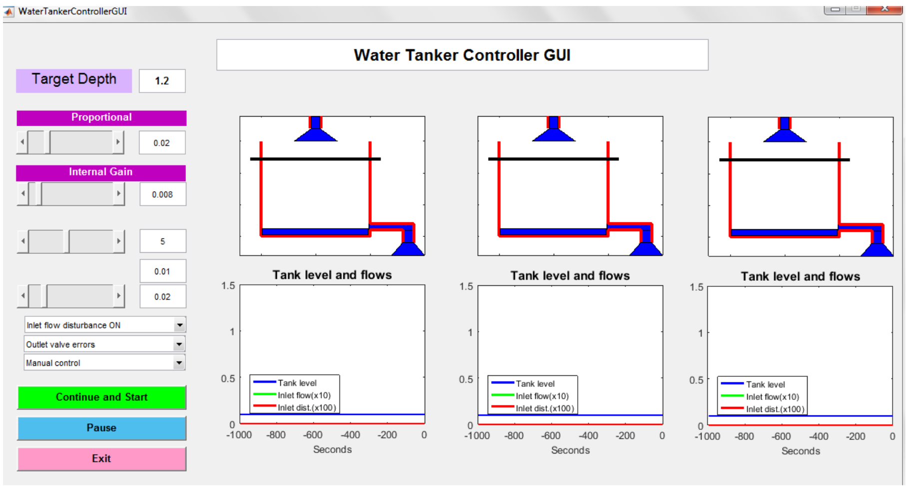


Figure 70: GUI general layout

18.5 Results

Following are the results for target depth 1 and flow rate also 1.

$$h\left(t\right) = 1 - 0.84e^{\text{-0.02t}} sin\left(0.06t + 1.25\right) \tag{66}$$

