Define permutation & Write its formula.

$$P(n,r) = rac{n!}{(n-r)!}$$

where n! (n factorial) is the product of all positive integers up to n.

Define Combination & Write its formula

$$C(n,r)=rac{n!}{r!(n-r)!}$$

where n! is the factorial of n, and r! is the factorial of r.

. Define Binomial coefficient & Write the general formula of binomial co-efficient

The binomial coefficient, denoted as $\binom{n}{k}$ or C(n,k), represents the number of ways to choose k elements from a set of n elements without regard to order. The general formula is:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

where n! is the factorial of n and k! is the factorial of k.

Define interpolation

Interpolation is a mathematical method used to estimate unknown values that fall within the range of known data points. By constructing new data points within the range of a discrete set of known data points, interpolation helps in predicting values at intermediate points. Common methods include linear, polynomial, and spline interpolation.

Define Probability



Probability is a measure of the likelihood that a particular event will occur. It is quantified as a number between 0 and 1, where 0 indicates impossibility and 1 indicates certainty. The formula for probability of an event \boldsymbol{A} is:

$$P(A) = rac{ ext{Number of favorable outcomes}}{ ext{Total number of possible outcomes}}$$

Write Newton's forward difference formula and Newton's Backward difference formula.

\$

Newton's Forward Difference Formula:

Newton's forward difference formula is used for interpolation of equally spaced data points. It is given by:

$$f(x) = f(x_0) + \Delta f(x_0) rac{(x-x_0)}{h} + rac{\Delta^2 f(x_0)}{2!} rac{(x-x_0)(x-x_1)}{h^2} + \cdots$$

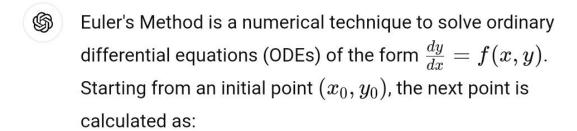
where Δ denotes the forward difference operator and h is the interval between data points.

Newton's Backward Difference Formula:

Newton's backward difference formula is also used for interpolation of equally spaced data points but starts from the end of the data set. It is given by:

$$f(x) = f(x_n) +
abla f(x_n) rac{(x-x_n)}{h} + rac{
abla^2 f(x_n)}{2!} rac{(x-x_n)(x-x_{n-1})}{h^2} + \cdots$$

where ∇ denotes the backward difference operator and h is the interval between data points.



$$y_{n+1} = y_n + hf(x_n, y_n)$$

Example:

For
$$rac{dy}{dx}=x+y$$
 , $y(0)=1$, with step size $h=0.1$:

$$y_1 = 1 + 0.1(0+1) = 1.1$$

State Simpson's 1/3 rule with an example.



Simpson's 1/3 rule is a numerical method to approximate the integral of a function. It is given by:

$$\int_a^b f(x)\,dx pprox rac{b-a}{6}[f(a)+4f(rac{a+b}{2})+f(b)]$$

Example:

Approximate $\int_0^2 (4-x^2) dx$:

$$a = 0, b = 2, \text{midpoint} = 1$$

$$pprox rac{2-0}{6} [f(0) + 4f(1) + f(2)]$$

$$=\frac{2}{6}[4+4(3)+0]=\frac{2}{6}[4+12]=\frac{32}{6}pprox 5.33$$

Define mean and standard deviation.

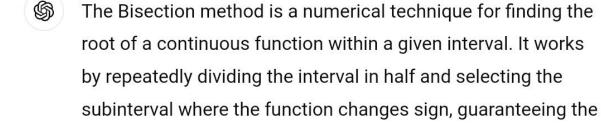


Mean: The mean, or average, of a set of numbers is the sum of all the numbers divided by the count of numbers. It represents the central tendency of a dataset.

Standard Deviation: The standard deviation measures the amount of variation or dispersion of a set of values. It quantifies the average distance of each data point from the mean, indicating the spread of data points around the mean.



