Lesson 0: Set Theory

Computer Science II
Mrs. Staffen

Set theory

1. Sets and Membership

2. Representation of Sets and Membership

3. Cardinality of sets

1. SETS and Membership

1.1 SETS (MOTIVATION)

Here are some examples of *sets* from daily life:

- Students in your class
- Cities in Texas
- Your favorite songs
- Characters in your favorite book

1. SETS and Membership

1.1 SETS (MOTIVATION)

Here are some examples of *sets* from daily life:

- Students in your class
- Cities in Texas
- Your favorite songs
- Characters in your favorite book

Can you tell us some of your favorite songs, and some characters in your favorite book?

What are some other examples of *sets*?

1.2 MEMBERSHIP (MOTIVATION)

Membership of sets matters.

- You are a *member* of the set of students in this class

- Lubbock is a *member* of the set of cities in Texas

1.2 MEMBERSHIP (MOTIVATION)

Membership of sets matters.

- You are a *member* of the set of students in this class
- Lubbock is a *member* of the set of cities in Texas

Tell us a *member* of the set of your favorite songs, and a *member* of the set of your favorite characters?

1.3 Informal Definition of sets and membership

Sets and *membership* form the basis of representing our world.

Sets and *membership* can be informally described as follows:

- A **set** is a collection of objects
- A **member** is an object contained in a set

1.3 Formal Definition and Informal Definition

- Note that these are not *formal* definitions of sets and membership because *collection*, *objects* and *contained* are not defined and depend on our intuitive understanding or examples.

- By *formal definition* of a concept, we mean that we use only *defined concepts* and *simple logic connectives* (e.g., conjunction (and) and disjunction (or)).

[Student discussion]

Set theory

1. Sets and Membership

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2. Representation of Sets and Membership

2.1 Use roster notation to represent a set

We have a formal way to represent a set, called **<u>roster notation</u>** (of a set):

- It is enclosed with { }
- Objects inside { } are separated by “,”

For example, the set of players of Houston Rockets can be represented by roster notation as

{Harden, Westbrook, Gordon, ...}

which is **read** as: a set consisting of Harden, Westbrook, Gordon and etc.

How do we read the set of {1, 3, 5, 7, 9}?

(Note: **…** means that we have more players but we don’t want to list them.)

2.1 Use roster notation to represent a set

We usually   to make it easy to refer to the set. For example, we can use a name *rocketPlayers* for the set above. It is written as

$$rocketPlayers = \{\text{Harden, Westbrook, Gordon, ...}\}$$

Which is **read** as *rocketPlayers* is a set consisting of Harden, Westbrook, Gordon and etc.

Note

- = means “is” or “is defined as”
- A set name is usually one word (i.e., no space inside it).

2.1 Use roster notation to represent a set

Exercise

- How do we read the following:

  $$A = \{1, 3, 5, 7, 9\}$$

- Using roster notation, write a few sets that you are interested in and give each of them a name.

2.2 Membership Representation

Once a set is defined, we can tell any member or element of that set.

Example. Assume we have a set *citiesOfTexas*:

citiesOfTexas = {Austin, Dallas, Lubbock, Bangs, Canyon, China, Happy, …}

Is Austin a member of the set citiesOfTexas? [Yes. How do we represent this membership?]

2.2 Membership Representation

Membership notation (∈). That Austin is a member of citiesOfTexas is denoted or represented by

$$Austin \in citiesOfTexas$$

Non-membership notation (∉). Is 5 a member of the set $A = \{ 1, 2, 3\}$?

No. It is represented by

$$5 \notin A \text{ or } 5 \notin \{1, 2, 3\}$$

2.2 Membership Representation (practice)

- Is 1 a member of the set A = { 1, 2, 3}? Write the expression?

- Is 6 a member of the set A = { 1, 2, 3}? Write the expression?

2.2 Membership Representation (practice)

- Is 1 a member of the set $A = \{1, 2, 3\}$? Write the expression?