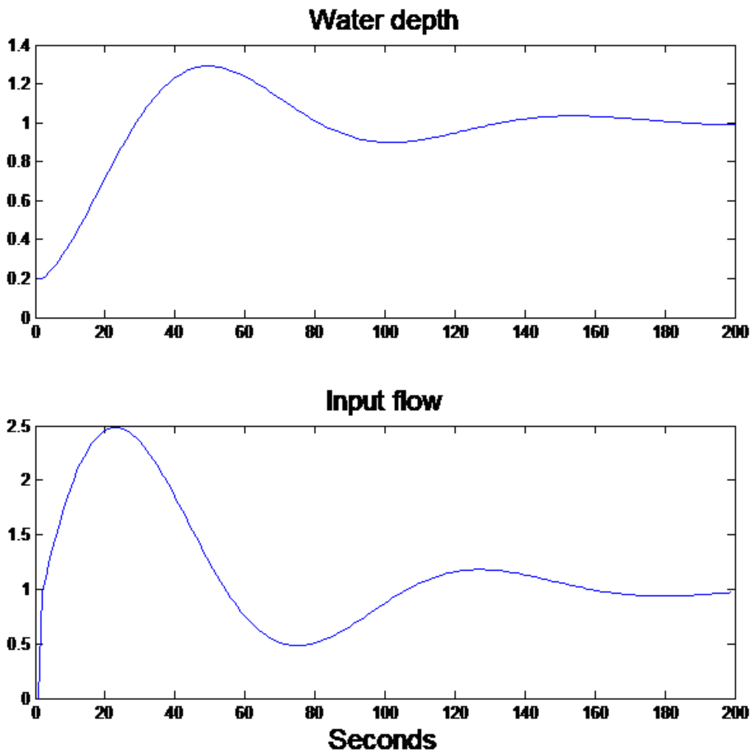


Figure 71: Basic water level and input flow

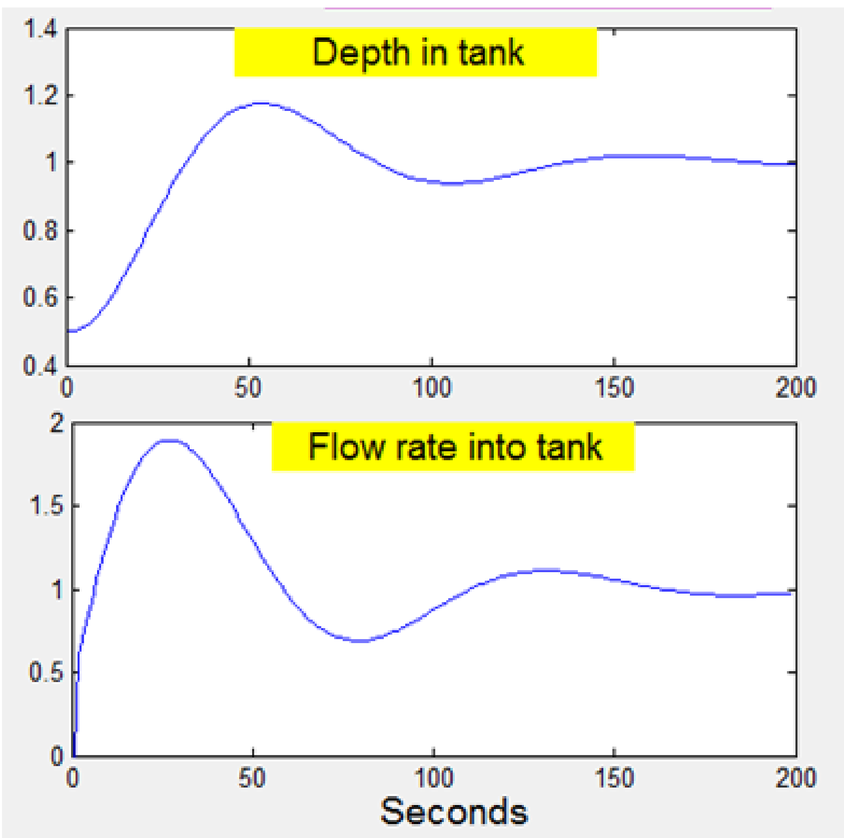


Figure 72: Depth and flow with different gain

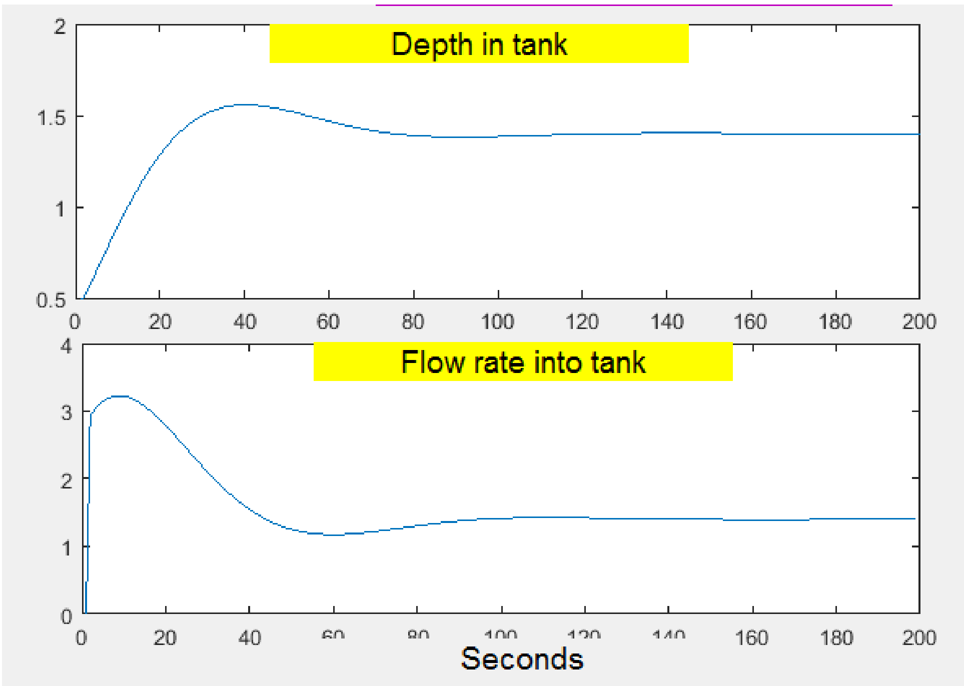


Figure 73: Target depth 1.5 and flow

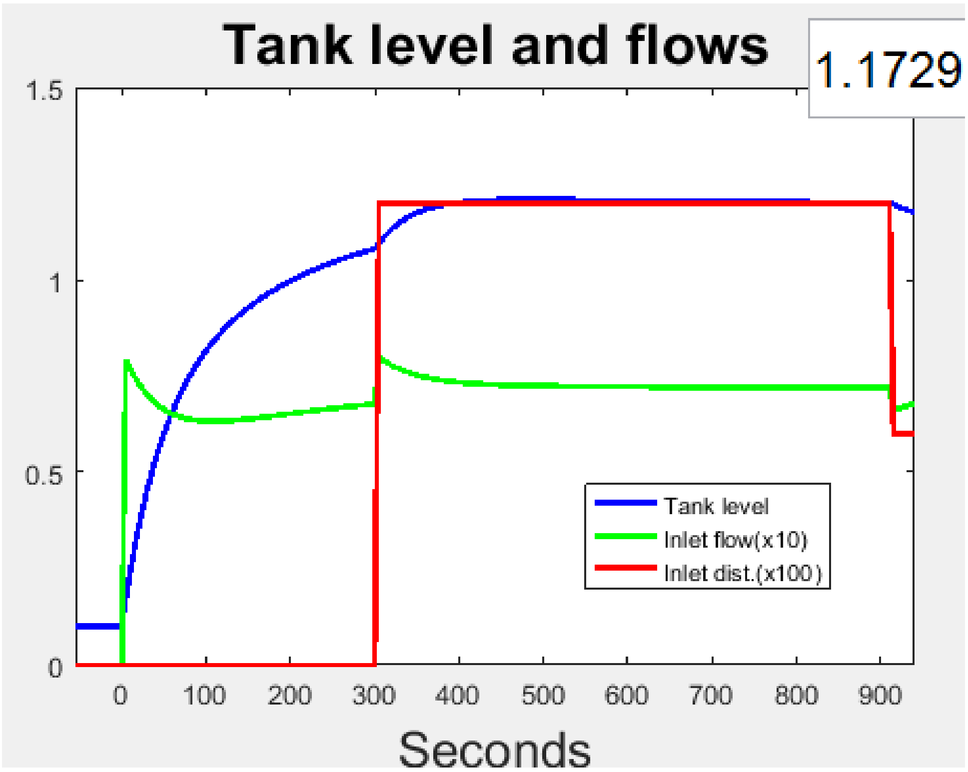


Figure 74: Fast response

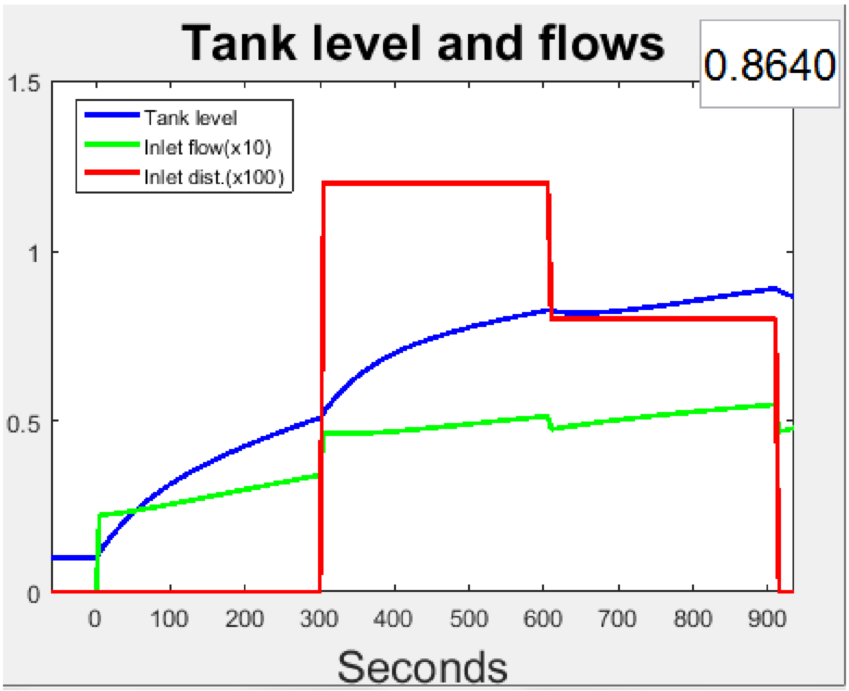


Figure 75: Slow response

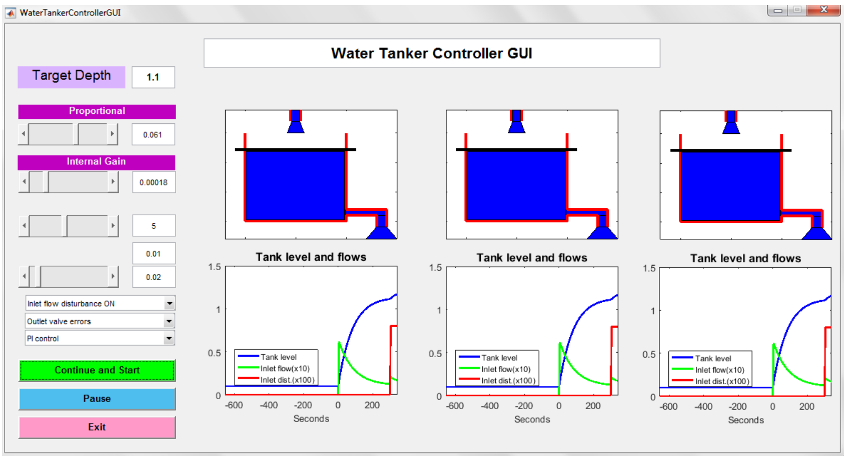


Figure 76: Final GUI with working loops

18.6 Conclusion

Clearly the dynamics depend upon the control law parameters which hence need to be selected appropriately to get the desired response. For practical reasons it is common place to ignore the differential of the target when the target undergoes step changes. With small R, the system time constant may be larger than 500 and level control will become difficult. With large R, you may need to exceed flow limits of 0.1 and thus not obtain target.