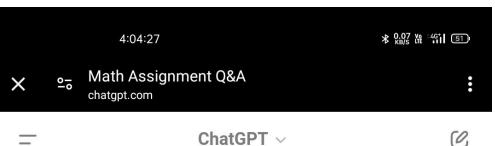
Define Bisection method.



existence of a root. This process continues until a sufficiently

accurate root is found.

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11. 10 different balls to put 4 different box, one box may cantain any no of ball such that only one box is empty. Find no. of ways.

Solve it

To solve the problem of distributing 10 different balls into 4 different boxes such that one box is empty, follow these steps:

1. Choose the Empty Box:

• There are 4 boxes, and we need to choose 1 box to be empty. This can be done in $\binom{4}{1}=4$ ways.

2. Distribute 10 Balls into 3 Non-Empty Boxes:

- Now, we need to distribute 10 different balls into 3 different boxes. Each ball has 3 choices of boxes.
- Since the balls are different, the number of ways to distribute them is 3^{10} .

Therefore, the total number of ways to distribute the 10 different balls into 4 boxes such that exactly one box is empty is:

$$4 imes 3^{10}$$

Now, let's calculate this value:

$$4\times 3^{10}=4\times 59049=236196$$

So, there are **236196** ways to distribute 10 different balls into 4 different boxes such that only one box is empty.



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Right answer is 96.

Let's recalculate the value using Newton's Backward Difference Formula carefully to ensure accuracy.

Given the data:

X	f(X)
1931	103
1921	93
1911	81
1901	66
1891	46

We need to find f(1925) using Newton's Backward Difference Formula.

Step 1: Construct the Backward Difference Table

Calculate the backward differences:

X	f(X)	$\Delta f(X)$	$\Delta^2 f(X)$	$\Delta^3 f(X)$	$\Delta^4 f(X)$
1931	103				
1921	93	10			
1911	81	12	2		
1901	66	15	3	1	
1891	46	20	5	2	1

Step 2: Apply Newton's Backward Difference Formula

Newton's Backward Difference Formula is:

$$f(x) = f(x_n) + rac{u}{1!} \Delta f(x_n) + rac{u(u+1)}{2!} \Delta^2 f(x_n) + rac{u(u+1)(u+2)}{3!} \Delta^3 f(x_n) + \cdots$$

where $u=rac{x-x_n}{h}$.

Given:

•
$$x_n = 1931$$

•
$$x = 1925$$

•
$$h = 10$$

Therefore:

$$u = \frac{1925 - 1931}{10} = -0.6$$

Substitute the values into the formula:

$$\begin{array}{l} f(1925) = f(1931) + \frac{u}{1!} \Delta f(1931) + \\ \frac{u(u+1)}{2!} \Delta^2 f(1931) + \frac{u(u+1)(u+2)}{3!} \Delta^3 f(1931) + \\ \frac{u(u+1)(u+2)(u+3)}{4!} \Delta^4 f(1931) \end{array}$$

Calculating step-by-step:

1.
$$f(1931) = 103$$

2.
$$\frac{u}{11}\Delta f(1931) = (-0.6)(10) = -6$$

3.
$$\frac{u(u+1)}{2!}\Delta^2 f(1931) = \frac{(-0.6)(0.4)}{2} \cdot 2 = -0.24$$

4.
$$\frac{u(u+1)(u+2)}{3!}\Delta^3 f(1931) = \frac{(-0.6)(0.4)(1.4)}{6} \cdot 1 = -0.056$$

5.
$$\frac{u(u+1)(u+2)(u+3)}{4!}\Delta^4 f(1931) = \frac{(-0.6)(0.4)(1.4)(2.4)}{24} \cdot 1 = -0.0336$$

Now, summing all these terms:

$$f(1925) = 103 - 6 - 0.24 - 0.056 - 0.0336 = 103 - 6 - 0.3296 = 96.6704 \approx 96$$

So, the value of f(x) at x=1925 is approximately **96**.