    Yes. It is represented by $1 \in A$ or $1 \in \{1, 2, 3\}$.

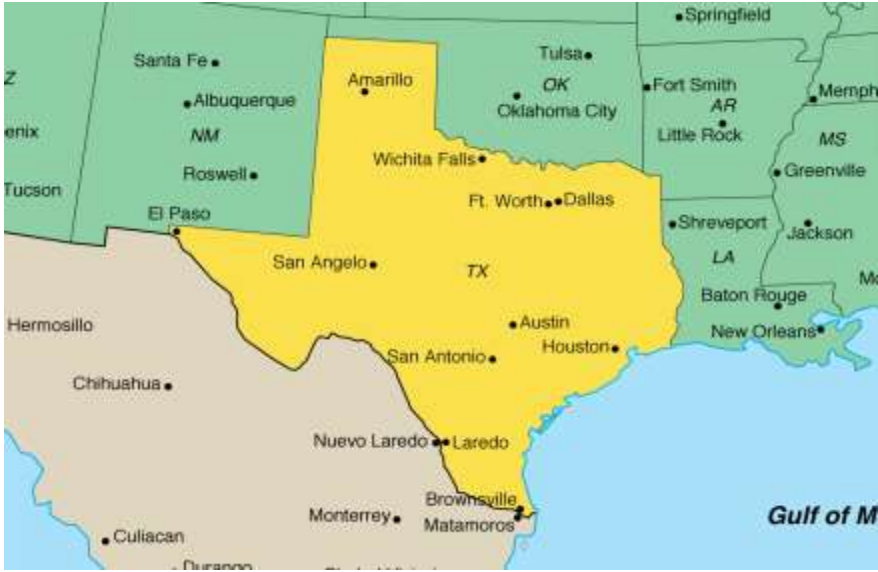
- Is 6 a member of the set $A = \{1, 2, 3\}$? Write the expression?

    No. It is represented by $6 \notin A$ or $6 \notin \{1, 2, 3\}$.

Check for Understanding

Assume *citiesOfTexas* is a set consisting of cities, strictly according to the map below.

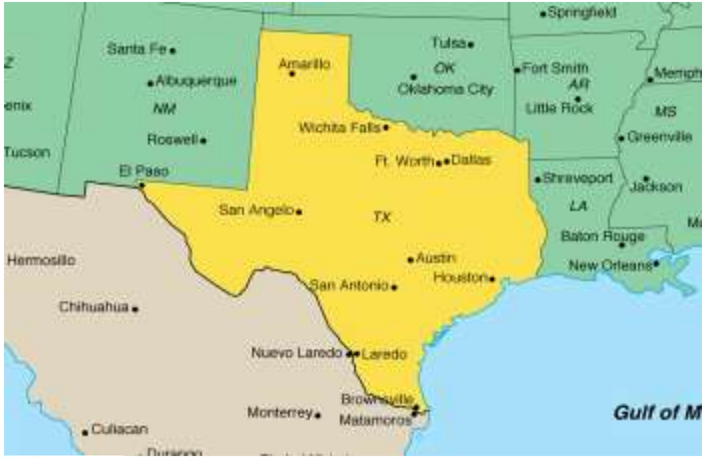
- Is Lubbock a member of *citiesOfTexas*? Write an expression to represent this membership?
- Is Amarillo a member of *citiesOfTexas*? Write an expression to represent this membership?



Check for Understanding

Assume *citiesOfTexas* is a set consisting of cities in the following map [??]

Is Lubbock a member of *citiesOfTexas*? How to represent it? [Lubbock $\notin$citiesOfTexas. Note this is counter-intuitive, but it illustrates that we have to follow the definition of a set]



Is Amarillo a member of *citiesOfTexas*? How to represent it? [Amarillo $\in$ citiesOfTexas]

Set theory

1. Sets and Membership

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3. Cardinality of Sets

3.1 MOTIVATION

- To buy a new iPad for each one of us, how many iPads do we need?