A Appendix

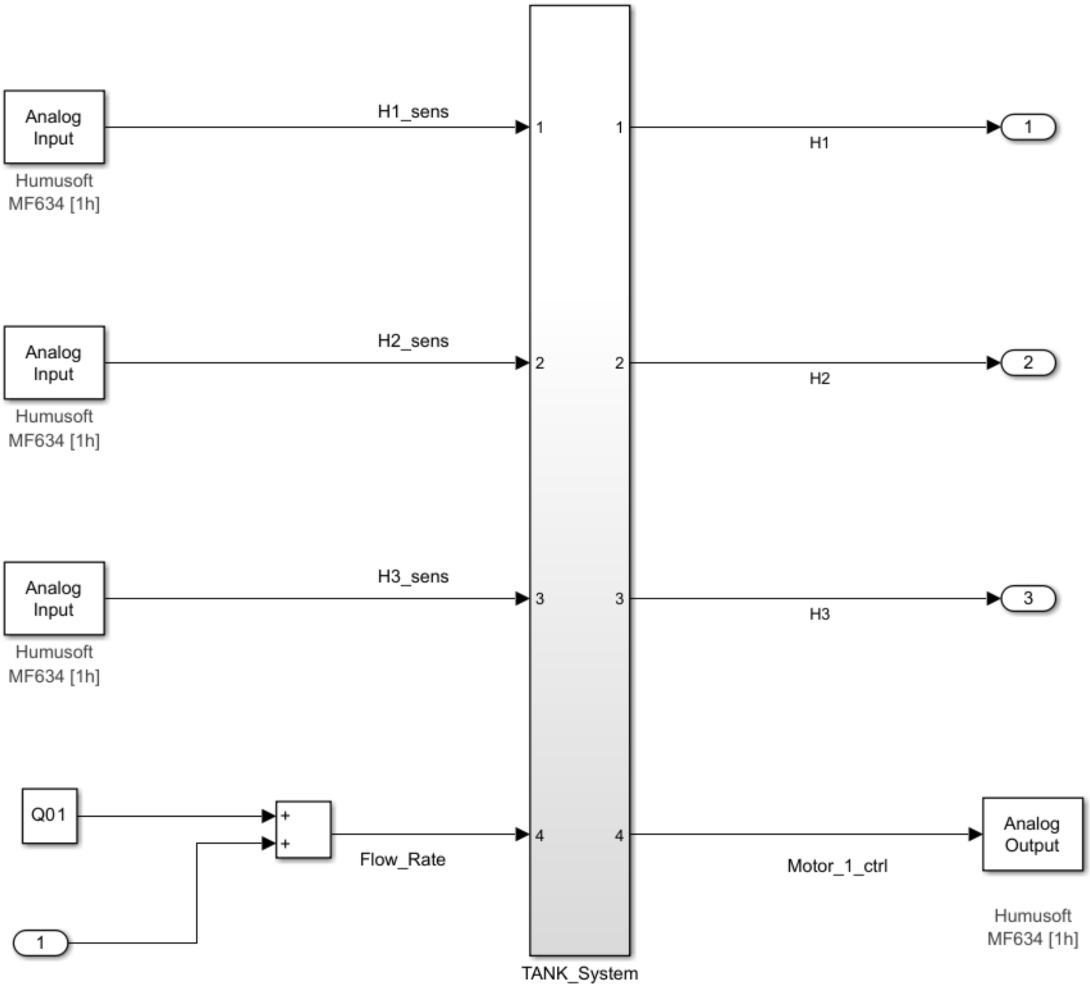


Figure 77: Tank System Model for Testing

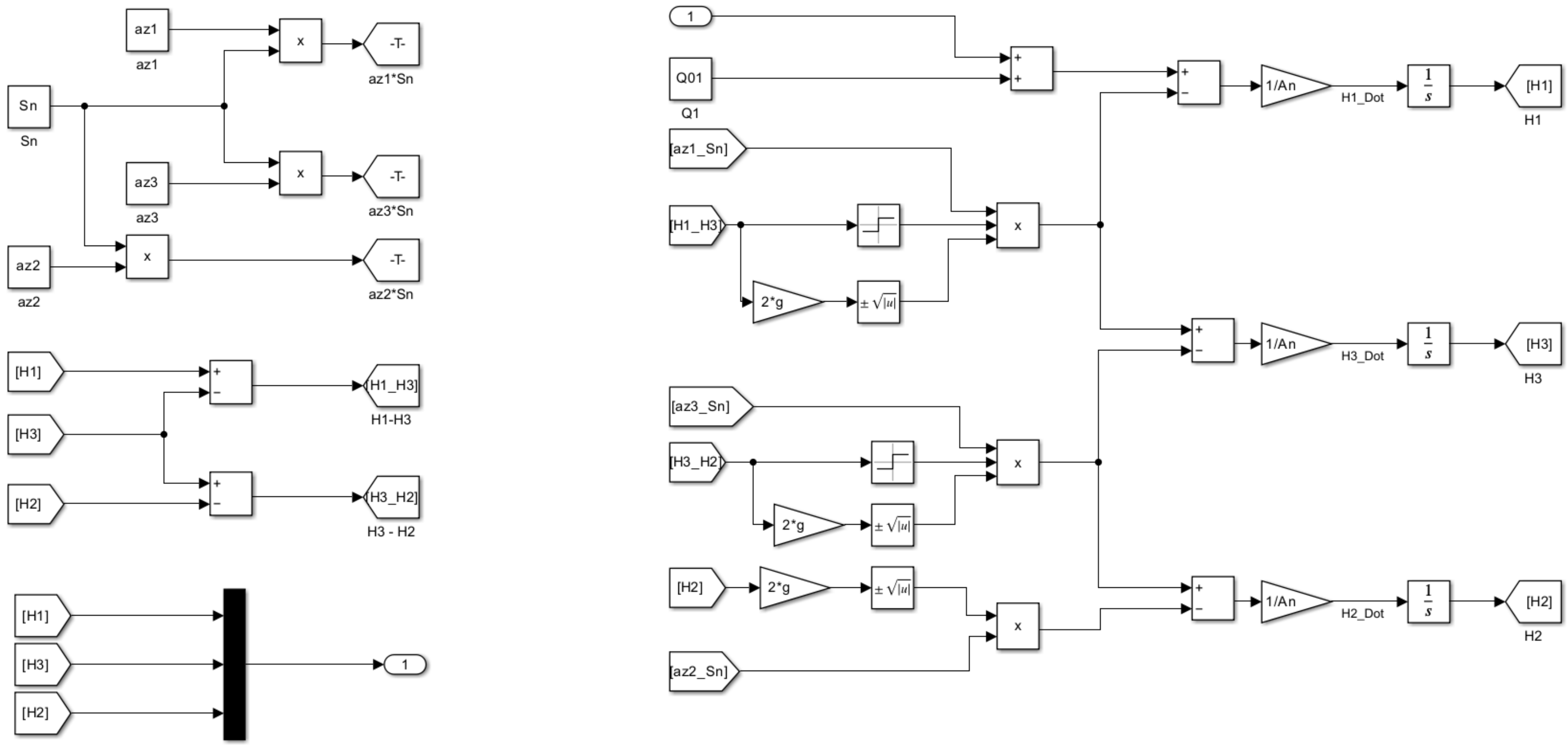


Figure 78: Simulink Model representation of non-linear SISO System