1979	8. F 8 3 D	(NO.114)	10/11/10	1 11 11 11		
1	X	F(a)	AF(X)	$\Delta^2 F(a)$		∆3 F(x)
1	300	2.4771	0.00145	7	61/01/12	-6369
1	304	2.4829	0.00145	1 - July 1 1/2	17 1 1 1 1 TH	
7	305	2.4843	0.0014	- 0.0000	6110)	111111111111111111111111111111111111111
	307	2.4871	0.0014	0	4	1042.85 X
9	101/19	84 - C 87 10 1 1 1 1	EUT A	The state of the s	ATT	
DIF	Fernce	interpocation:	d di	55	9/1/	
		Na 1 1001 3	Rughing.	1 = 5 = 16 11.		
	P(1)	= F(20) + (7-x0)	D (20,21)	+ (12-20) (72-21) . A:	2 (20,2,122)
	FOR MESS	= p(x0) + (x-x0) + (x-	20) (7-2	1) (2-22) 43	(20,7,002	, 2(3)
4						
1	P(2)	= 2.4771 + (4	-300)	0.00145 +	(x-300)	(2-304).
	1 8	= 2.4771 + (4	001) + (2	(-300) (2-3	104/12-30	128 24-1. (50
	HALL BY THE PARTY					

A Card from a pack of 52 Cards. From the remaining cards two are drawn randomly and found to

be both clubs. Find the mobability that the lost card is also a clubs.

To solve the problem of finding the probability that the lost card is a club, given that two randomly drawn cards from the remaining deck are clubs, we can use conditional probability and the concept of drawing cards without replacement.

Here's a step-by-step solution:

Step 1: Define Events

- ullet Let A be the event that the lost card is a club.
- Let B be the event that two drawn cards from the remaining deck are clubs.

Step 2: Determine Total Number of Clubs and Cards

Initially, the deck has:

- · 52 cards in total.
- 13 clubs.

If one card is lost:

Total cards left = 51.

Step 3: Calculate Probability of Event A

The probability that the lost card is a club (P(A)): $P(A)=rac{13}{52}=rac{1}{4}$

Step 4: Calculate Probability of Drawing Two Clubs Given Event A

If the lost card is a club, there are:

· 12 clubs left out of 51 cards.

The probability of drawing two clubs from these 51 cards without replacement:

$$P(B|A) = rac{inom{12}{2}}{inom{51}{2}} = rac{rac{12\cdot 11}{2}}{rac{51\cdot 50}{2}} = rac{12\cdot 11}{51\cdot 50} = rac{132}{2550} = rac{11}{212\cdot 5}$$

Step 5: Calculate Probability of Drawing Two Clubs Given Event Not A

Step 5: Calculate Probability of Drawing Two Clubs Given Event Not A

If the lost card is not a club, there are:

· 13 clubs left out of 51 cards.

The probability of drawing two clubs from these 51 cards without replacement:

$$P(B|A^c) = rac{inom{13}{2}}{inom{51}{2}} = rac{rac{13\cdot 12}{2}}{rac{51\cdot 50}{2}} = rac{13\cdot 12}{51\cdot 50} = rac{156}{2550} = rac{13}{212\cdot 5}$$

Step 6: Calculate Total Probability of Drawing Two Clubs

Using the law of total probability:

$$\begin{split} P(B) &= P(B|A)P(A) + P(B|A^c)P(A^c) \\ P(B) &= \left(\frac{11}{212.5}\right)\left(\frac{1}{4}\right) + \left(\frac{13}{212.5}\right)\left(\frac{3}{4}\right) \\ P(B) &= \left(\frac{11}{850}\right) + \left(\frac{39}{850}\right) \\ P(B) &= \frac{50}{850} = \frac{1}{17} \end{split}$$

Step 7: Calculate Conditional Probability P(A|B)

Using Bayes' theorem:

$$\begin{split} P(A|B) &= \frac{P(B|A)P(A)}{P(B)} \\ P(A|B) &= \frac{\left(\frac{1212.5}{212.5}\right)\left(\frac{1}{4}\right)}{\frac{17}{17}} \\ P(A|B) &= \frac{\frac{11.5}{17}}{\frac{17}{17}} = \frac{11}{850} \times 17 = \frac{11 \times 17}{850} = \frac{187}{850} = \frac{11}{50} \end{split}$$

Thus, the probability that the lost card is also a club, given that two drawn cards from the remaining deck are clubs, is $\frac{11}{50}$ or 0.22.