Here we need the concept of the number of objects in a set (i.e., the students in the question above), and this number is called *cardinality*.

3.2 Formation Definition

Definition (cardinality). The **cardinality** or **size** of a set is the number of elements in that set.

Cardinality notation. The cardinality of a set A is denoted as $|A|$. If A is a set, $|A|$ is **read** as the cardinality of A.

3.3 Examples of Cardinality

- What is the cardinality of the set below?

  someRocketPlayers = {Harden, Westbrook, Gordon}

- How do you read the following:

  |{Harden, Westbrook, Gordon}|

- What is its value?

- What is the cardinality of the set *rocketPlayers* = {Harden, Westbrook, Gordon, ...}?

3.3 Examples of Cardinality

- What is the cardinality of the set below? [Answer: 3]

  someRocketPlayers = {Harden, Westbrook, Gordon}

- How do you read the following:

  |{Harden, Westbrook, Gordon}|

- What is its value? [The cardinality of the set {Harden, Westbrook, Gordon}]
- What is the cardinality of the set *rocketPlayers* = {Harden, Westbrook, Gordon, ...}? [Answer: we don't know because … doesn't tell us how many there]

Checking for Understanding

What are your favorite pizza toppings, or your favorite songs on the radio?

Pair up with another student, then pick a category below or make one up!

- Movies
- Pizza Toppings
- Professional Athletes
- Songs
- TV Shows
- Video Games

- Write your favorite things for that category using roster notation!
- Then, compare your roster with your partner's!

○ Using the *membership notation*, write a member that is in your set but not in your partner's set.

○ Does your partner's roster have more or less elements than yours? Whose set has a larger cardinality?

4. Function outline

4.1 Motivation

4.2 What is a function

4.3 Representation of function

4.4 Use of function

4.5 Binary function

4.6 Define a function in intentional form

4. Function

4.1 Functions (MOTIVATION)

- *Functions* provide us a *short and precise form* (which is also called an *abstraction*) to *represent* and *reason* with *relations* among *objects* in our life.

- *Functions* also allow computers to do the reasoning automatically!

4.1 Motivation (continued)

4.1.1 Function in life

■ Problem. Given a family: Steve, a son of Tom, is married to Joan, and they have three kids: Alice, Bob, and Claudia. Who is the father of Alice?

■ Representation of knowledge (to answer the question) using function

- We introduce a function name *father* relating a person to his/her father
- Given a person *x*, we use the form *father(x)* to represent the father of *x*. *father(x)* is called an *expression*. In the expression, *x* is called an *input* to the function *father*.
- The father of Alice is represented as *father(Alice)*
- Write an expression to represent “the father of Bob” ?

4.1 Motivation (continued)

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- The father of Alice is represented as *father(Alice)*
- Write an expression to represent "the father of Bob" ?

  father(Bob)
- The *value* of expression *father(Bob)* is *Steve.*
- What is the *value* of expression *father(Steve)*?

4.1 Motivation (continued)

4.1.1 Function in life

- Problem. Given a family: Steve, a son of Tom, is married to Joan, and they have three kids: Alice, Bob, and Claudia. Who is the father of Alice?

- Representation of knowledge (to answer the question) using function
  - We introduce a function name *father* relating a person to his/her father
  - Given a person *x*, we use the form *father(x)* to represent the father of *x*. *father(x)* is called an *expression*. In the expression, *x* is called an *input* to the function *father*.
  - The father of Alice is represented as *father(Alice)*
  - Write an expression to represent "the father of Bob" ?

    father(Bob)
  - The *value* of expression *father(Bob)* is *Steve.*
  - What is the *value* of expression *father(Steve)*? *Tom*

4.1 Motivation (continued)

4.1.1 Function in life

■ Problem. Given a family: Steve, a son of Tom, is married to Joan, and they have three kids: Alice, Bob, and Claudia. Who is the father of Alice?