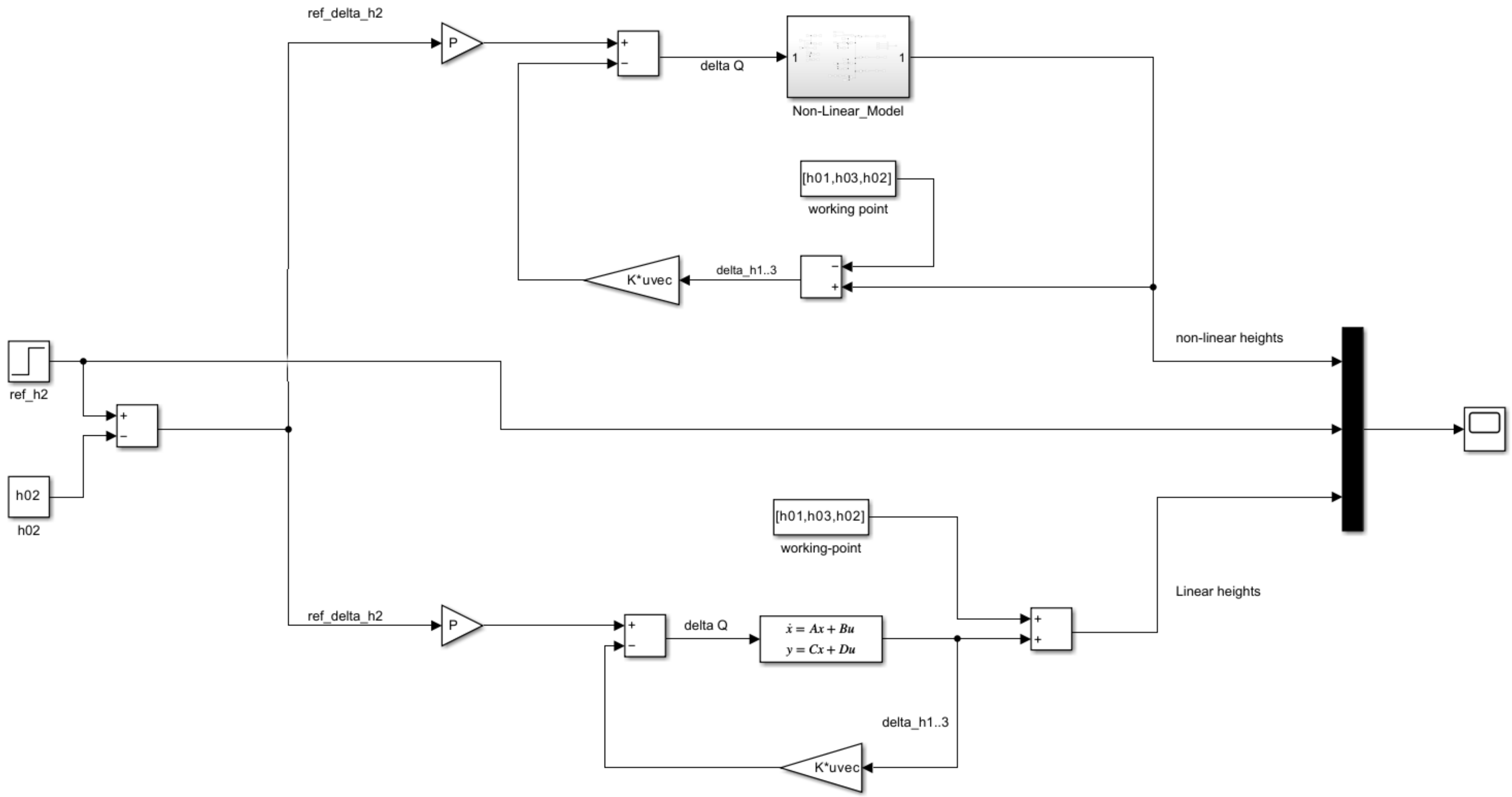


Figure 79: Simulink Model representation of p control feedback of SISO System

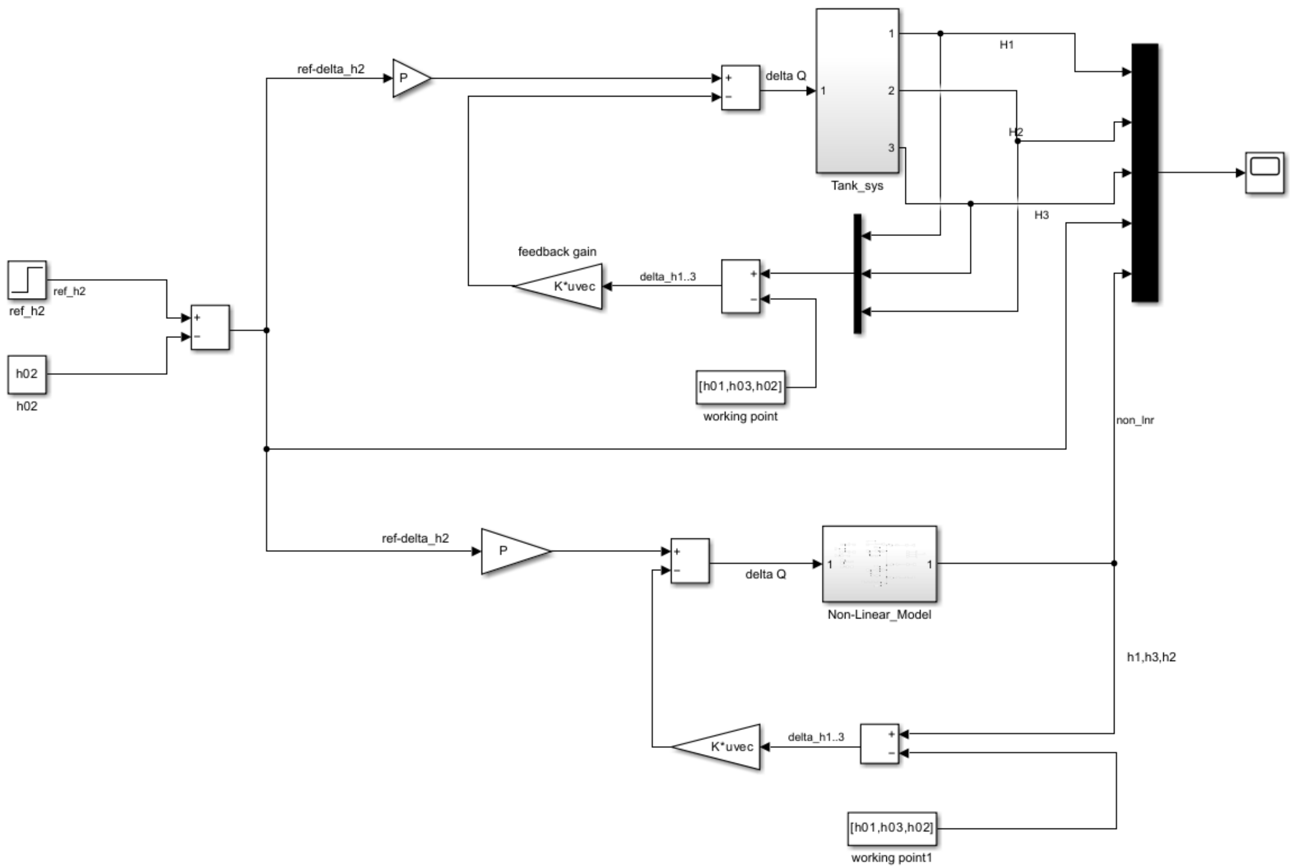


Figure 80: Simulink Model representation of p control feedback of SISO Test Stand