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19	. Find sol	ution using	Newton's forward difference formula.
	X	F(X)	
	1891	46	
	1901	66	
	4044	0.4	

Solve it?

To solve this problem using Newton's forward difference \$ formula, we'll first create the forward difference table and then use the formula to find the value at x=1895.

Given data:

•
$$x_0 = 1891, f(x_0) = 46$$

•
$$x_1 = 1901, f(x_1) = 66$$

•
$$x_2 = 1911$$
, $f(x_2) = 81$

•
$$x_3 = 1921, f(x_3) = 93$$

•
$$x_4 = 1931, f(x_4) = 101$$

The step size $h = x_1 - x_0 = 1901 - 1891 = 10$.

Step 1: Construct the Forward Difference Table

$oldsymbol{x}$	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	46				
1901	66	20			
1911	81	15	-5		
1921	93	12	-3	2	
1931	101	8	-4	-1	-3

Step 2: Use Newton's Forward Difference **Formula**

Newton's forward difference interpolation formula is:

$$P(x) = f(x_0) + u \Delta f(x_0) + rac{u(u-1)}{2!} \Delta^2 f(x_0) + rac{u(u-1)(u-2)}{3!} \Delta^3 f(x_0) + \cdots$$
 where $u = rac{x-x_0}{2!}$

where $u = \frac{x - x_0}{h}$.

For
$$x=1895$$
:

$$u = \frac{1895 - 1891}{10} = \frac{4}{10} = 0.4$$

Step 3: Calculate P(x) at x=1895

$$P(1895) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(x_0)$$

Substitute the values:

$$\begin{array}{l} P(1895) = 46 + 0.4 \cdot 20 + \frac{0.4(0.4 - 1)}{2} \cdot (-5) + \\ \frac{0.4(0.4 - 1)(0.4 - 2)}{6} \cdot 2 + \frac{0.4(0.4 - 1)(0.4 - 2)(0.4 - 3)}{24} \cdot (-3) \end{array}$$

Let's compute this step by step.

Step 4: Calculation

$$P(1895) = 46 + 0.4 \cdot 20 + \frac{0.4 \cdot (-0.6)}{2} \cdot (-5) + \frac{0.4 \cdot (-0.6) \cdot (-1.6)}{6} \cdot 2 + \frac{0.4 \cdot (-0.6) \cdot (-1.6) \cdot (-2.6)}{24} \cdot (-3)$$

First term:

46

Second term:

$$0.4\cdot 20=8$$

Third term:

$$\frac{0.4 \cdot (-0.6)}{2} \cdot (-5) = \frac{-0.24}{2} \cdot (-5) = -0.12 \cdot (-5) = 0.6$$

Fourth term:

$$\frac{0.4 \cdot (-0.6) \cdot (-1.6)}{6} \cdot 2 = \frac{-0.384}{6} \cdot 2 = -0.064 \cdot 2 = -0.128$$

$$\begin{array}{l} \frac{0.4\cdot(-0.6)\cdot(-1.6)\cdot(-2.6)}{24}\cdot(-3) = \frac{0.9984}{24}\cdot(-3) = 0.0416\cdot\\ (-3) = -0.1248 \end{array}$$

Summing these terms:

$$P(1895) = 46 + 8 + 0.6 - 0.128 - 0.1248 = 54.3472$$

So, the interpolated value at x=1895 is 54.35 .

Interpolated value of f chatgpt.com

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Let's calculate L(301) step by step using the Lagrange interpolation formula.

Given data:

•
$$x_0 = 300$$
, $f(x_0) = 2.4771$

•
$$x_1 = 304$$
, $f(x_1) = 2.4829$

•
$$x_2 = 305$$
, $f(x_2) = 2.4843$

•
$$x_3 = 307, f(x_3) = 2.4871$$

To find L(301), we need to calculate the basis polynomials $\ell_0(301)$, $\ell_1(301)$, $\ell_2(301)$, and $\ell_3(301)$.