■ Representation of knowledge (to answer the question) using function (continued)

- The content of the *father* function can be represented as table

x	$father(x)$
Alice	Steve
Bob	Steve
Claudia	Steve
Steve	Tom

■ Reasoning: the value of *father(Alice)* is *Steve* (by looking at the table of the function)

4.1 Function (Motivation)

4.1.2 "+" function

■ **Problem**. In a basketball game, Harden got 10 points in the first quarter and 20 points in the 2nd quarter. What is the total number of points Harden got in the first half?

■ Write an *expression* to represent the knowledge to answer the question:

4.1 Function (Motivation)

4.1.2 "+" function

- **Problem**. In a basketball game, Harden got 10 points in the first quarter and 20 points in the 2nd quarter. What is the total number of points Harden got in the first half?

- Write expression to represent the knowledge to answer the question : *10 + 20*

  We call 10 + 20 an *expression*.

  In fact, "+" is a function, following the *expression* in our life example,

  10+20 can be written as +(10, 20) which is called *prefix form* (the function is

  before the inputs)

4.1 Function (Motivation)

4.1.2 "+" function

■ **Problem**. In a basketball game, Harden got 10 points in the first quarter and 20 points in the 2nd quarter. What is the total number of points Harden got in the first half?

■ Representation of the knowledge to answer the question : $10 + 20$

■ The value of the expression is 30

Note the difference between the *expression* and *its value*.

In this course, we always distinguish these two concepts!

4.1 Function (Motivation)

4.1.2 "+" function

- **Problem.** In a game, Harden got 10 points in the first quarter, 20 points in the 2nd quarter, 15 points and 5 points in the 3rd and 4th quarter respectively. What is the total points Harden got in the whole game?

  - Represent the knowledge (or write an expression) to answer the question:

    $$(((10+20)+15)+5))$$

  - Reasoning: the value of the *expression* (applying the functions to numbers):

4.1 Function (Motivation)

4.1.2 “+” function

■ **Problem.** In a game, Harden got 10 points in the first quarter, 20 points in the 2nd quarter, 15 points and 5 points in the 3rd and 4th quarter respectively. What is the total points Harden got in the whole game?

- ■ Represent the knowledge (or write an expression) to answer the question:
- ■ Reasoning (applying the functions to numbers):

4.1 Function (Motivation)

4.1.2 "+" function

- **Problem.** In a game, Harden got 10 points in the first quarter, 20 points in the 2nd quarter, 15 points and 5 points in the 3rd and 4th quarter respectively. What is the total points Harden got in the whole game?

  - Represent the knowledge (or write an expression) to answer the question:

    (((10+20)+15)+5))

  - Reasoning: the value of the *expression* (applying the functions to numbers):

    50

4. Function outline

4.1 Motivation

4.2 What is a function

4.3 Representation of function

4.4 Use of function

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4.6 Define a function in intentional form

4.2 What is a function

A function has *signature* and *content*.

- Function signature

- Function content

4.2.1 function Signature (example)

The *signature* of a function involves a *name of the function*, and the kinds of objects the function relates.

Example Given a family: Steve, a son of Tom, is married to Joan, and they have three kids: Alice, Bob, and Claudia. Who is the father of Alice.

For the father function in the example,

- It relates people to people.
- We can give the function a name *father*

The signature of the father function is represented:

father: *people* -> *people*

Read: *People* (left one) is called the *domain* of the function, people (right one) is the *range* of the function.

4.2.2 Function Content (example)

A function has *signature* and *content*.

Example Given a family: Steve, a son of Tom, is married to Joan, and they have three kids: Alice, Bob, and Claudia. Who is the father of Alice?

The *content* of the function is specified as a table :

x	$father(x)$
Alice	Steve
Bob	Steve
Claudia	Steve
Steve	Tom

4.2.3 Definition of function signature

Definition (signature of a function, domain, range).

