```
In [24]: import numpy as np
import pandas as pd
import time
    from sklearn.model_selection import train_test_split
    from sklearn.preprocessing import OneHotEncoder, MinMaxScaler, OrdinalEncoder
    from sklearn.compose import ColumnTransformer
    from sklearn.pipeline import Pipeline
    from sklearn.linear_model import LinearRegression
    from sklearn.metrics import mean_absolute_error, mean_squared_error
    import matplotlib.pyplot as plt
    from category_encoders import BinaryEncoder
```

```
In [25]: #Load the dataset and print a sample number of rows.
# Load the dataset
insurance_data = pd.read_csv("insurance.csv")
# Print a sample number of rows
sample_rows = insurance_data.sample(5)
print("Sample of Rows:")
print(sample_rows)
```

```
Sample of Rows:
     Age Gender
                  BMI Children Smoker
                                          Region Expenses
1235
      26
            male
                  NaN
                              0
                                   no northwest
                                                  2699.57
840
      21
                              0
            male 31.1
                                   no southwest
                                                  1526.31
105
      20
            male 28.0
                              1
                                  yes
                                       northwest 17560.38
508
      24 female 25.3
                              0
                                       northeast
                                                  3044.21
                                   no
338
      50
            male 32.3
                                       northeast 41919.10
                                  ves
```

In [26]: #Find the number of rows with any missing values. Remove any row with a missin
#Find the number of rows with any missing values.
rows_with_missing_values = insurance_data[insurance_data.isnull().any(axis=1)]
print("Number of rows with missing values:", len(rows_with_missing_values))

#Remove any row with a missing value.
insurance_data = insurance_data.dropna()

Number of rows with missing values: 17

```
#Convert 'Gender', 'Smoker', and 'Region' into numerical values suitable for r
In [27]:
         ## Using one-hot encoding for 'Region' and binary encoding for 'sex' and 'smok
         encoder = ColumnTransformer(
              transformers=[
                  ('gender', OrdinalEncoder(categories=[['female', 'male']]), ['Gender']
                  ('smoker', OrdinalEncoder(categories=[['no', 'yes']]), ['Smoker']),
                  ('region', OneHotEncoder(), ['Region'])
              remainder='passthrough'
         insurance data encoded = pd.DataFrame(encoder.fit transform(insurance data), d
         print(insurance data encoded.head())
             gender Gender
                             smoker Smoker
                                              region Region northeast \
         0
                                         1.0
                                                                    0.0
                        0.0
          1
                        1.0
                                         0.0
                                                                    0.0
          2
                                         0.0
                                                                    0.0
                        1.0
          3
                        1.0
                                         0.0
                                                                    0.0
          4
                        1.0
                                         0.0
                                                                    0.0
                                       region Region southeast
             region Region northwest
         0
                                  0.0
                                                              0.0
          1
                                  0.0
                                                              1.0
          2
                                  0.0
                                                              1.0
          3
                                                              0.0
                                  1.0
          4
                                   1.0
                                                              0.0
             region__Region_southwest
                                        remainder Age remainder BMI \
         0
                                  1.0
                                                  19.0
                                                                   27.9
          1
                                  0.0
                                                  18.0
                                                                   33.8
          2
                                                                   33.0
                                  0.0
                                                  28.0
          3
                                  0.0
                                                  33.0
                                                                   22.7
          4
                                  0.0
                                                  32.0
                                                                   28.9
                                  remainder__Expenses
             remainder__Children
         0
                             0.0
                                              16884.92
          1
                             1.0
                                               1725.55
          2
                             3.0
                                               4449.46
          3
                             0.0
                                              21984.47
          4
                             0.0
                                               3866.86
```

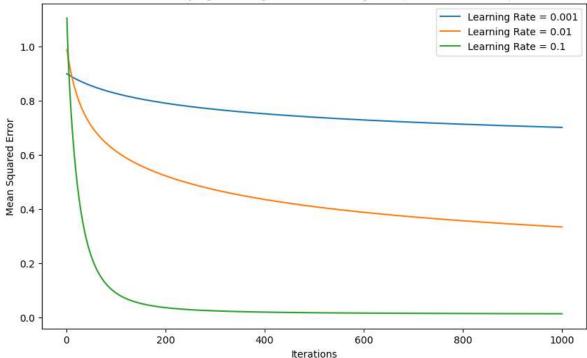
```
In [28]: #Normalize the features using Min-Max scaling.
    scaler = MinMaxScaler()
    insurance_data_normalized = pd.DataFrame(scaler.fit_transform(insurance_data_e)
    print(insurance_data_normalized.head())
```