Step 1: Calculate $\ell_0(301)$

$$\ell_0(301) = \frac{(301 - 304)(301 - 305)(301 - 307)}{(300 - 304)(300 - 305)(300 - 307)}$$

Substitute the values:

$$\ell_0(301) = \frac{(301 - 304)(301 - 305)(301 - 307)}{(300 - 304)(300 - 305)(300 - 307)}$$

$$\ell_0(301) = \frac{(-3)(-4)(-6)}{(-4)(-5)(-7)}$$

$$\ell_0(301) = \frac{-72}{140}$$

$$\ell_0(301) = -0.5143$$

Step 2: Calculate $\ell_1(301)$

$$\ell_1(301) = \frac{(301 - 300)(301 - 305)(301 - 307)}{(304 - 300)(304 - 305)(304 - 307)}$$

Substitute the values:

$$\ell_1(301) = \frac{(301 - 300)(301 - 305)(301 - 307)}{(304 - 300)(304 - 305)(304 - 307)}$$

$$\ell_1(301) = \frac{(1)(-4)(-6)}{(4)(-1)(-3)}$$

$$\ell_1(301) = \frac{24}{12}$$

$$\ell_1(301) = 2$$

Step 3: Calculate $\ell_2(301)$

$$\ell_2(301) = \frac{(301 - 300)(301 - 304)(301 - 307)}{(305 - 300)(305 - 304)(305 - 307)}$$

Substitute the values:

 $\ell_2(301) = -1.8$

$$\begin{array}{l} \ell_2(301) = \frac{(301-300)(301-304)(301-307)}{(305-300)(305-304)(305-307)} \\ \ell_2(301) = \frac{(1)(-3)(-6)}{(5)(1)(-2)} \\ \ell_2(301) = \frac{18}{-10} \end{array}$$

Step 4: Calculate $\ell_3(301)$

$$\ell_3(301) = \frac{(301 - 300)(301 - 304)(301 - 305)}{(307 - 300)(307 - 304)(307 - 305)}$$

Substitute the values:

$$\begin{array}{l} \ell_3(301) = \frac{(301 - 300)(301 - 304)(301 - 305)}{(307 - 300)(307 - 304)(307 - 305)} \\ \ell_3(301) = \frac{(1)(-3)(-4)}{(7)(3)(2)} \\ \ell_3(301) = \frac{12}{42} \end{array}$$

$$\ell_3(301) = \frac{(1)(-3)(-4)}{(7)(3)(2)}$$

$$\ell_3(301) = \frac{12}{42}$$

$$\ell_3(301) = 0.2857$$

Step 5: Combine to get L(301)

$$L(301) = f(x_0) \cdot \ell_0(301) + f(x_1) \cdot \ell_1(301) + f(x_2) \cdot$$

$$\ell_2(301) + f(x_3) \cdot \ell_3(301)$$

$$L(301) = 2.4771 \cdot (-0.5143) + 2.4829 \cdot 2 + 2.4843 \cdot$$

$$(-1.8) + 2.4871 \cdot 0.2857$$

Substitute the values:

$$L(301) = 2.4771 \cdot (-0.5143) + 2.4829 \cdot 2 + 2.4843 \cdot$$

$$(-1.8) + 2.4871 \cdot 0.2857$$

$$L(301) = -1.2736 + 4.9658 - 4.4717 + 0.7106$$

$$L(301) = 2.4786$$

So, the interpolated value at x=301 is $\left|2.4786\right|$

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22. Solve the following equations by Gauss-Seidel Method. $2x-7y-10Z=-17\\5x+y+3Z=14\\x+10\ y+9Z=7$

Solve it?