The **signature of a function**: *function name*, the *sets/sorts* whose elements are related by this Function. It is denoted by

functionName: *set1* -> *set2*

Where *functionName* can be any name, *set1* and *set2* are sets.

set1 is called the ***domain*** *of functionName, set2 the* ***range*** *of functionName.*

This type of function is called **unary** function.

4.2.3 Definition of function signature

functionName: *set1* -> *set2*

It is read: *functionName* maps every element of *set1* to a *unique* element of set *set2*

4.2.3 Definition of function signature

Example

Given a function signature

$$\text{Mother} : \{\text{John, Sara, Dian}\} \rightarrow \{\text{Laura, Mary}\}$$

- What are the function name, domain, range?
- How to read the function?
- Is this a unary function?

4.2.3 Definition of function signature

Example

Given a function signature

$$\text{Mother} : \{\text{John}, \text{Sara}, \text{Dian}\} \rightarrow \{\text{Laura}, \text{Mary}\}$$

- What is a the function name, domain, range?

  Function name: mother

  Domain: {John, Sara, Dian}

  Range: {Laura, Mary}

4.2.3 Definition of function signature

Example

Given a function signature

$$\text{Mother} : \{\text{John, Sara, Dian}\} \rightarrow \{\text{Laura, Mary}\}$$

- How to read the function?

Mother function maps every element of *set1* to a *unique* element of set *set2*

Set1 is {John, Sara, Dian}, *set2 is* {Laura, Mary}.

- Is this a unary function?

Yes. Because it relates **one** person to another person

4.2.4 Representation of function content by table

We use *table* to represent the *content* of the function.

x	*father(x)*
Alice	Steve
Bob	Steve
Claudia	Steve
Steve	Tom

Note the way to write the table head. First *column* contains the domain of the function, second contains the range of the function. We name to *domain* with a meaningful name such as *person*, while we directly use *father(person)* to name the *range* column.

4. Function outline

4.3 Representation of function (practice)

Problem.

- Many people like to ride roller coasters. Amusement parks try to increase attendance by building exciting new coasters.
- The following table displays data on several roller coasters that were opened in a recent year.

Roller coaster	Type	Height (ft)	Design	Speed (mph)	Duration (sec)
Wildfire	Wood	187.0	Sit down	70.2	120
Skyline	Steel	131.3	Inverted	50.0	90
Goliath	Wood	165.0	Sit down	72.0	105
Helix	Steel	134.5	Sit down	62.1	130
Banshee	Steel	167.0	Inverted	68.0	160
Black Hole	Steel	22.7	Sit down	25.5	75

Questions: What is the type of Helix? What its height? …

4.3 Representation of function (practice)

Roller coaster	Type	Height (ft)	Design	Speed (mph)	Duration (sec)
Wildfire	Wood	187.0	Sit down	70.2	120
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- To represent the knowledge in the problem to answer the questions there, can you give a function by giving the following:
    - What name do you want to give the function?
    - What is the domain of the function?
    - What is the range of the function
    - Write the signature of the function

4.3 Representation of function (practice)

Roller coaster	Type	Height (ft)	Design	Speed (mph)	Duration (sec)
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- To represent the knowledge in the problem to answer the questions there, can you give a function by giving the following:
  - What name do you want to give the function? *typeOf*
  - What is the domain of the function?
    {Wildfire, Skyline, Goliath, Helix, Ganshee, Black Hole}
  - What is the range of the function *{Wood, Steel}*
  - Write the signature of the function