0 1 2 3 4	genderGender smoker 0.0 1.0 1.0 1.0 1.0	_Smoker region 1.0 0.0 0.0 0.0 0.0	Region_northeast \
0 1 2 3 4	regionRegion_northwest 0.0 0.0 1.0 1.0	a a a a	n_southeast \
0 1 2 3 4	regionRegion_southwest 1.0 0.0 0.0 0.0 0.0	0 0.021739 0 0.000000 0 0.217391 0 0.326087	0.320755 0.479784 0.458221 0.180593
0 1 2 3 4	remainderChildren remain	mainderExpenses 0.251611 0.009636 0.053115 0.333016 0.043816	L 5 5

```
# Divide the data into "features" and "target" subsets.
In [29]:
         features = insurance_data_normalized.drop('remainder__Expenses', axis=1) # Fed
         target = insurance_data_normalized['remainder__Expenses'] # Target: 'Expenses'
         # Split the data into training and testing subsets
         # Here, I'm using a 80/20 split
         # it means that 80% of the data will be used for training model, and the remai
         X_train, X_test, y_train, y_test = train_test_split(features, target, test_siz
         # Display the shapes of the resulting subsets
         print(f"\nShapes of subsets:")
         print(f"X_train shape: {X_train.shape}") #feature train
         print(f"X test shape: {X test.shape}") #feature test
         print(f"y train shape: {y train.shape}") #target train
         print(f"y_test shape: {y_test.shape}") #target test
         Shapes of subsets:
         X train shape: (1056, 9)
         X test shape: (265, 9)
         y_train shape: (1056,)
         y_test shape: (265,)
In [30]:
         #exponential decay
         #Learning Rate = Initial Learning Rate/1+Decay Rate×Iteration Number
         #np.c --> concatenate array
         #np.ones ---> make a matrix with rows and colums having only 1's.
         #x = [1, 2, 3, 4, 5]
         #y = [2, 4, 6, 8, 10]
         #plt.plot(x, y)
```

```
In [31]: # Convert y_train to NumPy array
         y_train_np = np.array(y_train)
         # Gradient Descent
         def gradient_descent(X_train_b, y_train, alpha, n_iterations):
             m = X_train_b.shape[0]
             n_features = X_train_b.shape[1]
             W = np.random.randn(n_features, 1)
             loss = []
             decay_rate = 0.01 # Decay rate
             for iteration in range(n iterations):
                 alpha_decay = alpha / (1 + decay_rate * iteration) # Exponential deca
                 gradients = 1/m * X train b.T.dot(X train b.dot(W) - y train[:, np.new
                 W = W - alpha decay * gradients
                 predictions = X_train_b.dot(W)
                 loss.append(mean_squared_error(y_train, predictions))
             return loss
         # Varying Learning rates
         learning rates = [0.001, 0.01, 0.1]
         n_iterations = 1000
         plt.figure(figsize=(10, 6))
         for alpha in learning rates:
             X_b = np.c_[np.ones((X_train.shape[0], 1)), X_train] ## Add a column of on
             n features = X b.shape[1]
             W = np.random.randn(n_features, 1)
             start_time = time.time()
             loss = gradient_descent(X_b, y_train_np, alpha, n_iterations)
             end time = time.time()
             plt.plot(range(1, n_iterations + 1), loss, label=f'Learning Rate = {alpha}
         gradient descent time = end time - start time
         plt.title('Effect of Varying Learning Rate on Convergence (Gradient Descent)')
         plt.xlabel('Iterations')
         plt.ylabel('Mean Squared Error')
         plt.legend()
         plt.show()
```





```
In [32]: # Add a column of ones to the test set.
X_test_b = np.c_[np.ones((X_test.shape[0], 1)), X_test]