To solve the given system of equations using the Gauss-Seidel method, we follow an iterative approach. The system of equations is:

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1.
$$2x - 7y - 10z = -17$$

2.
$$5x + y + 3z = 14$$

3.
$$x + 10y + 9z = 7$$

Step-by-Step Solution:

First, rewrite the equations to express x, y, and z in terms of the other variables.

$$x=rac{-17+7y+10z}{2} \ y=14-5x-3z \ z=rac{7-x-10y}{9}$$

Initial Guess:

Let
$$x_0 = 0$$
, $y_0 = 0$, $z_0 = 0$.

Iteration Steps:

Let's iterate the equations using the Gauss-Seidel method.

teration 1:

1. Calculate x_1 :

$$x_1 = \frac{-17 + 7(0) + 10(0)}{2} = \frac{-17}{2} = -8.5$$

2. Calculate y_1 :

$$y_1 = 14 - 5(-8.5) - 3(0) = 14 + 42.5 = 56.5$$

3. Calculate z_1 :

$$z_1 = \frac{7 - (-8.5) - 10(56.5)}{9} = \frac{7 + 8.5 - 565}{9} = \frac{15.5 - 565}{9} = \frac{-549.5}{9} \approx -61.06$$

Iteration 2:

1. Calculate x_2 :

$$\begin{array}{l} x_2 = \frac{-17 + 7(56.5) + 10(-61.06)}{2} = \frac{-17 + 395.5 - 610.6}{2} = \\ \frac{-232.1}{2} = -116.05 \end{array}$$

2. Calculate y2:

$$y_2 = 14 - 5(-116.05) - 3(-61.06) = 14 + 580 + 183.18 = 777.18$$

3. Calculate z_2 :

$$z_2 = \frac{7 - (-116.05) - 10(777.18)}{9} = \frac{7 + 116.05 - 7771.8}{9} = \frac{123.05 - 7771.8}{9} = \frac{-7648.75}{9} \approx -849.86$$

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Solve it

- To solve $x^4 x 7 = 0$ using the Newtonmethod, we need to follow these steps:
 - 1. Function and Derivative:
 - $f(x) = x^4 x 7$
 - $f'(x) = 4x^3 1$
 - 2. Newton-Raphson Formula:

$$x_{n+1}=x_n-rac{f(x_n)}{f'(x_n)}$$

3. **Initial Guess**: Let's start with $x_0 = 2$.

Iterative Process:

Iteration 1:

$$x_0 = 2$$

$$f(x_0) = 2^4 - 2 - 7 = 16 - 2 -$$

$$f'(x_0) = 4(2^3) - 1 = 32 - 1$$

$$x_1 = 2 - \frac{7}{31} = 2 - 0.2258 \approx 1$$

 $x_1 \approx 1.7742$

 $f(x_1) \approx 1.7742^4 - 1.7742 - 7 = 9.9401 - 1.7742 - 7$

$$f'(x_1) pprox 4(1.7742^3) - 1 = 22.3248 - 1 pprox 21.3248$$

$$x_2 = 1.7742 - rac{1.1659}{21.3248} pprox 1.7742 - 0.0547 pprox 1.7195$$

Iteration 3:

$$x_2 \approx 1.7195$$

$$x_{n+1} = x_n - rac{f(x_n)}{f'(x_n)} \qquad \qquad f(x_2) pprox 1.7195^4 - 1.7195 - 7 = 8.7157$$

$$f'(x_2) \approx 4(1.7195^3) - 1 = 20.6106 - 1 \approx 19.6106$$

$$x_3 = 1.7195 - rac{-0.0038}{19.6106} pprox 1.7195 + 0.0002 pprox 1.7197$$

Iteration 4:

$$x_3 \approx 1.7197$$

$$f(x_3) \approx 1.7197^4 - 1.7197 - 7 = 8.7228 - 1.7197 - 7$$

$$f'(x_3) pprox 4(1.7197^3) - 1 = 20.6200 - 1 pprox 19.6200$$

$$x_4 = 1.7197 - rac{0.0031}{19.6200} pprox 1.7197 - 0.0002 pprox 1.7195$$

Since the value of x is converging towards 1.7195, we can conclude that:

The root of $x^4-x-7=0$, correct to six significant digits, is approximately 1.7195.

Iteration 2:

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