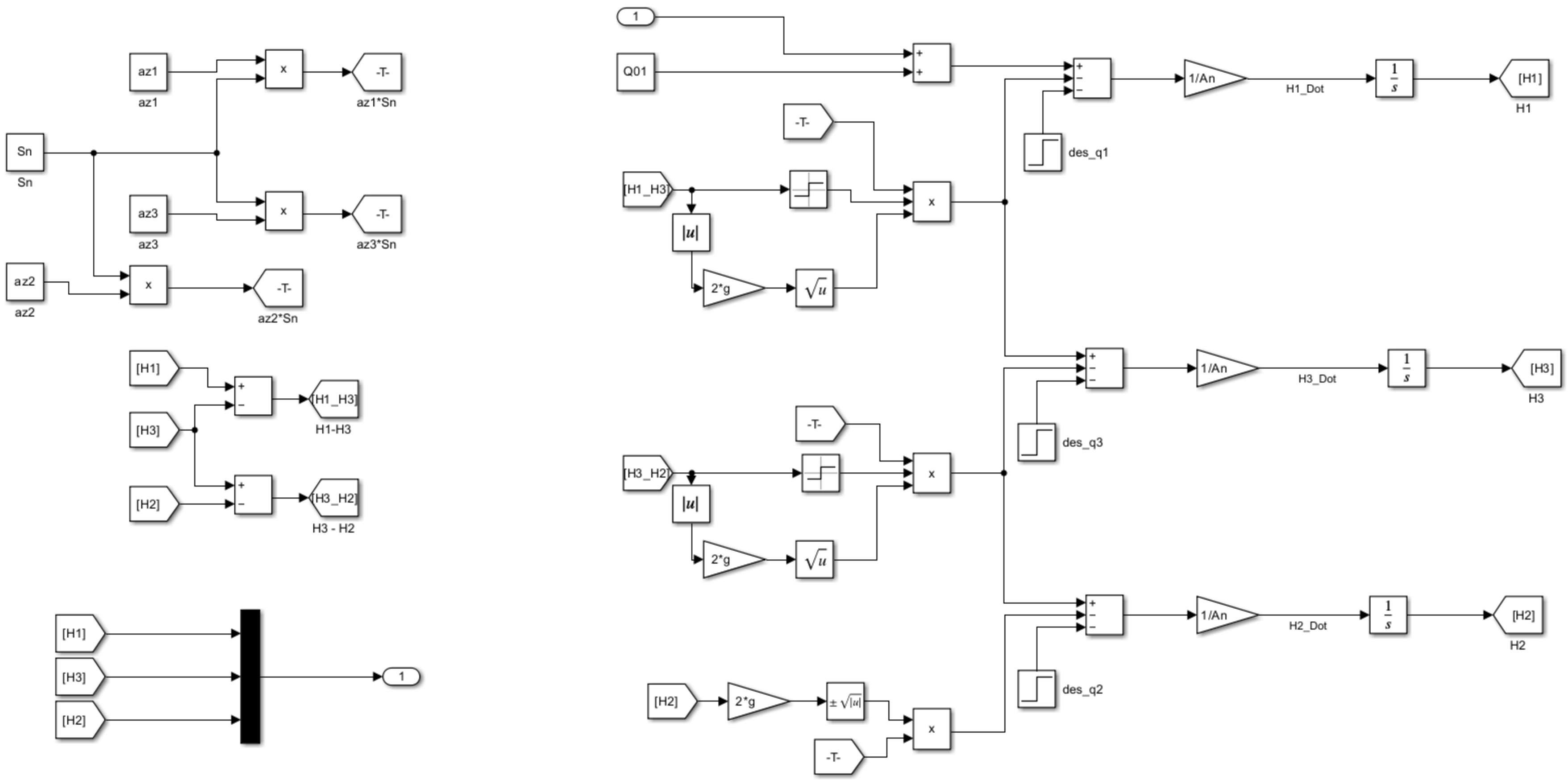


Figure 81: Simulink Model representation of non-linear equation of SISO System with leakage flow

