**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:-

MU(*μ)* = 45

STD DIV(*σ* ) = 8 mins

Total hour x = 60 mins

Work begins after 10 mins (µ) = 45 + 10 = 55

The probability completing commitment P(E) = Z = (x-µ)/*σ*

= (60-55)/8

= 5/8

Z = 0.625

P(E) = 0.7340145 or 73.40% The probability that the service manager cannot meet his commitment =1-P(E)

= 1 - 0.7340145

= 0.2659855

So Therefore ans is (B).because 0.2659855 is closet to 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:-

A). Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1

Z score for 38  = (38 - 38)/6 = 0

The probability for age below 44 = 0.8413447 = 84.13%

The probability for age below 38 = 0.5 = 50%

The probability for age between 38 & 44 = 0.3413447

People above 44 age = 100 - 84.13 =  15.87%

That means (15.87\*400)/100 = 6348/100 = 63.48

Hence that means age above 44 is 63 out of 400.

The probability of age between 38 & 44 = 34.13

That means (34.13\*400)/100 = 13,652/100 = 136.52

Hence that means age of 38 & 44 is 137 out of 400

**FALSE -** because More employees at the processing center are between 38 & 44. than older than 44

B). Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

The probability that employees under the age 30 = 0.09121127 = 9.12%

That means (9.12\*400)/100 = 3,648/100 = 36.48

Hence that means age under the 30 = 36

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:-

Given : Let X = *X1* ~ *N*(μ, σ2) &

Y = *X*2 ~ *N*(μ, σ2)

To Find : (a) 2 *X*1 ,

(b)*X*1 + *X*2 ?

Sol:-

As we know that if X ∼ N(µ1,1 σ2 ), and Y ∼ N(µ2, 2σ2) are two independent random variables where

X + Y ∼ N(µ1 + µ2, σ21 + σ22 ) , and

X − Y ∼ N(µ1 − µ2, σ21 + σ22 )

1. 2X1 = 2*N*(μ, σ2)

= N(2μ, 2σ2)

1. *X*1 + *X*2 = N(µ + µ, σ2 + σ2 ) OR

= N(2µ, 2σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:-

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area = 1-0.99 = 0.01

The Probability towards left from a =0.01/2 = -0.005

The Probability towards right from b = 0.01/2 = +0.005

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

Z = 100-99/20 =1/20 = 0.005

For Probability 0.005 the Z Value is-2.575829 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = (-(-2.575829))\*20+100 =-51.51658 + 100 =

151.5166

Z(+0.005)\*20+100 = (-2.575829)\*20+100 =-51.51658 + 100 = 48.48342

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:-

Let, X is the sum of two random variables having normal distribution.

E[X]= E[45\*(profit 1+profit 2)]= 45\*(5+7)=540 million rupees

SD[X]= SD[profit 1 +profit 2]=> 45\*()

= 45\*= 225 million rupees.

**Therefore, X~ N(540,)**

A). From the empirical rule, Approximately 95% of the data falls within two standard deviation of the mean.

μ ± 2σ = 540±2\*225=> (540-450, 540+450)=> **(90,990)**

B). μ - 1.5σ => 540-(1.5\*225) =>202.5 million rupees.

C). **Divisions 1**

For division1= Z score for a profit of zero: Z=(X-µ)/*σ = (0-5)/3 => -1.25 = 0.1056*

stats.norm.cdf(0, loc =5, scale = 3 )

0.04779035

Division1 has a lesser probability of making a loss.

**Divisions 2**

For division2= Z score for a profit of zero: Z=(X-µ)/ *σ*  =(0-7)/4 => -1.75= .0401

stats.norm.cdf(0, loc =7, scale = 4 )

0.04005916

Division2 has a higher probability of making a loss