4.3 Representation of function (practice)

Roller coaster	Type	Height (ft)	Design	Speed (mph)	Duration (sec)
Wildfire	Wood	187.0	Sit down	70.2	120
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- To represent the knowledge in the problem to answer the questions there, can you give a function by giving the following:
    - Write the signature of the function

typeOf: *rollerCoasters* -> *types* where

rollerCoasters = {Wildfire, Skyline, Goliath, Helix, Ganshee, Black Hole}

types = {Wood, Steel}

4.3 Representation of function (practice)

Roller coaster	Type	Height (ft)	Design	Speed (mph)	Duration (sec)
Wildfire	Wood	187.0	Sit down	70.2	120
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- To represent the knowledge in the problem to answer the questions there, can you give a function by giving the following:
    - Write the signature of the function

      type: rollerCoasters -> types where

      rollerCoaster = {Wildfire, Skyline, Goliath, Helix, Ganshee, Black Hole}

      types = {Wood, Steel}
- Can you write the content of function *type*?

4.3 Representation of function (practice)

Roller coaster	Type	Height (ft)	Design	Speed (mph)	Duration (sec)
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Black Hole	Steel	22.7	Sit down	25.5	75

- Can you write the content of function *type*?

rollerCoaster	*type(rollerCoaster)*
Wildfire	Wood
Skyline	Steel
Goliath	Wood
Helix	Steel
Banshee	Steel
Black Hole	Steel

4.3 Representation of function (practice)

Roller coaster	Type	Height (ft)	Design	Speed (mph)	Duration (sec)
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- To represent the knowledge in the problem to answer the questions there, can you give another function by giving
  - its signature, and
  - its function content?

4.3 Representation of function (practice)

Roller coaster	Type	Height (ft)	Design	Speed (mph)	Duration (sec)
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- To represent the knowledge in the problem to answer the questions there, can you give another function by giving
  - its signature,

    $design: rollerCoaster \rightarrow designs$

    where *rollerCoaster* is defined as before and *designs* = {sit down, inverted}
  - its function content?

4. Function outline

4.4 Use of a function

Problem. Given a family: Steve, a son of Tom, is married to Joan, and they have three kids: Alice, Bob, and Claudia. Who is the father of Alice?

- To represent "the father of Alice" using function *father*, we write *father(Alice)*. *Alice* is called an **argument** or **input** to the function *father*. Recall *father(Alice)* is called an **expression**.
- The *father* function is called **unary** (because it has only one argument). The "+" function is called **binary** because it has two arguments. (arguments in 1+3 are 1 and 3).
- The **value** or **output** of *father(Alice)* is *Steve* which the function relates *Alice* to.

4.4 Use of a function

- Given a unary function f and an object x, we use $f(x)$ to represent the object that function f maps to x.

- In expression $f(x)$, x is called an **input** or **argument** of the function f. The object that f maps x to is called the **output** or **value** of $f(x)$.

4.4 Use of a function

- **More examples**
  - Use the function *father*, write an expression to represent the grandfather of Alice?

.

4.4 Use of a function

- **More examples**
  - Use the function *father*, write an expression to represent the grandfather of Alice?
    - Expression:

      father(father(Alice))

      (or *father(mother(Alice))* if we have *mother* function).
    - (Reasoning) The *value* of *father(father(Alice))*?

      To find value we go from inside out:
      - Value of *father(Alice)*: *Steve*,
      - Then find *father(Steve)* whose value is *Tom*.

4. Function outline

4.5 Binary function (Motivation)

The table below shows the starting time of the courses taken by a set of students. For example, the start time of physics taken by Peter is 8am.

Persons	*Course*	*Start Time*
Sara	*Physics*	*8:00*
John	*CS II*	*10:00*
Sara	*CS II*	*12:00*

We can not use *unary* function to represent the information here. E.g., we cannot say *startTime(csII)* because *csII* has two start times. Can we say *startTime(Sara)*? (no, because there are two start times for Sara, depending on the course she takes)

4.5 Binary function (Motivation)

The table below shows the starting time of the courses taken by a set of students.
For example, the start time of physics taken by Peter is 8am.