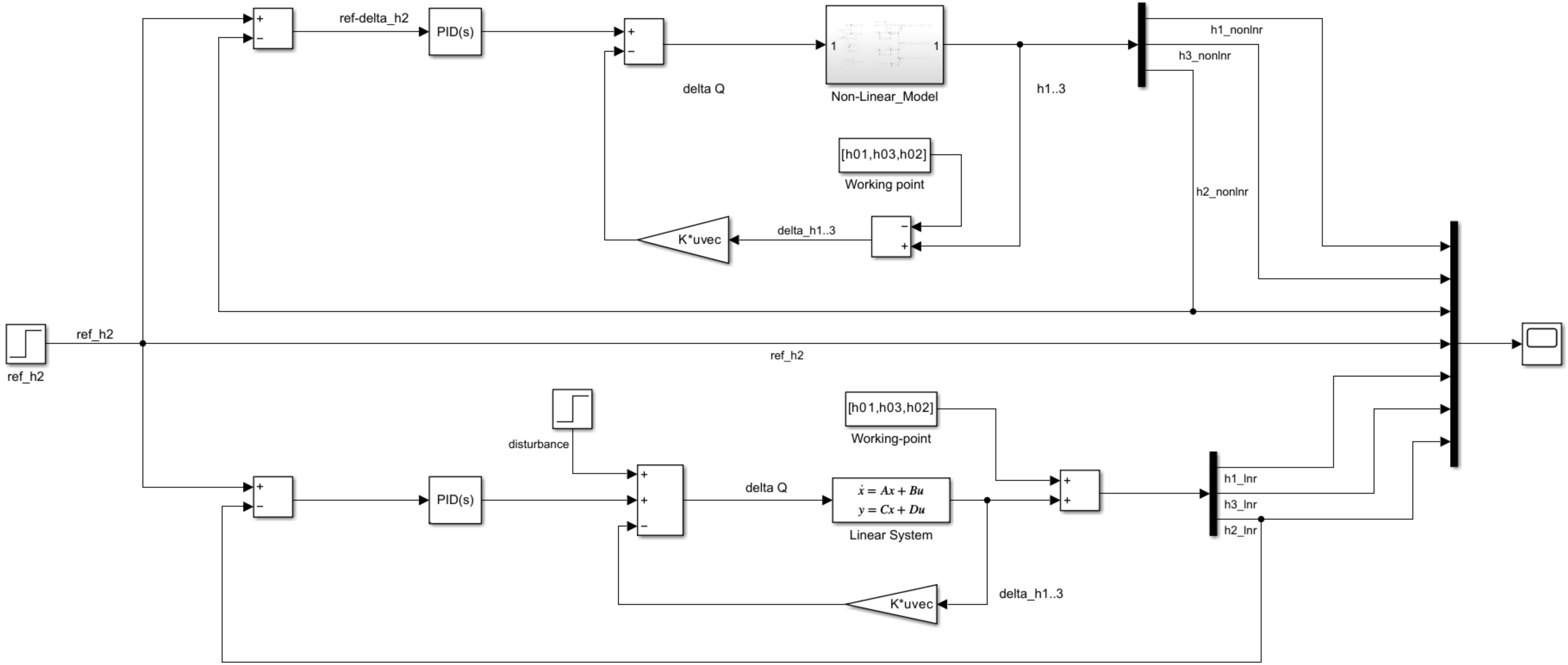


Figure 82: Simulink model with PI Control

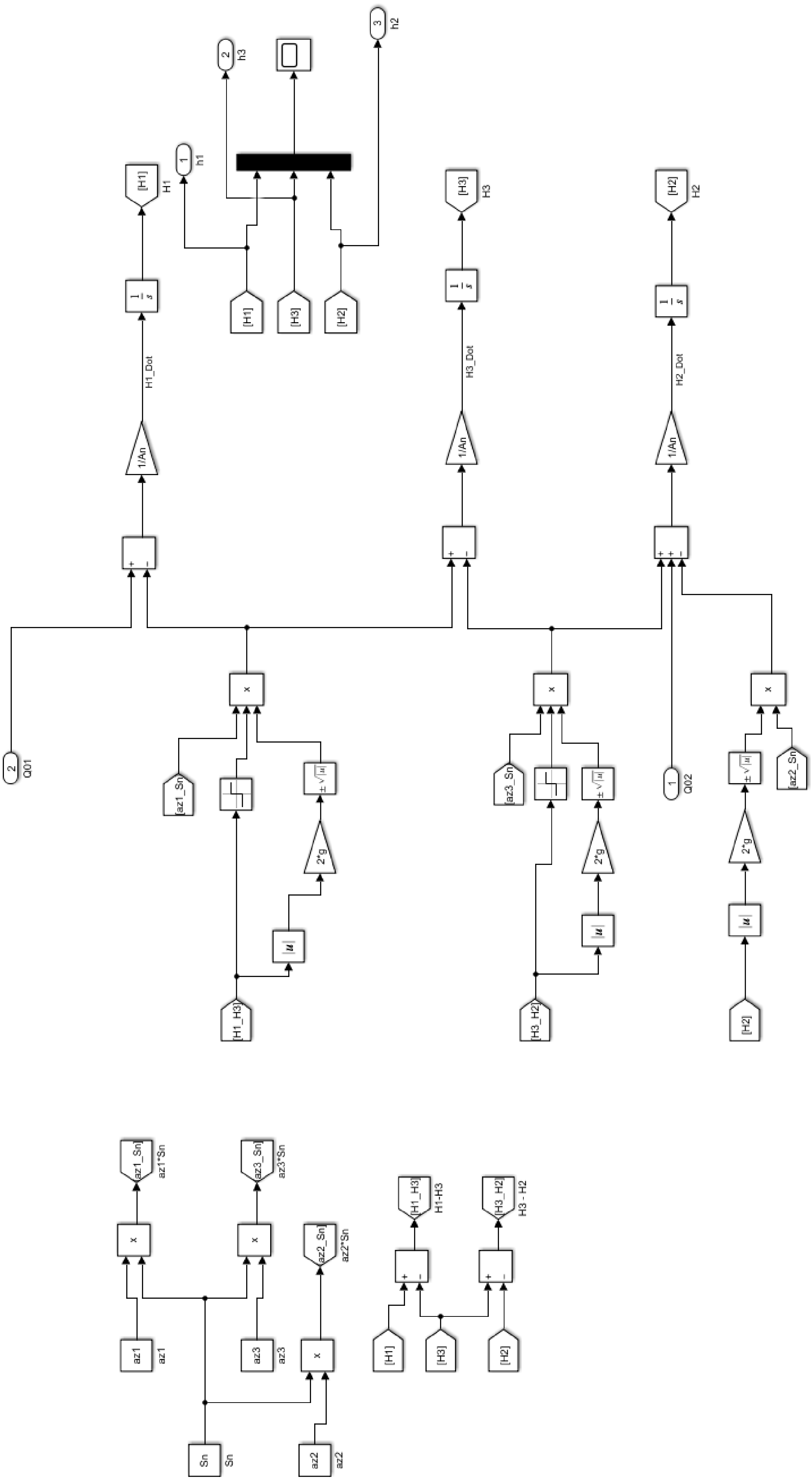


Figure 83: Nonlinear Model

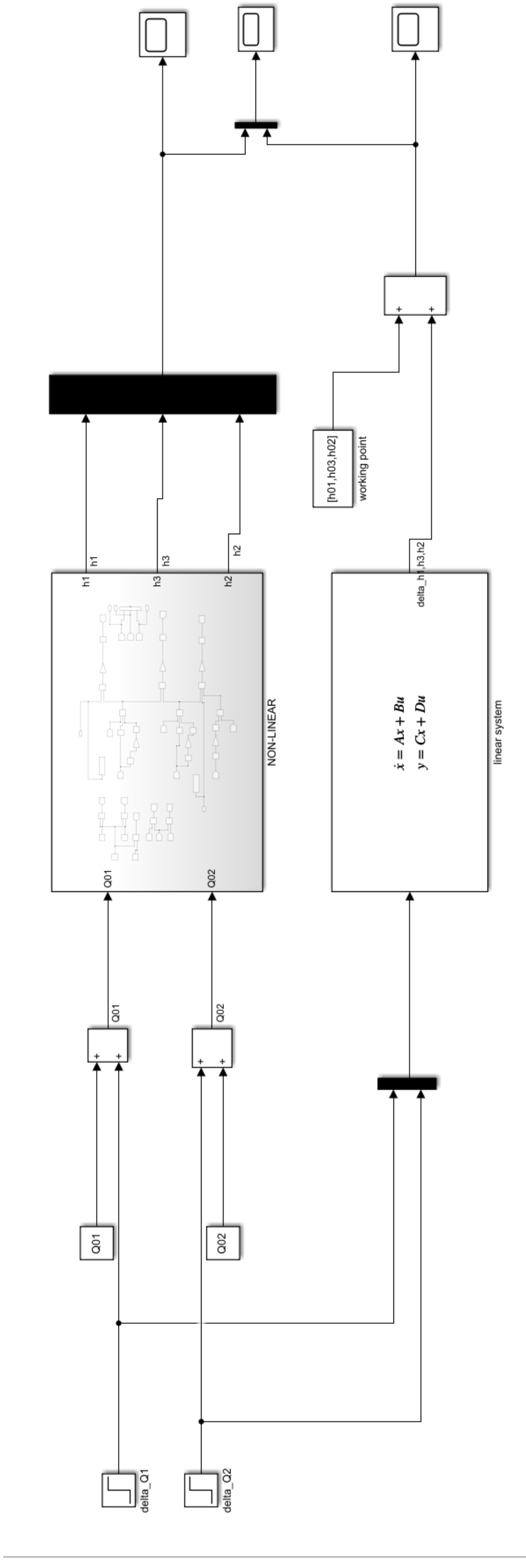


Figure 84: Linearization model