# Predict expenses using the trained model
predictions_gradient = X_test_b.dot(W).ravel()
```

```
In [33]: # Linear Regression from scikit-learn
    start_time = time.time()

linear_reg_model = LinearRegression()
    linear_reg_model.fit(X_train, y_train)
    predictions_linear_reg = linear_reg_model.predict(X_test)

end_time = time.time()
    linear_regression_time = end_time - start_time
```

```
In [34]: # Normal Equation
    start_time = time.time()

X_train_b = np.c_[np.ones((X_train.shape[0], 1)), X_train]
    X_test_b = np.c_[np.ones((X_test.shape[0], 1)), X_test]

theta_normal_eq = np.linalg.inv(X_train_b.T.dot(X_train_b)).dot(X_train_b.T).d
    predictions_normal_eq = X_test_b.dot(theta_normal_eq)

end_time = time.time()
    normal_equation_time = end_time - start_time
```

```
In [35]: # Calculate MAE and MSE
    mae_gradient_descent = mean_absolute_error(y_test, predictions_gradient)
    mse_gradient_descent = mean_squared_error(y_test, predictions_gradient)

mae_linear_reg = mean_absolute_error(y_test, predictions_linear_reg)
    mse_linear_reg = mean_squared_error(y_test, predictions_linear_reg)

mae_normal_eq = mean_absolute_error(y_test, predictions_normal_eq)
    mse_normal_eq = mean_squared_error(y_test, predictions_normal_eq)
```

```
In [36]: print("\nComparison of Solutions:")
    print(f"MAE (Gradient Descent): {mae_gradient_descent}")
    print(f"MSE (Gradient Descent): {mse_gradient_descent}")
    print(f"Computational Time (Gradient Descent): {gradient_descent_time:.4f} sec

    print(f"MAE (Linear Regression): {mae_linear_reg}")
    print(f"MSE (Linear Regression): {mse_linear_reg}")
    print(f"Computational Time (Linear Regression): {linear_regression_time:.4f} s

    print(f"MAE (Normal Equation): {mae_normal_eq}")
    print(f"MSE (Normal Equation): {mse_normal_eq}")
    print(f"Computational Time (Normal Equation): {normal_equation_time:.4f} secon
```

```
Comparison of Solutions:

MAE (Gradient Descent): 0.9899742433107207

MSE (Gradient Descent): 1.4355745147611263

Computational Time (Gradient Descent): 1.3936 seconds

MAE (Linear Regression): 0.07322090456300645

MSE (Linear Regression): 0.010997654911521038

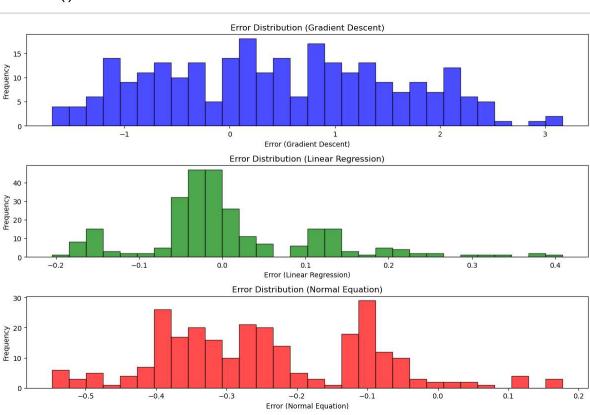
Computational Time (Linear Regression): 0.0081 seconds

MAE (Normal Equation): 0.2520891234248898

MSE (Normal Equation): 0.08121139853953759

Computational Time (Normal Equation): 0.0042 seconds
```

```
# Calculate errors
In [37]:
         error_gradient_descent = y_test - predictions_gradient
         error_linear_reg = y_test - predictions_linear_reg
         error_normal_eq = y_test - predictions_normal_eq
         # Plot histograms
         plt.figure(figsize=(12, 8))
         plt.subplot(3, 1, 1)
         plt.hist(error gradient descent, bins=30, edgecolor='black', color='blue', alp
         plt.xlabel('Error (Gradient Descent)')
         plt.ylabel('Frequency')
         plt.title('Error Distribution (Gradient Descent)')
         plt.subplot(3, 1, 2)
         plt.hist(error linear reg, bins=30, edgecolor='black', color='green', alpha=0.
         plt.xlabel('Error (Linear Regression)')
         plt.ylabel('Frequency')
         plt.title('Error Distribution (Linear Regression)')
         plt.subplot(3, 1, 3)
         plt.hist(error_normal_eq, bins=30, edgecolor='black', color='red', alpha=0.7)
         plt.xlabel('Error (Normal Equation)')
         plt.ylabel('Frequency')
         plt.title('Error Distribution (Normal Equation)')
         plt.tight_layout()
         plt.show()
```



In []:	
In []:	
In []:	