Persons	*Course*	*Start Time*
Sara	*Physics*	*8:00*
John	*CS II*	*10:00*
Sara	*CS II*	*12:00*

However, the start time of a person and a course would be unique! So, we have a function with two arguments or inputs: e.g., *startTime(Sara, Physics)*

4.5 Binary function (Motivation)

Persons	Course	Start Time
Sara	Physics	8:00
John	CS II	10:00
Sara	CS II	12:00

More formally, we have a function with signature

startTime: person, course -> time

where *person= {peter, John, Sara},*

course={Physics, Computer Science II},

time = {8:00, 10:00, 12:00 }.

4.5 Binary function (Motivation)

Persons	*Course*	*Start Time*
Sara	*Physics*	*8:00*
John	*CS II*	*10:00*
Sara	*CS II*	*12:00*

The function signature

startTime: person, course -> time

is **read** as the function *startTime* maps a person and a course to a time, or more generally, *startTime* maps an object of set *person*, and an object of set course to an object of *time*.

4.5 Binary function (Motivation)

Persons	*Course*	*Start Time*
Sara	*Physics*	*8:00*
John	*CS II*	*10:00*
Sara	*CS II*	*12:00*

- *Signature.*

  startTime: person, course -> Time (intuitive reading?)

- Use of function

  We write *expression startTime(Sara, physics)* to represent "the start time of Sara's physics class"

  What is the *value* of *startTime(Sara, physics)* by the table?

4.5 Binary function (Motivation)

Persons	*Course*	*Start Time*
Sara	*Physics*	*8:00*
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Sara	*CS II*	*12:00*

- *Signature.*

  startTime: person, course -> Time (intuitive reading?)

- Use of function

  We write *expression startTime(Sara, physics)* to represent “the start time of Sara’s physics class”

  What is the *value* of *startTime(Sara, physics)* by the table? 8:00

4.5 Binary function - Definition

A **binary function** has a signature of the form

functionName : *set1*, *set2* -> *set3*

where *functionName* is a name, *set1*, *set2*, and *set3* are sets.

How to read: *functionName* maps a value from *set1* and a value from *set2* to a value of *set3*

4.5 Binary function - Example

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Example

Is + function a binary function?

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Function + takes two arguments 1 and 2, and it maps them to 3.

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Function + takes two arguments 1 and 2, and it maps them to 3.

Now can you write the signature of +?

+: *numbers, numbers* → *numbers*

where *numbers* are set of numbers.

4.5 Binary function - Example

Example

Given the signature of +

$+: numbers, numbers \longrightarrow numbers$

Following the way of the use of general function (e.g., *startTime(Sara, csII)*,

use + to write an expression for "addition of 10 and 3":

4.5 Binary function - Example

Example

Given the signature of +

+: *numbers, numbers* $\longrightarrow$ *numbers*

Following the way of the use of general function (e.g., *startTime(Sara, csII)*,

use + to write an expression for “addition of 10 and 3”:

+*(10, 3)*

In fact, this form is called ***prefix form*** (the function name is in the front).

The normal writing 10+3 is called ***infix form*** (the function name is in the middle).

4.5 Binary function - Example

Example

Given the signature of +

+: *numbers*, *numbers* $\rightarrow$ *numbers*

Do you feel that “2+4+6” is strange now? + is a binary function. Is the 4 shared by the first + and second +? No. For 2nd +, it is really the value of 2+4 is its first argument and 6 is the second. In this understanding + has always two arguments.

Can you write an expression in *prefix form* to represent “2+4+6”?

4.5 Binary function - Example

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Can you write an expression in *prefix form* to represent "2+4+6"?

We write +(2, 4) first, then add it and 6: +($\underline{+(2, 4)}$, 6)
(Recall similar use of function is *father(father(Alice))*.