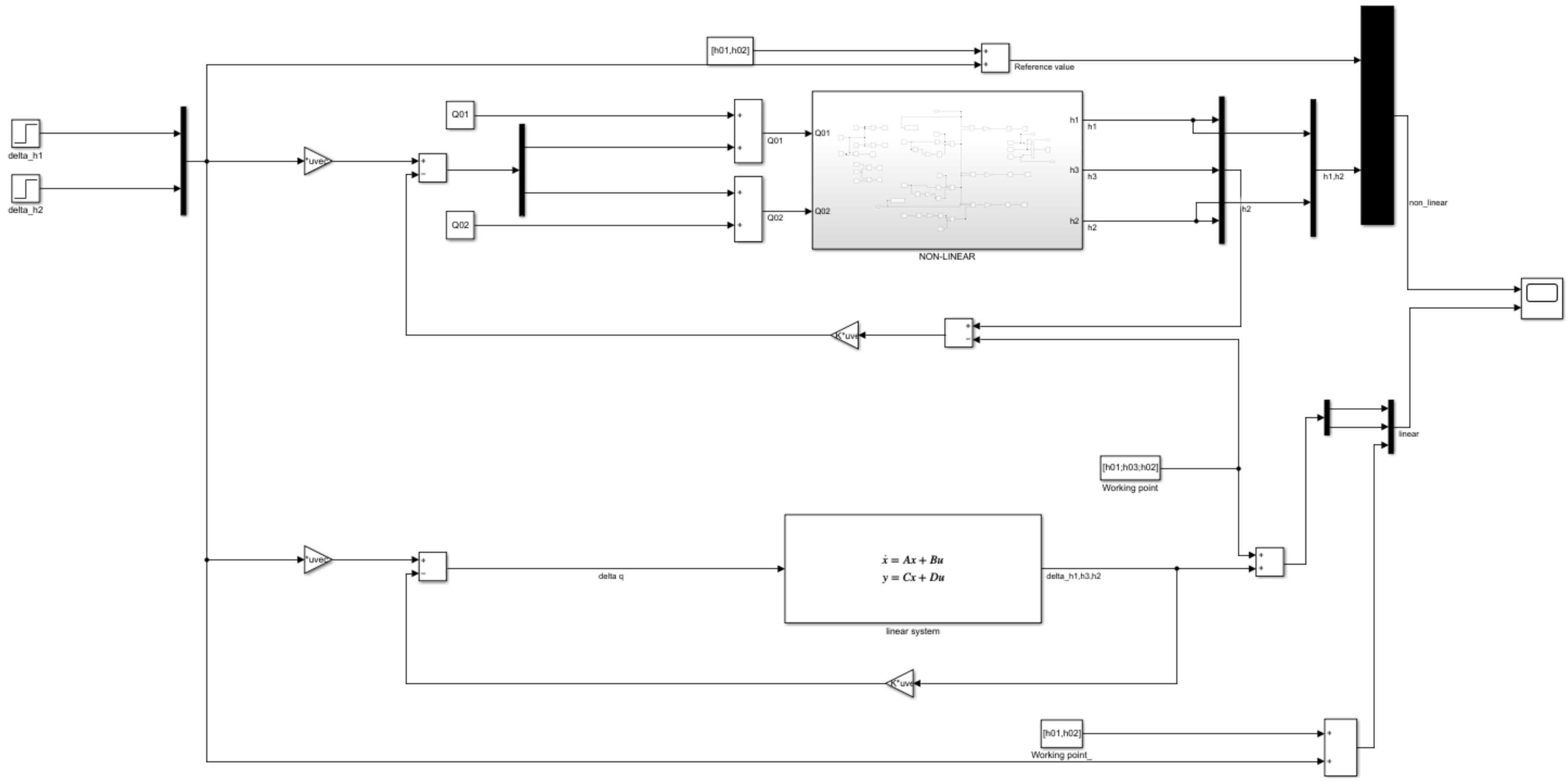


Figure 85: Simulink model of LQR Control in MIMO

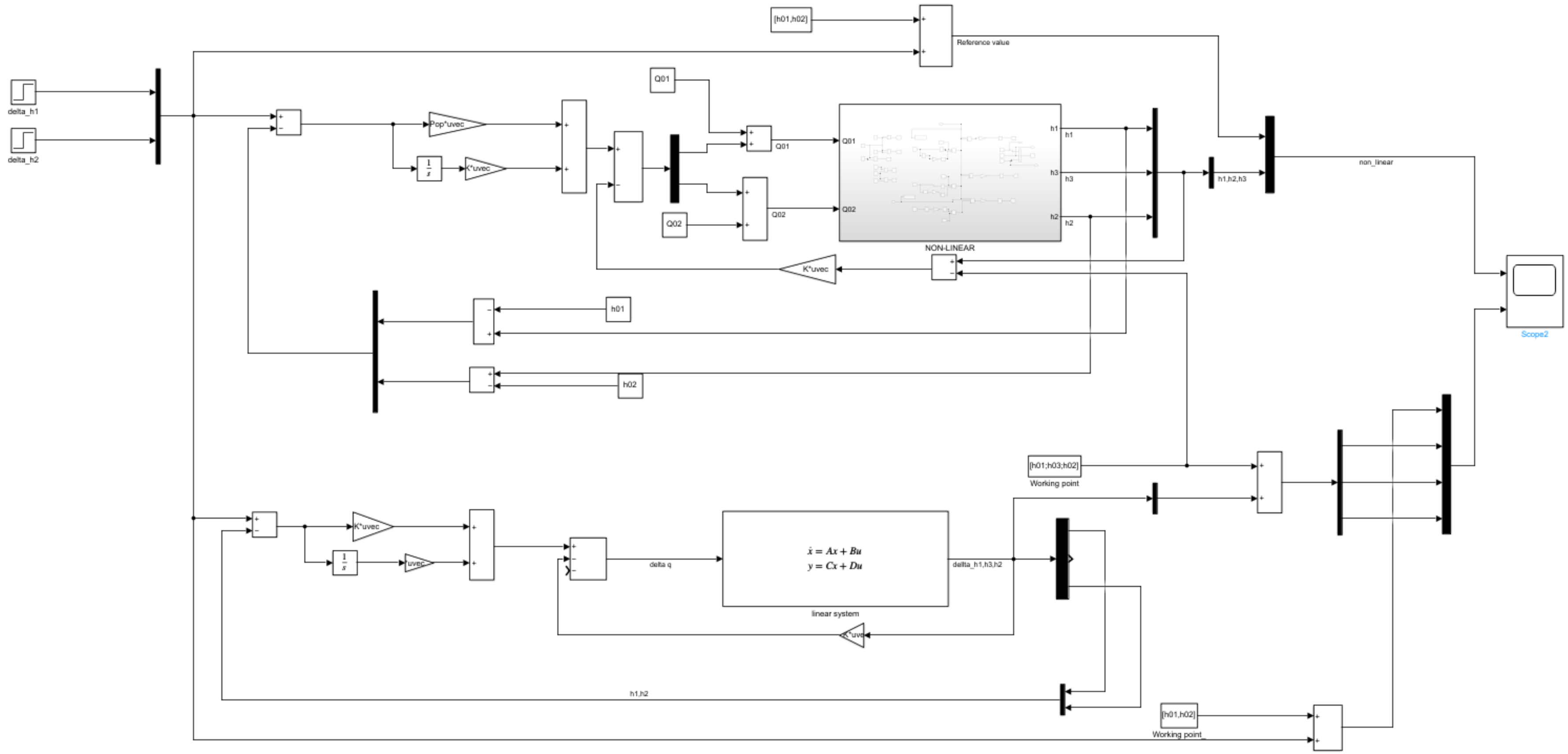


Figure 86: Simulink Model of PI LQR Control in MIMO

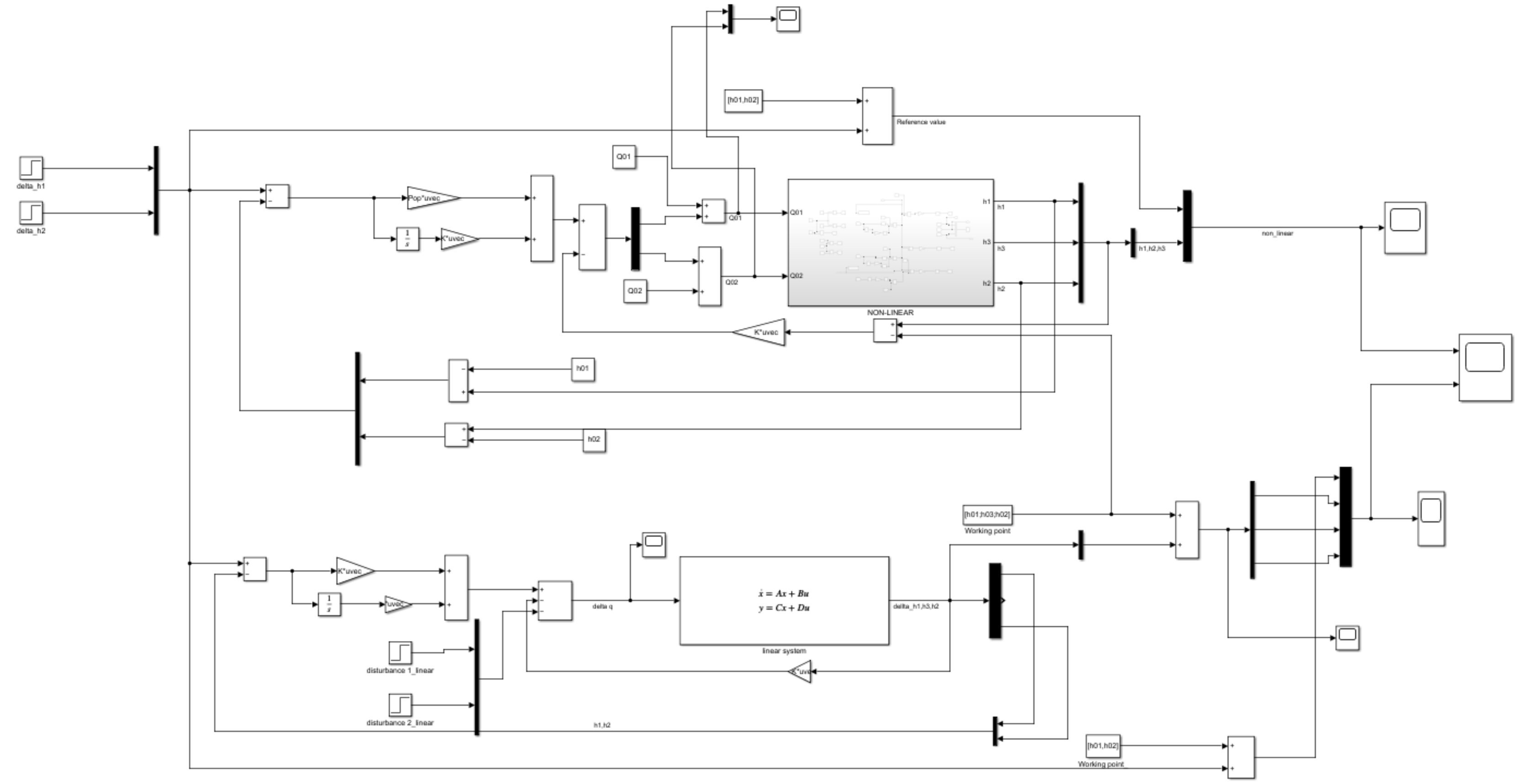