4.5 Binary function - Checking Understanding

Example

Now consider + and multiplication *:

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4.5 Binary function

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1. Using *, can you write an expression in *prefix form* to represent "2*4*6"?

$$*(*(2, 4), 6)$$

1. Using *, can you write an expression in *prefix form* to represent "2+4*6"? (note we have to do * first)

$$+(2, *(4, 6))$$

4.5 Binary function

Is = a binary or unary function? Why? [General student discussion here]

4.5 Binary function

Is = a binary or unary function? Why?

A binary function. = is to say TWO things are equal. Two inputs are involved and = function does output a value. E.g., expression 5 = 5 involves two input: 5 and 5. It does have a value *true*. Expression 5 = 6 also involves two inputs 5 and 6. It’s value is *false*. The values *true* and *false* are called **logical values** or **Boolean values**.

4.5 Binary function

Are you able to write the signature of = function?

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$$=: set1, set2 \rightarrow \{\text{true}, \text{false}\}$$

Where *set1* and *set2* are arbitrary sets.

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Using function *father* and =, write an expression for “Alice and Bob have the same father:”

- Write expression for “Alice’s father”:
- Write expression for “Bob’s father”:

4.5 Binary function

Using function *father* and =, write an expression for "Alice and Bob have the same father:"

- Write expression for "Alice's father": *father(Alice)*
- Write expression for "Bob's father": *father(Bob)*
- Write expression for "Alice and Bob have the same father":

4.5 Binary function

Using function *father* and =, write an expression for “Alice and Bob have the same father:”

- Write expression for “Alice’s father”: *father(Alice)*
- Write expression for “Bob’s father”: *father(Bob)*
- Write expression for “Alice and Bob have the same father”:

$$father(Alice) = father(Bob)$$

- (Reasoning) What’s its value? (we calculate the value of subexpression first)
  - *father(Alice)* is *Steve*
  - *father(Bob)* is *Steve*
  - Substitute value to the original expression *Steve* = *Steve* whose value is *true* (Here we use the understanding that the value of $x = y$ is true if the value of x and the value of y are identical.)

4.5 Binary function

For function =, is the expression *father(Alice)* = *father(Bob)* in prefix form or infix form?

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Can you write it into prefix form?

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Infix form!

Can you write it into prefix form?

= (*father(Alice)*, *father(Bob)*)

Can you write an expression for “Alice and Steve have the same father”?

4.5 Binary function

Can you write an expression for “Alice and Steve have the same father”?

$$=(father(Alice), father(Steve))$$

What is its value? (Recall Alice’s father is Steve and Steve’s father is Tom)

4.5 Binary function

Can you write an expression for "Alice and Steve have the same father"?

$=(\textit{father}(\textit{Alice}), \textit{father}(\textit{Steve}))$

What is its value? (Recall Alice's father is Steve and Steve's father is Tom)

false!

Recall the range of = function is {true, false}. The false is a value of the range.

4. Function outline

4.1 Motivation

4.2 What is a function

4.3 Representation of function

4.4 Use of function

4.5 Binary function

4.6 Define a function in intentional form

4.6 Define a function in intentional form

Sometimes, we don't want to give the *signature* and *content* of a function, but we still want to give people some idea of the function so that they can use the function. One way to do so is called an *intentional notation*.

Recall we use the following before for function =

The value $=(x, y)$ (or in infix form $x = y$) is true if the value of x is identical to value of y.

It is an intentional form which is sufficient for us to use the function or answer questions about the function. E.g., what is the value of $=(5, 6)$?

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4.6 Define a function in intentional form

An ***intentional notations*** for a function are illustrated by examples.

- To define *father* function in an intentional way,
  The *output* or the *value* of *father(x)* is the father of *x*.
- To define a function *likes* to tell us who likes which sport,
  the *output* or the *value* of *like(x, y)* is true if *x* likes sport *y*.

Example

Can you write the weight function (of a person) using intentional notation?

4.6 Summary of functions

- What is a function
- Representation of function
- Use of function
- Binary function
- Define a function in intentional form