Figure 87: Simulation model of Leakage applied on the MIMO system

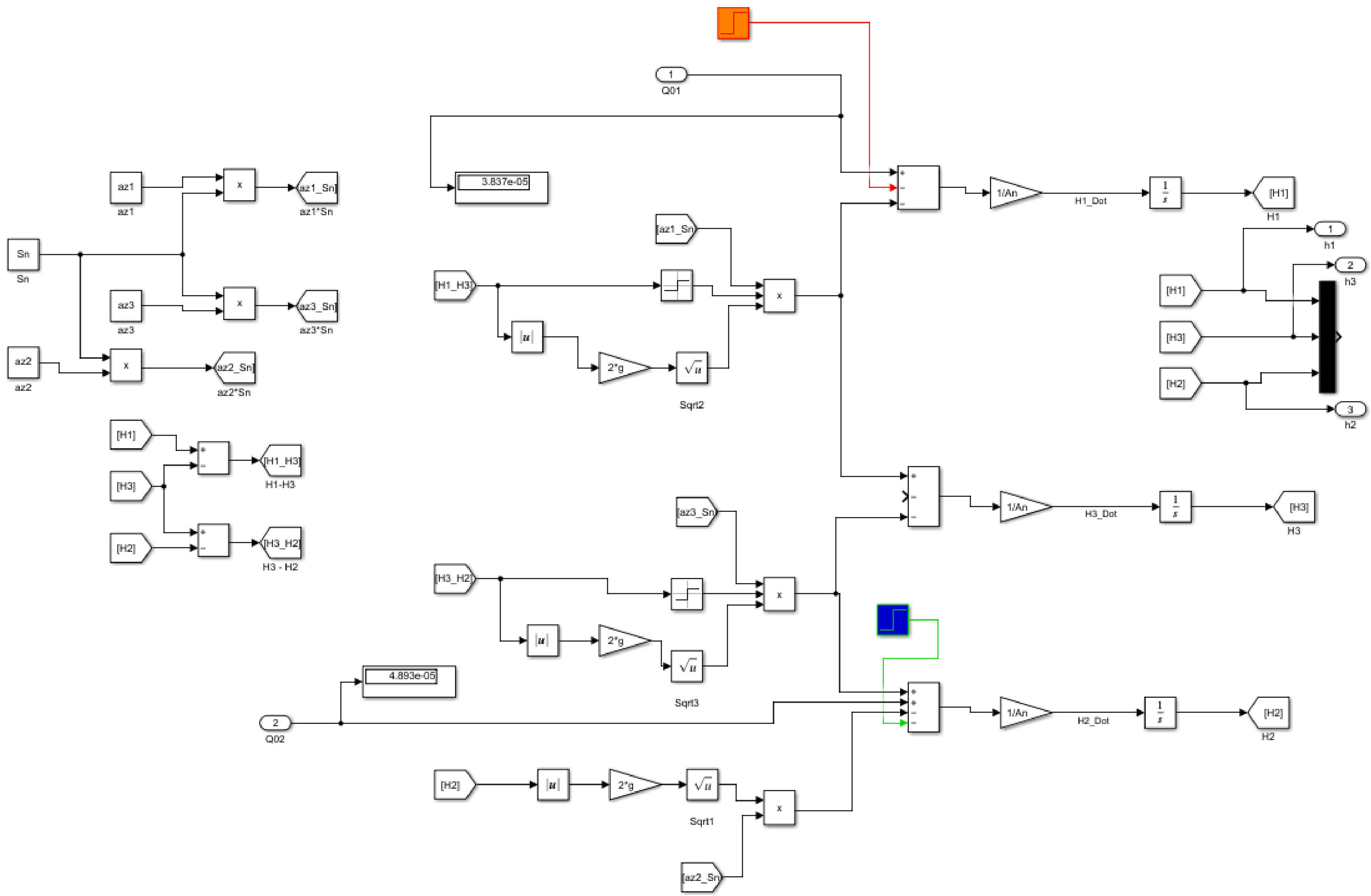


Figure 88: Leakages of Non-linear Model

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